CHAPTER 1—PROBLEM SOLUTIONS

1.1 (a) $I = \dfrac{V}{R} = \dfrac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$

(b) $R = \dfrac{V}{I} = \dfrac{10\text{ V}}{1\text{ mA}} = 10\text{ k}\Omega$

(c) $V = IR = 10\text{ mA} \times 10\text{ k}\Omega = 100\text{ V}$

(d) $I = \dfrac{V}{R} = \dfrac{10\text{ V}}{100\ \Omega} = 0.1\text{ A}$

Note: Volts, milliamps, and kilo-ohms constitute a consistent set of units.

1.2 (a) $P = I^2R = (30 \times 10^{-3})^2 \times 1 \times 10^3 = 0.9\text{ W}$
Thus, R should have a 1-W rating.

(b) $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3 = 1.6\text{ W}$
Thus, the resistor should have a 2-W rating.

(c) $P = I^2R = (3 \times 10^{-3})^2 \times 10 \times 10^3 = 0.09\text{ W}$
Thus, the resistor should have a $\frac{1}{8}$-W rating.

(d) $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3 = 0.16\text{ W}$
Thus, the resistor should have a $\frac{1}{4}$-W rating.

(e) $P = V^2/R = 20^2/(1 \times 10^3) = 0.4\text{ W}$
Thus, the resistor should have a $\frac{1}{2}$-W rating.

(f) $P = V^2/R = 11^2/(1 \times 10^3) = 0.121\text{ W}$

Thus, a rating of $\frac{1}{8}$ W should theoretically suffice though $\frac{1}{4}$ W would be prudent to allow for consistent tolerances and measurement errors.

1.3 (a) $V = IR = 10\text{ mA} \times 1\text{ k}\Omega = 10\text{ V}$

$P = I^2R = (10\text{ mA})^2 \times 1\text{ k}\Omega = 100\text{ mW}$

(b) $R = V/I = 10\text{ V}/1\text{ mA} = 10\text{ k}\Omega$

$P = VI = 10\text{ V} \times 1\text{ mA} = 10\text{ mW}$

(c) $I = P/V = 1\text{ W}/10\text{ V} = 0.1\text{ A}$

$R = V/I = 10\text{ V}/0.1\text{ A} = 100\ \Omega$

(d) $V = P/I = 0.1\text{ W}/10\text{ mA} = 100\text{ mW}/10\text{ mA} = 10\text{ V}$

$R = V/I = 10\text{ V}/10\text{ mA} = 1\text{ k}\Omega$

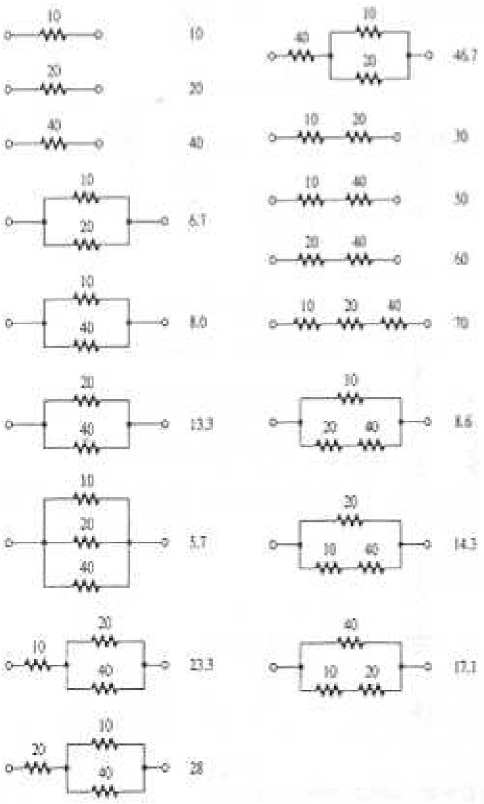
(e) $P = I^2R \Rightarrow I = \sqrt{P/R}$

$I = \sqrt{1000\text{ mW}/1\text{ k}\Omega} = 31.6\text{ mA}$

$V = IR = 31.6\text{ mA} \times 1\text{ k}\Omega = 31.6\text{ V}$

Note: V, mA, kΩ, and mW constitute a consistent set of units.

1.4



Thus, there are 17 possible resistance values.

1.5 Shunting the 10 kΩ by a resistor of value R result in the combination having a resistance R_{eq},

$$R_{eq} = \frac{10R}{R + 10}$$

Thus, for a 1% reduction,

$$\frac{R}{R+10} = 0.99 \Rightarrow R = 990\text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R+10} = 0.95 \Rightarrow R = 190\text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R+10} = 0.90 \Rightarrow R = 90\text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R+10} = 0.50 \Rightarrow R = 10\text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ result in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9\text{ k}\Omega\text{, a 1\% reduction;}$$

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09\text{ k}\Omega\text{, a 9.1\% reduction;}$$

(c) 10 kΩ results in

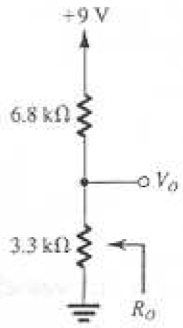
$$R_{eq} = \frac{10}{10 + 10} = 5\text{ k}\Omega\text{, a 50\% reduction.}$$

1.6 $V_O = V_{DD}\dfrac{R_2}{R_1 + R_2}$

To find R_O, we short circuit V_{DD} and look back into node X,

$$R_O = R_2 \,/\!/\, R_1 = \frac{R_1 R_2}{R_1 + R_2}$$

1.7



$$V_O = 9\frac{3.3}{3.3+6.8}$$

$$= 2.94\text{ V}$$

$$R_O = 2.22\text{ k}\Omega$$

For ±5% resistor tolerance the extreme values of V_O are

$$V_{O\text{-low}} = 9\frac{3.3(1-0.05)}{3.3(1-0.05)+6.8(1+0.05)}$$

$$= 2.75\text{ V}$$

$$V_{O\text{-high}} = 9\frac{3.3(1+0.05)}{3(1+0.05)+6.8(1-0.05)}$$

$$= 3.14\text{ V}$$

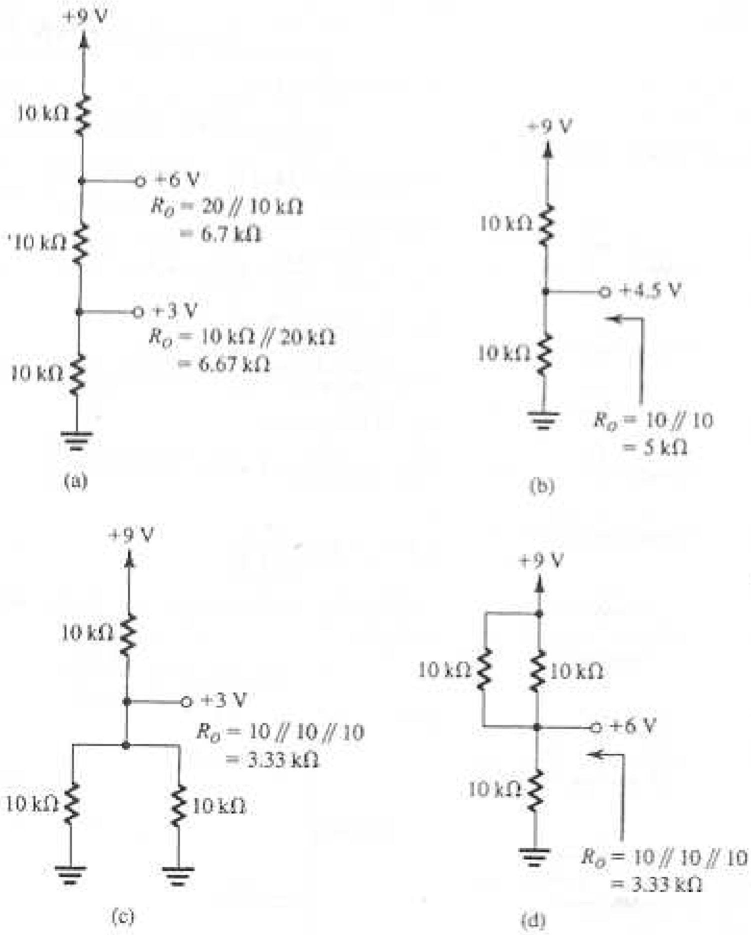
The extreme values of R_O are

$$R_{O\text{-low}} = \frac{3.3(1-0.05)\times 6.8(1-0.05)}{3.3(1-0.05)+6.8(1-0.05)}$$

$$= 2.22(1-0.05) = 2.11\text{ k}\Omega$$

$$R_{O\text{high}} = 2.22(1+0.05) = 2.33\text{ k}\Omega$$

1.8



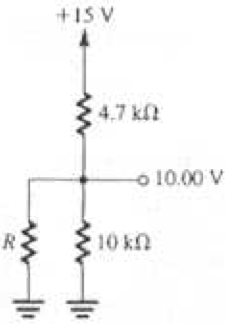
Voltages generated:

+3 V	(two ways: (a) and (c) with (c) having lower output resistance)
+4.5 V	(b)
+6 V	(two ways: (a) and (d) with (d) having a lower output resistance)

1.9

$$V_O = 15\frac{10}{10+4.7} = 10.2 \text{ V}$$

To reduce V_O to 10.00 V we shunt the 10-kΩ resistor by a resistor R whose value is such that $10 \,/\!/\, R = 2 \times 4.7$.



Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

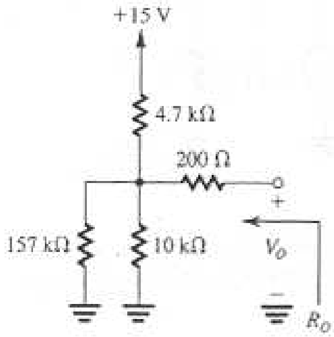
$$\Rightarrow R = 156.7 \simeq 157 \text{ k}\Omega$$

Now,

$$R_O = 10 \text{ k}\Omega \,/\!/\, R \,/\!/\, 4.7 \text{ k}\Omega$$

$$= 9.4 \,/\!/\, 4.7 = \frac{9.4}{3} = 3.133 \text{ k}\Omega$$

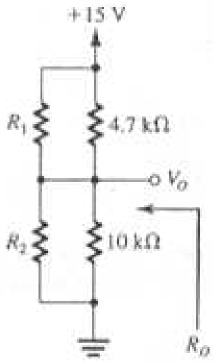
To make $R_O = 3.33$ we add a series resistance of approximately 200 Ω, as shown.



To obtain $V_O = 10.00$ V and $R_O = 3$ kΩ we have to shunt both the 4.7-kΩ and the 10-kΩ resistors as shown. To yield an output voltage $V_O = 10.00$ V we must have

$$\underbrace{(R_2 \,/\!/\, 10)}_{R_2'} = \underbrace{2(R_1 \,/\!/\, 4.7)}_{R_1'}$$

$$R_2' = 2R_1' \qquad (1)$$



For $R_O = 3$ kΩ we must have

$$R_1' \,/\!/\, R_2'' = 3 \qquad (2)$$

Solving (1) and (2) yields

$$R_1' = 4.5 \text{ k}\Omega$$

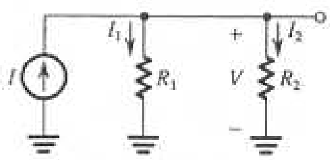
$$R_2' = 9.0 \text{ k}\Omega$$

which can be used to find R_1 and R_2 respectively,

$$R_1 = 157 \text{ k}\Omega$$

$$R_2 = 90 \text{ k}\Omega$$

1.10



$$V = I(R_1 \,/\!/\, R_2)$$

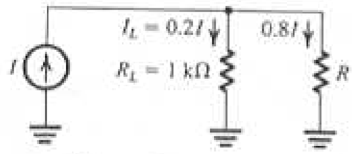
$$= I\frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = I\frac{R_2}{R_1 + R_2}$$

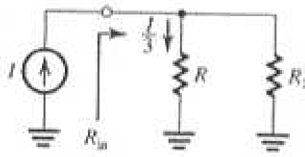
$$I_2 = \frac{V}{R_2} = I\frac{R_1}{R_1 + R_2}$$

1.11 Connect a resistor R in parallel with R_L. To make $I_L = 0.2I$ (and thus the current through R, $0.8I$), R should be such

$$0.2I \times 1\ \text{k}\Omega = 0.8IR$$
$$\Rightarrow R = 250\ \Omega$$



1.12



To make the current through R equal to $I/3$ we shunt R by a resistance R_1 of value such that the current through it will be $2I/3$; thus

$$\frac{I}{3}R = \frac{2I}{3}R_1 \Rightarrow R_1 = \frac{R}{2}$$

The input resistance of the divider, R_{in}, is

$$R_{in} = R \mathbin{/\!/} R_1 = R \mathbin{/\!/} \frac{R}{2} = \frac{1}{3}R$$

Now if R_1 is 10% too high, i.e.,

$$R_1 = 1.1\frac{R}{2}$$

the problem can be solved in two ways:

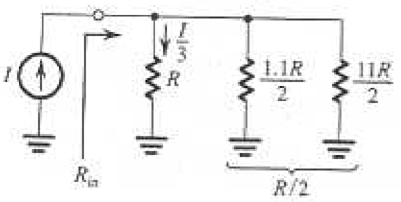
(a) Connect a resistor R_2 across R_1 of value such that $R_1 \mathbin{/\!/} R_2 = R/2$, thus

$$\frac{R_2(1.1R/2)}{R_2 + (1.1R/2)} = \frac{R}{2}$$

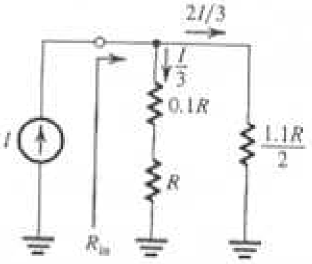
$$1.1R_2 = R_2 + \frac{1.1R}{2}$$

$$\Rightarrow R_2 = \frac{11R}{2} = 5.5R$$

$$R_{in} = R \mathbin{/\!/} \frac{1.1R}{2} \mathbin{/\!/} \frac{11R}{2}$$
$$= R \mathbin{/\!/} \frac{R}{2} = \frac{R}{3}$$



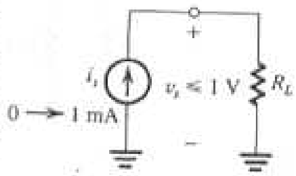
(b) Connect a resistor in series with the load resistor R so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of R i.e., $0.1R$.



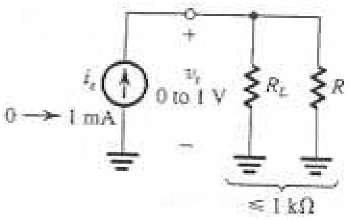
$$R_{in} = 1.1R \mathbin{/\!/} \frac{1.1R}{2}$$
$$= \frac{1.1R}{3}$$

i.e., 10% higher than in case (a).

1.13 If $R_L = 10\ \text{k}\Omega$ then a voltage of 0 to 10 V may develop across the source. To limit the voltage to the specified maximum of 1 V, we have to shunt R_L with a resistor R whose value is such that the parallel combination of R_L and R is $\le 1\ \text{k}\Omega$. Thus,

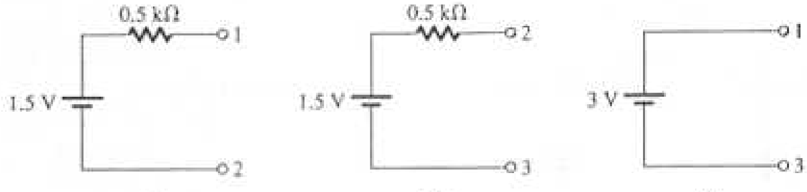


$$\frac{RR_L}{R + R_L} \le 1$$
$$R \le 1.111\ \text{k}\Omega$$
$$\Rightarrow R = 1.1\ \text{k}\Omega$$



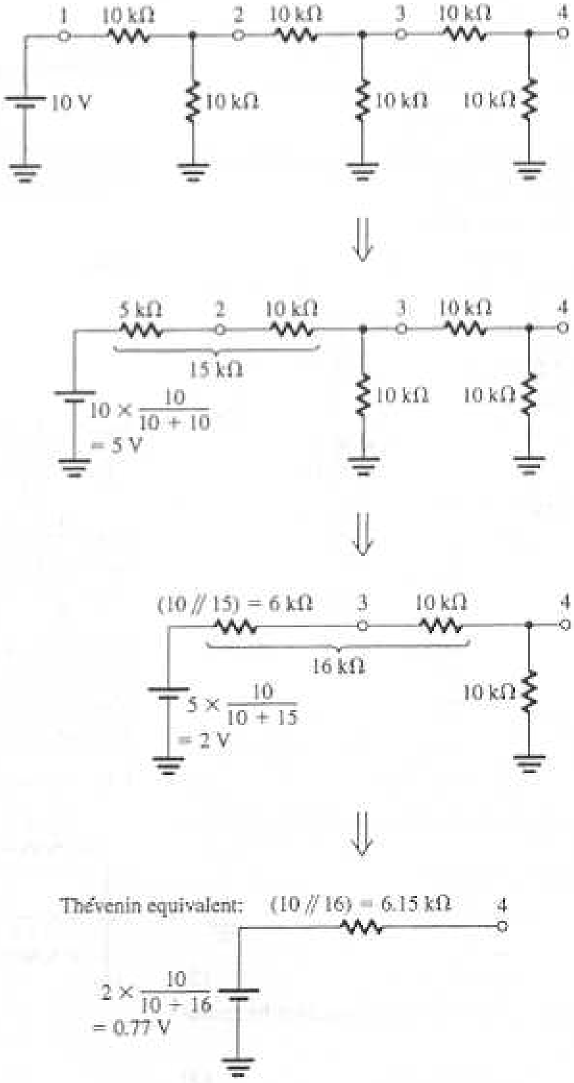
The resulting circuit, utilizing only one additional resistor of value 1.1 kΩ creates a current divider across the source.

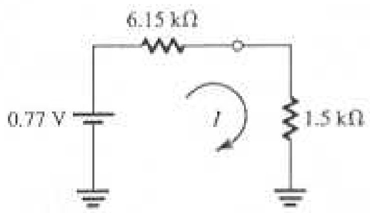
1.14


0.5 kΩ
1.5 V
1
2
0.5 kΩ
1.5 V
2
3
3 V
1
3

(a) (b) (c)

1.15


1
10 kΩ
2
10 kΩ
3
10 kΩ
4
10 V
10 kΩ
10 kΩ
10 kΩ
⇓
5 kΩ
2
10 kΩ
3
10 kΩ
4
15 kΩ
$10 \times \frac{10}{10 + 10}$
= 5 V
10 kΩ
10 kΩ
⇓
(10 // 15) = 6 kΩ
3
10 kΩ
4
16 kΩ
$5 \times \frac{10}{10 + 15}$
= 2 V
10 kΩ
⇓
Thévenin equivalent:
(10 // 16) = 6.15 kΩ
4
$2 \times \frac{10}{10 + 16}$
= 0.77 V



Now, when a resistance of 1.5 kΩ is connected between 4 and ground,

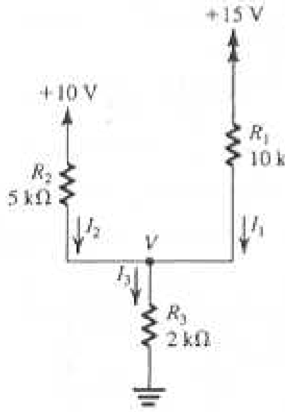
$$I = \frac{0.77}{6.15 + 1.5}$$
$$= 0.1 \text{ mA}$$

1.16 (a) Node equation at the common node yields

$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across R_1 and R_3 equals 15 V, we write

$$15 = I_1R_1 + I_3R_3$$
$$= 10I_1 + (I_1 + I_2) \times 2$$
$$= 12I_1 + 2I_2$$



That is,

$$12I_1 + 2I_2 = 15 \quad (1)$$

Similarly, the voltage drops across R_2 and R_3 add up to 10 V, thus

$$10 = I_2R_2 + I_3R_3$$
$$= 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 10 \quad (2)$$

Equations (1) and (2) can be solved together by multiplying (2) by 6,

$$12I_1 + 42I_2 = 60 \quad (3)$$

Now, subtracting (1) from (3) yields

$$40I_2 = 45$$
$$\Rightarrow I_2 = 1.125 \text{ mA}$$

Substituting in (2) gives

$$2I_1 = 10 - 7 \times 1.125 \text{ mA}$$
$$\Rightarrow I_1 = 1.0625 \text{ mA}$$
$$I_3 = I_1 + I_2$$
$$= 1.0625 + 1.1250$$
$$= 1.1875 \text{ mA}$$
$$V = I_3R_3$$
$$= 1.1875 \times 2 = 2.3750 \text{ V}$$

To summarize:

$$I_1 \simeq 1.06 \text{ mA} \qquad I_2 \simeq 1.13 \text{ mA}$$
$$I_3 \simeq 1.19 \text{ mA} \qquad V \simeq 2.38 \text{ V}$$

(b) A node equation at the common node can be written in terms of V as

$$\frac{15 - V}{R_1} + \frac{10 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{15 - V}{10} + \frac{10 - V}{5} = \frac{V}{2}$$
$$\Rightarrow 0.8V = 3.5$$
$$\Rightarrow V = 2.375 \text{ V}$$

Now, I_1, I_2, and I_3 can be easily found as

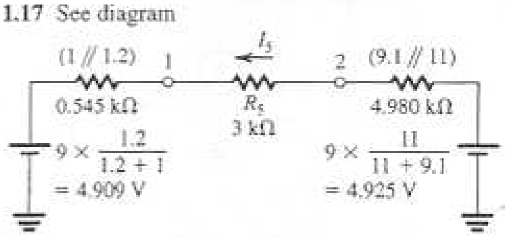
$$I_1 = \frac{15 - V}{10} = \frac{15 - 2.375}{10} = 1.0625 \text{ mA} \simeq 1.06 \text{ mA}$$

$$I_2 = \frac{10 - V}{5} = \frac{10 - 2.375}{5} = 1.125 \text{ mA} \simeq 1.13 \text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{2.375}{2} = 1.1875 \text{ mA} \simeq 1.19 \text{ mA}$$

Method (b) is much preferred; faster, more insightful and less prone to errors. In general, one attempts to identify the least possible number of variables and write the corresponding minimum number of equations.

1.17 See diagram

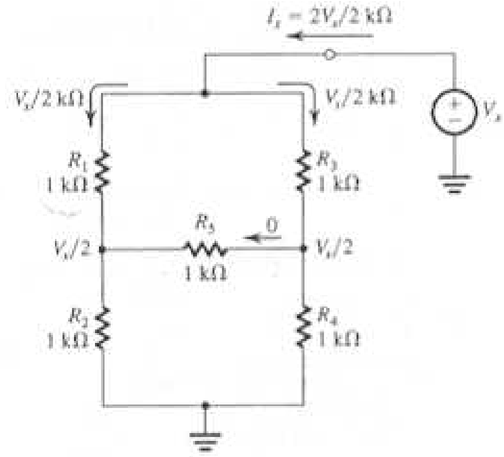


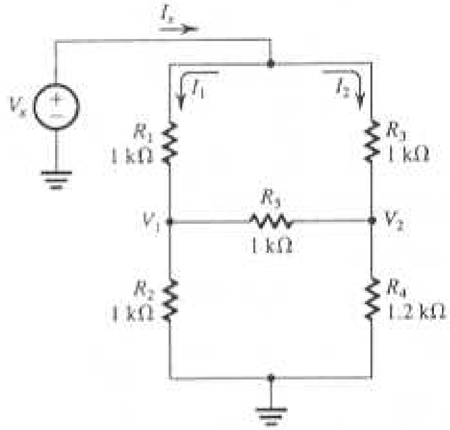
$$I_5 = \frac{4.925 - 4.909}{4.98 + 3 + 0.545} = 1.88 \ \mu\text{A}$$

$$V_5 = 1.88 \ \mu\text{A} \times 3 \text{ k}\Omega = 5.64 \text{ mV}$$

1.18 From the symmetry of the circuit, there will be no current in R_5. (Otherwise the symmetry would be violated.) Thus each branch will carry a current $V_x/2\text{ k}\Omega$ and I_x will be the sum of the two current.

$$I_x = \frac{2V_x}{2\text{ k}\Omega} = \frac{V_x}{1\text{ k}\Omega}$$





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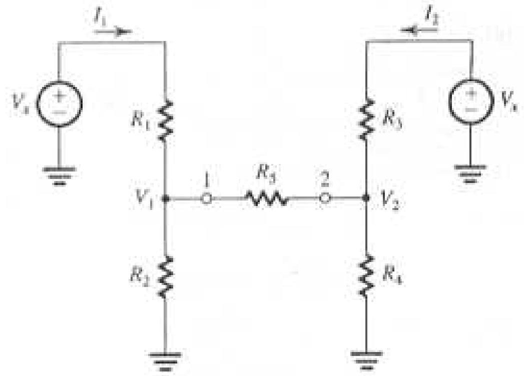

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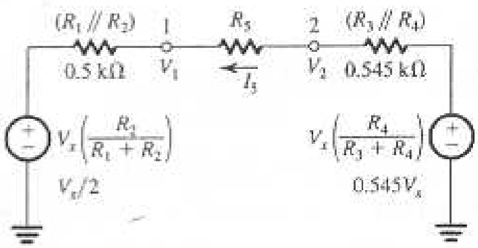


Thus,

$$R_{eq} \equiv \frac{V_x}{I_x} = 1\text{ k}\Omega$$

Now, if R_4 is raised to 1.2 kΩ the symmetry will be broken. To find I_x we use Thévenin's theorem as follows:

$$I_5 = \frac{0.545V_x - 0.5V_x}{0.5 + 1 + 0.545} = 0.022V_x$$

$$V_1 = \frac{V_x}{2} + 0.022V_x \times 0.5$$

$$= 0.5V_x \times 1.022 = 0.511V_x$$

$$V_2 = V_1 + I_5R_5 = 0.533V_x$$

$$I_1 = \frac{V_x - V_1}{1\text{ k}\Omega} = 0.489V_x$$

$$I_2 = \frac{V_x - V_2}{1\text{ k}\Omega} = 0.467V_x$$

$$I_x = I_1 + I_2 = 0.956V_x$$

$$\Rightarrow\ R_{eq} \equiv \frac{V_x}{I_x} = 1.05\text{ k}\Omega$$

1.19 (a) $T = 10^{-4}$ ms $= 10^{-7}$ s

$$f = \frac{1}{T} = 10^7 \text{ Hz}$$
$$\omega = 2\pi f = 6.28 \times 10^7 \text{ Hz}$$

(b) $f = 1$ GHz $= 10^9$ Hz

$$T = \frac{1}{f} = 10^{-9} \text{ s}$$
$$\omega = 2\pi f = 6.28 \times 10^9 \text{ rad/s}$$

(c) $\omega = 6.28 \times 10^2$ rad/s

$$f = \frac{\omega}{2\pi} = 10^2 \text{ Hz}$$
$$T = \frac{1}{f} = 10^{-2} \text{ s}$$

(d) $T = 10$ s

$$f = \frac{1}{T} = 10^{-1} \text{ Hz}$$
$$\omega = 2\pi f = 6.28 \times 10^{-1} \text{ rad/s}$$

(e) $f = 60$ Hz

$$T = \frac{1}{f} = 1.67 \times 10^{-2} \text{ s}$$
$$\omega = 2\pi f = 3.77 \times 10^2 \text{ rad/s}$$

(f) $\omega = 1$ krad/s $= 10^3$ rad/s

$$f = \frac{\omega}{2\pi} = 1.59 \times 10^2 \text{ Hz}$$
$$T = \frac{1}{f} = 6.28 \times 10^{-3} \text{ s}$$

(g) $f = 1900$ MHz $= 1.9 \times 10^9$ Hz

$$T = \frac{1}{f} = 0.526 \times 10^{-9} \text{ s}$$
$$\omega = 2\pi f = 1.194 \times 10^9 \text{ rad/s}$$

1.20 (a) $Z = 1$ kΩ at all frequencies

(b) $Z = 1/j\omega C = -j\dfrac{1}{2\pi f \times 10 \times 10^{-9}}$

At $f = 60$ Hz, $Z = -j265$ kΩ
At $f = 100$ kHz, $Z = -j159$ Ω
At $f = 1$ GHz, $Z = -j0.016$ Ω

(c) $Z = 1/j\omega C = -j\dfrac{1}{2\pi f \times 2 \times 10^{-12}}$

At $f = 60$ Hz, $Z = -j1.33$ GΩ
At $f = 100$ kHz, $Z = -j0.8$ MΩ
At $f = 1$ GHz, $Z = -j79.6$ Ω

(d) $Z = j\omega L = j2\pi f L = j2\pi f \times 10 \times 10^{-3}$

At $f = 60$ Hz, $Z = j3.77$ Ω
At $f = 100$ kHz, $Z = j6.28$ kΩ
At $f = 1$ GHz, $Z = j62.8$ MΩ

1.21 (a) $Z = R + \dfrac{1}{j\omega C}$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$
$$= (1 - j1.59) \text{ k}\Omega$$

(b) $Y = \dfrac{1}{R} + j\omega C$

$$= \frac{1}{10^3} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}$$
$$= 10^{-3}(1 + j0.628) \text{ ℧}$$
$$Z = \frac{1}{Y} = \frac{1000}{1 + j0.628}$$
$$= \frac{1000(1 - j0.628)}{1 + 0.628^2}$$
$$= (717.2 - j450.4) \ \Omega$$

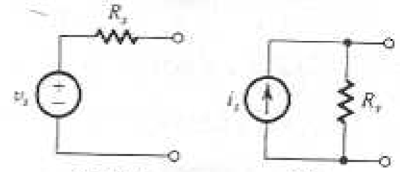
(c) $Y = \dfrac{1}{R} + j\omega C$

$$= \frac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12}$$
$$= 10^{-5}(1 + j0.628)$$
$$Z = \frac{10^5}{1 + j0.628}$$
$$= (71.72 - j45.04) \text{ k}\Omega$$

(d) $Z = R + j\omega L$

$$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$$
$$= 100 + j6.28 \times 100$$
$$= (100 + j628) \ \Omega$$

1.22



Thévenin Equivalent

Norton Equivalent

$$v_{oc} = v_s$$
$$i_{sc} = i_s$$
$$v_s = i_s R_s$$

Thus,

$$R_s = \frac{v_{oc}}{i_{sc}}$$

(a) $v_s = v_{oc} = 10\text{ V}$

$i_s = i_{sc} = 100\ \mu\text{A}$

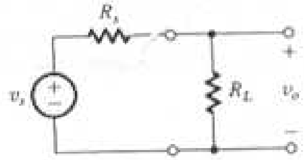
$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{10\text{ V}}{100\ \mu\text{A}} = 0.1\text{ M}\Omega = 100\text{ k}\Omega$$

(b) $v_s = v_{oc} = 0.1\text{ V}$

$i_s = i_{sc} = 10\ \mu\text{A}$

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{0.1\text{V}}{10\ \mu\text{A}} = 0.01\text{ M}\Omega = 10\text{ k}\Omega$$

1.23



$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s}$$

$$v_o = v_s \Big/ \left(1 + \frac{R_s}{R_L}\right)$$

Thus,

$$\frac{v_s}{1 + \dfrac{R_s}{100}} = 30 \quad (1)$$

and

$$\frac{v_s}{1 + \dfrac{R_s}{10}} = 10 \quad (2)$$

Dividing (1) by (2) gives

$$\frac{1 + (R_s/10)}{1 + (R_s/100)} = 3$$

$$\Rightarrow R_s = 28.6\text{ k}\Omega$$

Substituting in (2) gives

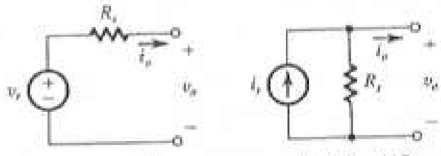
$$v_s = 38.6\text{ mV}$$

The Norton current i_s can be found as

$$i_s = \frac{v_s}{R_s} = \frac{38.6\text{ mV}}{28.6\text{ k}\Omega} = 1.35\ \mu\text{A}$$

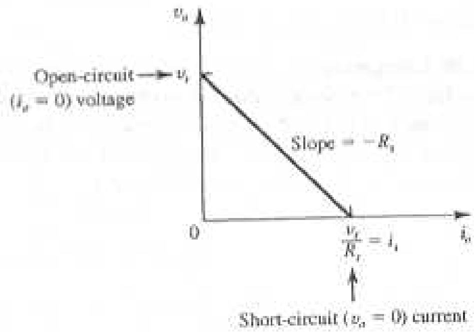
1.24 The observed output voltage is 1 mV/°C which is one half the voltage specified by the sensor, presumably under open-circuit conditions that is without a load connected. It follows that that sensor internal resistance must be equal to R_L, i.e., 10 kΩ.

1.25

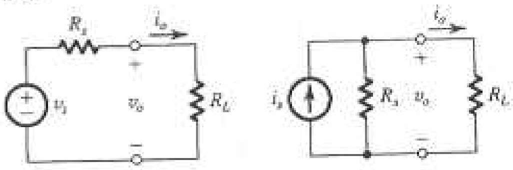


$v_o = v_s - i_o R_s$

$v_o = (i_s - i_o)R_s$
$= i_s R_s - i_o R_s$
$v_o = v_s - i_o R_s$



1.26



R_L represents the input resistance of the processor

For $v_o = 0.9\, v_s$,

$$0.9 = \frac{R_L}{R_L + R_s} \quad \Rightarrow \quad R_L = 9R_s$$

For $i_o = 0.9 i_s$,

$$0.9 = \frac{R_s}{R_s + R_L} \quad \Rightarrow \quad R_L = R_s/9$$

1.27

Case	ω (rad/s)	f (Hz)	T (s)
a	6.28×10^9	1×10^9	1×10^{-9}
b	1×10^9	1.59×10^8	6.28×10^{-9}
c	6.28×10^{10}	1×10^{10}	1×10^{-10}
d	3.77×10^2	60	1.67×10^{-2}
e	6.28×10^3	1×10^3	1×10^{-3}
f	6.28×10^6	1×10^6	1×10^{-6}

1.28 (a) $V_{peak} = 117 \times \sqrt{2} = 165$ V

(b) $V_{rms} = 33.9/\sqrt{2} = 24$ V

(c) $V_{peak} = 220 \times \sqrt{2} = 311$ V

(d) $V_{peak} = 220 \times \sqrt{2} = 311$ kV

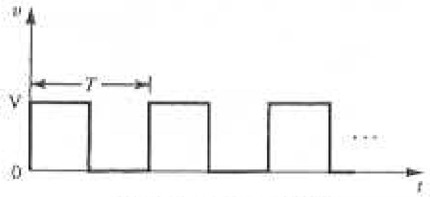
1.29 (a) $v = 10 \sin(2\pi \times 10^4 t)$, V

(b) $v = 120\sqrt{2} \sin(2\pi \times 60)$, V

(c) $v = 0.1 \sin(1000t)$, V

(d) $v = 0.1 \sin(2\pi \times 10^{+3} t)$, V

1.30 Comparing the given waveform to that described by Eq. 1.2 we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform look as follows



Average value = 0.5 V
Peak-to-peak value = 1 V
Lowest value = 0 V
Highest value = 1 V

$$\text{Period } T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3}\,\text{s}$$

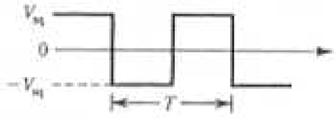
1.31 The two harmonics have the ratio $126/98 = 9/7$. Thus, these are the 7th and 9th harmonics. From Eq. 1.2 we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of 98/7 or 14 kHz and peak amplitude of $63 \times 7 = 441$ mV. The rms value of the fundamental will be $441/\sqrt{2} = 312$ mV. To find the peak-to-peak amplitude of the square wave we note that $4V/\pi = 441$ mV. Thus,

$$\text{Peak-to-peak amplitude} = 2V = 441 \times \frac{\pi}{2} = 693 \text{ mV}$$

$$\text{Period } T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4\ \mu\text{s}$$

1.32 To be barely audible by a relatively young listener, the 5th harmonic must be limited to 20 kHz; thus the fundamental will be 4 kHz. At the low end, hearing extends down to about 20 Hz. For the fifth and higher to be audible the fifth must be no lower than 20 Hz. Correspondingly, the fundamental will be at 4 Hz.

1.33 If the amplitude of the square wave is V_{sq} then the power delivered by the square wave to



a resistance R will be V_{sq}^2/R. If this power is to equal that delivered by a sine wave of peak amplitude $\hat{V}$ then

$$\frac{V_{sq}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus, $V_{sq} = \hat{V}/\sqrt{2}$. This result is independent of frequency.

1.34

Decimal	Binary
0	0
5	101
8	1000
25	11001
57	111001

1.35

$b_3\ b_2\ b_1\ b_0$	Value Represented
0 0 0 0	+0
0 0 0 1	+1
0 0 1 0	+2
0 0 1 1	+3
0 1 0 0	+4
0 1 0 1	+5
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	−0
1 0 0 1	−1
1 0 1 0	−2
1 0 1 1	−3
1 1 0 0	−4
1 1 0 1	−5
1 1 1 0	−6
1 1 1 1	−7

Note that there are two possible representation of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range ±3.5 V can be represented

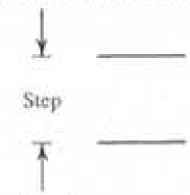
Input	Steps	Code
+2.5 V	+5	0101
−3.0 V	−6	1110
+2.7	+5	0101
−2.8	−6	1110

1.36 (a) For N bits there will be 2^N possible levels, from 0 to V_{FS}. Thus there will be $(2^N - 1)$ discrete steps from 0 to V_{FS} with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is



$$\frac{1}{2} \times \text{step size} = \frac{1}{2}\,\frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

(c)
$$\frac{10\text{ V}}{2^N - 1} \le 5\text{ mV}$$
$$2^N - 1 \ge 2000$$
$$2^N \ge 2001 \;\Rightarrow\; N = 11$$

For $N = 11$,

$$\text{Resolution} = \frac{10}{2^{11} - 1} = 4.9\text{ mV}$$

$$\text{Quantization error} = \frac{4.9}{2} = 2.4\text{ mV}$$

1.37 When $b_i = 1$, the ith switch is in position 1 and a current $(V_{ref}/2^iR)$ flows to the output. Thus i_O will be the sum of all the currents corresponding to "1" bits, i.e.,

$$i_O = \frac{V_{ref}}{R}\left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N}\right)$$

(b) b_N is the LSB
b_1 is the MSB

(c) $$i_{O\max} = \frac{10\text{ V}}{5\text{ k}\Omega}\left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}\right)$$
$$= 1.96875\text{ mA}$$

Corresponding to the LSB changing from 0 to 1 the output changes by $10/5 \times 1/2^6 = 0.03125$ mA.

1.38 There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be $44{,}100 \times 16 = 7.056 \times 10^5$ bits per second.

1.39 (a) $A_v = \frac{v_o}{v_i} = \frac{10\text{ V}}{100\text{ mV}} = 100$ V/V

or, 20 log 100 = 40 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{10\text{ V}/100\ \Omega}{100\ \mu\text{A}} = \frac{0.1\text{ A}}{100\ \mu\text{A}}$$
$$= 1000\text{ A/A}$$

or, 20 log 1000 = 60 dB

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i} = 100 \times 1000$$
$$= 10^5\text{ W/W}$$

or 10 log 10^5 = 50 dB

(b) $A_v = \frac{v_o}{v_i} = \frac{2\text{ V}}{10\ \mu\text{V}} = 2 \times 10^5$ V/V

or, 20 log 2×10^5 = 106 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{2\text{ V}/10\text{ k}\Omega}{100\text{ nA}}$$
$$= \frac{0.2\text{ mA}}{100\text{ nA}} = \frac{0.2 \times 10^{-3}}{100 \times 10^{-9}} = 2000\text{ A/A}$$

or 20 log A_i = 66 dB

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$
$$= 2 \times 10^5 \times 2000$$
$$= 4 \times 10^8\text{ W/W}$$

or 10 log A_p = 86 dB

(c) $A_v = \frac{v_o}{v_i} = \frac{10\text{ V}}{1\text{ V}} = 10$ V/V

or, 20 log 10 = 20 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{10\text{ V}/10\ \Omega}{1\text{ mA}}$$
$$= \frac{1\text{ A}}{1\text{ mA}} = 1000\text{ A/A}$$

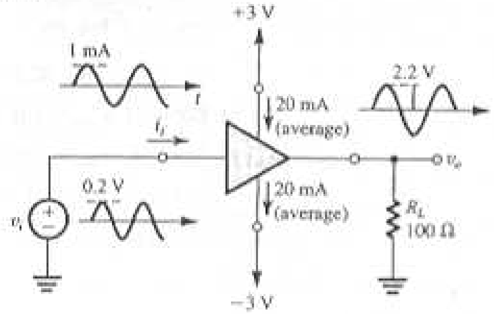
or, 20 log 1000 = 60 dB

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$

$$= 10 \times 1000 = 10^4 \text{ W/W}$$

or $10 \log_{10} A_p = 40$ dB

1.40



$$A_v = \frac{v_o}{v_i} = \frac{2.2}{0.2}$$

$$= 11 \text{ V/V}$$

or 20 log 11 = 20.8 dB

$$A_i = \frac{i_o}{i_i} = \frac{2.2 \text{ V}/100\ \Omega}{1 \text{ mA}}$$

$$= \frac{22 \text{ mA}}{1 \text{ mA}} = 22 \text{ A/A}$$

or, $20 \log A_i = 26.8$ dB

$$A_p = \frac{p_o}{p_i} = \frac{(2.2/\sqrt{2})^2/100}{\dfrac{0.2}{\sqrt{2}} \times \dfrac{10^{-3}}{\sqrt{2}}}$$

$$= 242 \text{ W/W}$$

or, $10 \log A_p = 23.8$ dB

Supply power = 2 × 3 V × 20 mA = 120 mW

$$\text{Output power} = \frac{v_{orms}^2}{R_L} = \frac{(2.2/\sqrt{2})^2}{100\ \Omega} = 24.2 \text{ mW}$$

$$\text{Input power} = \frac{24.2}{242} = 0.1 \text{ mW} \quad \text{(negligible)}$$

Amplifier dissipation ≃ Supply power − Output power

$$= 120 - 24.2 = 95.8 \text{ mW}$$

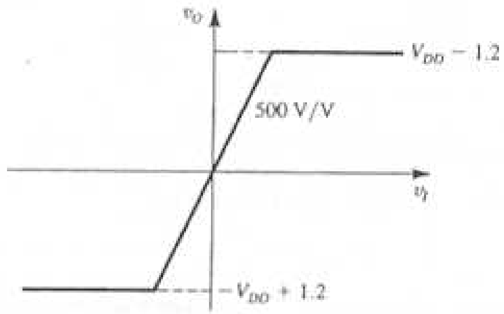
$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{Supply power}} \times 100$$

$$= \frac{24.2}{120} \times 100 = 20.2\%$$

1.41 For $V_{DD} = 5$ V:

The largest undistorted sine-wave output is of 3.8-V peak amplitude or $3.8/\sqrt{2} = 2.7 V_{rms}$. Input needed is 5.4 mV_{rms}.

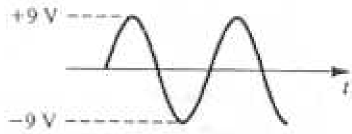
Supplies are V_{DD} and $-V_{DD}$



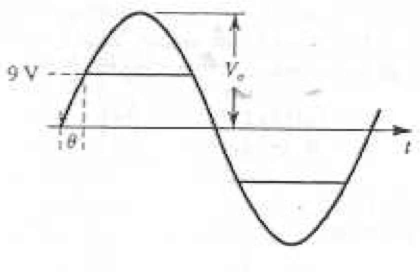
For $V_{DD} = 10$ V, the largest undistorted sine-wave output is of 8.8-V peak amplitude or 6.2 V_{rms}. Input needed is 12.4 mV_{rms}.

For $V_{DD} = 15$ V, the largest undistorted sine-wave output is of 13.8-V peak amplitude or 9.8 V_{rms}. The input needed is 9.8 V/500 = 19.6 mV_{rms}.

1.42 (a) For an output whose extremes are just at the edge of clipping, i.e., an output of 9-V_{peak}, the input must have 9 V/1000 = 9 mV_{peak}.



(b) For an output that is clipping 90% of the time, $\theta = 0.1 \times 90° = 9°$ and $V_p \sin 9° = 9 \text{ V} \Rightarrow V_p = 57.5$ V which of course does not occur as the output saturates at ±9 V. To produce this result, the input peak must be 57.5/1000 = 57.5 mV.

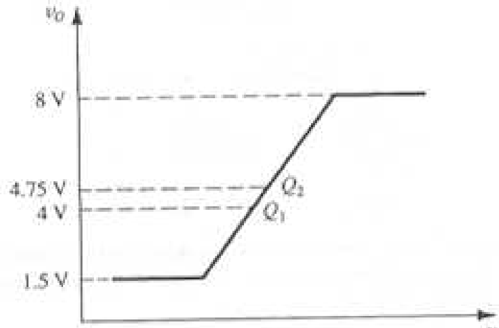


(c) For an output that is clipping 99% of the time, $\theta = 0.01 \times 90° = 0.9°$

$$V_p \sin 0.9° = 9 \text{ V}$$
$$\Rightarrow V_p = 573 \text{ V}$$

and the input must be 573 V/1000 or 0.573 V_{peak}.

1.43 When the amplifier is biased at 4 V (i.e., at point Q_1), the maximum possible amplitude of a sine-wave output without clipping is $(4 - 1.5) = 2.5\ V_{peak}$.

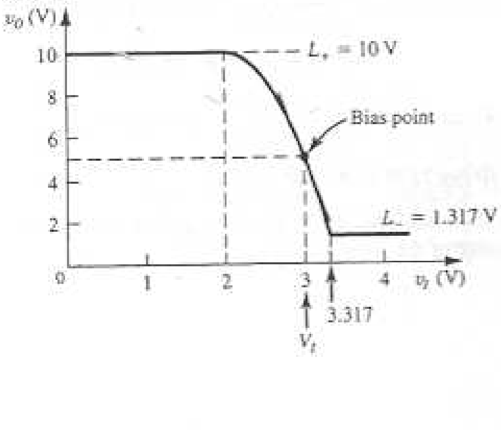


To obtain the largest undistorted sine wave output possible with this amplifier, it must be biased halfway between the saturation levels, i.e., at $V_O = (8 + 1.5)/2 = 4.75$ V (point Q_2) and the resulting output will have a peak value of $8 - 4.75 = 4.75 - 1.50 = 3.25$ V.

1.44 $v_O = 10 - 5(v_I - 2)^2$, $\quad 2 \le v_I \le v_O + 2$, $\quad v_O \ge 0$

(a) For $v_I \le 2$ V, $v_O = 10$ V.

The upper limit on v_I is found by substituting $v_I = v_O + 2$, that is, $v_O = v_I - 2$ in the transfer characteristic. The result is $v_O = 10 - 5v_O^2$, whose solution is $v_O = 1.317$ V and the corresponding $v_I = 3.317$ V. To obtain a sketch of v_O versus v_I, we evaluate v_O for values of v_I in the range 2 V to 3.317 V. The result is the following sketch:



(b) To obtain $V_O = 5$ V we bias at $V_I = 3$ V.

(c) Small-signal gain at bias point $= \left.\dfrac{\partial v_O}{\partial v_I}\right|_{V_I = 3\text{V}}$

$$= -5 \times 2(V_I - 2) = -10 \text{ V/V}$$

(d) $v_I = 3 + V_i \cos \omega t$

$$v_O = 10 - 5(3 + V_i \cos \omega t - 2)^2$$
$$= 10 - 5(1 + 2V_i \cos \omega t + V_i^2 \cos^2 \omega t)$$
$$= 5 - 10V_i \cos \omega t - 5V_i^2\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega t\right)$$

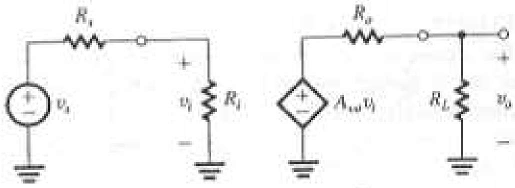
$$= \underbrace{(5 - 2.5V_i^2)}_{\text{dc}} - \underbrace{10V_i \cos \omega t}_{\text{Fundamental}} - \underbrace{2.5V_i^2 \cos 2\omega t}_{\text{2nd harmonic}}$$

For 1% second-harmonic distortion: $2.5V_i^2/10V_i = 0.01$

Thus,

$$V_i = \frac{10 \times 0.01}{2.5} \text{ V} = 40 \text{ mV}$$

1.45



$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \times A_{v_o} \times \frac{R_L}{R_L + R_o}$$

(a) $$\frac{v_o}{v_s} = \frac{10R_s}{10R_s + R_s} \times A_{v_o} \times \frac{10R_o}{10R_o + R_o}$$
$$= \frac{10}{11} \times 10 \times \frac{10}{11} = 8.26 \text{ V/V}$$

or, 20 log 8.26 = 18.3 dB

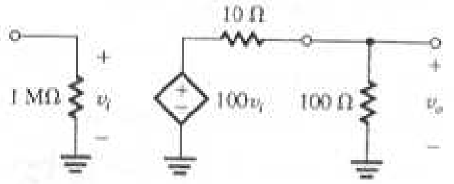
(b) $$\frac{v_o}{v_s} = \frac{R_s}{R_s + R_s} \times A_{v_o} \times \frac{R_o}{R_o + R_o}$$
$$= 0.5 \times 10 \times 0.5 = 2.5 \text{ V/V}$$

or, 20 log 2.5 = 8 dB

(c) $$\frac{v_o}{v_s} = \frac{R_s/10}{(R_s/10) + R_s} \times A_{v_o} \times \frac{R_o/10}{(R_o/10) + R_o}$$
$$= \frac{1}{11} \times 10 \times \frac{1}{11} = 0.083 \text{ V/V}$$

or 20 log 0.083 = −21.6 dB

1.46 $20 \log A_{v_o} = 40$ dB $\Rightarrow A_{v_o} = 100$ V/V



$$A_v = \frac{v_o}{v_i}$$

$$= 100 \times \frac{100}{100+10}$$

$$= 90.9 \text{ V/V}$$

or, $20 \log 90.9 = 39.1$ dB

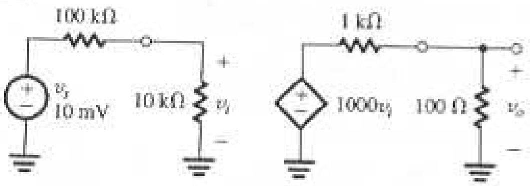
$$A_p = \frac{v_o^2/100\ \Omega}{v_i^2/1\ \text{M}\Omega} = A_v^2 \times 10^4 = 8.3 \times 10^7 \text{ W/W}$$

or $10 \log (8.3 \times 10^7) = 79.1$ dB.

For a peak output sine-wave current of 100 Ω, the peak output voltage will be 100 mA × 100 Ω = 10 V. Correspondingly v_i will be a sine wave with a peak value of 10 V/A_v = 10/90.9 or an rms value of $10/(90.9 \times \sqrt{2}) = 0.08$ V.

Corresponding output power $= (10/\sqrt{2})^2/100\ \Omega$
$= 0.5$ W

1.47



$$\frac{v_o}{v_s} = \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 100\ \text{k}\Omega} \times 1000 \times \frac{100\ \Omega}{100\ \Omega + 1\ \text{k}\Omega}$$

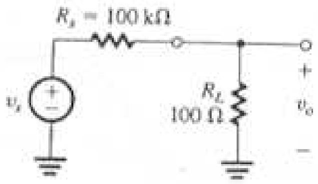
$$= \frac{10}{110} \times 1000 \times \frac{100}{1100} = 8.26 \text{ V/V}$$

The signal loses about 90% of its strength when connected to the amplifier input (because $R_i = R_s/10$). Also, the output signal of the amplifier loses approximately 90% of its strength when the load is connected (because $R_L = R_o/10$). Not a good design! Nevertheless, if the source were connected directly to the load,

$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s}$$

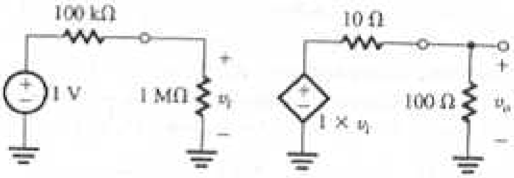
$$= \frac{100\ \Omega}{100\ \Omega + 100\ \text{k}\Omega}$$

$$= 0.001 \text{ V/V}$$



which is clearly a much worse situation. Indeed inserting the amplifier increases the gain by a factor $8.3/0.001 = 8300$.

1.48



$$v_o = 1\text{V} \times \frac{1\ \text{M}\Omega}{1\ \text{M}\Omega + 100\ \text{k}\Omega} \times 1 \times \frac{100\ \Omega}{100\ \Omega + 10\ \Omega}$$

$$= \frac{1}{1.1} \times \frac{100}{110} = 0.83 \text{ V}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = 0.83 \text{ V/V} \quad \text{or} \quad -1.6 \text{ dB}$$

$$\text{Current gain} = \frac{v_o/100\ \Omega}{v_s/1.1\ \text{M}\Omega} = 0.83 \times 1.1 \times 10^4$$

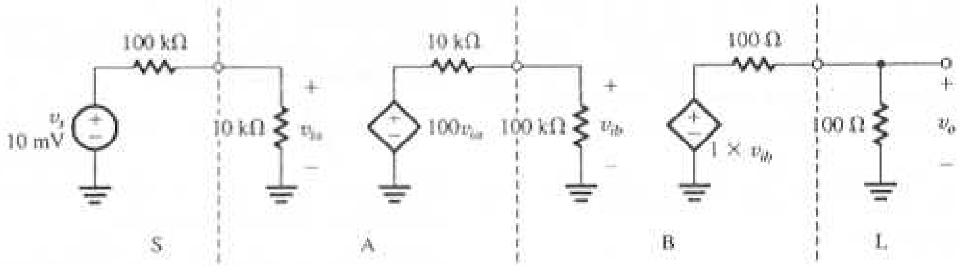
$$= 9091 \text{ A/A} \quad \text{or} \quad 79.2 \text{ dB}$$

$$\text{Power gain} = \frac{v_o^2/100\ \Omega}{v_s^2/1.1\ \text{M}\Omega} = 7578 \text{ W/W}$$

or $10 \log 7578 = 38.8$ dB

(This takes into acct. the power dissipated in the internal resistance of the source.)

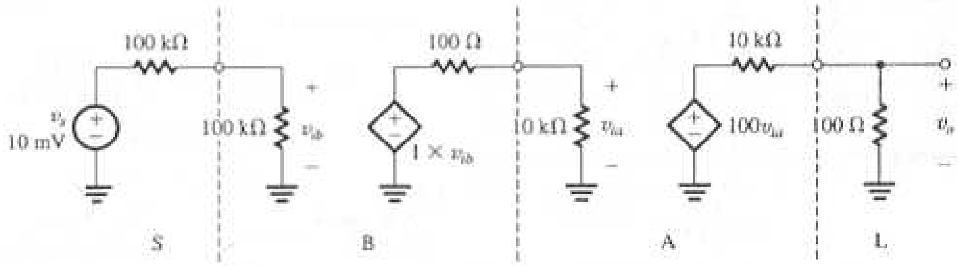
1.50 Case (a) S-A-B-L



$$\frac{v_o}{v_s} = \frac{10}{10+100} \times 100 \times \frac{100}{100+10} \times 1 \times \frac{100}{100+100}$$
$$= 4.1 \text{ V/V}$$

or $\quad 20 \log 4.1 = 12.3$ dB

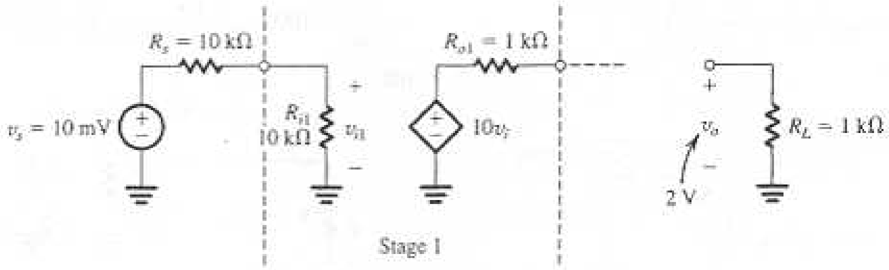
Case (b) S-B-A-L



$$\frac{v_o}{v_s} = \frac{100}{100+100} \times 1 \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 100\ \Omega} \times 100 \times \frac{100\ \Omega}{100\ \Omega + 10\text{ k}\Omega}$$
$$= 0.5 \times \frac{10}{10.1} \times 100 \times \frac{0.1}{10.1}$$
$$= 0.5 \text{ V/V} \quad \text{or} \quad -6 \text{ dB}$$

Thus, obviously case (a) i.e., SABL is preferred.

1.51

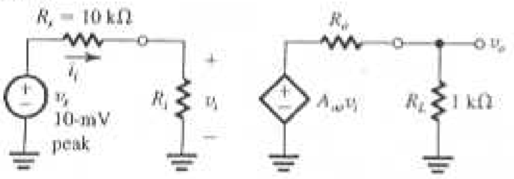


Required overall voltage gain = 2 V/10 mV = 200 V/V. Each stage is capable of providing a *maximum* voltage gain of 10 (the open-circuit gain value). For n stages in cascade the maximum (unattainable) voltage gain in 10^n. We thus see that we need at least 3 stages. For 3 stages, the overall voltage gain obtained is

$$\frac{v_o}{v_i} = \frac{10}{10+10} \times 10 \times \frac{10}{1+10} \times 10 \times \frac{10}{1+10} \times 10 \times \frac{1}{1+1}$$
$$= 206.6 \text{ V/V}$$

Thus, three stages suffice and provide a gain slightly larger than required. The output voltage actually obtained is 10 mV × 206.6 = 2.07 V.

1.53



(a) Required voltage gain $\equiv \frac{v_o}{v_s}$

$$= \frac{3 \text{ V}}{0.01 \text{ V}} = 300 \text{ V/V}$$

(b) The smallest R_i allowed is obtained from

$$0.1\ \mu\text{A} = \frac{10 \text{ mV}}{R_s + R_i} \Rightarrow R_s + R_i = 100 \text{ k}\Omega$$

Thus $R_i = 90$ kΩ.

For $R_i = 90$ kΩ, $i_i = 0.1\ \mu$A peak, and

$$\text{Overall current gain} = \frac{v_o/R_L}{i_i}$$
$$= \frac{3 \text{ mA}}{0.1\ \mu\text{A}} = 3 \times 10^4 \text{A/A}$$

$$\text{Overall power gain} \equiv \frac{v^2_{o\text{rms}}/R_L}{v_{s(\text{rms})} \times i_{i(\text{rms})}}$$

$$= \frac{\left(\frac{3}{\sqrt{2}}\right)^2 / 1000}{\left(\frac{10 \times 10^{-3}}{\sqrt{2}}\right) \times \left(\frac{0.1 \times 10^{-6}}{\sqrt{2}}\right)}$$

$$= 9 \times 10^6 \text{ W/W}$$

(This takes into acct. the power dissipated in the internal resistance of the source.)

(c) If $(A_{v_o} v_i)$ has its peak value limited to 5 V, the largest value of R_o is found from

$$5 \times \frac{R_L}{R_L + R_o} = 3 \Rightarrow R_o = \frac{2}{3} R_L = 667\ \Omega$$

(If R_o were greater than this value, the output voltage across R_L would be less than 3 V.)

(d) For $R_i = 90$ kΩ and $R_o = 667$ Ω, the required value of A_{v_o} can be found from

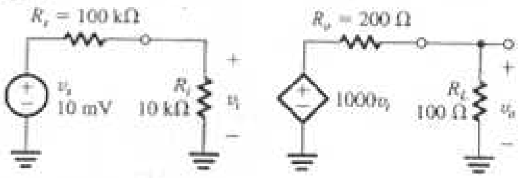
$$300 \text{ V/V} = \frac{90}{90+10} \times A_{v_o} \times \frac{1}{1+0.667}$$
$$\Rightarrow A_{v_o} = 555.7 \text{ V/V}$$

(e) $R_i = 100$ kΩ $(1 \times 10^5\ \Omega)$

$R_o = 100$ Ω $(1 \times 10^2\ \Omega)$

$$300 = \frac{100}{100+10} \times A_{v_o} \times \frac{1000}{1000+100}$$
$$\Rightarrow A_{v_o} = 363 \text{ V/V}$$

1.54



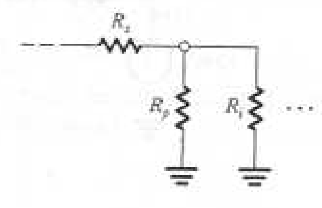
(a) $v_o = 10 \text{ mV} \times \frac{10}{10+100} \times 1000 \times \frac{100}{100+200}$

$= 303$ mV

(b) $\frac{v_o}{v_s} = \frac{303 \text{ mV}}{10 \text{ mV}} = 30.3$ V/V

(c) $\frac{v_o}{v_i} = 1000 \times \frac{100}{100+200} = 333.3$ V/V

(d)



Connect a resistance R_p in parallel with the input and select its value from

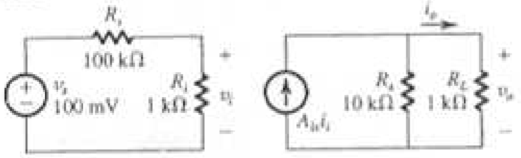
$$\frac{(R_p \,\|\, R_i)}{(R_p \,\|\, R_i)+R_s} = \frac{1}{2}\,\frac{R_i}{R_i+R_s}$$

$$\Rightarrow\ 1+\frac{R_s}{R_p \,\|\, R_i} = 22\ \Rightarrow\ R_p \,\|\, R_i = \frac{R_s}{21} = \frac{100}{21}$$

$$\Rightarrow\ \frac{1}{R_p}+\frac{1}{R_i} = \frac{21}{100}$$

$$R_p = \frac{1}{0.21-0.1} = 9.1\ \text{k}\Omega$$

1.55



(a) Current gain $= \dfrac{i_o}{i_i}$

$$= A_{is}\,\frac{R_o}{R_o+R_L}$$

$$= 100\frac{10}{11}$$

$$= 90.9\ \frac{\text{A}}{\text{A}} = 39.2\ \text{dB}$$

(b) Voltage gain $= \dfrac{v_o}{v_s}$

$$= \frac{i_o}{i_i}\,\frac{R_L}{R_s+R_i}$$

$$= 90.9\times\frac{1}{101}$$

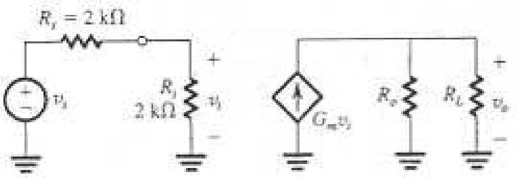
$$= 0.9\ \text{V/V} = -0.9\ \text{dB}$$

(c) Power gain $= A_p = \dfrac{v_o i_o}{v_i i_i}$

$$= 0.9\times 90.9$$

$$= 81.8\ \text{W/W} = 19.1\ \text{dB}$$

1.56



$G_m = 40$ mA/V

$R_o = 20$ kΩ

$R_L = 1$ kΩ

$$v_i = v_s\frac{R_i}{R_s+R_i}$$

$$= v_s\frac{2}{2+2} = \frac{v_s}{2}$$

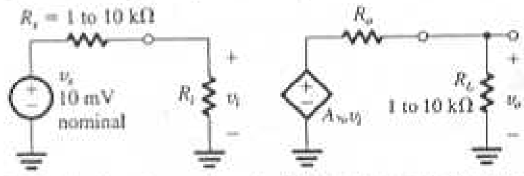
$$v_o = G_m v_i\,(R_L \,\|\, R_o)$$

$$= 40\,\frac{20\times 1}{20+1}\,v_i$$

$$= 40\frac{20}{21}\frac{v_s}{2}$$

Overall voltage gain $\equiv \dfrac{v_o}{v_s} = 19.05$ V/V

1.57 A voltage amplifier is required.



To limit the change in v_o to 10% as R_s varies from 1 to 10 kΩ we select R_i sufficiently large;

$$R_i \ge 10R_{s\max}$$

Thus $R_i = 100$ kΩ.

To limit the change in v_o corresponding to R_L varying in the range 1 to 10 kΩ, to 10%, we select R_o sufficiently small;

$$R_o \le R_{L\min}/10$$

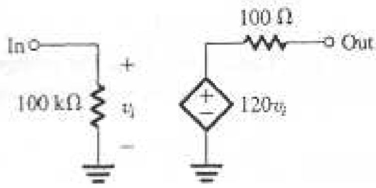
Thus,

$$R_o = 100\ \Omega$$

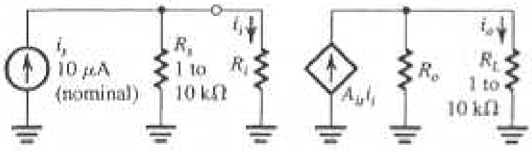
$$v_{o\min} = v_s\frac{R_i}{R_i+R_{s\max}}\,A_{v_o}\,\frac{R_{L\min}}{R_{L\min}+R_o}$$

$$1 = 0.01\,\frac{100}{100+10}\,A_{v_o}\,\frac{1000}{1000+100}$$

$$\Rightarrow\ A_{v_o} = 121\ \text{V/V}$$



1.58 Current amplifier.



To limit the change in i_o resulting from R_s varying over the range 1 to 10 kΩ, to 10% we select R_i sufficiently low so that,

$$R_i \leq R_{s\min}/10$$

Thus, $R_i = 100\ \Omega$

To limit the change in i_o as R_L changes from 1 to 10 kΩ, to 10% we select R_o sufficiently large:

$$R_o \geq 10R_{L\max}$$

Thus, $R_o = 100\ \text{k}\Omega$

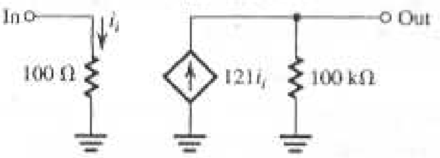
Now for $i_s = 10\ \mu\text{A}$,

$$i_{o\min} = 10^{-5}\frac{R_{s\min}}{R_{s\min}+R_i}A_{is}\frac{R_o}{R_o+R_{L\max}}$$

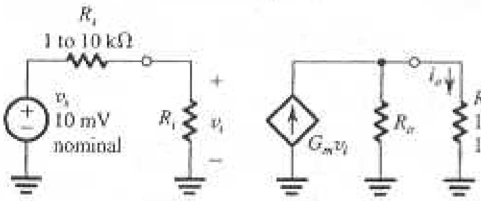
Thus,

$$10^{-3} = 10^{-5}\frac{1000}{1000+100}A_{is}\frac{100}{100+10}$$

$$\Rightarrow A_{is} = 121\ \text{A/A}$$



1.59 Transconductance amplifier.



For R_s varying in the range 1 to 10 kΩ, and Δi_o limited to 10% we have to select R_i sufficiently large;

$$R_i \geq 10R_{s\max}$$

$$R_i = 100\ \text{k}\Omega$$

For R_L varying in the range 1 to 10 kΩ, the change in i_o can be kept to 10% if R_o is selected sufficiently large;

$$R_o \geq R_{L\max}$$

Thus $R_o = 100\ \text{k}\Omega$

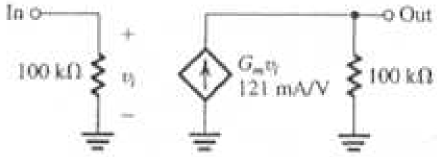
For $v_s = 10\ \text{mV}$,

$$i_{o\min} = 10^{-2}\frac{R_i}{R_i+R_{s\max}}G_m\frac{R_o}{R_o+R_{L\max}}$$

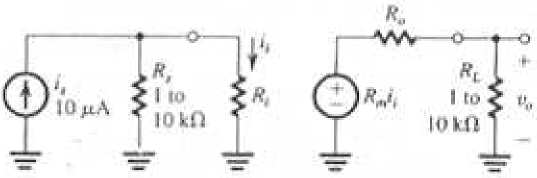
$$10^{-3} = 10^{-2}\frac{100}{100+10}G_m\frac{100}{100+10}$$

$$G_m = 1.21\times10^{-1}\ \text{A/V}$$

$$= 121\ \text{mA/V}$$



1.60 Transresistance amplifier



To limit Δv_o to 10% corresponding to R_s varying in the range 1 to 10 kΩ, we select R_i sufficiently low;

$$R_i \leq \frac{R_{s\min}}{10}$$

Thus, $R_i = 100\ \Omega$

To limit Δv_o to 10% while R_L varies over the range 1 to 10 kΩ, we select R_o sufficiently low;

$$R_o \leq R_{L\min}/10$$

Thus, $R_o = 100\ \Omega$

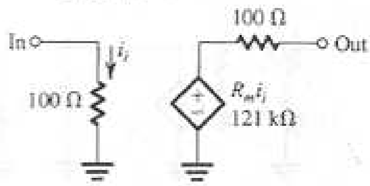
Now, for $i_s = 10\ \mu\text{A}$,

$$v_{o\min} = 10^{-5}\frac{R_{s\min}}{R_{s\min}+R_i}R_m\frac{R_{L\min}}{R_{L\min}+R_o}$$

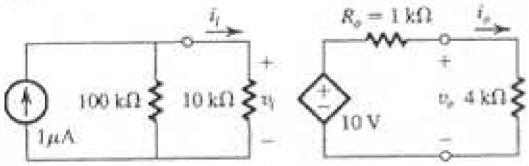
$$1 = 10^{-5}\frac{1000}{1000+100}R_m\frac{1000}{1000+100}$$

$$\Rightarrow R_m = 1.21\times10^5$$

$$= 121\ \text{k}\Omega$$



1.64



$$R_o = \frac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \frac{10\ \text{V}}{10\ \text{mA}} = 1\ \text{k}\Omega$$

$$v_o = 10 \times \frac{4}{1+4} = 8\ \text{V}$$

$$A_v = \frac{v_o}{v_i} = \frac{8}{1 \times 10^{-3} \times (100 \,/\!/\, 10) \times 10^3}$$

$$= 888\ \text{V/V} \quad \text{or} \quad 58.9\ \text{dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{10^{-3} \times \dfrac{100}{100+10}} = \frac{8/(4\times 10^3)}{10^{-3} \times \dfrac{100}{110}}$$

$$= 2200\ \text{A/A} \quad \text{or} \quad 66.8\ \text{dB}$$

$$A_p = \frac{v_o^2/R_L}{i_i^2 R_i} = \frac{8^2/(4\times 10^3)}{\left(10^{-3} \times \dfrac{100}{100+10}\right)^2 10 \times 10^3}$$

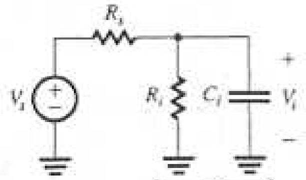
$$= 19.36 \times 10^5\ \text{W/W} \quad \text{or} \quad 62.9\ \text{dB}$$

$$\text{Overall current gain} \equiv \frac{i_o}{1\mu\text{A}}$$

$$= \frac{v_o/R_L}{1\mu\text{A}} = \frac{8/(4\times 10^3)}{10^{-3}}$$

$$= 2000\ \text{A/A} \quad \text{or} \quad 66\ \text{dB}$$

1.65 Using the voltage divider rule



$$Z_i = R_i \,/\!/\, \frac{1}{sC_i}$$

$$Y_i = \frac{1}{R_i} + sC_i$$

$$\frac{V_i}{V_s} = \frac{Z_i}{Z_i + R_s}$$

$$= \frac{1}{1 + R_s Y_i}$$

$$= \frac{1}{1 + R_s\left(\dfrac{1}{R_i} + sC_i\right)}$$

$$= \frac{1}{1 + \dfrac{R_s}{R_i} + sC_i R_s} = \frac{1\Big/\left(1+\dfrac{R_s}{R_i}\right)}{1 + sC_i \dfrac{R_s}{1 + \dfrac{R_s}{R_i}}}$$

$$= \frac{1}{1 + \dfrac{R_s}{R_i}}\,\frac{1}{1 + sC_i\left(\dfrac{R_s R_i}{R_s + R_i}\right)}$$

$$= \frac{R_i}{R_i + R_s}\,\frac{1}{1 + sC_i(R_i \,/\!/\, R_s)}$$

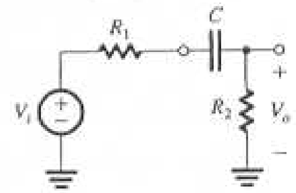
This transfer function is of the STC low-pass type with a dc gain $K = R_i/(R_i + R_s)$ and a 3-dB frequency $\omega_0 = 1/C_i(R_i \,/\!/\, R_s)$.

For $R_s = 20\ \text{k}\Omega$, $R_i = 80\ \text{k}\Omega$, and $C_i = 5$ pF,

$$\omega_0 = \frac{1}{5 \times 10^{-12} \times \dfrac{20 \times 80}{20 + 80} \times 10^3} = 1.25 \times 10^7\ \text{rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1.25 \times 10^7}{2\pi} \simeq 2\ \text{MHz}$$

1.66 Using the voltage-divider rule,



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \dfrac{1}{sC}}$$

$$= \frac{R_2}{R_1 + R_2}\,\frac{s}{s + \dfrac{1}{C(R_1 + R_2)}}$$

which from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2}$$

and

$$\omega_0 = \frac{1}{C(R_1 + R_2)}$$

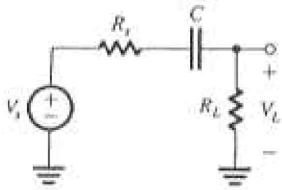
As a further verification that this is a high-pass network and $T(s)$ is a high-pass transfer function we observe as at $s = 0$, $T(s) = 0$; and that as $s \to \infty$, $T(s) = R_2/(R_1 + R_2)$. Also, from the circuit observe as at $s \to \infty$, $(1/sC) \to 0$ and $V_o/V_i = R_2/(R_1 + R_2)$. Now, for $R_1 = 10$ kΩ, $R_2 = 40$ kΩ, and $C = 0.1$ μF,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 0.1 \times 10^{-6}(10+40) \times 10^3}$$

$$= 31.8 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10+40}\frac{1}{\sqrt{2}} = 0.57 \text{ V/V}$$

1.67 Using the voltage divider rule,



$$\frac{V_o}{V_s} = \frac{R_L}{R_L + R_s + \frac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_s} \frac{s}{s + \frac{1}{C(R_L + R_s)}}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_s} \qquad \omega_0 = \frac{1}{C(R_L + R_s)}$$

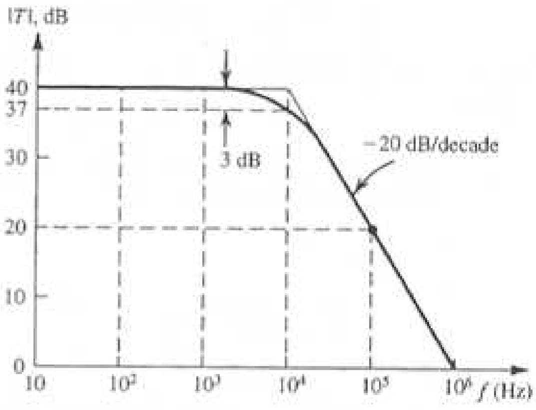
For $f_0 \le 10$ Hz

$$\frac{1}{2\pi C(R_L + R_s)} \le 10$$

$$\Rightarrow \quad C \ge \frac{1}{2\pi \times 10(20+5) \times 10^3}$$

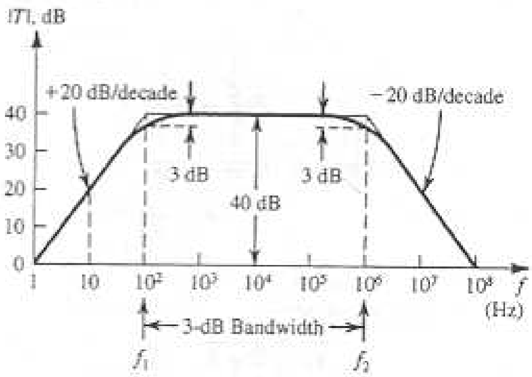
Thus, the smallest value of C that will do the job is $C = 0.64$ μF.

1.68 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of 10^4 Hz. From our knowledge of the Bode plots for low-pass STC networks (Figure . . .) we can complete the Table entries and sketch the amplifier frequency response.



f (Hz)	|T| (dB)	∠T (degrees)
1000	40	−5.7°
10^4	37	−45°
10^5	20	−84.3°
10^6	0	−90°

1.69 From our knowledge of the Bode plots of STC low-pass and high-pass networks we see that this amplifier has a mid-band gain of 40 dB, a low-frequency response of the high-pass STC type with $f_{3dB} = 10^2$ Hz, and a high-frequency response of the low-pass STC type with $f_{3dB} = 10^6$ Hz. We thus can sketch the amplifier frequency response and complete the table entries as follows



f (Hz)	1	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
|T| (dB)	0	20	37	40	40	40	37	20	0

1.72 Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects since the buffer amplifiers have input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2}} = -1$$

$$\Rightarrow 1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{dB}} = 0.51\,\omega_0$$

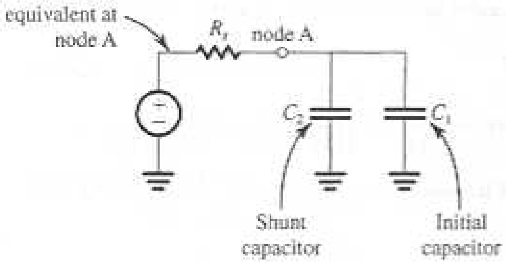
$$\omega_{1\text{dB}} = 0.51/CR$$

1.73 $R_s = 100$ kΩ, since the 3-dB frequency is reduced by a very high factor (from 6 MHz to 120 kHz) C_2 must be much larger than C_1. Thus, neglecting C_1 we find C_2 from

$$120 \text{ kHz} = \frac{1}{2\pi C_2 R_s}$$

$$= \frac{1}{2\pi C_2 \times 10^5}$$

$$\Rightarrow C_2 = 13.3 \text{ pF}$$



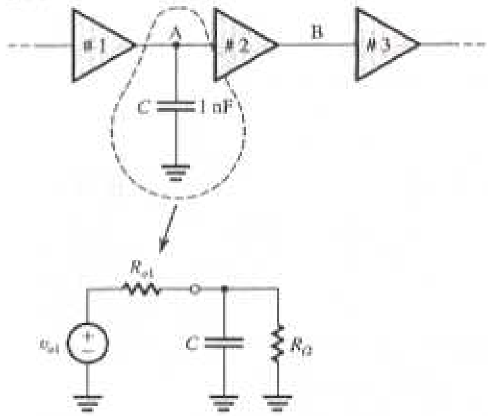
If the original 3-dB frequency (6 MHz) is attributable to C_1 then

$$6 \text{ MHz} = \frac{1}{2\pi C_1 R_s}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 6 \times 10^6 \times 10^5}$$

$$= 0.26 \text{ pF}$$

1.74



Since when C is connected the 3-dB frequency is reduced by a large factor, the value of C must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that C is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$150 \text{ kHz} = \frac{1}{2\pi C(R_{o1} \,/\!/\, R_{i2})}$$

$$\Rightarrow (R_{o1} \,/\!/\, R_{i2}) = \frac{1}{2\pi \times 150 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 1.06 \text{ k}\Omega$$

Now $R_{i2} = 100$ kΩ,

Thus $R_{o1} = 1.07$ kΩ

Similarly, for node B,

$$15 \text{ kHz} = \frac{1}{2\pi C(R_{o2} \,/\!/\, R_{i3})}$$

$$\Rightarrow R_{o2} \,/\!/\, R_{i3} = \frac{1}{2\pi \times 15 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 10.6 \text{ k}\Omega$$

$R_{o2} = 11.9$ kΩ

She should connect a capacitor of value C_p to node B where C_p can be found from,

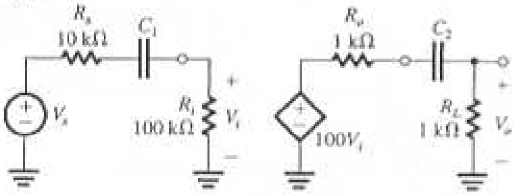
$$10 \text{ kHz} = \frac{1}{2\pi C_p (R_{o2} \parallel R_{i3})}$$

$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 10.6 \times 10^3}$$

$$= 1.5 \text{ nF}$$

Note that if she chooses to use node A she would need to connect a capacitor 10 time larger!

1.75



For the input circuit, the corner frequency f_{01} is found from

$$f_{01} = \frac{1}{2\pi C_1 (R_s + R_i)}$$

For $f_{01} \le 100$ Hz,

$$\frac{1}{2\pi C_1 (10 + 100) \times 10^3} \le 100$$

$$\Rightarrow C_1 \ge \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 4.4 \times 10^{-8}$$

Thus we select $C_1 = 1 \times 10^{-7}$ F $= 0.1$ μF. The actual corner frequency resulting from C_1 will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2 (R_o + R_L)}$$

For $f_{02} \le 100$ Hz,

$$\frac{1}{2\pi C_2 (1 + 1) \times 10^3} \le 100$$

$$\Rightarrow C_2 \ge \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

Select $C_2 = 1 \times 10^{-6} = 1$ μF

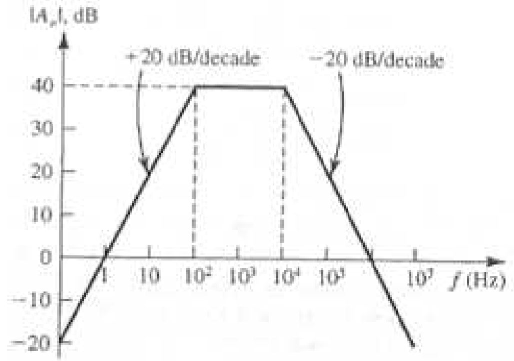
This will place the corner frequency at

$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80 \text{ Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right)} \frac{s}{\left(1 + \frac{s}{2\pi f_{02}}\right)}$$

1.76 The LP factor $1/(1 + jf/10^4)$ results in a Bode plot like that in Fig. 1.23(a) with the 3 dB frequency $f_0 = 10^4$ Hz. The high-pass factor $1/(1 + 10^4/jf)$ results in a Bode plot like that in Fig. 1.24(a) with the 3 dB frequency $f_0 = 10^4$ Hz.

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then raising the resulting plot vertically by 40 dB (corresponding to the factor 100 in the numerator). The result is as follows:



$f =$ 10 10^2 10^3 10^4 10^5 10^6 10^7 (Hz)

$|A_v| \simeq$ 20 40 40 40 20 0 −20 (dB)

37 37

Better approximation
(3-dB frequencies)

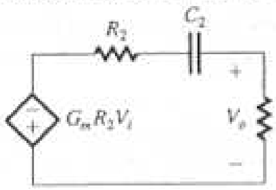
Bandwidth $= 10^4 - 10^2 = 9900$ Hz

1.77

$$T_i(s) = \frac{V_i(s)}{V_s(s)} = \frac{1/sC_1}{1/sC_1 + R_1} = \frac{1}{sC_1R_1 + 1} \quad \text{LP}$$

$$\text{3 dB frequency} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^6} = 15.9 \text{ Hz}$$

For $T_o(s)$, the following equivalent circuit can be used:



$$T_o(s) = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2}$$

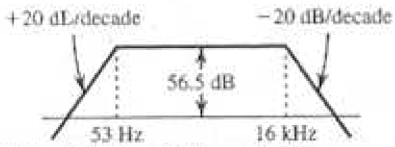
$$= -G_m(R_2 \,//\, R_3)\frac{s}{s + \dfrac{1}{C_2(R_2 + R_3)}}$$

$$3 \text{ dB frequency} = \frac{1}{2\pi C_2(R_2 + R_3)}$$

$$= \frac{1}{2\pi 100 \times 10^{-9} \times 30 \times 10^3} = 53 \text{ Hz}$$

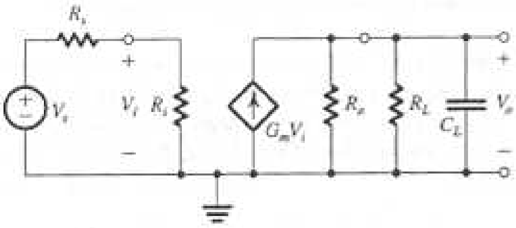
$$\therefore\ T(s) = T_i(s)T_o(s)$$

$$= \frac{1}{1 + \dfrac{s}{2\pi \times 15.9 \times 10^3}} \times -666.7 \times \frac{s}{s + (2\pi \times 53)}$$



Bandwidth = 16 kHz − 53 Hz ≃ 16 kHz

1.78



$$V_i = V_s \frac{R_i}{R_s + R_i} \qquad (1)$$

To satisfy constraint (1), namely

$$V_i \geq \left(1 - \frac{x}{100}\right)V_s$$

We substitute in Eq.(1) to obtain

$$\frac{R_i}{R_s + R_i} \geq 1 - \frac{x}{100}$$

Thus

$$\frac{R_s + R_i}{R_i} \leq \frac{1}{1 - \dfrac{x}{100}}$$

$$\frac{R_s}{R_i} \leq \frac{1}{1 - \dfrac{x}{100}} - 1 = \frac{\dfrac{x}{100}}{1 - \dfrac{x}{100}}$$

which can be expressed as

$$\frac{R_i}{R_s} \geq \frac{1 - \dfrac{x}{100}}{\dfrac{x}{100}}$$

resulting in

$$R_i \geq R_s\left(\frac{100}{x} - 1\right) \qquad (1)$$

The 3-dB frequency is determined by the parallel RC circuit at the output.

$$f_0 = \frac{1}{2\pi}\omega_0 = \frac{1}{2\pi}\frac{1}{C_L(R_L \,//\, R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_o}\right)$$

To obtain a value for f_0 greater than a specified value $f_{3\text{dB}}$ we select R_o so that

$$\frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_o}\right) \geq f_{3\text{dB}}$$

$$\frac{1}{R_L} + \frac{1}{R_o} \geq 2\pi C_L f_{3\text{dB}}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3\text{dB}} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3\text{dB}} C_L - \dfrac{1}{R_L}} \qquad (2)$$

To satisfy constraint (3), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_s + R_i} G_m(R_o \,//\, R_L)$$

For the dc gain to be greater than a specified value A_0,

$$\frac{R_i}{R_s + R_i} G_m(R_o \,//\, R_L) \geq A_0$$

The first factor on the LHS is (from constraint (1))

greater or equal to $(1 - x/100)$. Thus

$$G_m \geq \frac{A_0}{\left(1-\frac{x}{100}\right)(R_o \mathbin{//} R_L)} \quad (3)$$

Substituting $R_s = 10\ \text{k}\Omega$ and $x = 20\%$ in (1) results in

$$R_i \geq 10\left(\frac{100}{20} - 1\right) = 40\ \text{k}\Omega$$

Substituting $f_{3dB} = 3$ MHz, $C_L = 10$ pF and $R_L = 10\ \text{k}\Omega$ in Eq. (2) results in

$$R_o \leq \frac{1}{2\pi \times 3 \times 10^6 \times 10 \times 10^{-12} - \frac{1}{10^4}} = 11.3\ \text{k}\Omega$$

Substituting $A_0 = 80$, $x = 20\%$, $R_L = 10\ \text{k}\Omega$, and $R_o = 11.3\ \text{k}\Omega$, eq. (3) results in

$$G_m \geq \frac{80}{\left(1-\frac{20}{100}\right)(10 \mathbin{//} 11.3) \times 10^3} = 18.85\ \text{mA/V}$$

If the more practical value of $R_o = 10\ \text{k}\Omega$ is used then

$$G_m \geq \frac{80}{\left(1-\frac{20}{100}\right)(10 \mathbin{//} 10) \times 10^3} = 20\ \text{mA/V}$$

1.79 Using the voltage-divider rule we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \mathbin{//} \frac{1}{sC_1} \quad \text{and} \quad Z_2 = R_2 \mathbin{//} \frac{1}{sC_2}.$$

It is obviously more convenient to work in terms of admittances. Therefore we express V_o/V_i in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute $Y_1 = (1/R_1) + sC_1$ and $Y_2 = (1/R_2) + sC_2$ to obtain

$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)}$$

$$= \frac{C_1}{C_1 + C_2} \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

This transfer function will be independent of frequency (s) if the second factor reduces to unity. This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

which can be simplified as follows

$$\frac{C_1 + C_2}{C_2} = R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1 R_1 = C_2 R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1) above can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ($C_1 R_1 = C_2 R_2$) its transmission can be determined either by its two resistors R_1, R_2 or by its two capacitors. C_1, C_2, and the transmission is *not* a function of frequency.

1.80 The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of 11.4° at $f = 100$ Hz. Using the equation for $\angle T(j\omega)$ from Table 1.2 we obtain

$$\tan^{-1}\frac{f_0}{100} = 11.4° \Rightarrow f_0 = 20.16\ \text{Hz}.$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of −11.4° at $f = 1$ kHz. Using the relationship for $\angle T(j\omega)$ given in Table 1.2 we obtain for the LP STC circuit.

$$-\tan^{-1}\frac{10^3}{f_0} = -11.4° \Rightarrow f_0 = 4959.4\ \text{Hz}$$

At $f = 100$ Hz the drop in gain is due to the HP STC network, and thus its value is

$$20\log\frac{1}{\sqrt{1+\left(\frac{20.16}{100}\right)^2}} = -0.17 \text{ dB}$$

Similarly, at $f = 1$ kHz the drop in gain is caused by the LP STC network. The drop in gain is

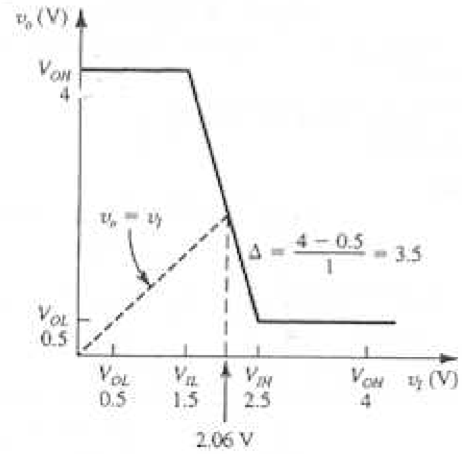
$$20\log\frac{1}{\sqrt{1+\left(\frac{1000}{4959.4}\right)^2}} = -0.17 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at $f = 20.16$ Hz and $f = 4959.4$ Hz.

1.81 $NM_H = V_{OH} - V_{IH} = 3.3 - 1.7 = 1.6$ V

$NM_L = V_{IL} - V_{OL} = 1.3 - 0 = 1.3$ V

1.82



(a) $NM_H = V_{OH} - V_{IH} = 4 - 2.5 = 1.5$ V

$NM_L = V_{IL} - V_{OL} = 1.5 - 0.5 = 1$ V

(b) In the transition region

$$V_O = 4 - 3.5(V_I - 1.5)$$
$$= 9.25 - 3.5V_I$$

If

$$V_O = V_I \Rightarrow 4.5V_O = 9.25$$
$$V_O = V_I = 2.06 \text{ V}$$

(c) Slope = −3.5 V/V

1.83

$NM_H = V_{OH} - V_{IH} = 0.8\,V_{DD} - 0.6\,V_{DD} = 0.2V_{DD}$

$NM_L = V_{IL} - V_{OL} = (0.4 - 0.1)V_{DD} = 0.3V_{DD}$

width of transition region $= V_{IH} - V_{IL} = 0.2V_{DD}$

for a minimum NM of 1 V $\Rightarrow 0.2V_{DD} = 1$

$V_{DD} = 5$ V

1.84

(a) Worse case $NM_H = V_{OH,\min} - V_{IH} = 2.4 - 2 = 0.4$ V

Worse case $NM_L = V_{IL,\max} - V_{OL} = 0.8 - 0.4 = 0.4$ V

(b) $P_{D_{av}} = \frac{1}{20}[5\times 3 + 5\times 1] = 10$ mW

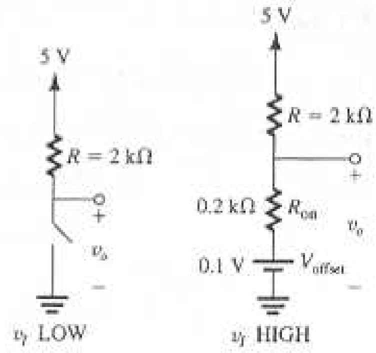
(c) Dynamic power dissipation $= f_C V_{DD}^2 =$

$10^6 \times 45 \times 10^{-12} \times 25 = 1.13$ mW

(d) $t_P(\text{typical}) = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(7 + 11) = 9$ ns

$t_P(\text{maximum}) = \frac{1}{2}(15 + 22) = 18.5$ ns

1.85



$v_{IL} = 1$ V $\quad v_{IH} = 2$ V

(a) $V_{OL} = \frac{5 - 0.1}{2.2} = 0.2 + 0.1 = 0.545$ V

$V_{OH} = 5$ V

$NM_H = V_{OH} - V_{IH} = 3$ V

$NM_L = V_{IL} - V_{OL} = 0.455$ V

(b) $V_{OH} = 5 - N(0.2 \times 10^{-3})R = 5 - 0.4N$

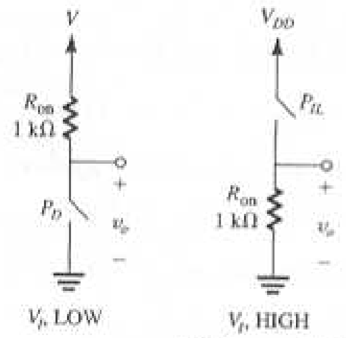
$NM_H = 5 - 0.4N - 2 = 3 - 0.4N = 0.455 \quad \therefore N = 6$

(c) (i) $P_{D_{v_I,LOW}} = (5-0.1)^2/2.2\ k\Omega = 10.9\ mW$

(ii) $P_{D_{v_I,HIGH}} = 5 \times (0.2 \times 6) = 6\ mW$

1.86 (a) $V_{OL} = 0 \quad V_{OH} = 5 \quad NM_L = V_{IL} - V_{OL} = 2.5 - 0 = 2.5\ V$

$NM_H = V_{OH} - V_{IH} = 5 - 2.5 = 2.5\ V$



(b) $V_O(t) = 0 - (0-5)e^{-t/R_{on}C} = 5e^{-t/R_{on}C}$

For $t_{PHL} \Rightarrow V_O(t) = 5e^{-t_{PHL}/R_{on}C} = \frac{1}{2}(5) = 2.5$

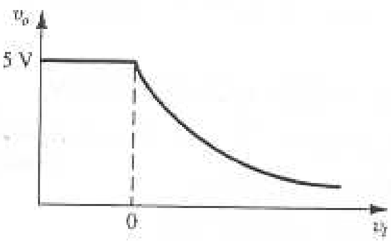
$t_{PHL} = -(10^3)(10^{-n}) \ln \frac{2.5}{5} = 0.69\ ns$

For t_{THL} $V_O(t) = 5e^{-t_H/R_{on}C} = 4.5\ V$

$t_1 = 0.01\ ns \quad V_O(t) = 5e^{-t_L/R_{on}C} = 0.5\ V$

$t_2 = 2.3\ ns$

$\therefore \quad t_{THL} = t_2 - t_1 = 2.2\ ns$



(c) $V_O(t) = 5 - (5-0)e^{-t/R_{on}C} = 5 - 5e^{-t/R_{on}C}$

$V_O = 5 - 5e^{-t_{PLH}/R_{on}C} = 2.5\ ns$

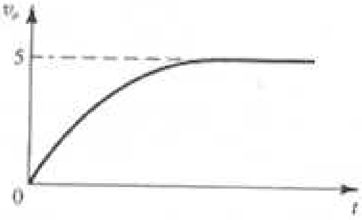
$t_{PLH} = 0.69\ ns$

For t_{TLH},

$V_O(t) = 5 - 5e^{-t_1/R_{on}C} = 0.5 \Rightarrow t_1 = 0.10\ ns.$

$V_O(t) = 5 - 5e^{-t_2/R_{on}C} = 4.5 \Rightarrow t_2 = 2.3\ ns.$

$t_{TLH} = 2.3 - 0.1 = 2.2\ ns$



1.87

$V_{OH} = 5\ V$

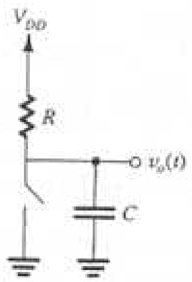
$V_{OL} = 5 - 2 \times 1 = 3\ V$

1.88 $P_{dynamic} = fCV_{DD}^2 = 100 \times 10^6 \times 10 \times 10^{-12} \times 25 = 25\ mW$

$P = V_{DD} I_{avg} = 5I_{avg} = 25\ mW$

$I_{avg} = 5\ mA$

1.89



$v_O(t)$ begins at V_{OL} and rises toward V_{OH} (in this case $V_{OH} = V_{DD}$) according to

$v_O(t) = v_\infty - (v_\infty - v_{0+})e^{-t/CR}$

$= V_{OH} - (V_{OH} - V_{OL})\, e^{-t/CR}$

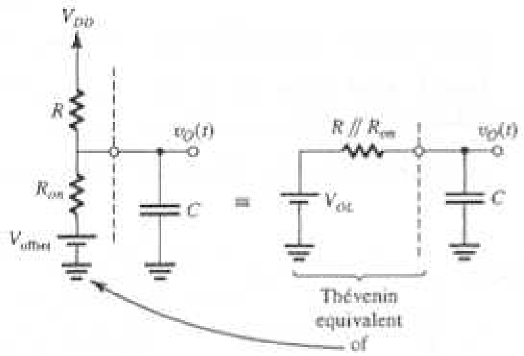
$= V_{OH} - (V_{OH} - V_{OL})e^{-t/\tau_1},\ \tau_1 = CR$ Q.E.D.

$v_O(t)$ reaches $\frac{1}{2}(V_{OH} + V_{OL})$ at $t = t_{PLH}$,

$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL})e^{-t_{PLH}/\tau_1}$

$\Rightarrow t_{PLH} = \tau_1 \ln 2 = 0.69CR$ Q.E.D.

(b)



$$V_{OL} = V_{DD}\frac{R_{on}}{R+R_{on}} + V_{offset}\frac{R}{R+R_{on}}$$

$$v_O(t) = v_\infty - (v_\infty - v_{o+})e^{-t/\tau_2},\ \tau_2 = C(R \,\|\, R_{on})$$

$$= V_{OL} - (V_{OL} - V_{OH})e^{-t/\tau_2}$$

$$= V_{OL} + (V_{OH} - V_{OL})e^{-t/\tau_2} \qquad \text{Q.E.D.}$$

$$v_O(t_{PHL}) = \frac{1}{2}(V_{OL} + V_{OH})$$

Thus,

$$t_{PHL} = \tau_2 \ln 2 = 0.69C\,(R \,\|\, R_{on})$$

$$\simeq 0.69\, CR_{on}, \qquad R_{on} \ll R \qquad \text{Q.E.D.}$$

(c) $$\tau_P = \frac{1}{2}(t_{PLH} + t_{PLH}) = \frac{1}{2} \times 0.69(CR + CR_{on})$$

$$\simeq 0.35CR, \quad \text{for} \quad R_{on} \ll R \qquad \text{Q.E.D.}$$

(d) Static power is dissipated only when the output is low, in which case

$$P = \frac{V_{DD}^2}{R + R_{on}} \simeq \frac{V_{DD}^2}{R}, \quad \text{for } R_{on} \ll R,$$

and assuming that $V_{offset} \ll V_{DD}$. Thus if the inverter spends only half the time in this low-output state,

$$P = \frac{1}{2}\frac{V_{DD}^2}{R}$$

(e) Large R results in low P but high τ_P and vice-versa. For $V_{DD} = 5$ V and $C = 10$ pF,

- $\tau_P \le 10$ ns $\Rightarrow$ $0.35CR \le 10$ ns
  $\Rightarrow R \le 2875\ \Omega$
- $P \le 10$ mW $\Rightarrow$ $\frac{1}{2} \times \frac{25}{R} \le 10 \times 10^{-3}$
  $\Rightarrow R \ge 1250\ \Omega$

Thus,

$$1250 \le R \le 2875\ \Omega$$

Selecting $R = 2$ kΩ,

$$\tau_P = 0.35 \times 10 \times 10^{-12} \times 2 \times 10^3 = 7\text{ ns}$$

$$P = \frac{1}{2} \times \frac{25}{2} = 6.25\text{ mW}$$

Unnumbered 1.48

$$\frac{v_{i1}}{v_i} = 0.5 \qquad \frac{v_{i2}}{v_{i1}} = 100\,\frac{1000}{1001} \simeq 100$$

$$\frac{v_{i3}}{v_{i2}} = 10 \times \frac{10}{11} = 9.09$$

$$\frac{v_L}{v_{i3}} = \frac{10}{11} = 0.909$$

$$A_v = 826.3$$

$$\frac{v_o}{v_i} = 413.1$$

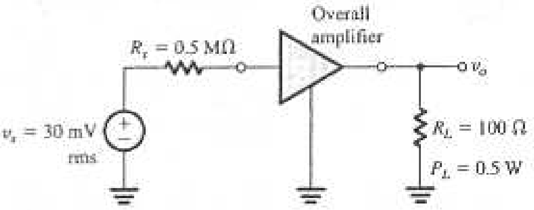
$$0.5 \times 100 \times 10 \times \frac{10}{11} = 450$$

$$P_L = 0.5\text{ W} = v_o^2/R_L$$

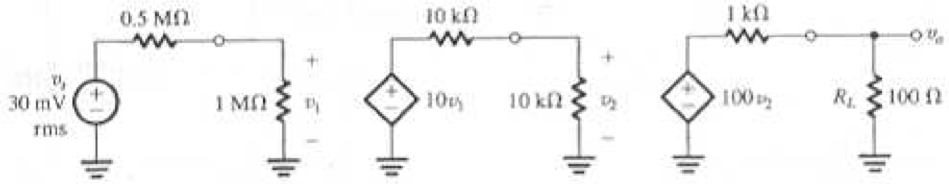
$$v_o = \sqrt{0.5 \times 100} = 7.07\text{ V}$$

Unnumbered 1.50

Required overall voltage gain = 7.07/0.03 = 235.7 = 235 V/V.



In order to avoid the loss of more than 2/3 of the signal strength in coupling the source to the first stage of the amplifier, we have use the type (1), high-input resistance amplifier as the input stage. Both type 2 and type 3 do not satisfy this requirement. The type (1) amplifier has an open-circuit voltage gain of 10 and thus connect by itself satisfy the overall gain requirement.



We next consider cascading the type (1) input stage with the type (2), high-gain stages. The result would be

Overall voltage gain

$$= \frac{1}{1+0.5} \times 10 \times \frac{10}{10+10} \times 100 \times \frac{100}{100+1000}$$

$$= 30.3 \text{ V/V}$$

Thus, this cascade amplifier does not meet specs. The reason is obvious from the gain calculation: the output resistance, 1 kΩ, is too high for feeding a load of 100 Ω and indeed results in a loss of gain by a factor of 11!

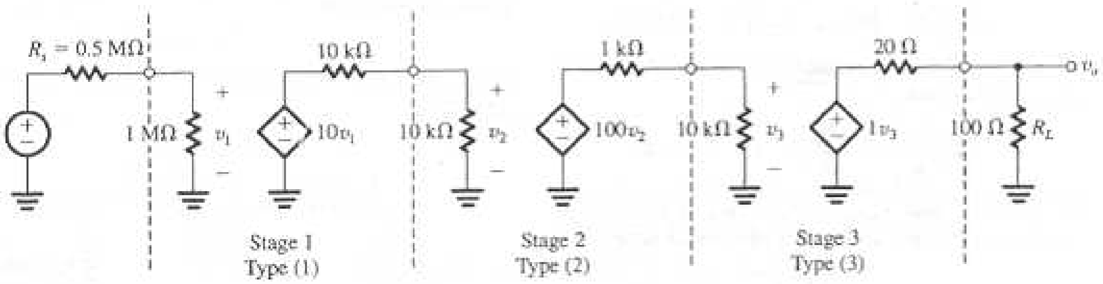
That's where the type (3) amplifier stage can be beneficial. While its open-circuit gain is only 1, its output resistance is 20 Ω, five times lower than the 100 Ω load. The overall amplifier would then look as follows:

Overall voltage gain $= \frac{v_o}{v_s}$

$$= \frac{1}{1+0.5} \times 10 \times \frac{10}{10+10} \times 100 \times \frac{10}{10+1} \times 1 \times \frac{100}{100+20}$$

$= 252.5$ V/V

which meets the specified gain of 235 V/V.



Chapter 2 – Problems

2.1

The minimum number of pins required by dual-op-amp is 8. Each op-amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power.
Similarly, the minimum number of pins required by quad-op-amp is 14:
$4\times2+4\times1+2=14$

2.2

Refer to Fig. P2.2. $v_+ = v_3 \dfrac{1K\Omega}{1M\Omega+1K\Omega} = \dfrac{4}{1001}V$

$v_o = Av_+ \Rightarrow A = \dfrac{4}{4/1001} = 1001$

2.3

The voltage at the positive input has to be $-3.000V$.

$v_+ = -3.020V$, $A = \dfrac{v_o}{(v_+ - v_-)} = \dfrac{-2}{-3.020-(-3)} = 100$

2.4

#	v_1	v_2	$v_d = v_2 - v_1$	v_o	v_o/v_d
1	0.00	0.00	0.00	0.00	–
2	1.00	1.00	0.00	0.00	–
3	ⓐ	1.00	ⓑ	1.00	
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	-0.01	-0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	ⓒ	ⓓ	-5.10	

experiments 4, 5, 6 show that the gain is approximately 100 V/V. The missing entry for experiment #3 can be predicted as follows:

ⓑ $v_d = \dfrac{v_o}{A} = \dfrac{1.00}{100} = 0.01V.$

ⓐ $v_1 = v_2 - v_d = 1.00 - 0.01 = 0.99V$

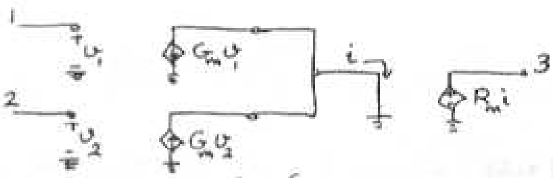
The missing entries for experiment #7:

ⓓ $v_d = \dfrac{-5.10}{100} = -0.051V$

ⓒ $v_2 = v_1 + v_d = 5.10 - 0.051 = 5.049V$

All the results seem to be reasonable.

2.5



$A = G_m R_m = 100\times10^{-3}\times10^{6} = 100{,}000$ V/V

2.6

$v_{CM} = 1V \sin(2\pi 60)t \quad = \frac{1}{2}(v_1+v_2)$

$v_d = 0.01 \sin(2\pi 1000)t \; = v_1 - v_2$

$v_1 = v_{CM} - v_d/2 = \sin(120\pi)t - 0.005 \sin 2000\pi t$

$v_2 = v_{CM} + v_d/2 = \sin 120\pi t + 0.005 \sin 2000\pi t$

2.7

$v_d = R(G_{m2}v_2 - G_{m1}v_1)$ Refer to Fig. 2.4.

$v_o = v_3 = \mu v_d = \mu R(G_{m2}v_2 - G_{m1}v_1)$

$v_o = \mu R(G_m v_2 + \frac{1}{2}\Delta G_m v_2 - G_m v_1 + \frac{1}{2}\Delta G_m v_1)$

$v_o = \mu R G_m \underbrace{(v_2 - v_1)}_{v_{Id}} + \frac{1}{2}\mu R \Delta G_m \underbrace{(v_1+v_2)}_{2v_{ICM}}$

we have $v_o = A_d v_{Id} + A_{CM} v_{ICM}$

$\Rightarrow A_d = \mu R G_m \quad , \quad A_{CM} = \mu R \Delta G_m$

$CMRR = 20\log\left|\dfrac{A_d}{A_{CM}}\right| = 20\log\dfrac{G_m}{\Delta G_m}$

cont.

$20\log_{10} A_d = 80\,dB \Rightarrow A_d = 10^4$

$\frac{A_{cm}}{A_d} = \frac{\Delta G_m}{G_m} \Rightarrow A_{cm} = 10^4 \times \frac{0.1}{100} = 10$

$CMRR = 20\log\frac{G_m}{\Delta G_m} = 20\log\frac{1}{0.1/100} = 60$

2.8

circuit	v_o/v_i (V/V)	Rin (kΩ)	
a	$\frac{-100}{10} = -10$	10	
b	-10	10	
c	-10	10	virtual ground
d	-10	10	no current in 10kΩ

2.9

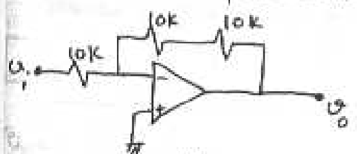
closed loop gain $= -1\,V/V$. For $v_i = 5V \Rightarrow v_o = -5\,V$

Gain would be in the range of $\frac{-0.95}{1.05}$ to $\frac{-1.05}{0.95}$: $-0.9 < G < -1.1$

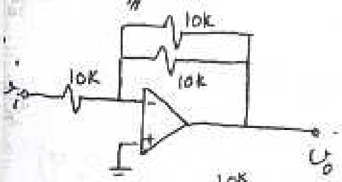
For $v_i = 5 \Rightarrow -4.5 < v_o < -5.5V$

2.10

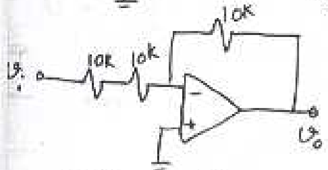
There are four possibilities:



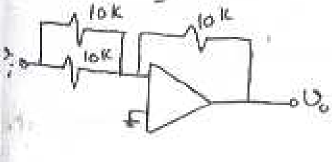
$\frac{v_o}{v_i} = -2\,V/V \quad R_{in} = 10k\Omega$



$\frac{v_o}{v_i} = -0.5\,V/V \quad R_{in} = 10\,k\Omega$



$\frac{v_o}{v_i} = -0.5\,V/V \quad R_{in} = 20\,k\Omega$



$\frac{v_o}{v_i} = -2\,V/V \quad R_{in} = 5\,k\Omega$

2.11

a. $G = -1\,V/V$

b. $G = -10\,V/V$

c. $G = -0.1\,V/V$

d. $G = -100\,V/V$

e. $G = -10\,V/V$

2.12

a. $G = -1\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10K\Omega$

b. $G = -2\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = 10K\Omega,\ R_2 = 20K\Omega$

c. $G = -0.5\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = 20K\Omega,\ R_2 = 10K\Omega$

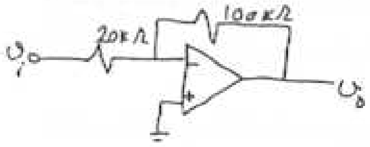
d. $G = -100\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = 10K\Omega,\ R_2 = 1M\Omega$

2.13

$\frac{v_o}{v_i} = -5 = -\frac{R_2}{R_1} \Rightarrow R_2 = 5R_1$

$R_1 + R_2 = 120K\Omega \Rightarrow 5R_1 + R_1 = 120K\Omega \Rightarrow$

$R_1 = 20K\Omega \Rightarrow R_2 = 100K\Omega$



2.14

$20\log|G| = 26\,dB \Rightarrow G = -19.95\,V/V = \frac{v_o}{v_i} = -\frac{R_2}{R_1}$

$\Rightarrow R_2 = 19.95 R_1 \leq 10M\Omega$

For largest possible input resistance, select

$R_2 = 10M\Omega \Rightarrow R_1 \simeq 500K\Omega$

$R_{in} = 500K\Omega$

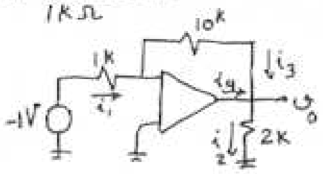
2.15

$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1} = -\frac{100}{10} = -10$

$v_{low} = -10V,\ v_{high} = 0,\ v_{avg} = -5V$

2.16

$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow v_o = -1 \times -\frac{10k\Omega}{1k\Omega} = 10V$



$i_2 = \frac{v_o}{2k\Omega} = 5mA$

$i_1 = i_3 = \frac{v_o}{10k\Omega} = 1mA$

$i_4 = i_2 - i_3 = 4mA$ This additional current comes from the output of the op.amp.

2.17

$|Gain| = \frac{R_2}{R_1} = \frac{R_2(1+x/100)}{R_1(1+x/100)} \simeq \frac{R_2}{R_1}\left(1 \pm \frac{2x}{100}\right)$ for small x

$\Rightarrow 2x\%$ is the tolerance on the closed loop gain (G).

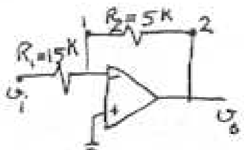
$G = -100\,V/V$, $x = 5 \Rightarrow -110 < G < -90$

or more precisely: $-100 \times \frac{105}{95} < G < -100\,\frac{95}{105}$

$-110.5 < G < -90.5$

2.18

$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = \frac{5}{15}$



$v_1 = 0V$, $v_2 = v_o = 5V$

For $\pm 1\%$ on R_1, R_2: $R_1 = 15 \pm 0.15\,k\Omega$

$R_2 = 5 \pm 0.05\,k\Omega$

$v_o = v_i \frac{-R_2}{R_1} = 15\frac{R_2}{R_1} \Rightarrow 15 \times \frac{4.95}{15.15} \leq v_o \leq 15 \times \frac{5.05}{14.85}$

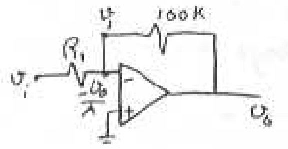
$\Rightarrow 4.9V \leq v_o \leq 5.1V$

For $v_i = -15 \pm 0.15\,V$ $14.85 \times \frac{4.95}{15.15} \leq v_o \leq 15.15 \times \frac{5.05}{14.85}$

$\Rightarrow 4.85V \leq v_o \leq 5.15V$

2.19

$v_i = -\frac{v_o}{A} = -\frac{v_o}{200}$



$\frac{v_o}{v_i} = -50\,V/V$

$\frac{v_i - \left(-\frac{v_o}{A}\right)}{R_1} = \frac{\left(-\frac{v_o}{A} - v_o\right)}{100k\Omega} \Rightarrow R_1 = 100K \times \frac{\frac{v_o}{200} - \frac{v_o}{50}}{-\frac{v_o}{200} - v_o}$

$\Rightarrow R_1 = 100K \times \frac{3}{201} = 1.49k\Omega$

shunt Resistor R_a: $R_a \parallel 2k\Omega = 1.49K$

$\frac{R_a \times 2}{R_a + 2} = 1.49 \Rightarrow R_a = 5.84k\Omega$

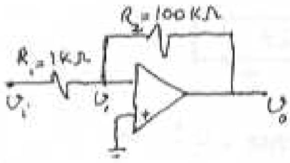
2.20

(a)

$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow -100\,V/V = \frac{-R_2}{1k\Omega} \Rightarrow R_2 = 100k\Omega$

(b) $A = 1000\,V/V$

$v_1 = \frac{-v_o}{A}$



$\frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_2}$

$\frac{v_o}{v_i} = \frac{-R_2/R_1}{1 + (1 + \frac{R_2}{R_1})/A} = \frac{-100}{1 + \frac{101}{1000}} = -90.8\,V/V$

$\Rightarrow \frac{v_o}{v_i} = 90.8\,V/V$

(c) Assume $R_1' = R_x \parallel R_1$ when $R_1 = 1k\Omega$

$\frac{v_o}{v_i} = -100\,V/V$

$\frac{v_i - v_1}{R_1'} = \frac{v_1 - v_o}{R_2} \Rightarrow R_1' = R_2 \times \left(\frac{-v_o}{100} - \frac{-v_o}{1000}\right) / \left(\frac{-v_o}{1000} - v_o\right)$

$R_1' = \frac{1 - 0.1}{1.001} = 0.899\,k\Omega = \frac{R_1 R_x}{R_1 + R_x} = \frac{R_x}{1 + R_x}$

$\Rightarrow R_x = 8.9\,k\Omega \simeq 8.87\,k\Omega \pm 1\%$

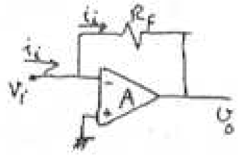
2.21

Voltage of the inverting input terminal

Cont.

will vary from $\frac{-10V}{1000}$ to $\frac{+10V}{1000}$. Thus the virtual ground will depart from the ideal voltage of zero by a maximum of $\pm 10\,mV$.

2.22



a) For $A = \infty$: $v_i = 0$

$v_o = -i_i R_F$

$R_m = \frac{v_o}{i_i} = -R_F$

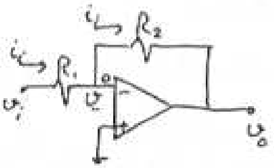
$R_{in} = \frac{v_i}{i_i} = 0$

b) For A = Finite : $v_i = -\frac{v_o}{A}$, $v_o = v_i - i_i R_F$

$\Rightarrow v_o = \frac{-v_o}{A} - i_i R_F \Rightarrow R_m = \frac{v_o}{i_i} = -\frac{R_F}{1+\frac{1}{A}}$

$R_i = \frac{v_i}{i_i} = \frac{R_F}{1+A}$ $\quad R_i = \frac{v_i}{i_i} = \frac{-V_o/A}{i_i} = \frac{-\frac{v_i}{A} + i_i R_f}{i_i}$

2.23



$v_o = -A v_- = v_- - i_i R_2$

$i_i R_2 = (1+A) v_-$

$v_- = \frac{i_i R_2}{1+A}$

Now: $v_i = i_i R_1 + v_- = i_i R_1 + i_i \frac{R_2}{1+A}$

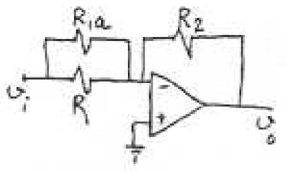
$R_{in} = \frac{v_i}{i_i} = R_1 + \frac{R_2}{1+A}$

2.24

$G = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A}}$ $\quad$ Gain Error $\epsilon = \left(1+\frac{R_2}{R_1}\right)/A \times 100$

ϵ	0.1%	1%	10%
A	$1000(1+\frac{R_2}{R_1})$	$100(1+\frac{R_2}{R_1})$	$10(1+\frac{R_2}{R_1})$

Gain with R_{1a} :



$G \simeq \frac{\frac{R_2}{R_1}\left(1+\frac{R_1}{R_{1a}}\right)}{1+\frac{1+R_2/R_1}{A}}$

where we have neglected the effect of R_{1a} on the error on the denominator. To restore the gain to its nominal value of R_2/R_1 we use:

$\frac{R_1}{R_{1a}} = \frac{1+R_2/R_1}{A} = \frac{\epsilon}{100} \Rightarrow R_{1a} = \frac{100 R_1}{\epsilon}$

ϵ	0.1%	1%	10%
R_{1a}	$1000R_1$	$100R_1$	$10R_1$

2.25

$R_1' = R_1 \parallel R_c$ $\quad G' = \frac{-R_2/R_1'}{1+\frac{1+R_2/R_1'}{A}}$

$G = -\frac{R_2}{R_1}$

In order for $G' = G$: $G = \frac{-R_2/R_1'}{1+\frac{1+R_2/R_1'}{A}} = \frac{-R_2}{R_1}$

$R_1' = \frac{R_1 R_c}{R_1 + R_c}$

$\Rightarrow \frac{R_1+R_c}{R_1 R_c} = \frac{1}{R_1}\left(1+\frac{1+\frac{R_2(R_1+R_c)}{R_1 R_c}}{A}\right)$

$(R_1+R_c)A = AR_c + R_c + \frac{R_2}{R_1}(R_1+R_c)$

$R_1 A = R_c + GR_1 + GR_c$

$\frac{R_c}{R_1} = \frac{A-G}{1+G}$

2.26

$G = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A}}$ $\quad G_{nominal} = -\frac{R_2}{R_1}$

$\epsilon = \left|\frac{G - G_{nominal}}{G_{nominal}}\right| = \left|\frac{G}{G_{nominal}} - 1\right|$

$\epsilon = \left|\frac{1}{1+\frac{1+R_2/R_1}{A}} - 1\right| = \left|\frac{-\frac{1+R_2/R_1}{A}}{1+\frac{1+R_2/R_1}{A}}\right| = \frac{1}{\frac{A}{1+\frac{R_2}{R_1}}+1}$

which can be rearranged to yield:

$\frac{A}{1+\frac{R_2}{R_1}} + 1 = \frac{1}{\epsilon} \Rightarrow A = \left(1+\frac{R_2}{R_1}\right)\left(\frac{1}{\epsilon} - 1\right)$

or $A = (1 - G_{nominal})\left(\frac{1}{\epsilon} - 1\right)$

For $G_{nominal} = -100$ V/V and $\epsilon = 10\% = 0.1$

$A = (1+100)\left(\frac{1}{0.1} - 1\right) = 909$ V/V

This is the minimum required value for A.

2.27

$$|G| = \frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A}} \qquad A \longrightarrow A\left(1-\frac{x}{100}\right) \qquad G \longrightarrow G'$$

$$|G'| = \frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A(1-\frac{x}{100})}}$$

For $|G'| = |G|\left(1-\frac{x}{100K}\right)$

$$\frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A(1-x/100)}} = \frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A}}\left(1-\frac{x}{100K}\right)$$

$$1+\frac{1+R_2/R_1}{A(1-\frac{x}{100})} = \left(1+\frac{1+R_2/R_1}{A}\right)\Big/\left(1-\frac{x}{100K}\right)$$

$$1-\frac{x}{100K} + \frac{1+R_2/R_1}{A}\,\frac{1-x/100K}{1-x/100} = 1+\frac{1+R_2/R_1}{A}$$

$$\frac{1+R_2/R_1}{A}\,\frac{1-x/100K-1+x/100}{1-x/100} = \frac{x}{100K}$$

$$A = \frac{-1+K}{1-\frac{x}{100}}\left(1+R_2/R_1\right) = \left(\frac{K-1}{1-\frac{x}{100}}\right)\left(1+\frac{R_2}{R_1}\right)$$

For $\frac{R_2}{R_1}=100$ $\quad x=50 \quad K=100$: $A = \frac{99}{0.5}\times 101 = 19998$

$A \simeq 2\times 10^4$ V/V

Thus for $A = 2\times10^4$ V/V, a reduction of 50% results in only 0.5% reduction of the closed loop gain whose nominal value is $\frac{R_2}{R_1}$ (100).

2.28

From the results of example 2.2, the gain of the circuit in Fig. 2.8 is given by:

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)$$

For $R_1 = R_2 = R_4 = 1M\Omega \longrightarrow \frac{v_o}{v_i} = -(1+1+\frac{1}{R_3})$

a) $\frac{v_o}{v_i} = -10$ V/V $\Rightarrow 10 = 2+\frac{1}{R_3} \longrightarrow R_3 = \frac{1}{8}M\Omega = 125\ k\Omega$

b) $\frac{v_o}{v_i} = -100$ V/V $\Rightarrow 100 = 2+\frac{1}{R_3} \longrightarrow R_3 = \frac{1}{98}M\Omega = 10.2\ k\Omega$

c) $\frac{v_o}{v_i} = -2$ V/V $\Rightarrow 2 = 2+\frac{1}{R_3} \longrightarrow R_3 = \infty$: eliminate R_3.

2.29

$R_2/R_1 = 1000$, $R_2 = 100k\Omega \Rightarrow R_1 = 100\Omega$

a) $R_{in} = R_1 = 100\Omega$

b) $\frac{v_o}{v_i} = \frac{-R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right) = -1000$

If $R_2 = R_1 = R_4 = 100k \Rightarrow R_3 = \frac{100K}{1000-2} \simeq 100\ \Omega$

$R_{in} = R_1 = 100k\Omega$

2.30

$v_x = 0 - i_1R_2$, $i_1 = \frac{v_I}{R_1} \Rightarrow v_x = -v_I\frac{R_2}{R_1}$

$\frac{v_x}{v_I} = -\frac{R_2}{R_1}$

$$v_x = v_o\,\frac{R_2\|R_3}{R_2\|R_3+R_4} = v_o\,\frac{R_2R_3}{R_2R_3+R_4R_2+R_4R_3}$$

$$\frac{v_o}{v_x} = \frac{R_2R_3+R_2R_4+R_3R_4}{R_2R_3} = 1+\frac{R_4}{R_3}+\frac{R_4}{R_2}$$

$$\frac{v_o/v_x}{v_I/v_x} = \frac{v_o}{v_i} = \frac{(1+R_4/R_3+R_4/R_2)}{-R_1/R_2} \Rightarrow$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}+\frac{R_4}{R_2}\right)$$

2.31

a) $R_1 = R$

$R_2 = R\|R + \frac{R}{2} = \frac{R}{2}+\frac{R}{2} = R$

$R_3 = R_2\|R + \frac{R}{2} = R\|R+\frac{R}{2} = R$

$R_4 = R_3\|R + \frac{R}{2} = R\|R+\frac{R}{2} = R$

b) $v_1 = RI = RI_1 \Rightarrow I_1 = I$

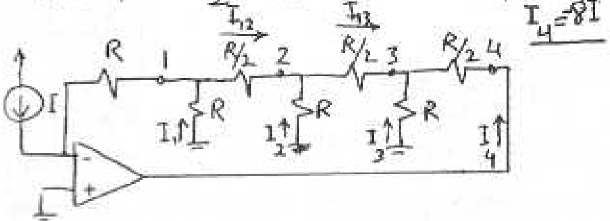
$I_{12} = I + I = 2I \Rightarrow v_1 + 2I\times\frac{R}{2} = RI_2$

$RI + RI = RI_2 \Rightarrow I_2 = 2I$

$I_3 = I_2 + I_{12} = 4I \Rightarrow v_2 + 4I\times\frac{R}{2} = RI_3$

$R\times 2I + 4I\times\frac{R}{2} = RI_3 \Rightarrow I_3 = 4I$, $I_4 = -(4I+4I)$

$I_4 = 8I$



Cont.

c) $v_1 = I_1 R = {}^-IR$

$v_2 = {}^-I_2 R = {}^-2IR$

$v_3 = {}^-I_3 R = {}^-4IR$

$v_4 = {}^-I_3 R + I_4 \frac{R}{2} = -4IR - 8I\frac{R}{2} = -8IR$

2.32

a) $I_1 = \frac{1V}{10k\Omega} = 0.1mA$

$I_2 = I_1 = 0.1\ mA$, $I_2 \times 10k\Omega = I_3 \times 100\Omega \Rightarrow I_3 = 10^{mA}$

$v_x = 10mA \times 100\Omega = 1V$

b) $v_x = R_L I_L + v_o$, $I_L = I_2 + I_3 = 10.1mA$

$1V = R_L \times 10.1mA + v_o$

$R_L = \frac{1 - v_o}{10.1} \Rightarrow R_{Lmax} = \frac{1 - v_{omin}}{10.1} = \frac{14}{10.1}$

$R_{Lmax} =$

c) $100\Omega \leq R_L \leq 1k\Omega$

I_L stays fixed at $10.1mA$

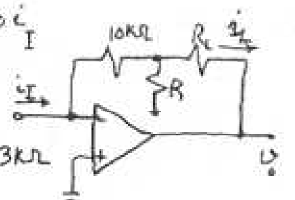
$v_o = v_x - R_L I_L = 1 - R_L \times 10.1 \Rightarrow -9.1^V \leq v_o \leq -0.01^V$

2.33

a) $\frac{i_L}{i_i} = 20 \Rightarrow i_L = 20 i_I$

$-10k\Omega \times i_I = R(i_I - i_L)$

$R = \frac{10k\Omega \times i_I}{20 i_I - i_I} = 0.53k\Omega$



b) $R_L = 1k\Omega$ $-12^V \leq v_o \leq 12^V$

$v_o = R_L i_L + 10k\Omega \times i_I = i_I (1k\Omega \times \frac{i_L}{i_I} + 10^{k\Omega})$

$v_o = i_I (1 \times 20 + 10) = 30 i_I$

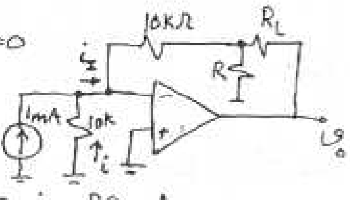
$i_I = \frac{v_o}{30} \Rightarrow \frac{-12}{30} \leq i_I \leq \frac{12}{30} \rightarrow -0.4_{mA} \leq i_I \leq 0.4_{mA}$

c) $R_I = \frac{v_I}{i_I} = \frac{0}{i_I} = 0$

$v_- = 0 \Rightarrow i = 0$

$\Rightarrow i_I = 1mA$

From part a: $i_L = 20 \times i_I = 20\ mA$



2.34

$R_2 \gg R_3$, if we ignore the current across R_2 : $v_A = \frac{v_o R_3}{R_3 + R_4}$

$\frac{v_I}{R_1} = \frac{0 - v_A}{R_2} \Rightarrow v_A = -\frac{R_2 v_I}{R_1}$

$v_o \frac{R_3}{R_3 + R_4} = -\frac{R_2}{R_1} \cdot v_I \Rightarrow \frac{v_o}{v_I} = \frac{-R_2}{R_1}\left(1 + \frac{R_4}{R_3}\right)$

Now if we recalculate v_A considering that there is a voltage divider between R_4 and $R_3 \| R_2$:

$$v_A = v_o \frac{R_3 \| R_2}{R_4 + R_3 \| R_2} = v_o \frac{R_3 R_2}{R_4(R_3 + R_2) + R_2 R_3}$$

$$v_A = v_o \frac{R_2 R_3}{R_3 R_4 + R_2 R_4 + R_2 R_3}$$

$$v_A = v_o \frac{1}{R_4/R_2 + \frac{R_4}{R_3} + 1}$$

$v_A = -\frac{R_2}{R_1} \cdot v_I \Rightarrow \frac{v_o}{v_I} = \frac{-R_2}{R_1}\left(\frac{R_4}{R_2} + \frac{R_4}{R_3} + 1\right)$

Same as example 2.2.

2.35

$R_I = 100k\Omega$ $-10 \leq \frac{v_o}{v_i} \leq -1\ V/V$

$R_I = R_1 = 100k\Omega$

$\frac{v_o}{v_i} = \frac{-R_2}{R_1}\left(\frac{R_4}{R_3} + \frac{R_4}{R_2} + 1\right)$

$R_4 = 0 \Rightarrow \frac{v_o}{v_i} = \frac{-R_2}{R_1} = -1 \Rightarrow R_2 = 100k\Omega$

$R_4 = 10k\Omega \Rightarrow \frac{v_o}{v_i} = -10 = -1 \times \left(\frac{10k\Omega}{R_3} + \frac{10k\Omega}{100k\Omega} + 1\right)$

$+10 = \left(\frac{10}{R_3} + 1.1\right) \Rightarrow R_3 = 1.12k\Omega$

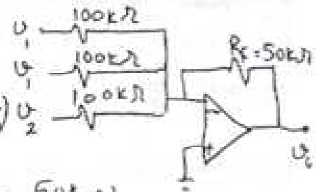
Potentiometer in the middle: $\frac{v_o}{v_i} = -1\left(\frac{5}{5 + R_3} + \frac{5}{100} + 1\right)$

$\frac{v_o}{v_i} = -1.87 V/V$

2.36

According to eq. 2.7:

$v_o = -\left(\frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_n}v_2\right)$



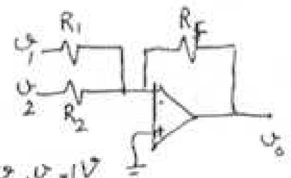
$v_o = -\left(\frac{50k}{100k}v_1 + \frac{50k}{100k}v_1 + \frac{50k}{100k}v_2\right)$

$v_o = -\left(v_1 + \frac{v_2}{2}\right) \qquad v_1 = 3, v_2 = -3 \Rightarrow v_o = -1.5v$

2.37

we choose the weighted Sommer configuration

$v_o = -\left[4v_1 + \frac{v_2}{3}\right]$



$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_2}{R_2}$

$i_1, i_2 \leq 0.1\,mA$ for $v_1, v_2 = 1V$

$R_1, R_2 \geq 10K\Omega$

$\frac{R_F}{R_1} = 4$, if $R_1 = 10K\Omega \Rightarrow R_f = 40K\Omega$

$\frac{R_F}{R_2} = \frac{1}{3} \Rightarrow R_2 = 120K\Omega$

2.38

$v_o = -(2v_1 + 4v_2 + 8v_3)$

$R_1, R_2, R_3 \geq 10K\Omega$

$\frac{R_F}{R_1} = 2 \quad , \frac{R_F}{R_2} = 4 \qquad \frac{R_F}{R_3} = 8$

$R_3 = 10K\Omega \Rightarrow R_f = 80K\Omega$

$R_2 = 20K\Omega$

$R_1 = 40K\Omega$

2.39

a) $v_o = -(v_1 + 2v_2 + 3v_3)$

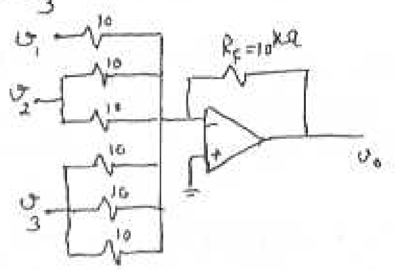
$\frac{R_F}{R_1} = 1 \Rightarrow R_1 = 10K\Omega \quad , \frac{R_F}{R_2} = 2 \Rightarrow R_2 = 5K\Omega$

$\frac{R_F}{R_3} = 3 \Rightarrow R_3 = \frac{10}{3}K\Omega$

$R_{I1} = 10K\Omega$

$R_{I2} = 5K\Omega$

$R_{I3} = 3.3K$



b) $v_o = -(v_1 + v_2 + 2v_3 + 2v_4)$

$\frac{R_F}{R_1} = 1 \Rightarrow R_1 = 10K\Omega$

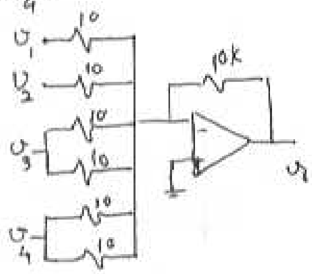
$\frac{R_F}{R_2} = 1 \Rightarrow R_2 = 10K\Omega$

$\frac{R_F}{R_3} = 2 \Rightarrow R_3 = \frac{10}{2}K\Omega$

$\frac{R_F}{R_4} = 2 \Rightarrow R_4 = \frac{10}{2}K\Omega$

$R_{I1} = R_{I2} = 10K\Omega$

$R_{I3} = R_{I4} = 5K\Omega$



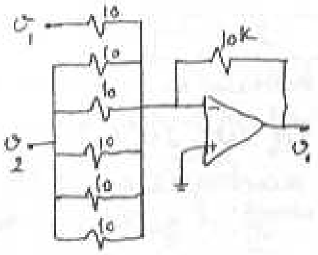
c) $v_o = -(v_1 + 5v_2)$

$R_1 = 10K$

$R_2 = \frac{10K}{5}$

$R_{I1} = 10K\Omega$

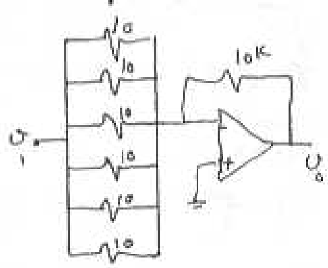
$R_{I2} = 2K\Omega$



d) $v_o = -6v_1$

$R_1 = \frac{10K}{6}$

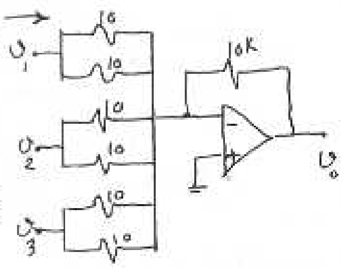
$R_{I1} = 1.67K\Omega$

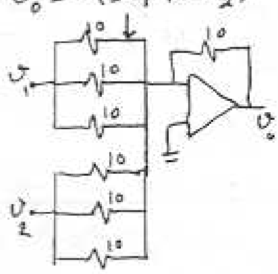


Suggested configurations:

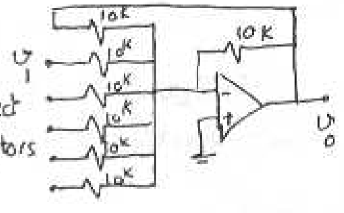
$v_o = -(2v_1 + 2v_2 + 2v_3)$

$v_o = -(3v_1 + 3v_2)$





In order to have coeficient = 0.5, connect one of the input resistors to v_o. $\frac{v_o}{v_1} = 0.5$



2.40

The output signal should be:

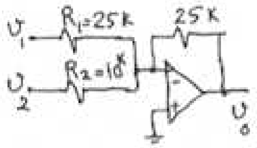
$v_o = -5\sin\omega t - 5$

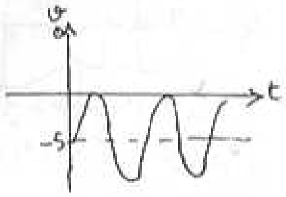
if we assume: $v_1 = 5\sin\omega t$, $v_2 = 2V$ $\Big\}\ v_o = -(v_1 + 2.5 v_2)$

In a weighted summer configuration:

$\frac{R_F}{R_1} = +1 \qquad \frac{R_F}{R_2} = 2.5$

$R_2 = 10K\Omega \Rightarrow R_F = 25K = R_1$





2.41

$v_o = v_1 + 2v_2 - 3v_3 - 4v_4$: Consider Fig. 2.11.

According to eq. 2.8 for a weighted summer circuit:

$v_o = v_1 \frac{R_a}{R_1}\frac{R_c}{R_b} + v_2 \frac{R_a}{R_2}\frac{R_c}{R_b} - v_3 \frac{R_c}{R_3} - v_4 \frac{R_c}{R_4}$

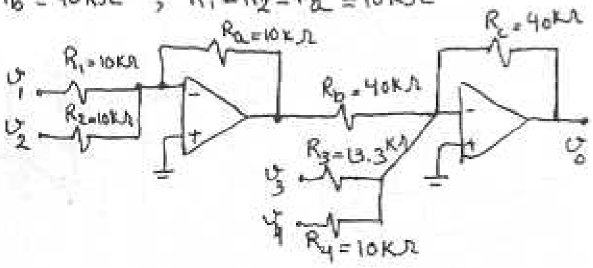
$\frac{R_a}{R_1}\frac{R_c}{R_b} = 1, \quad \frac{R_a}{R_2}\frac{R_c}{R_b} = 1, \quad \frac{R_c}{R_3} = 3, \quad \frac{R_c}{R_4} = 4$

assume:

$R_4 = 10K\Omega \Rightarrow R_c = 40^{K\Omega} \Rightarrow R_3 = \frac{40}{3} = 13.3^{K\Omega}$

$\frac{R_a}{R_1} \times \frac{40}{R_b} = 1 \qquad \frac{R_a}{R_2} \times \frac{40}{R_b} = 1$

$R_b = 40K\Omega, \quad R_1 = R_2 = R_a = 10K\Omega$



2.42

$v_1 = 3\sin(2\pi \times 60t) + 0.01\sin(2\pi \times 1000t)$

$v_2 = 3\sin(2\pi \times 60t) - 0.01\sin(2\pi \times 1000t)$

we want to have: $v_o = 10v_1 - 10v_2$

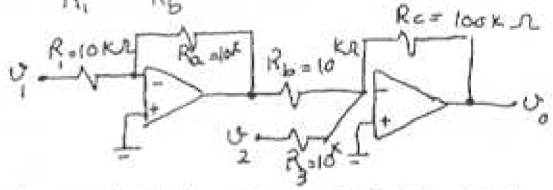
we use the circuit in Fig. 2.11.

According to Eq. 2.8:

$v_o = v_1 \frac{R_a}{R_1}\frac{R_c}{R_b} - v_3 \frac{R_c}{R_3}$

$\frac{R_a}{R_1}\frac{R_c}{R_b} = 10, \quad \frac{R_c}{R_3} = 10$, if $R_3 = 10^{K\Omega} \Rightarrow R_c = 100^{K\Omega}$

$\Rightarrow \frac{R_a}{R_1} \times \frac{100^{K\Omega}}{R_b} = 10 \Rightarrow R_a = R_1 = R_b = 10K\Omega$



$v_o = 10v_1 - 10v_2 = 10 \times 0.02\sin 2\pi \times 1000t$

$v_o = 0.2\sin(2\pi \times 1000t) \quad -0.2^V \leq v_o \leq 0.2^V$

2.43

This is a weighted summer circuit:

$v_o = -\left(\frac{R_F}{R_0}v_0 + \frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_3}v_3\right)$

we may write: $v_0 = 5^V \times a_0, \quad v_2 = 5^V \times a_2$
$v_1 = 5^V \times a_1, \quad v_3 = 5^V \times a_3$

$v_o = -R_F\left(\frac{5a_0}{80K} + \frac{5}{40K}a_1 + \frac{5}{20K}a_2 + \frac{5}{10K}a_3\right)$

$v_o = -R_F\left(\frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2}\right)$

$v_o = -\frac{R_F}{16}(2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3)$

$-12^V \leq v_o \leq 0 \Rightarrow \frac{R_F}{16}(2^0 \times 1 + 2 \times 1 + 2^2 \times 1 + 2^3 \times 1) =$

$= \frac{15 R_F}{16} = 12$ when $a_0 = a_1 = a_2 = a_3 = 1$ we have the peak value at v_o.

$\Rightarrow R_F = 12.8K\Omega$

2.44

a) $\frac{v_o}{v_i} = 1 = 1 + \frac{R_2}{R_1} \Rightarrow R_2 = 0, R_1 = 10K\Omega$

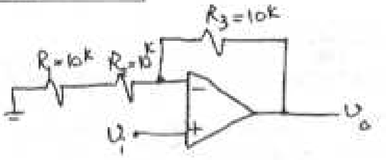
b) $\frac{v_o'}{v_i} = 2 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10K\Omega$

Cont.

c) $\frac{v_o}{v_i} = 101\ V/V = 1 + \frac{R_2}{R_1} \Rightarrow$ if $R_1 = 10^{k\Omega} \Rightarrow R_2 = 1^{M\Omega}$

d) $\frac{v_o}{v_i} = 100\ V/V = 1 + \frac{R_2}{R_1} \Rightarrow$ if $R_1 = 10^{k\Omega} \Rightarrow R_2 = 990\ k\Omega$

2.45

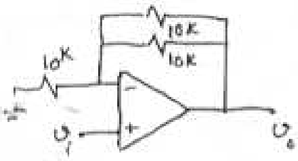


short-circuit R_2:

$\frac{v_o}{v_i} = 2$

Short circuit R_3:

$\frac{v_o}{v_i} = 1$



2.46

$v_+ = v_- = V = R \times i$, $i = 100\mu A$ when $V = 10^v$

$\Rightarrow R = \frac{10}{0.1mA} = 100\,k\Omega$

As indicated, i only depends on R and V and the meter resistance does not affect i.

2.47

Refer to the circuit in P2.47:

a) using superposition, we first set $v_{P_1} = v_{P_2} = \dots = 0$

The output voltage that results in response to $v_{N_1}, v_{N_2}, \dots v_{N_n}$ is:

$$v_{ON} = -\left[\frac{R_F}{R_{N_1}} v_{N_1} + \frac{R_F}{R_{N_2}} v_{N_2} + \dots + \frac{R_F}{R_{N_n}} v_{N_n}\right]$$

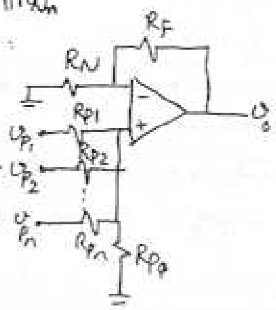
Then we set $v_{N_1} = v_{N_2} = \dots = 0$, then:

$R_N = R_{N_1} \,//\, R_{N_2} \,||\, R_{N_3} \,||\, \dots \,||\, R_{N_n}$

The circuit simplifies to:

$v_{OP} = \left(1 + \frac{R_F}{R_N}\right) \times$

$$\left(v_{P_1} \frac{1/R_{P_1}}{\frac{1}{R_{P_1}} + \frac{1}{R_{P_2}} + \dots + \frac{1}{R_{P_n}}} + v_{P_2} \frac{1/R_{P_2}}{\frac{1}{R_{P_1}} + \dots + \frac{1}{R_{P_n}}} + \dots v_{P_n} \frac{1/R_{P_n}}{1/R_{P_1} + \dots + \frac{1}{R_{P_n}}}\right)$$



$$v_{op} = \left(1 + \frac{R_F}{R_N}\right)\left(v_{P_1} \frac{R_P}{R_{P_1}} + v_{P_2} \frac{R_P}{R_{P_2}} + \dots + \frac{R_P}{R_{P_n}} v_{P_n}\right)$$

where:

$$R_P = R_{P_1} \,||\, R_{P_2} \,||\, \dots \,||\, R_{P_n}$$

when all inputs are present:

$$v_o = v_{ON} + v_{op} = -\left(\frac{R_F}{R_{N_1}} v_{N_1} + \frac{R_F}{R_{N_2}} v_{N_2} + \dots\right) + \left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P_1}} v_{N_1} + \frac{R_P}{R_{P_2}} v_{N_2} + \dots\right)$$

b) $v_o = -2v_{N_1} + v_{P_1} + 2v_{P_2}$

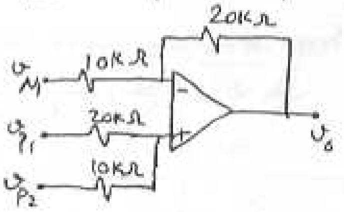
$\frac{R_F}{R_{N_1}} = 2 \quad R_{N_1} = 10K\Omega \Rightarrow R_F = 20K\Omega$

$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P_1}}\right) = 1 \Rightarrow 3\frac{R_P}{R_{P_1}} = 1 \Rightarrow R_{P_2} = \frac{R_{P_1}}{2}$

$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P_2}}\right) = 2 \Rightarrow 3\frac{R_P}{R_{P_2}} = 2 \Rightarrow R_{P_2} = \frac{R_{P_1}}{2}$

where $R_P = \frac{R_{P_1} \times R_{P_2}}{R_{P_1} + R_{P_2}}$ (ignoring $R_{P\phi}$)

Note that if the results from the last 2 constraints differ, we would use an additional resistor connected from the positive input to ground. ($R_{P\phi}$)



2.48

$v_o = v_{I1} + 3v_{I2} - 2(v_{I3} + 3v_{I4})$

Refer to P2.47.

$\frac{R_F}{R_{N3}} = 2$ if $R_{N3} = 10k\Omega \Rightarrow R_F = 20k\Omega$

$\frac{R_F}{R_{N4}} = 6 \Rightarrow R_{N4} = \frac{20}{6} = 3.3^{k\Omega}$

$R_N = R_{N3} \,||\, R_{N4} = 10K \,||\, 3.3^K = 2.48^{k\Omega}$

$\left(1 + \frac{R_F}{R_N}\right) \frac{R_P}{R_{P_1}} = 1 \Rightarrow \left(1 + \frac{20}{2.48}\right) \frac{R_P}{R_{P_1}} = 1 \Rightarrow 9.06 R_P = R_{P_1}$

$R_P = R_{P_1} \,||\, R_{P_2} \,||\, R_{P\phi} \Rightarrow R_P = \frac{1}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P\phi}}}$

$\left(1 + \frac{R_F}{R_N}\right) \frac{R_P}{R_{P_2}} = 3 \Rightarrow 9.06 \frac{R_P}{R_{P2}} = 3 \Rightarrow R_{P_2} \simeq 3R_P$

$R_{P1} \,//\, R_{P2} = \frac{9 \times 3R_P}{9+3} = 2.25 R_P$, $R_P = 2.25 R_P \,//\, R_{P\phi}$

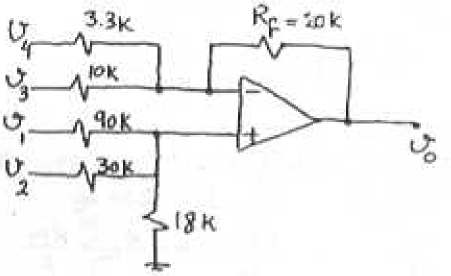
Cont.

$2.25 R_p + R_{P_0} = 2.25 R_{P_0} \Rightarrow R_{P_0} = 1.8 R_p$

if $R_p = 10 K\Omega \Rightarrow R_{P_0} = 18 K\Omega$

$R_{P_1} = 9 \times 10K = 90 K\Omega$

$R_{P_2} = 3 \times 10K = 30 K\Omega$



2.49

$$v_+ = v_I \frac{R_4}{R_3 + R_4} = v_-$$

$$\frac{v_-}{R_1} = \frac{v_o - v_-}{R_2} \Rightarrow v_o = v_-\left(1 + \frac{R_2}{R_1}\right)$$

from the two above equations :

$$\frac{v_o}{v_I} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.50

Refer to Fig. 2.50. Setting $v_2 = 0$, we obtain the output component due to v_1 as:

$v_{o1} = -20v$

Setting $v_1 = 0$, we obtain the output component due to v_2 as:

$$v_{o2} = v_2\left(1 + \frac{20R}{R}\right)\left(\frac{20R}{20R + R}\right) = 20 v_2$$

The total output voltage is :

$v_o = v_{o1} + v_{o2} = 20(v_2 - v_1)$

For $v_1 = 10 \sin 2\pi \times 60t - 0.1 \sin(2\pi \times 1000t)$

$v_2 = 10 \sin 2\pi \times 60t + 0.1 \sin(2\pi \times 1000t)$

$v_o = 4 \sin(2\pi \times 1000t)$

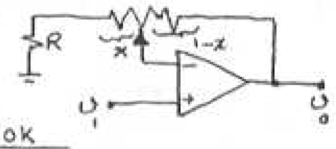
2.51

$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{(1-x)}{x} = 1 + \frac{1}{x} - 1 = \frac{1}{x}$$

$0 \leq x \leq 1 \Rightarrow 1 \leq \frac{v_o}{v_i} \leq \infty$

if we add a resistor on the ground path :

$$\frac{v_o}{v_i} = 1 + \frac{(1-x)\times 10K}{x \times 10K + R}$$

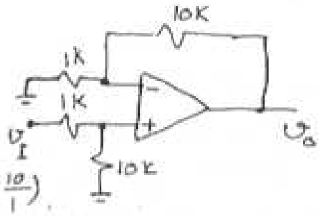


$\text{Gain}_{max} = 21$ when

$x = 0 \Rightarrow 21 = 1 + \frac{10K}{R}$

$\Rightarrow R = \frac{10K}{20} = 0.5 K\Omega$

2.52



$$v_o = v_I \frac{10}{1+10}\left(1 + \frac{10}{1}\right)$$

$v_o = 10 v_I$

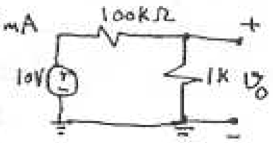
2.53

a) Source is connected directly.

$v_o = 10 \times \frac{1}{101} = 0.099V$

$i_L = \frac{v_o}{1K} = \frac{0.099}{1} = 0.099 mA$

current supplied by the source is 0.099 mA.

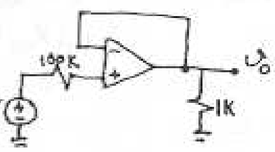


b) inserting a buffer

$v_o = 10V$

$i_L = \frac{10V}{1K} = 10 mA$

current supplied by the source is 0.

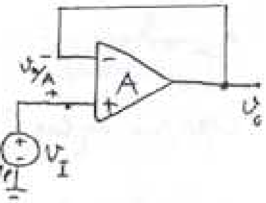


The load current i_L comes from the power supply of the op-amp.

2.54

$v_O = v_I - \frac{v_O}{A}$

$\frac{v_O}{v_I} = \frac{1}{1+\frac{1}{A}}$



error of Gain magnitude

$$\frac{\left|\frac{v_O}{v_I}\right| - 1}{1} = -\frac{1}{A+1}$$

A (V/V)	1000	100	10
$\frac{v_O}{v_I}$ (V/V)	0.999	0.990	0.909
Gain Error	-0.1%	-1%	-9.1%

2.55

for an inverting amplifier:

$R_i = R_1$, $G = -\frac{R_2}{R_1}$

for a non-inverting amplifier:

$R_i = \infty$ $G = 1 + \frac{R_2}{R_1}$

Case	Gain V/V	Rin	R_1	R_2
a	-10	10K	10K	100K
b	-1	100K	100K	100K
c	-2	50K	50K	100K
d	+1	∞	10K	10K
e	+2	∞	10K	10K
f	+11	∞	10K	100K
g	-0.5	10K	10K	5K

2.56

$A = 50\,V/V$ $1 + \frac{R_2}{R_1} = 10\,V/V$

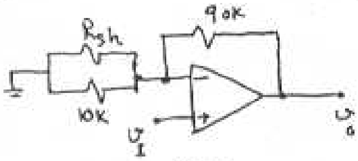
if $R_1 = 10k\Omega \Rightarrow R_2 = 90K$

According to Eq. 2.11: $G = \frac{v_O}{v_I} = \frac{1+\frac{R_2}{R_1}}{1+\frac{1+R_2/R_1}{A}}$

$G = \frac{1+90/10}{1+\frac{1+90/10}{50}} = \frac{10}{1.2} = 8.33\,V/V$

In order to compensate the gain drop, we can shunt a resistor with R_1.

Compensated:



R_{sh}: $10 = \frac{1+\left(\frac{90}{10}+\frac{90}{R_{sh}}\right)}{1+\frac{1+\frac{90}{10}+\frac{90}{R_{sh}}}{50}} \Rightarrow$

$10\times(510R_{sh} + 90R_{sh} + 900) = 50\times(10R_{sh} + 90R_{sh} + 900)$

$100R_{sh} = 3600 \Rightarrow R_{sh} = 36K\Omega$

if $A = 100$ then:

$G_{uncompensated} = \frac{1+\frac{90}{10}}{1+\frac{1+90/10}{100}} = \frac{10}{1.1} = 9.09\,V/V$

$G_{compensated} = \frac{1+\frac{90}{10}+\frac{90}{36}}{1+\frac{1+\frac{90}{10}+\frac{90}{36}}{100}} = \frac{12.5}{1.125} = 11.1\,V/V$

2.57

$G = \frac{G_0}{1+\frac{G_0}{A}}$, $\frac{G_0 - G}{G_0}\times 100 = \frac{G_0/A \times 100}{1+\frac{G_0}{A}} \le x$

or $\frac{1+\frac{G_0}{A}}{G_0/A} \ge \frac{100}{x} \Rightarrow \frac{A}{G_0} \ge \underbrace{\left(\frac{100}{x} - 1\right)}_{F}$

$\Rightarrow A \ge G_0 F$ where $F = \frac{100}{x} - 1 \simeq \frac{100}{x}$

x	0.01	0.1	1	10
F	10^4	10^3	10^2	10

Thus for:

$x = 0.01$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10^4	10^5	10^6	10^7	10^8

10^7, 10^8: too high to be practical

$x = 0.1$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10^3	10^4	10^5	10^6	10^7
$x = 1$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10^2	10^3	10^4	10^5	10^6
$x = 10$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10	10^2	10^3	10^4	10^5

2.58

for non-inverting amplifier, Eq. 2.11:

$G = \frac{G_0}{1+\frac{G_0}{A}}$, $\varepsilon = \frac{G_0 - G}{G_0} \times 100$

for inverting amplifier, Eq. 2.5:

$G = \frac{G_0}{1+\frac{1-G_0}{A}}$, $\varepsilon = \frac{G_0 - G}{G_0} \times 100$

Case	G_0 (V/V)	A (V/V)	G (V/V)	ε %
a	−1	10	−0.83	16
b	1	10	0.91	9
c	−1	100	−0.98	2
d	10	10	5	50
e	−10	100	−9	10
f	−10	1000	−9.89	1.1
g	+1	2	0.67	33

2.59

Refer to fig. P2.59, when potentiometer is set to the bottom:

$v_o = v_+ = -15 + \frac{30 \times 20}{20+100+20} = -10.714V$

when set to the top: $v_o = -15 + \frac{30 \times 120}{20+100+20} = 10.714V$

$\Rightarrow -10.714 \leq v_o \leq +10.714$

pot has 20 turn, each turn: $\Delta v_o = \frac{2 \times 10.714}{20} = 1.07V$

2.60

Refer to Fig. 2.16. Notice that similar to eq. 2.15 we have: $\frac{R_4}{R_3} = \frac{R_2}{R_1} = \frac{100}{10}$. therefore according to 2.16:

$v_o = \frac{R_2}{R_1} v_{Id} \Rightarrow A = \frac{R_2}{R_1} = 10 \ V/V$

According to 2.20: $R_{id} = 2R_1 = 20k\Omega$

If $\frac{R_2}{R_1}$, $\frac{R_4}{R_3}$ were different by 1%:

$\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$

Refer to eq. 2.19: $A_{cm} = \frac{R_4}{R_4+R_3}\left(1 - \frac{R_2}{R_1}\cdot\frac{R_3}{R_4}\right)$

$A_{cm} = \frac{100}{100+10}(1-0.99) = 0.009$

$CMRR = 20 \log \frac{|A_d|}{|A_{cm}|}$, so let's calculate A_d

$A_d = \frac{v_o}{v_{id}}$ if we apply superposition:

$v_{o1} = -\frac{R_2}{R_1} v_{I_1}$ $\quad v_{o2} = v_{I2} \frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)$

$v_o = v_{o2} + v_{o1} = v_{I2} \frac{R_4/R_3}{1+\frac{R_4}{R_3}}\left(1+\frac{R_2}{R_1}\right) - \frac{R_2}{R_1} v_{I_1}$

Replace $\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3} \Rightarrow \frac{R_2}{R_1} = 0.99 \times \frac{100}{10} = 9.9$

$v_o = v_{I2} \frac{10}{1+10}(1+9.9) - v_{I_1} 9.9 = 9.9(v_{I2} - v_{I_1})$

$\frac{v_o}{v_{Id}} = 9.9 = A_d \Rightarrow CMRR = 20 \log \frac{9.9}{0.009} = 60.8$

CMRR = 60.8

2.61

If we assume $R_3 = R_1$, $R_4 = R_2$, then

eq. 2.20: $R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10k\Omega$

(Refer to Fig. 2.15)

a) $A_d = \frac{R_2}{R_1} = 1 \ V/V \Rightarrow R_2 = 10k\Omega$

$R_1 = R_2 = R_3 = R_4 = 10k\Omega$

b) $A_d = \frac{R_2}{R_1} = 2 \ V/V \Rightarrow R_2 = 20k\Omega = R_4$

$R_1 = R_3 = 10k\Omega$

c) $A_d = \frac{R_2}{R_1} = 100 \ V/V \Rightarrow R_2 = 1M\Omega = R_4$

$R_1 = R_3 = 10k\Omega$

d) $A_d = \frac{R_2}{R_1} = 0.5 \ V/V \Rightarrow R_2 = 5k\Omega = R_4$

$R_1 = R_3 = 10k\Omega$

2.62

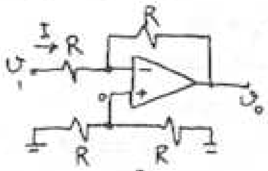
Refer to Fig P2.62:

Cont.

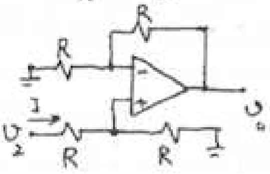
Considering that $v_- = v_+$:

$v_1 + \frac{v_o - v_1}{2} = \frac{v_2}{2} \Rightarrow v_o = v_2 - v_1$

v_1 only: $R_I = \frac{v_1}{I} = R$



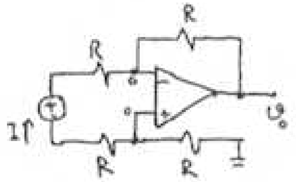
v_2 only: $R_I = \frac{v_2}{I} = 2R$



v_s between 2 terminals:

$R_I = \frac{v}{I} = 2R$

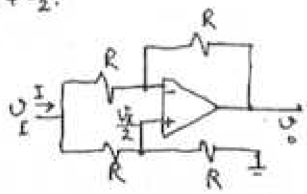
$v_+ = v_- = 0$



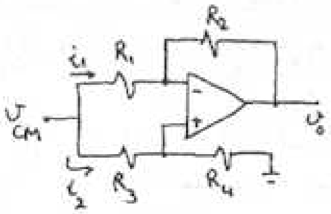
v_s connected to both v_1 & v_2:

$R_I = \frac{v}{I} = R$

$v_+ = v_- = \frac{v_I}{2}$



2.63



$v_+ = v_{CM}\frac{R_4}{R_3+R_4}$

$v_+ = v_-$

$i_2 = \frac{v_{CM}}{R_3+R_4}$

$i_1 = \frac{v_{CM}}{R_1} - \frac{v_{CM} R_4}{R_3+R_4}\cdot\frac{1}{R_1} = \frac{v_{CM}}{R_1}\frac{R_3}{R_3+R_4}$

$i = i_1+i_2 = \frac{v_{CM}}{R_1}\frac{R_3}{R_3+R_4} + \frac{v_{CM}}{R_3+R_4}$

$\frac{1}{R_I} = \frac{i}{v_{CM}} = \frac{1}{R_1}\frac{1}{\frac{R_4}{R_3}+1} + \frac{1}{R_3+R_4}$

if we replace $\frac{R_4}{R_3}$ with $\frac{R_2}{R_1}$: $\left(\frac{R_4}{R_3} = \frac{R_2}{R_1}\right)$

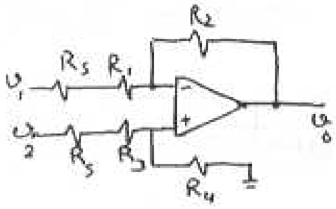
$\frac{1}{R_I} = \frac{1}{R_1+R_2} + \frac{1}{R_3+R_4} \Rightarrow R_I = (R_1+R_2)\,||\,(R_3+R_4)$

2.64

In order to have an ideal differential amp:

$\frac{R_s+R_1}{R_2} = \frac{R_s+R_3}{R_4}$

$\frac{R_s/R_1+1}{R_2/R_1} = \frac{R_s/R_3+1}{R_4/R_3}$



Since $\frac{R_2}{R_1} = \frac{R_4}{R_3}$:

$\frac{R_s}{R_1}+1 = \frac{R_s}{R_3}+1 \Rightarrow R_1 = R_3 \Rightarrow R_2 = R_4$

2.65

Refer to eq. 2.19 and Fig. P2.62:

$A_{cm} = \frac{v_o}{v_{ICM}} = \frac{R_4}{R_3+R_4}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$

The worst case is when A_{cm} has its maximum value.

$A_{cm} = \frac{1}{\frac{R_3}{R_4}+1}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$

Max $A_{cm} \Rightarrow \frac{R_3}{R_4}$ has to be at its minimum value and also $\frac{R_2}{R_1}$ has to be minimum.

$\frac{100-x}{100+x} \le \frac{R_3}{R_4} \le \frac{100+x}{100-x}$ $\quad$ $\frac{100-x}{100+x} \le \frac{R_2}{R_1} \le \frac{100+x}{100-x}$

so if $\frac{R_3}{R_4} = \frac{100-x}{100+x}$ & $\frac{R_2}{R_1} = \frac{100-x}{100+x}$

$A_{cm\,Max} = \frac{1}{\frac{100-x}{100+x}+1}\left(1 - \frac{100-x}{100+x}\frac{100-x}{100+x}\right)$

$A_{cm\,Max} = \frac{1}{200}\frac{(100+x)^2-(100-x)^2}{100+x} = \frac{2x}{100+x} \simeq \frac{x}{50}$

x	0.1	1	5
$A_{cm\,Max}$	0.002	0.02	0.1

CMRR $= 20\log\left|\frac{A_d}{A_{cm}}\right|$. Now we have to calculate A_d based on values we chose for $R_1 - R_4$ that gave us $A_{cm\,Max}$.

$R_2 = R_3 = 100 - x$ $\quad$ $R_1 = R_4 = 100 + x$

$v_o = v_{o1} + v_{o2}$ by applying superposition

$v_o = -\frac{R_2}{R_1}v_1 + v_2\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)$

$v_o = -\frac{100-x}{100+x}v_1 + v_2\frac{100+x}{200}\left(1+\frac{100-x}{100+x}\right)$

$v_o = -\frac{100-x}{100+x}v_1 + v_2$

if we consider $\frac{100-x}{100+x} \simeq 1 \Rightarrow \frac{v_o}{v_{id}} \simeq 1$

Cont.

$$CMRR = 20\log\frac{A_d}{A_{cm}} = 20\log\frac{1}{x/50} = 20\log\frac{50}{x}$$

x	0.1	1	5
CMRR	54db	34db	20db

2.66

Refer to Fig. 2.16 and eq. 2.19:

$$A_{cm} = \frac{R_4}{R_3+R_4}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

In order to calculate A_d, we use superposition principle:

$$v_o = v_{o1} + v_{o2} = -\frac{R_2}{R_1}v_1 + v_2\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)$$

then replace $v_1 = v_{cm} - \frac{v_d}{2}$

$v_2 = v_{cm} + \frac{v_d}{2}$

$$v_o = -\frac{R_2}{R_1}v_{cm} + \frac{R_2}{R_1}v_d/2 + v_{cm}\frac{1+R_2/R_1}{1+\frac{R_3}{R_4}} + \frac{v_d}{2}\frac{1+R_2/R_1}{1+\frac{R_3}{R_4}}$$

$$v_o = \underbrace{\frac{R_2}{2R_1}\left[1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]}_{A_d}v_d + \frac{R_2}{R_1}\left[-1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]v_{cm}$$

$$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log\frac{\frac{R_2}{2R_1}\left[1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]}{\frac{1}{\frac{R_3}{R_4}+1}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)}$$

$$CMRR = 20\log\left|\frac{\frac{1}{2}\frac{R_2}{R_1}\left[2+\frac{R_1}{R_2}+R_3/R_4\right]}{1-\frac{R_2}{R_1}\cdot\frac{R_3}{R_4}}\right|$$

$$CMRR = 20\log\left|\frac{1+\frac{1}{2}\frac{R_1}{R_2}+\frac{1}{2}\frac{R_3}{R_4}}{\frac{R_1}{R_2}-\frac{R_3}{R_4}}\right|$$

for worst case, or minimum CMRR we have to maximize the denominator, which means:

$R_1 = R_{1n}(1+\varepsilon)$ $\quad R_3 = R_{3n}(1-\varepsilon)$

$R_2 = R_{2n}(1-\varepsilon)$ $\quad R_4 = R_{4n}(1+\varepsilon)$

also: $\frac{R_{2n}}{R_{1n}} = \frac{R_{4n}}{R_{3n}} = K$

$$CMRR = 20\log\left|K\,\frac{1+\frac{1}{2K}\frac{1+\varepsilon}{1-\varepsilon}+\frac{1}{2K}\frac{1-\varepsilon}{1+\varepsilon}}{\frac{1+\varepsilon}{1-\varepsilon}-\frac{1-\varepsilon}{1+\varepsilon}}\right|$$

$$CMRR = 20\log\left|\frac{K(1-\varepsilon^2)+(1+\varepsilon^2)}{4\varepsilon}\right| \simeq 20\log\left|\frac{K+1}{4\varepsilon}\right|$$

for $\varepsilon^2 \ll 1$.

if $K = A_{d\,\text{ideal}} = 100$, $\varepsilon = 0.01$

$$CMRR = 20\log\frac{101}{0.04} \simeq 68\text{db}$$

2.67

$A_d = 100$

we assume $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ then $A_d = \frac{R_2}{R_1} = K$

$K \simeq 100$

$R_{id} = 2R_1 = 20k\Omega \rightarrow R_1 = 10k\Omega$

$CMRR = 80db = 20\log\frac{A_d}{A_{cm}} \rightarrow \frac{A_d}{A_{cm}} = 10^4$

$\Rightarrow A_{cm} = 0.01$

$A_d = 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 1M\Omega$

Refer to P2.66: $\quad CMRR = 20\log\frac{K+1}{4\varepsilon}$

$CMRR = 10^4$

$\Rightarrow \varepsilon = 10^{-2}\times 0.25$

we assumed earlier $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ then

$\frac{R_4}{R_3} \simeq 100 \Rightarrow$ if $R_3 = 10k \pm \varepsilon$

$\Rightarrow R_4 = 1M\Omega \pm \varepsilon$ $\qquad \varepsilon = 0.25\%$

$R_2 = 1M\Omega \pm \varepsilon$

$R_1 = 10k \pm \varepsilon$

2.68

Refer to Fig. P2.68 and Eq. 2.19:

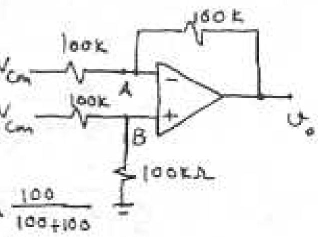
$$A_{cm} = \frac{R_4}{R_3+R_4}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right) = \frac{100}{100+100}\left(1-\frac{100.100}{100.100}\right)$$

$A_{cm} = 0$

Refer to 2.17: $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

$\rightarrow A_d = \frac{R_2}{R_1} = 1$

b) Since $A_{cm} = 0$, then if we apply V_{cm} to V_{I1} and V_{I2}, $v_o = 0$.



Therefore, $V_A = v_{cm}\frac{100}{100+100}$

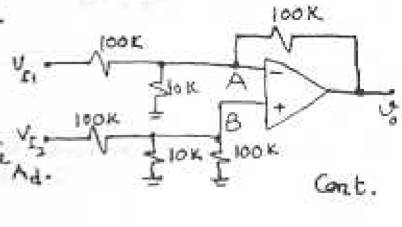
$V_A = \frac{v_{cm}}{2}$

Similarly, $v_B = \frac{v_{cm}}{2}$

We know $V_A = V_B$ and $-2.5 \leq v_A \leq 2.5$

$\Rightarrow -5 \leq v_{cm} \leq 5$

c) we apply the superposition principle to calculate A_d.



Cont.

v_{01} is the output voltage when $v_{I2}=0$
v_{02} is the output voltage when $v_{I1}=0$

$v_0 = v_{01} + v_{02}$

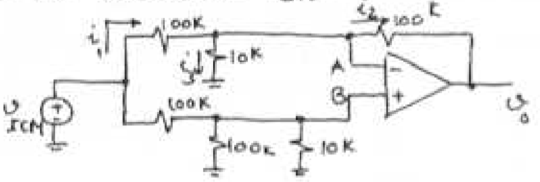
$v_{01} = \frac{-R_2}{R_1} v_{I1} = -v_{I1}$

$v_{02} = v_{I2} \frac{100K\Omega \| 10K}{100K \| 10K + 100} \left(1 + \frac{100K}{100K \| 10K}\right)$

$v_{02} = v_{I2} \times 1$

$\Rightarrow v_0 = v_{01} + v_{02} = -v_{I1} + v_{I2} \Rightarrow A_d = 1$

Now we calculate A_{cm}:



$v_B = v_{ICM} \frac{100K \| 10K}{100K \| 10K + 100K}$, $v_A = v_B$

$i_1 = \frac{v_{ICM} - v_A}{100K}$

$v_0 = v_A - 100K \times i_2$ and $i_2 = i_1 - i_3 = i_1 - \frac{v_A}{10K}$

$v_0 = v_A - 100K \times i_1 + 10 \times v_A$

$v_0 = v_A - v_{ICM} + v_A + 10 \times v_A$

$v_A = v_B \Rightarrow v_0 = v_{ICM} \left(-1 + 12 \frac{100K \| 10K}{100 \| 10K + 100}\right)$

$\frac{v_0}{v_{ICM}} = A_{cm} = 0$

Now we calculate v_{ICM} range:

$-2.5 \leq v_B \leq 2.5 \Rightarrow -2.5 \leq v_{ICM} \times \frac{100K \| 10K}{100K \| 10K + 100K} \leq 2.5$

$-30^V \leq v_{ICM} \leq 30^V$

2.69

Refer to Fig. P2.69; we use superposition:

$v_0 = v_{01} + v_{02}$

calculate v_{01}: $v_+ = \frac{\beta v_{01}}{2} = v_-$

$\frac{v_1 - \frac{\beta v_{01}}{2}}{R} = \frac{\frac{\beta v_{01}}{2} - v_0}{R} \Rightarrow v_{01} = \frac{v_1}{\beta - 1}$

calculate v_{02}:

$v_- = \frac{v_{02}}{2} = v_+ \Rightarrow v_2 - \frac{v_{02}}{2} = \frac{v_{02}}{2} - \beta v_{02}$

$\Rightarrow v_{02} = \frac{v_2}{1-\beta}$

$v_0 = v_{01} + v_{02} = \frac{v_1}{\beta - 1} + \frac{v_2}{1-\beta} = \frac{1}{1-\beta}(v_2 - v_1)$

$A_d = \frac{v_0}{v_2 - v_1} = \frac{1}{1-\beta}$

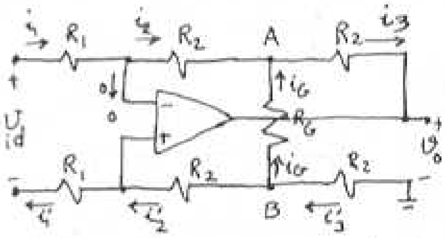
$A_d = 10^V\!/\!_V \Rightarrow \beta = 0.9 = \frac{R5}{R5 + R6}$

$R_{id} = 2R = 2M\Omega \Rightarrow R = 1M\Omega$

$R_5 + R_6 \leq \frac{R}{100} \Rightarrow R_5 + R_6 \leq 10K\Omega$

$R_5 = 6.8K\Omega$ $R_6 = 680\Omega$ $\Rightarrow \beta = \frac{6.8}{6.8 + 0.68} \simeq 0.9$

2.70



$v_+ = v_-$ So we can consider v_+, v_- a virtual short: $i_1 = v_{id}/2R_1 \Rightarrow i_2 = \frac{v_{id}}{2R_1}$

$i_1' = i_2' = \frac{v_{id}}{2R_1}$

then: $i_2 R_2 + v_{AB} + i_2' R_2 = 0 \Rightarrow v_{AB} = \frac{-v_{id}}{R_1} R_2$

$i_G = \frac{v_{id}}{R_G} \times \frac{R_2}{R_1}$

$i_3 = i_2 + i_G = \frac{v_{id}}{2R_1} + \frac{v_{id}}{R_G} \frac{R_2}{R_1}$

$i_3' = i_G + i_2' = i_3$

$\Rightarrow v_0 = -[i_3' R_2 + v_{BA} + i_3 R_2]$

$v_0 = -[2 i_3 R_2 + v_{BA}]$

$v_0 = -\left[\frac{2 v_{id}}{2R_1} R_2 + 2 v_{id} \frac{R_2}{R_1} \frac{R_2}{R_G} + \frac{v_{id}}{R_1} R_2\right]$

$\frac{v_0}{v_{id}} = A_d = -2 \frac{R_2}{R_1}\left[1 + \frac{R_2}{R_G}\right]$

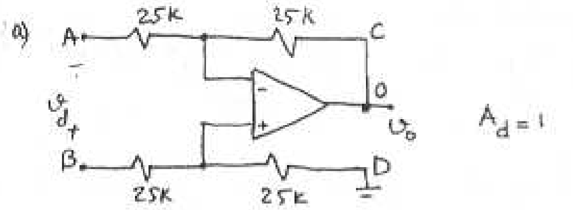
2.71

a)

Refer to Eq. 2.17: $A_d = \frac{R_2}{R_1} = 1$. Connect C and D together.

Cont.

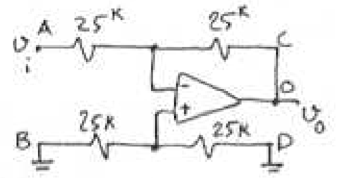
a)



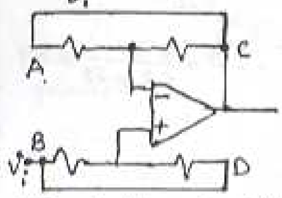
$A_d = 1$

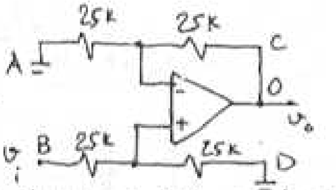
b) $\frac{v_o}{v_i} = -1\ V/V$

i)



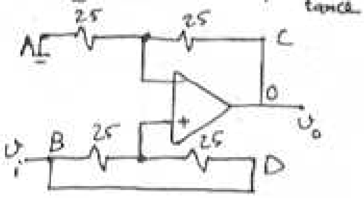
ii) $\frac{v_o}{v_i} = +1\ V/V$





The circuit on the left ideally has infinite input resistance

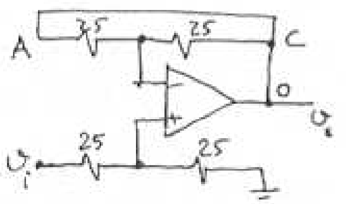
iii) $\frac{v_o}{v_i} = +2\ V/V$



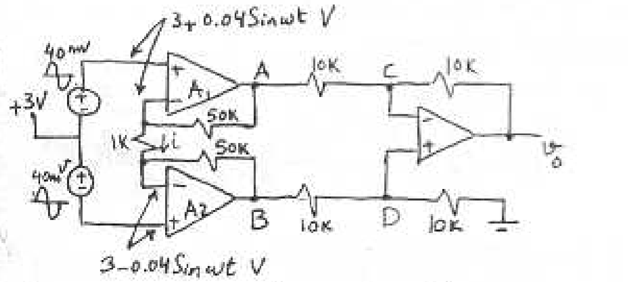
iv) $\frac{v_o}{v_i} = +\frac{1}{2}\ V/V$

$v_+ = \frac{v_i}{2} = v_o$

$\Rightarrow \frac{v_o}{v_i} = \frac{1}{2}$



2.72



$i = \frac{3+0.04\sin\omega t - (3-0.04\sin\omega t)}{1K} = 0.08\sin\omega t,\ mA$

$v_A = 3+0.04\sin\omega t + 50K \times i = 3+4.04\sin\omega t,\ V$

$v_B = 3-0.04\sin\omega t - 50K \times i = 3-4.04\sin\omega t,\ V$

$v_C = v_D = \frac{1}{2}v_B = 1.5 - 2.02\sin\omega t,\ V$

$v_o = v_B - v_A = -8.08\sin\omega t,\ V$

2.73

Refer to Fig. 2.20.a.

The gain of the first stage is: $(1+\frac{R_2}{R_1}) = 101$

If the opamps of the first stage saturate at $\pm 14\ V$: $-14 \leq v_1 \leq +14V \Rightarrow -14 \leq 101\, v_{icm} \leq +14$

$\Rightarrow -0.14 \leq v_{ICM} \leq 0.14$

As explained in the text, the disadvantage of circuit in fig. 2.20.a is that v_{ICM} is amplified by a gain equal to vid, $(1+\frac{R_2}{R_1})$ in the first stage and therefore a very small v_{icm} range is acceptable to avoid saturation.

b) In fig. 2.20b, when v_{ICM} is applied, v for both A_1 & A_2 is the same and therefore no current flows through $2R_1$. This means voltage at the output of A_1 and A_2 is the same as V_{ICM}.

$-14 \leq v_o \leq 14 \Rightarrow -14 \leq v_{ICM} \leq 14$

This circuit allows for bigger range of V_{icm}.

2.74

$v_{i1} = v_{cm} - v_d/2$

$v_{i2} = v_{cm} + v_d/2$

Refer to fig. 2.20.a.

output of the first stage: $(1+\frac{R_2}{R_1})(v_{cm} - \frac{v_d}{2})$

$v_{o1} = (1+\frac{R_2}{R_1})(v_{cm} - v_d/2)$

$v_{o2} = (1+\frac{R_2}{R_1})(v_{cm} + v_d/2)$

$v_{o2} - v_{o1} = (1+\frac{R_2}{R_1})v_d \quad \Rightarrow A_{d(1)} = 1+\frac{R_2}{R_1}$

$\frac{v_{o2}+v_{o1}}{2} = (1+\frac{R_2}{R_1})v_{cm} \quad \Rightarrow A_{cm(1)} = 1+\frac{R_2}{R_1}$

CMRR $= 20\log\left|\frac{A_d}{A_{cm}}\right| = 0$ (First stage)

Now Consider Fig. 2.20.b

$v_{o1} = v_{i1} + R_2 \times \frac{(v_{i1}-v_{i2})}{2R_1}$

$v_{o1} = v_{cm} - v_d/2 + \frac{R_2}{2R_1}(-v_d)$

Cont.

$$v_{o1} = v_{cm} - \frac{v_d}{2}\left(1+\frac{R_2}{R_1}\right)$$

$$v_{o2} = v_{i2} - R_2 \times \frac{v_{i1}-v_{i2}}{2R_1} = v_{cm} + \frac{v_d}{2} + R_2 \frac{v_d}{2R_1}$$

$$v_{o2} = v_{cm} + \frac{v_d}{2}\left(1+\frac{R_2}{R_1}\right)$$

$$v_{o2} - v_{o1} = v_d\left(1+\frac{R_2}{R_1}\right) \Rightarrow A_{d(1)} = 1+\frac{R_2}{R_1}$$

$$\frac{v_{o2}+v_{o1}}{2} = v_{cm}. \Rightarrow A_{cm(1)} = 1$$

$$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log\left(1+\frac{R_2}{R_1}\right)$$

In 2.20.b, the common mode voltage is not amplified and it is only propagated to the outputs of the first stage.

2.75

Refer to eq. 2.22:

$$A_d = \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right) = \frac{100^k}{100k}\left(1+\frac{100k}{5^k}\right) = 21\ V/V$$

$A_{cm} = 0$

$$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = \infty$$

If all resistors are $\pm 1\%$:

$A_d \simeq 21$

In order to calculate A_{cm}, apply V_{cm} to both inputs and note that V_{cm} will appear at both output terminals of the first stage. Now we can evaluate v_o by analyzing the second stage as was done in problem 2.65. In P2.65 we showed that if each 100k resistor has $\pm x\%$ tolerance, A_{cm} of the differential amplifier is: $A_{cm} = \frac{v_o}{v_{cm}} = \frac{x}{50}$. Therefore the overall A_{cm} is also $\frac{x}{50}$.

$$x = 1 \Rightarrow A_{cm} = \frac{1}{50} = 0.02$$

$$CMRR = 20\log\frac{21}{0.02} = \underline{60db}$$

If $2R_1 = 1k\Omega$: $A_d = \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right) = 201\ V/V$

$A_{cm} = 0.02$ unchanged

$$CMRR = 20\log\frac{201}{0.02} = \underline{80db}$$

Conclusion: Large CMRR can be achieved by having relatively Large A_d in the first stage.

2.76

$A_{d(2)}$ of the second stage is $\frac{R_4}{R_3} = 0.5$

$R_4 = 100k\Omega$, $R_3 = 200k\Omega$

we use a series configuration of R_{1f} and R_1 (pot): R_1 (Fixed) $= 100k$ pot

$$\text{Minimum gain} = 0.5\left(1+\frac{R_2}{R_1}\right) = 0.5\left(1+\frac{R_2}{\frac{100^k+R_1}{2}}\right)$$

$$1 \leqslant A_d \leqslant 100 \Rightarrow 1 = 0.5\left(1+\frac{2R_2}{R_{1f}+100k}\right)$$

$$\Rightarrow R_{1f} + 100 = 2R_2 \quad ①$$

$$\text{Maximum gain} = 100 = 0.5\left(1+\frac{R_2}{R_{1f}/2}\right) \Rightarrow$$

$$2R_2 = 199R_{1f} \quad ②$$

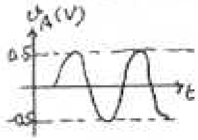
$$①, ② \Rightarrow R_{1f} = 0.505\,k\Omega \simeq 0.5k\Omega$$

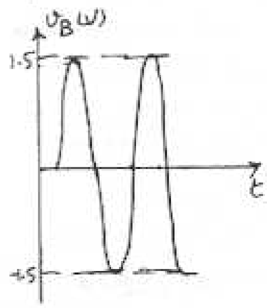
$$R_2 = 50.25\,k\Omega \simeq 50k\Omega$$

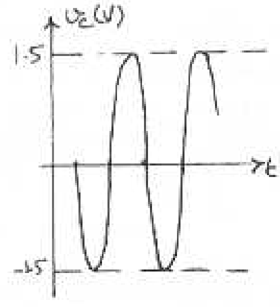
2.77

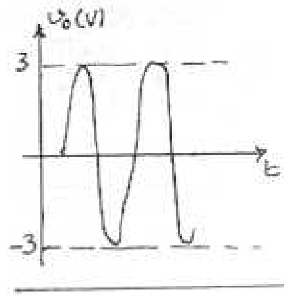
a) $\frac{v_B}{v_A} = 1 + \frac{20}{10} = 3\ V/V$, $\frac{v_C}{v_A} = -\frac{30}{10} = -3\ V/V$

b) $v_o = v_B - v_C = 6\,v_A \Rightarrow \frac{v_o}{v_A} = 6\ V/V$









c) v_B and v_C can be $\pm 14V$ or $28V$ p-p.

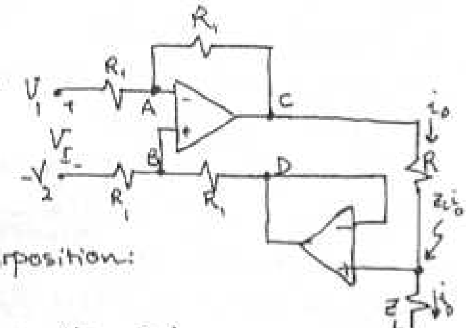
$-28 \leqslant v_o \leqslant 28$ or $\underline{56\,v\,p\text{-}p}$.

$$v_{o\,rms} = \underline{19.8\ V} = \frac{28}{\sqrt{2}}$$

2.78

Refer to Fig. P.2.78.a:
Since the inputs of the op-amp donot draw any current, V_I appears across R,
$i_o = \frac{V_I}{R}$

Fig. P2.78.B



$V_D = Z_L i_o$

we use superposition:
$V_I = V_1 - V_2$

V_1 only: $V_B = \frac{V_D}{2} = \frac{Z_L i_{o1}}{2}$

$$\frac{V_1 - \frac{Z_L i_{o1}}{2}}{R_1} = \frac{\frac{Z_L i_{o1}}{2} - i_{o1}(Z_L + R)}{R_1}$$

$\Rightarrow V_1 = i_{o1} R \Rightarrow i_{o1} = \frac{V_1}{R}$

Now if only $(-V_2)$ is applied:

$V_B = \frac{-V_2 + Z_L i_{o2}}{2}$, $V_A = \frac{i_{o2} \times (R + Z_L)}{2}$

$V_A = V_B \Rightarrow -V_2 + Z_L i_{o2} = i_{o2} R + i_{o2} Z_L$

$-V_2 = i_{o2} R \Rightarrow i_{o2} = \frac{-V_2}{R}$

The total current due to both sources is:

$i_o = i_{o1} + i_{o2} = \frac{V_1}{R} - \frac{V_2}{R} = \frac{V_I}{R}$

The circuit in Figure P2.78(a) has ideally infinite input resistance, and it requires that both terminals of Z_L be availabe, while the other circuit has finite input resistance with one side of Z_L grounded.

2.79

A_o	f_b (HZ)	f_t (HZ)
10^5	10^2	10^7
10^6	1	10^6
10^5	10^3	10^8
10^7	10^{-1}	10^6
2×10^5	10	2×10^6

eq. 2.28:
$\omega_t = A_o \omega_b$
$\Rightarrow f_t = A_o f_b$

2.80

Eq. 2.25 : $A = \frac{A_o}{1 + j\,\omega/\omega_b} \Rightarrow |A| = \frac{|A_o|}{\sqrt{1 + (\frac{f}{f_b})^2}}$

$A_o = 86\text{db}$, $A = 40\text{db}$ @ $f = 100\text{KHZ}$

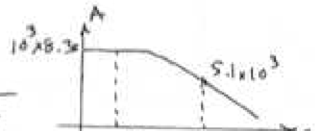
$20\log\sqrt{1+(\frac{f}{f_b})^2} = 20\log\frac{|A_o|}{|A|} = 20\log A_o - 20\log A$
$= 86 - 40 = 46\text{db}$

$1 + (\frac{100^{KHZ}}{f_b})^2 = (199.5)^2 \Rightarrow f_b = 0.501\text{KHZ}$
$f_b = 501\text{ HZ}$

$f_t = A_o f_b = \underbrace{1.995\times10^4}_{86\text{db}} \times 501 = 9.998\text{ MHZ} \simeq 10\text{MHZ}$

2.81

$A_o = 8.3\times10^3\ V/V$



Eq. 2.25: $A = \frac{A_o}{1 + j\frac{f}{f_b}}$

$f_t = A_o f_b$

$5.1\times10^3 = \frac{8.3\times10^3}{\sqrt{1 + (\frac{100KH}{f_b})^2}} \Rightarrow 1 + (\frac{100^{KHZ}}{f_b})^2 = 2.65$

$f_b = 60.7$ KHZ

$f_t = A_o f_b = 8.3\times10^3 \times 60.7 = 503$ MHZ

2.82

we have:
$A_{o(db)} = 20\text{db} + A_{(db)}$ $20\text{db} = 20\log 10 \Rightarrow A_o = 10 \times A$

a) $A_o = 10 \times 3\times10^5 = 3\times10^6\ V/V$

$A = \frac{A_o}{1 + jf/f_b} \Rightarrow |1 + j\frac{f}{f_b}| = \frac{A_o}{A} = 10 \Rightarrow \frac{6\times10^2}{f_b} = \sqrt{99}$

$\Rightarrow f_b = 60.3$ HZ

$f_t = A_o f_b = 3\times10^6 \times 60.3 = 180.9$ MHZ

b) $A = 50\times10^5\ V/V \Rightarrow A_o = 10 \times 50\times10^5 = 50\times10^6\ V/V$

$|1 + j\frac{f}{f_b}| = \frac{A_o}{A} = 10 \Rightarrow \frac{10^{HZ}}{f_b} = \sqrt{99} \Rightarrow f_b = 1$ HZ

$f_t = A_o f_b = 50$ MHZ

c) $A = 1500\ V/V \Rightarrow A_o = 15000\ V/V$

Cont.

$\left|1+\frac{jf}{f_b}\right|=10 \Rightarrow \frac{0.1\times10^6}{f_b}=\sqrt{99} \Rightarrow f_b=10KHZ$

$f_t=15000\times10^K=150MHZ$

d) $A_o=10\times100=1000\ V/V_q$

$\left|1+\frac{jf}{f_b}\right|=10 \Rightarrow \frac{0.1\times10^6}{f_b}=\sqrt{99} \Rightarrow f_b=10\ MHZ$

$f_t=1000\times10^{MHZ}=100GHZ$

e) $A_o=25\ V/_{mV}\times10=25\times10^4\ V/V$

$\left|1+\frac{jf}{f_b}\right|=10 \Rightarrow \frac{25KHZ}{f_b}=\sqrt{99} \Rightarrow f_b=2.51^{KHZ}$

$f_t=A_of_b=25\times10^4\times2.51\times10^3=627.5MHZ$

2.83

$G_{nom}=-\frac{R_2}{R_1}=-20 \qquad A_o=10^4\ V/V \qquad f_b=10^6HZ$

Eq. 2.35 : $\omega_{3db}=\frac{\omega_b}{1+R_2/R_1}=\frac{2\pi\times10^6}{1+20}=2\pi\times47.6\ KHZ$

$f_{3db}=47.6KHZ$

Eq. 2.34 : $\frac{V_o}{V_i}\simeq\frac{-R_2/R_1}{1+\frac{s}{\omega_t/(1+\frac{R_2}{R_1})}}=\frac{-20}{1+\frac{2\pi s}{2\pi\times10^6}}$

$f=0.1f_{3db} \Rightarrow \left|\frac{V_o}{V_i}\right|=\frac{-20}{\sqrt{1+(0.1)^2}}=-19.9\ V/V$

$f=10f_{3db} \Rightarrow \left|\frac{V_o}{V_i}\right|=\frac{-20}{\sqrt{1+100}}=-1.99\ V/V$

2.84

$1+\frac{R_2}{R_1}=100\ V/V \quad , f_t=20MHZ$

$f_{3db}=\frac{f_t}{1+\frac{R_2}{R_1}}=200\ KHZ$

$G(j\omega)=\frac{100}{1+j\ f/f_{3db}} \Rightarrow \varphi=-\tan^{-1}\frac{f}{f_{3db}}=$

$\varphi=-6^\circ \Rightarrow f=f_{3db}\times\tan6^\circ=21KHZ$

$\varphi=-84^\circ \Rightarrow f=f_{3db}\times\tan84^\circ=1.9MHZ$

2.85

a) $-\frac{R_2}{R_1}=-100\ V/V \quad , f_{3db}=100KHZ$

Eq. 2.35 : $\omega_t=\omega_{3db}\left(1+\frac{R_2}{R_1}\right) \Rightarrow f_t=100^K\times101=10.1^{MHZ}$

b) $1+\frac{R_2}{R_1}=100\ V/V \qquad f_{3db}=100KHZ$

$f_t=f_{3db}\left(1+\frac{R_2}{R_1}\right)=10\ MHZ$

c) $1+\frac{R_2}{R_1}=2\ V/V \qquad f_{3db}=10MHZ$

$f_t=10MHZ\times2=20\ MHZ$

d) $-\frac{R_2}{R_1}=-2\ V/V \qquad f_{3db}=10MHZ$

$f_t=10\ MHZ(1+2)=30\ MHZ$

e) $-\frac{R_2}{R_1}=-1000V/V \qquad f_{3db}=20KHZ$

$f_t=20K(1+1000)=20.02MHZ$

f) $1+\frac{R_2}{R_1}=1\ V/V \qquad f_{3db}=1MHZ$

$f_t=1M\times1=1MHZ$

g) $-\frac{R_2}{R_1}=-1 \qquad f_{3db}=1MHZ$

$f_t=1M(1+1)=2MHZ$

2.86

$1+\frac{R_2}{R_1}=100\ V/V \qquad f_{3db}=8KHZ$

$f_t=8\times100=800KHZ$

for $f_{3db}=20KHZ$: $G_o=\frac{800}{20}=40\ V/V$

2.87

$f_{3db}=f_t=1MHZ$

$|G|=\frac{1}{\sqrt{1+\left(\frac{f}{f_{3db}}\right)^2}}=\frac{1}{\sqrt{1+f^2}} \qquad f\ in\ MHZ$

$|G|=0.99 \Rightarrow f=0.142\ MHZ$

The follower behaves like a low-pass STC circuit with a time constant $\tau=\frac{1}{\omega_{3db}}$

Thus: $\tau=\frac{1}{2\pi\times10^6}=\frac{1}{2\pi}\ \mu s$

$t_r=2.2\tau=0.35\mu s$ (Refer to Appendix F)

2.88

$1+\frac{R_2}{R_1}=10\,V/V \qquad R_1=1K\Omega \quad R_2=9K\Omega$

If we consider 5τ the time that it takes for the output voltage to reach 99% of its final value, then: $5\tau=100ns \Rightarrow \tau=20\,ns$

$\tau=\frac{1}{\omega_{3db}} \Rightarrow \omega_{3db}=50\times10^6 \Rightarrow f_{3db}=7.96MHz$

$f_t=(1+\frac{R_2}{R_1})f_{3db}=10\times7.96=79.6\,MHz$

2.89

a) Assume two identical stages, each with a gain function: $G=\frac{G_0}{1+j\frac{\omega}{\omega_1}}=\frac{G_0}{1+jf/f_1}$

$G=\frac{G_0}{\sqrt{1+(\frac{f}{f_1})^2}}$

Overall gain of the cascade is $\frac{G_0^2}{1+(\frac{f}{f_1})^2}$

The gain will drop by 3db when: $1+(\frac{f_{3db}}{f_1})^2=\sqrt{2}$, Note $3db=20\log\sqrt{2}$

$f_{3db}=f_1\sqrt{\sqrt{2}-1}$

b) $40db=20\log G_0 \Rightarrow G_0=100=1+\frac{R_2}{R_1}$

$f_{3db}=\frac{f_t}{1+\frac{R_2}{R_1}}=\frac{1\,MHz}{100}=10KHz$

c) Each stage should have 20db gain or $1+\frac{R_2}{R_1}=10$ and therefore a 3db frequency of: $f_1=\frac{10^6}{10}=10^5\,Hz$.

The overall $f_{3db}=10^5\sqrt{\sqrt{2}-1}=64.35\,KHz$

which is 6 times greater than the bandwidth achieved using single op-amp. (case b above)

2.90

$f_t=100\times5=500MHz$. if single op-amp is used.

with op-amp that has only $f_t=40MHz$, the possible closed loop gain at 5MHz is: $\frac{40}{5}=8\,V/V$.

To obtain an overall gain of 100, three such amplifiers cascaded, would be required. Now, if each of the 3 stages, has a low-frequency (d) closed loop gain K, then its 3db frequency will be $\frac{40}{K}$ MHz. Thus for each stage the closed loop gain is: $|G|=\frac{K}{\sqrt{1+\left(\frac{f}{\frac{40}{K}}\right)^2}}$

which at $f=5MHz$ becomes:

$|G_{5MHz}|=\frac{K}{\sqrt{1+(\frac{K}{8})^2}}$

The overall gain of 100: $100=\left[\frac{K}{\sqrt{1+(\frac{K}{8})^2}}\right]^3$

$K=5.7$

Thus for each cascade stage: $f_{3db}=\frac{40}{5.7}$

$f_{3db}=7MHz$

The 3-db frequency of the overall amplifiers f_1, can be calculated as:

$\left[\frac{5.7}{\sqrt{1+(\frac{f}{7})^2}}\right]^3=\frac{(5.7)^3}{\sqrt{2}} \Rightarrow f_1=3.6MHz$

2.91

a) $\frac{R_2}{R_1}=K \qquad f_{3db}=\frac{f_t}{1+\frac{R_2}{R_1}}=\frac{f_t}{1+K}$

$GBP=Gain\times f_{3db}$

$GBP=K\frac{f_t}{1+K}$

b) $1+\frac{R_2}{R_1}=K \qquad f_{3db}=\frac{f_t}{K}$

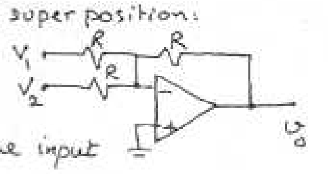
$GBP=K\frac{f_t}{K}=f_t$

The non-inverting amplifier realizes a higher GBP and it's independent of K.

2.92

To find f_{3db} we use superposition:

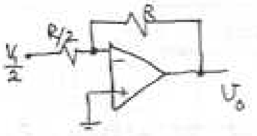
set $V2=0$



Now using Thevenin's Theorem to simplify the input circuit results in:

Cont.

$$\frac{V_o}{V_{1/2}} = \frac{-R/R/2}{1+s\,\frac{1+R/R/2}{\omega_t}}$$



Thevenin's equivalent

which gives:

$$\frac{V_o}{V_1} = \frac{-1}{1+s/(\omega_t/3)}$$

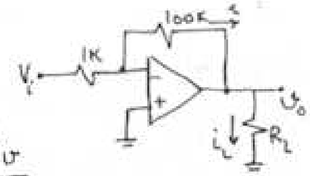
$f_{3dB} = \frac{f_t}{3}$. Similar results can be obtained for $\frac{V_o}{V_2}$.

2.93

The peak value of the largest possible sine wave that can be applied at the input without output clipping is: $\frac{\pm 12V}{100} = 0.12V = 120mV$

rms value $= \frac{120}{\sqrt{2}} = 85mV$

2.94



a) $R_L = 1K\Omega$

for $V_{omax} = 10^V$: $V_p = \frac{10}{100}$

$V_p = 0.1V$

when output is at its peak, $i_L = \frac{10}{1K} = 10mA$

$i = \frac{10}{100K} = 0.1mA$. therefore $i_o = 10+0.1 = 10.1\,mA$

is well under $i_{omax} = 20mA$.

b) $R_L = 100\Omega$

If output is at its peak: $i_L = \frac{10^V}{0.1} = 100mA$

which exceeds $i_{omax} = 20mA$. Therefore V_o cannot go as high as 10^V. instead:

$20mA = \frac{V_o}{100\Omega} + \frac{V_o}{100^K} \Rightarrow V_o = \frac{20}{10.01} = 2^V$

$V_p = \frac{2}{100} = 0.02V = \underline{20mV}$

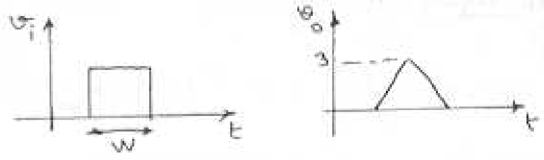
c) $R_L = ?$ $\quad i_{omax} = 20mA = \frac{10^V}{R_{Lmin}} + \frac{10^V}{100^K}$

$20-0.1 = \frac{10}{R_{Lmin}} \Rightarrow R_{Lmin} = 502\Omega$

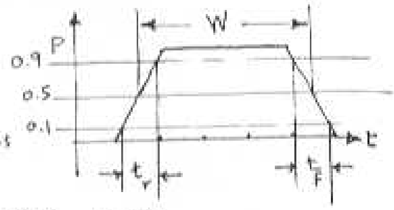
2.95

The output is triangular with the slew rate of $20^V/\mu s$. In order to reach $3V$, it takes $\frac{3}{20}\,\mu s = 0.15\mu s = 150ns$.

Therefore the minimum pulse width is 150^{ns}.



2.96



$W = 2\mu s$

$t_r + t_f = 0.2W = 0.4\mu s$

$t_r = t_f = 0.2\mu s$

$SR = \frac{(0.9-0.1)P}{t_r} = \frac{0.8\times 10}{0.2} = 40\,V/\mu s$

2.97

Slope of the triangle wave $= \frac{20V}{T/2} = SR$

Thus $\frac{20}{T}\times 2 = 10V/\mu s$

$\Rightarrow T = 4\mu s$ or $f = \frac{1}{T} = 250KHz$

For a sine wave $v_o = \hat{v}_o \sin(2\pi\times 250\times 10^3 t)$

$\left.\frac{dv_o}{dt}\right|_{max} = 2\pi\times 250\times 10^3\,\hat{v}_o = SR$

$\Rightarrow \hat{v}_o = \frac{10\times 10^6}{2\pi\times 10^3\times 250} = 6.37V$

2.98

$v_o = 10\sin\omega t \Rightarrow \frac{dv_o}{dt} = 10\omega\cos\omega t \Rightarrow \left.\frac{dv_o}{dt}\right|_{max} = 10\omega$

The highest frequency at which this output is possible is that for which:

$\left.\frac{dv_o}{dt}\right|_{max} = SR \Rightarrow 10\omega_{max} = 60\times 10^{+6} \Rightarrow \omega_{max} = 6\pi 10^5$

$\Rightarrow f_{max} = 45.5\,KHz$.

2.99

a) $v_i = 0.5$, $V_o = 10\times 0.5 = 5V$

Cont.

Output distortion will be due to slew Rate limitation and will occur at the frequency for which $\left.\frac{dv_o}{dt}\right|_{max} = SR$

$$\omega_{max} \times 5 = \frac{1}{10^{-6}} = 2\times10^{5}\ rad/s \Rightarrow f_{max} = 31.8KHz$$

b) The output will distort at the value of V_i that results in $\left.\frac{dv_o}{dt}\right|_{max} = SR$.

$v_o = 10\, v_i \sin 2\pi \times 20\times10^3$

$$\left.\frac{dv_o}{dt}\right|_{max} = 10\, v_i \times 2\pi \times 20\times10^3$$

$$\text{Thus } v_i = \frac{1/10^{-6}}{10\times 2\pi \times 20\times10^3} = 0.795\ V$$

c) $v_i = 50mV$ $V_o = 500mV = 0.5V$

Slew rate begins at the frequency for which $\omega \times 0.5 = SR$

which gives $\omega = \frac{1/10^{-6}}{0.5} = 2\times10^6\ rad/s$ or $f = 318.3\ KHz$

However the small signal 3db frequency is

$$f_{3db} = \frac{f_t}{1+\frac{R_2}{R_1}} = \frac{2\times10^6}{10} = 200KHz$$

Thus the useful frequency range is limited at 200KHz.

d) for $f = 5KHz$, the slew Rate limitation occurs at the value of v_i given by

$$\omega \times 10\, V_i = SR \Rightarrow V_i = \frac{1/10^{-6}}{2\pi \times 5\times10^3 \times 10} = 3.18\ V$$

Such an input voltage, however would ideally result in an output of 31.8V which exceeds V_{omax}. Thus $V_{imax} = \frac{V_{omax}}{10} = 1V$ peak.

2.100

$$v_o = v_{os}\left(1+\frac{R_2}{R_1}\right) \Rightarrow -0.3 = V_{os}\left(1+\frac{100}{1}\right) \approx 3mV$$

2.101

$v_{os} = \pm 2mV$

$$v_o = 0.01 \sin\omega t \times 200 + v_{os} \times 200 = 2\sin\omega t \pm 0.4\ V$$

2.102

Output DC offset, $v_{os} = 3mV \times 1000 = 3V$

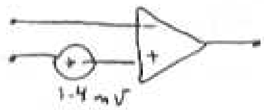
Therefore the maximum amplitude of an input sinusoid is the one that results in an output peak amplitude of $13-3 = 10V \Rightarrow v_i = \frac{10}{1000} = 10\ mV$

If the amplifier is capacity coupled, then:

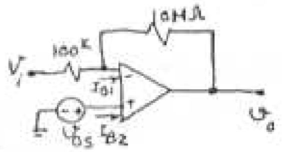
$$v_{imax} = \frac{13}{1000} = 13mV$$

2.103

$$v_{os} = \frac{1.4}{100} = 1.4\ mV$$



2.104



a) $I_B = (I_{B1} + I_{B2})/2$

open input:

$$v_o = V_+ + R_2 I_{B_1} = v_{os} + R_2 I_{B1}$$

$$9.31 = v_{os} + 10000\, I_{B1} \quad (1)$$

input connected to ground:

$$v_o = v_+ + R_2\left(I_{B1} + \frac{v_{os}}{R_1}\right) = v_{os}\left(1+\frac{R_2}{R_1}\right) + R_2 I_{B1}$$

$$9.09 = v_{os} \times 101 + 10000\, I_{B1} \quad (2)$$

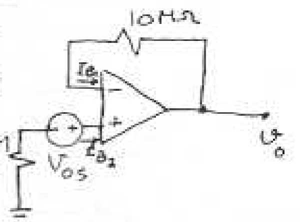
(1), (2) $\Rightarrow 100 v_{os} = -0.22 \Rightarrow v_{os} = -2.2mV$

$\Rightarrow I_{B_1} = 930nA$

$I_B \simeq I_{B_1} = 930nA$

b) $v_{os} = -2.2\ mV$

c) In this case, Since R is too large, we may ignore v_{os} compare to the voltage drop across R.

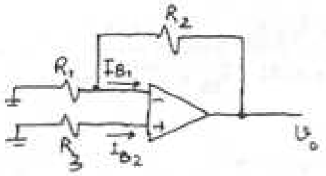


$v_{os} \ll R I_B$, Also Eq. 2.46 holds; $R_3 = R_1 \| R_2$

therefore From Eq. 2.47: $v_o = I_{os} \times R_2 \Rightarrow I_{os} = \frac{-0.8}{10^M}$

$I_{os} = -80\ nA$

2.105



$R_2 = 100k\Omega$

$R_1 = \frac{100k\Omega}{9}$

$R_3 = 5k\Omega$

$I_{B1} = 1 \pm 0.05\ \mu A$, $V_{os} = 0$

$I_{B2} = 1 \mp 0.05\ \mu A$

From Eq. 2.45 : $V_o = -I_{B2}R_3 + R_2(I_{B1} - I_{B2}\frac{R_3}{R_1})$

a) For $I_{B_1} = 1.05\mu A$, $I_{B_2} = 0.95\mu A$

$V_o = -0.95 \times 5 + 100(1.05 - 0.95 \times \frac{5}{100} \times 9) = 57.5mV$

b) For $I_{B_1} = 0.95\mu A$, $I_{B_2} = 1.05\mu A$

$V_o = -1.05 \times 5 + 100(0.95 - 1.05 \times \frac{5}{100} \times 9) = 42.5mV$

$\Rightarrow 42.5^{mV} \leq V_o \leq 57.5^{mV}$

From the discussion in the text we know that to minimize the dc output voltage resulting from the input bias current, we should make the total DC resistance in the inputs of the op-amp equal. Currently, the negative input sees a resistance of $R_1 \| R_2 = \frac{100}{9} \| 100 = 10k\Omega$ while the positive input terminal sees $5k\Omega$ source resistance. Therefore we should add $5k\Omega$ series resistor to the positive input terminal to make the effective Resistance $5k\Omega + 5k\Omega = 10^{k\Omega}$

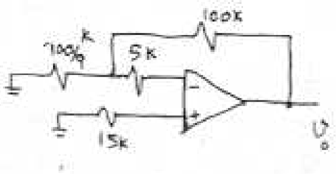
The resulting V_o can be found as follows:

$V_o = -I_{B2} \times 10 + 100(I_{B_1} - I_{B2}\frac{10}{100/9}) = (I_{B_1} - I_{B_2}) \times 100$

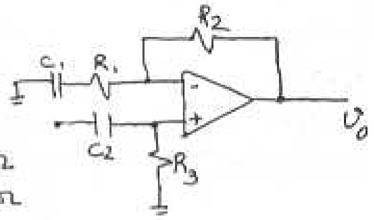
$V_o = I_{os} \times 100 = \pm 0.1 \times 100 = \pm 10mV$

$V_o = \pm 10mV$

If the signal source resistance is $15k\Omega$, then the resistances can be equalized by adding a $5k\Omega$ resistor in series with the negative input load of the op-amp.



2.106



$R_2 = R_3 = 100k\Omega$

$1 + \frac{R_2}{R_1} = 200$

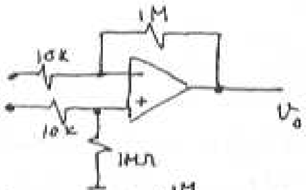
$R_1 = \frac{100k}{199} = 502\Omega \simeq 500\Omega$

$\frac{1}{R_1C_1} = 2\pi \times 100 \Rightarrow C_1 = \frac{1}{500 \times 2\pi \times 100} = 3.18\ \mu F$

$\frac{1}{R_3C_2} = 2\pi \times 10 \Rightarrow C_2 = \frac{1}{100^k \times 2\pi \times 10} = 0.16\ \mu F$

2.107

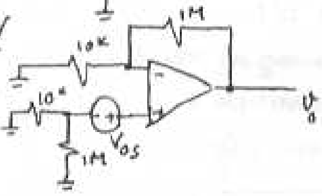
The output component due to V_{os} is :



$V_{o1} = V_{os}(1 + \frac{1M}{10^k})$

$V_{o1} = 4(1 + 100) = 404mV$

The output component due to I_B or input bias current is :



$I_{B_1} = I_B + \frac{I_{os}}{2}$, $I_{B2} = I_B - \frac{I_{os}}{2}$

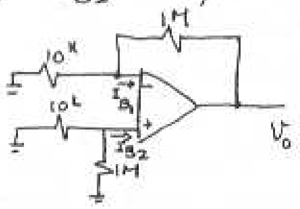
$I_{B_1} = 0.3 + \frac{0.05}{2} = 0.325\mu A$ $\quad I_{B2} = 0.275\ \mu A$

$V_+ = -I_{B_2} \times (10^k \| 1M)$

$V_+ = -2.72mV$

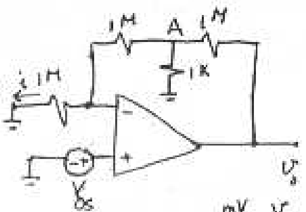
$V_{o2} = V_+ + (1^M \times (I_{B_1} + \frac{V_+}{10^k}))$

$V_{o2} = 50\ mV$



The worst case (largest) DC offset voltage at the output is $404 + 50 = 454mV$

2.108



$V_- = V_+ = V_{os} \Rightarrow V_A = 2V_{os} = 8mV$

$i = \frac{V_{os}}{1M} = V_{os}\ (\mu A)$

$V_o = V_A + 1M \times (i + \frac{V_A}{1K})$

$V_o = 2V_{os} + 1^M(\frac{V_{os}}{1M} + \frac{2V_{os}}{1K}) = 2003V_{os} = 2003 \times 4^{mV} = 8^V$

$V_o = 8V$

Cont.

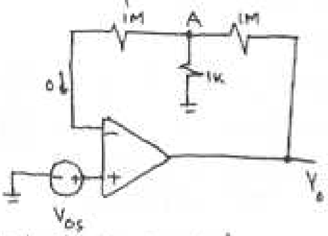
for capacitively coupled input:

$V_+ = V_- = V_{os}$

$V_A = V_{os}$

$V_0 = V_A + 1M \times \frac{V_{os}}{1^k}$

$V_0 = 1001\, V_{os} = \pm 4.004^V$



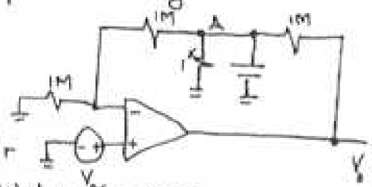
for capacitively coupled 1^K to ground:

$V_+ = V_- = V_{os}$

$V_A = 2V_{os}$

$V_0 = 3V_{os} = \pm 12mV$

This is much smaller than capacitively coupled input case.



2.109

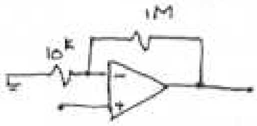
At $0^\circ C$, we expect $\pm 10 \times 25 \times 1000^\mu = \pm 250mV$

At $75^\circ C$, we expect $\pm 10 \times 50 \times 1000^\mu = \pm 500mV$

We expect these quantities to have opposite polarities.

2.110

$100 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = 10.1^{K\Omega}$



a) $V_0 = 100 \times 10^{-9} \times 1 \times 10^6 = 0.1^V$

b) Largest output offset is:

$V_0 = 1^{mV} \times 100 + 0.1^V = 200mV = 0.2^V$

c) For bias current compensation we connect a resistor R_3 in series with the positive input terminal of the op-amp, with: $R_3 = R_1 \| R_2$

$I_{os} = \frac{100}{10} = 10nA$ $\qquad R_3 = 10^K \| 1^M = 10K\Omega$

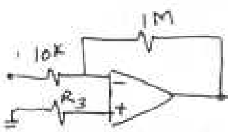
The offset current alone results in an output offset voltage of $I_{os} \times R_2 = 10 \times 10^{-9} \times 1 \times 10^6 = 10^{mV}$

d) $V_0 = 100mV + 10mV = 110mV$

2.111

$R_3 = R_1 \| R_2 = 9.9^{K\Omega}$

(Refer to 2.46)

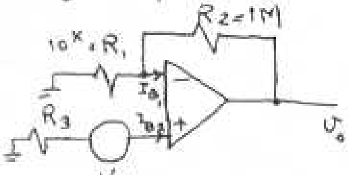


$V_0 = I_{os} R_2$ Eq. 2.47

$V_0 = 0.21 = I_{os} \times 1^M \Rightarrow I_{os} = 0.21 \mu A$

If $V_{os} = 1mV$

$V_+ = -I_{B2} R_3 \mp V_{os}$



$$I_{B_1} = \frac{R_3 I_{B_2} \pm V_{os}}{R_1} + \frac{0.21 + R_3 I_{B2} \pm V_{os}}{R_2}$$

$$I_{B_1} = R_3 I_{B_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \pm V_{os}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow I_{B_1} - I_{B_2} = \pm V_{os}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\Rightarrow I_{os} = \pm \frac{1^{mV}}{9.9^K} = \pm 0.1 \mu A$$

If we apply the same current as I_{os} to the other end of R_3, then it will cancel out the offset current effect on the output. $\pm 0.1\mu A$

Now if we use $\pm 15V$ supplies:

2.112

$$\frac{V_o}{V_i} = \frac{-1}{sCR} = \frac{-1}{j\omega CR} = \frac{1}{-j\omega \times 10 \times 10^{-9} \times 100 \times 10^3}$$

$$\frac{V_o}{V_i} = -\frac{10^3}{j\omega}$$

a) $\frac{V_o}{V_i} = 1 \Rightarrow \omega = 1^{K\,rad/s} \Rightarrow f = 159Hz$

b) $\frac{1}{j}$ indicates 90° lag, but since it's -1, it results in output leading the input by 90°

c) $\frac{V_o}{V_i} = \frac{-10^3}{j\omega}$ if frequency is lowered by a factor of 10, then the output would increase by a factor of 10.

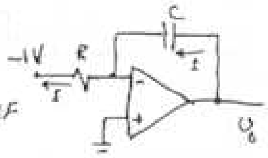
Cont.

d) The phase doesnot change and the output still leads the input by $90°$

2.113

$R_{in} = R = 100K\Omega$

$CR = 1s \Rightarrow C = \frac{1}{100\times10^3} = 10\mu F$

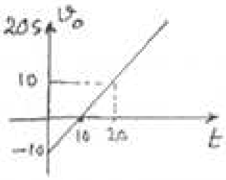


with a $-1V$ dc input applied, the capacitor charges with a constant current:

$I = \frac{1V}{R} = 0.01\,mA$ and its voltage rises linearly:

$v_o(t) = -10 + \frac{1}{C}\int_0^t I dt = -10 + \frac{I}{C}t = -10 + \frac{t}{RC}$

the voltage reaches $0V$ at $t = 10RC = 10s$ and it reaches $10V$ at $t = 20s$



2.114

$|T| = \frac{1}{\omega RC}$ If $|T| = 100V/_V$ for $f = 1^{KHZ}$, then for $|T| = 1V/_V$, f has to be $1^K \times 100 = 100KHZ$.

Also $RC = \frac{1}{\omega \cdot T} = \frac{1}{2\pi \times 1^K \times 100} = \underline{1.59\mu s}$

2.115

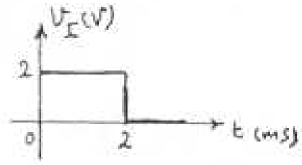
$R_{in} = R$, Thus $R = 100K\Omega$.

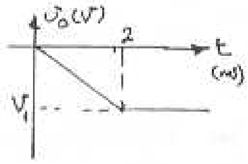
$|T| = \frac{1}{\omega RC} = 1$ at $\omega = \frac{1}{RC}$.

$\omega = 1000 = \frac{1}{RC} \Rightarrow C = \frac{1}{1000\times100^K} = 10^{nF}$

with a 2V_2ms pulse at the input, the output falls linearly until $t = 2ms$ at which $v_o = V_1$, $v_o = \frac{-I}{C}t = \frac{-2}{RC}t = -2t$ Volts where t in ms

Thus $\underline{V_1 = -4V}$





with $V_I = 2\sin 1000t$ applied at the input,

$v_o(t) = 2 \cdot \frac{1}{1000\times10^{-3}} \sin(1000t + 90°)$

$v_o(t) = 2\sin(1000t + 90°)$

2.116

$R_{in} = R = \underline{20K\Omega}$

$|T| = \frac{1}{\omega RC} = 1$ at $\omega = 2\pi\times10^{KHZ} \Rightarrow C = \frac{1}{2\pi\times10^K\times20^K}$

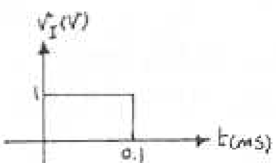
$\underline{C = 0.796\ nF}$

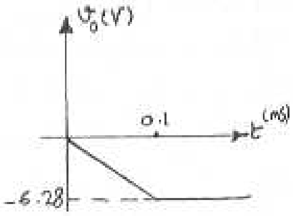
Refer to discussion in page 110:

$\frac{V_o}{V_i} = \frac{R_F/R}{1+sCR_F}$ and the finite dc gain is $\frac{-R_F}{R}$. There fore for 40db gain or equivalently $100V/_V$ we have: $\frac{-R_F}{R} = -100\,V/_V$

$\Rightarrow R_F = 100\times20K = 2M\Omega$

The corner frequency $\frac{1}{CR_F}$ is: $\frac{1}{0.796^n\times2^M} = 628^{HZ}$





a) when no R_F

$v_o(t) = \frac{-1}{RC}\int^t 1.dt = -62.8t \qquad 0 \leq t \leq 0.1ms$

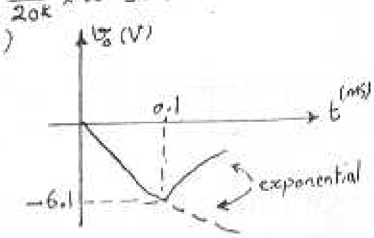
$v_o(0.1) = -6.28V$

b) with R_F: $v_o(t) = V_o(\infty)(1 - e^{-t/CR_F})$

(Refer to pg. 112)

$v_o(\infty) = -I\times R_F = -\frac{1V}{20K}\times2^M = -100V$

$v_o(t) = -100(1 - e^{-t/1.6})$

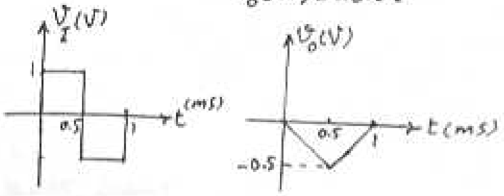


2.117

For $0 \le t \le 0.5\,ms$: $v_o(t) = v_o(0) - \frac{1}{RC}\int_0^t v_I\,dt$

$v_o(t) = 0 - \frac{t}{RC} = -\frac{t}{1\,ms}$

$v_o(0.5) = -0.5V$



For $0.5 \le t \le 1\,ms$: $v_o(t) = v_o(0.5) - \frac{1}{RC}\int_{0.5}^{t} -1\,dt$

$v_o(t) = -0.5 + \frac{1}{RC}(t - 0.5)$

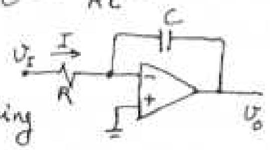
$v_o(1^{ms}) = -0.5 + \frac{0.5}{1} = 0V$

Another way of thinking about this circuit is as follows:

For $0 \le t \le 0.5ms$ a current $I = \frac{1^V}{R}$ flows through R and C in the direction indicated on the diagram. At time t we write:

$I.t = -C\,v_o(t) \Rightarrow v_o(t) = \frac{-I}{C}t = \frac{-1}{RC}t$

which indicates that the output voltage is linearly decreased, reaching -0.5V at t=0.5ms.



Then for $0.5 \le t \le 1ms$, the current flows in the opposite direction and v_o rises linearly reaching 0^V at t=1ms.

For $v_I = \pm 2V$: we obtain the following waveform: (assuming time constant is the same)

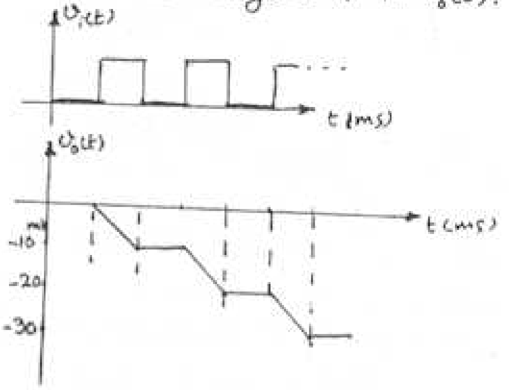
If RC is also doubled, then the waveform becomes the same as the first case where $v_I = \pm 1V$ and RC = 1ms.

2.118

Each pulse lowers the output voltage by:

$$\Delta v_o = \frac{1}{RC}\int_0^{10\,\mu s} 1.dt = \frac{10\,\mu s}{RC} = \frac{10\,\mu s}{1\,ms} = 10\,mV$$

Therefore a total of 100 pulses are required to cause a change of $1V$ in $v_o(t)$.



2.119

Refer to fig. P2.119.

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{1/R_2 + sC} = -\frac{R_2/R_1}{1 + sCR_2}$$

which is an STC LP circuit with a dc gain of $\frac{-R_2}{R_1}$ and a 3-db frequency $\omega_0 = \frac{1}{CR_2}$.

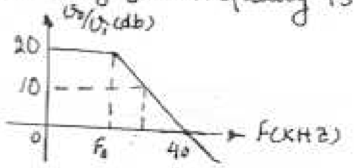
The input resistance equal to R_1. So for:

$R_i = 1K \Rightarrow R_1 = 1K\Omega$ and for dc gain of 20^{db} or 10 : $\frac{R_2}{R_1} = 10 \Rightarrow R_2 = 10\ K\Omega$

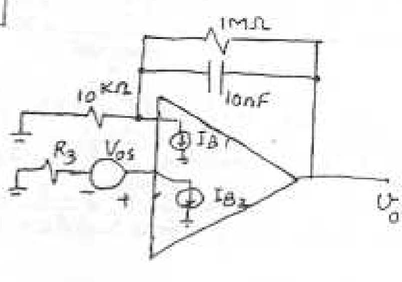
For 3db frequency of 4KHZ: $\omega_0 = 2\pi \times 4 \times 10^3 = \frac{1}{CR_2}$

$\Rightarrow C \cong 4nF$

the unity gain frequency is (0db) is 40 KHZ

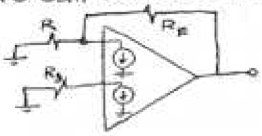


2.120



Cont.

a) To compensate for the effect of dc bias current I_B, we can consider the following model



Similar to the discussion leading to equation(2.46) we have: $R_3 = R \| R_F = 10k\Omega \| 1M\Omega \Rightarrow R_3 = 9.9k\Omega$

(b) As discussed in Section 2.8.2 the dc output voltage of the integrator when the input is grounded is: $V_o = V_{os}\left(1 + \frac{R_F}{R}\right) + I_{os} R_F$

$V_o = 3mV\left(1 + \frac{1M\Omega}{10k\Omega}\right) + 10nA \times 1M\Omega = 0.303V + 0.01V$

$V_o = 0.313V$

2.121

$\frac{V_o}{V_i}(s) = -SRC = -S \times 0.01 \times 10^{-6} \times 10 \times 10^{3} = -10^{-4} S$

$\frac{V_o}{V_i}(j\omega) = -j\omega \times 10^{-4} \Rightarrow \left|\frac{V_o}{V_i}\right| = -\omega \times 10^{-4} \Rightarrow$

$\left|\frac{V_o}{V_i}\right| = 1$ when $\omega = 10^4$ Rad/s or $f = 1.59KHz$

For an input 10 times this frequency, the output will be 10 times as large as the input: 10V peak-to-peak. The $(-j)$ indicates that the output lags the input by 90°. Thus $v_o(t) = -5\sin(10^5 t + 90^\circ)$ volts

2.122

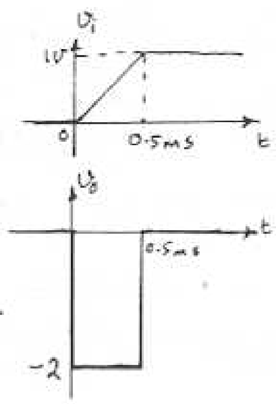
$v_o = -CR\frac{dv_i}{dt}$

therefore:

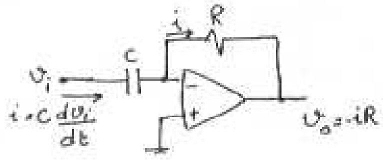
for $0 \le t \le 0.5$:

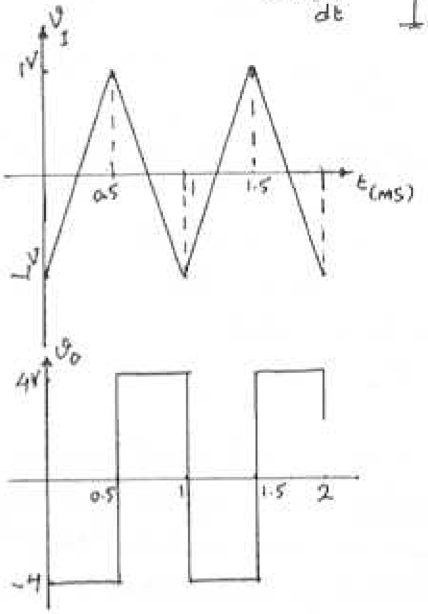
$v_o = -1ms \times \frac{1V}{0.5ms} = -2V$

and $v_o = 0$ otherwise



2.123





Slope $= \frac{2V}{0.5ms} = 4V/ms$

$C\frac{dv_i}{dt} = 0.1 \times 10^{-6} \times \frac{4}{10^{-3}} = 0.4mA$

Thus the peak value of the output square wave is $0.4mA \times 10^{k\Omega} = 4V$. The frequency of the output is the same as the input (1KHz).

The average value of the output is 0.

To increase the value of the output to 10V, R has to be increased to $\frac{10}{4} = 2.5$, i.e $25^{k\Omega}$.

When a 1-KHz, 1V peak input sine wave is applied

$v_i = \sin(2\pi \times 1000t)$

a sinusoidal signal appears at the output. It can be determined by one of the following methods:

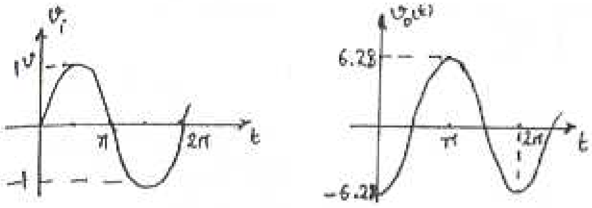
a) $v_o(t) = -RC\frac{dv_i}{dt} = -0.1 \times 10^{-6} \times 10 \times 10^{3}\frac{dv_i}{dt} = -10^{-3}\frac{dv_i}{dt}$

$v_o(t) = -10^{-3} \times 2\pi \times 1000 \times \cos(2\pi \times 1000t)$

$v_o(t) = -2\pi\cos(2\pi \times 1000t)$

Thus the peak amplitude is 6.28V and the negative peaks occur at $t = 0, \frac{2\pi}{2\pi \times 1000}, \ldots$

Cont.



b) $\frac{v_o}{v_i} = -SRC \Rightarrow \frac{v_o}{v_i}(j\omega) = -j\omega RC \Rightarrow v_o(t) = -j\omega RC\, v_i(t)$

the output is inverted and has $90°$ phase shift, due to $(-j)$ factor.

$v_o(t) = -(\omega RC) \times 1 \sin(2\pi \times 1000t + 90°)$

$v_o(t) = -6.28 \sin(2\pi \times 1000t + 90°)$

$v_o(t) = -6.28 \cos(2\pi \times 1000t)$

Same as before.

c) The peaks of the output waveform are equal to $RC \times$(maximum slope of input wave). Since the maximum slope occurs at the zero crossings, its value is $2\pi \times 1000$. Thus the peak output $= 2\pi \times 1000 \times RC = 6.28V$

The negative peak occurs at $\omega t = 0, 2\pi, \ldots$

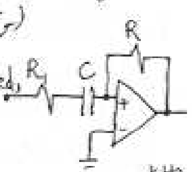
2.124

$RC = 10^{-3}s$ when $C = 10^{nF} \Rightarrow R = 100K\Omega$

$\frac{v_o}{v_i} = -SRC \quad \frac{v_o}{v_i}(j\omega) = -j\omega RC \quad \varphi = -90°$ always

$\left|\frac{v_o}{v_i}\right| = 1 \Rightarrow \omega_{unity} = \frac{1}{RC} = 1\,\text{krad/s}$. Gain is 10 times the unity gain, when the frequency is 10 times the unity gain frequency. Similarly for $\omega = \frac{1}{10}$ krad/s, gain is $0.1\,V/V$. (for $\omega = 10$krad/s, gain $= 10\,V/V$)

for high frequencies C is short-circuited,



$\frac{v_o}{v_i} = \frac{-R}{R_1} = -100 \Rightarrow R_1 = 1K\Omega$

$\frac{v_o}{v_i} = \frac{-RCs}{R_1CS+1} = \frac{-10^{-3}S}{10^{-5}S+1} \Rightarrow \omega_{3db} = 100$Krad/s or $f_{3db} = 15.9$ KHz

for unity gain: $|10^{-3}S| = |10^{-5}S + 1| \Rightarrow \omega_H = 1.01$ Krad/s

if $\omega = 10.1$ Krad/s : $\left|\frac{v_o}{v_i}\right| = \frac{10.1}{1.01} = 10, \quad \varphi = -95.77°$

2.125

Refer to Fig. P2.125:

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = \frac{-R_2}{R_1 + \frac{1}{sC}} = \frac{-(\frac{R_2}{R_1})s}{s + \frac{1}{R_1C}}$$

which is the transfer function of an STC HP filter with a high frequency gain $K = -\frac{R_2}{R_1}$ and a 3-db frequency $\omega_0 = \frac{1}{R_1C}$

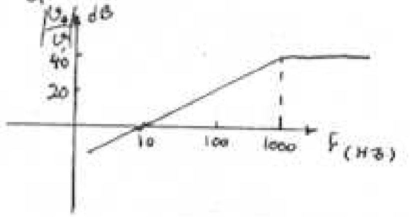
The high-frequency input impedance approaches R_1. (as $\frac{1}{j\omega C}$ becomes negligibly small) So we can select $R_1 = 10K\Omega$

To obtain a high-frequency gain of 40db (i.e. 100): $\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 1M\Omega$.

for a 3-db frequency of 1000 Hz:

$\frac{1}{R_1C} = 2\pi \times 1000 \Rightarrow C = 15.9nF$

from the Bode-diagram below, we see that $\left|\frac{v_o}{v_i}\right|$ reduces to unity at $f = 0.01 f_0 = 10$ Hz



2.126

Refer to the circuit in Fig. P2.126:

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2} = -\frac{1}{(R_1 + \frac{1}{sC_1})(\frac{1}{R_2} + sC_2)}$$

$$\frac{v_o}{v_i} = -\frac{R_2/R_1}{(1 + \frac{1}{R_1C_1s})(1 + sR_2C_2)}$$

$$\frac{v_o}{v_i}(j\omega) = \frac{-R_2/R_1}{(1 + \frac{1}{j\omega R_1C_1})(1 + j\omega R_2C_2)} = \frac{-R_2/R_1}{(1 + \frac{\omega_1}{j\omega})(1 + j\frac{\omega}{\omega_2})}$$

where $\omega_1 = \frac{1}{R_1C_1}$, $\omega_2 = \frac{1}{R_2C_2}$

a) for $\omega \ll \omega_1 \ll \omega_2$

$$\frac{v_o}{v_i}(j\omega) \propto \frac{-R_2/R_1}{(1 + \frac{\omega_1}{j\omega})} \simeq \frac{-R_2/R_1}{\omega_1/j\omega} = -j\frac{R_2}{R_1}\frac{\omega}{\omega_1}$$

Cont.

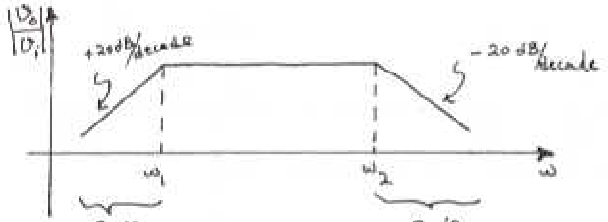
b) for $\omega_1 << \omega << \omega_2$

$$\frac{V_o}{V_i}(j\omega) \simeq \frac{-R_2}{R_1}$$

c) for $\omega >> \omega_2$ and $\omega_2 >> \omega_1$:

$$\frac{V_o}{V_i}(j\omega) \simeq \frac{-R_2/R_1}{1+j\omega/\omega_2} \simeq \frac{-R_2/R_1}{j\,\omega/\omega_2} = j\left(\frac{R_2}{R_1}\right)\left(\frac{\omega_2}{\omega}\right)$$

from the results of a), b) and c) we can draw the Bode-plot:



$\underbrace{\qquad}$ $\dfrac{R_2/R_1}{1+\frac{\omega_1}{j\omega}}$ HP STC

$\underbrace{\qquad}$ $\dfrac{R_2/R_1}{1+j\omega/\omega_2}$ LP STC

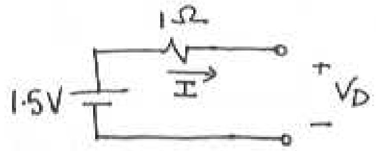
Design: $\frac{R_2}{R_1} = 1000$ (60dB gain in the mid-frequency range)

Rin for $\omega >> \omega_1$ $= \underline{R_1 = 1K\Omega} \Rightarrow \underline{R_2 = 1M\Omega}$

$f_1 = 100HZ \Rightarrow \omega_1 = 2\pi \times 100 = \frac{1}{R_1 C_1} \Rightarrow C_1 = 1.59\mu F$

$f_2 = 10KHZ \Rightarrow \omega_2 = 2\pi \times 10 \times 10^3 = \frac{1}{R_2 C_2} \Rightarrow C_2 = 15.9\,pF$

3.1



The diode can be reverse-biased and thus no current would flow, or forward-biased where current would flow.

(a) Reverse biased $I = 0A$ $V_D = 1.5V$

(b) Forward biased $I = 1.5A$ $V_D = 0V$

3.2

(a) Diode is conducting and thus has a 0V drop across it. Consequently

$V = \underline{\underline{-3V}}$

$$I = \frac{3-(-3)}{10k\Omega} = \underline{\underline{0.6mA}}$$

(b) Diode is cut off.

$V = \underline{\underline{3V}}$ $\quad I = \underline{\underline{0A}}$

(c) Diode is conducting

$V = \underline{\underline{3V}}$

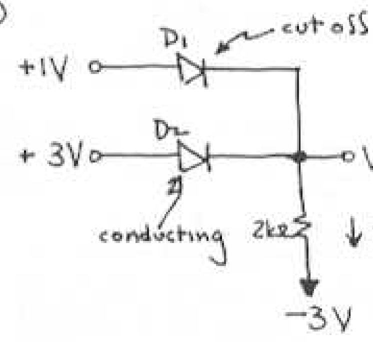
$$I = \frac{3-(-3)}{10k\Omega} = \underline{\underline{0.6mA}}$$

(d) Diode is cut off.

$V = \underline{\underline{-3V}}$ $\quad I = \underline{\underline{0A}}$

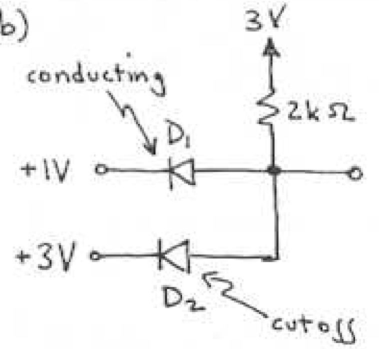
3.3

(a)



$$\downarrow I = \frac{3-(-3)}{2k\Omega} = \underline{\underline{3mA}}$$

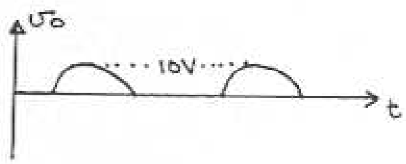
(b)



$$\downarrow I = \frac{3-(-1)}{2} = \underline{\underline{1mA}}$$

3.4

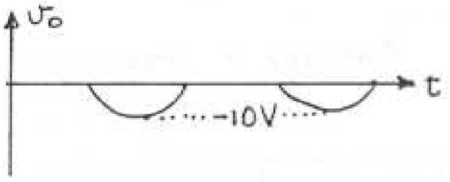
(a)



$V_{p+} = \underline{\underline{10V}}$ $\quad V_{p-} = \underline{\underline{0V}}$

$f = 1kHz$.

(b)



$V_{p+} = \underline{\underline{0V}}$ $V_{p-} = \underline{\underline{-10V}}$

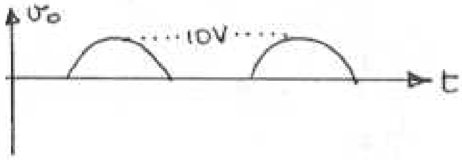
$f = 1kHz$

(c)

$v_o = \underline{\underline{0V}}$

Neither D_1 nor D_2 conducts so there is no output.

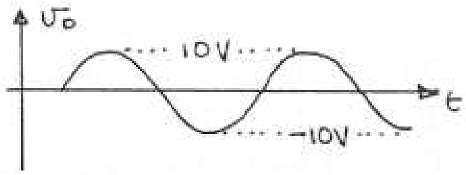
(d)



$V_{p+} = \underline{\underline{10V}}$ $V_{p-} = \underline{\underline{0V}}$ $f = 1kHz$

Both D_1 and D_2 conduct when $v_I > 0$

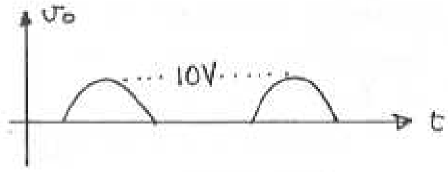
(e)



$V_{p+} = \underline{\underline{10V}}$ $V_{p-} = \underline{\underline{-10V}}$ $f = 1kHz$

D_1 conducts when $v_I > 0$ and D_2 conducts when $v_I < 0$. Thus the output follows the input.

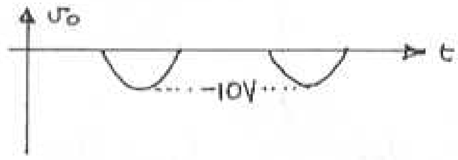
(f)



$V_{p+} = \underline{\underline{10V}}$ $V_{p-} = \underline{\underline{0V}}$ $f = 1kHz$

- D_1 is cutoff when $v_I < 0$

(g)



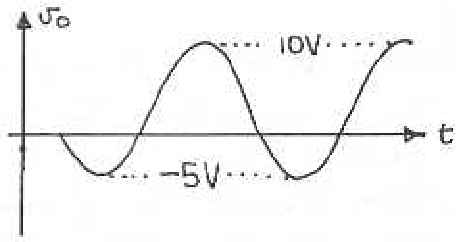
$V_{p+} = \underline{\underline{0V}}$ $V_{p-} = \underline{\underline{-10V}}$ $f = 1kHz$

D_1 shorts to ground when $v_I > 0$ and is cut off when $v_I < 0$ whereby the output follows v_I.

(h)

$v_o = \underline{\underline{0V}}$ ~ The output is always shorted to ground as D_1 conducts when $v_I > 0$ and D_2 conducts when $v_I < 0$.

(i)



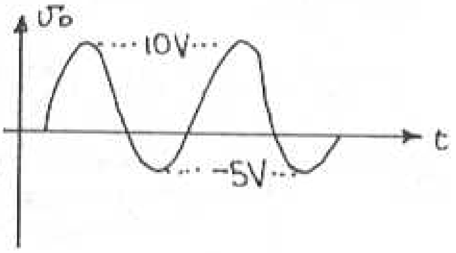
$V_{p+} = \underline{\underline{10V}}$ $V_{p-} = \underline{\underline{-5V}}$ $f = 1kHz$

- When $v_I > 0$, D_1 is cutoff and v_o follows v_I.

- When $v_I < 0$, D_1 is conducting and the circuit becomes a voltage divider where the negative peak is

$$\frac{1k\Omega}{1k\Omega + 1k\Omega} \cdot -10V = -5V$$

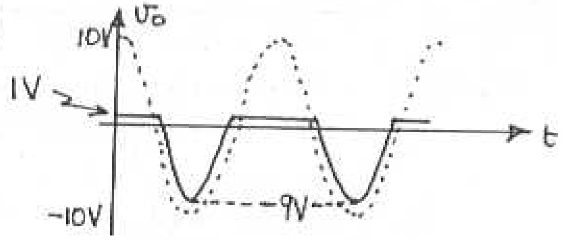
(j)



$V_{p+} = \underline{\underline{10V}}$ $V_{p-} = \underline{\underline{-5V}}$ $f = 1kHz$

- When $v_I > 0$, the output follows the input as D_1 is conducting.
- When $v_I < 0$, D_1 is cut off and the circuit becomes a voltage divider.

(k)



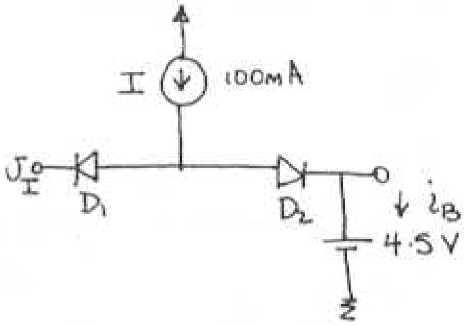
$V_{p+} = \underline{\underline{1V}}$ $V_{p-} = \underline{\underline{-9V}}$ $f = 1kHz$

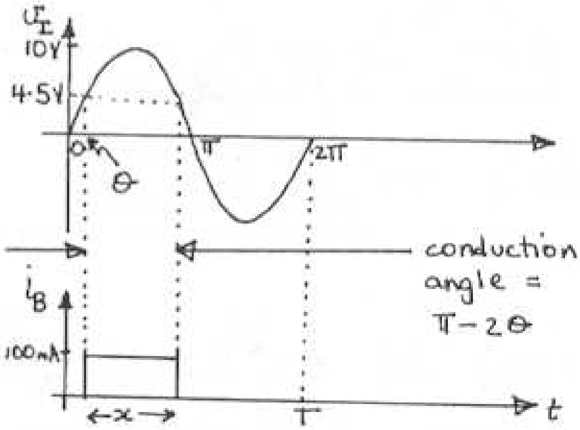
- When $v_I > 0$, D_1 is cutoff and D_2 is conducting. The output becomes 1V.
- When $v_I < 0$, D_1 is conducting and D_2 is cutoff. The output becomes :-

$$v_O = v_I + 1V.$$

CHAPTER 3 PROBLEMS

3.5





- When $v_I < 4.5V$ D_1 conducts and D_2 is cutoff so $i_B = 0A$. For $v_I > 4.5V$ D_2 conducts and D_1 is cutoff thus disconnecting the input v_I. All of the current then flows through the battery.

$$10 \sin\theta = 4.5V$$
$$\theta = \sin^{-1}(4.5/10)$$
$$\text{conduction angle} = \pi - 2\theta$$

fraction of cycle that $i_B = \underline{\underline{100mA}}$ is given by :-

$$x = \frac{\pi - 2\theta}{2\pi} = 0.35$$

$$i_{B\,avg} = \frac{1}{T}\int_T i_B \, dt$$

$$= \frac{1}{T}\left[100 \cdot 0.35T\right]$$

$$= \underline{\underline{35\,mA}}$$

If v_I is reduced by 10% the peak value of i_B remains the same

$$i_{B\,peak} = \underline{100\,mA}$$

but the fraction of the cycle for conduction changes

$$x = \frac{\pi - 2\theta}{2\pi} = \frac{\pi - 2\sin^{-1}(4.5/9)}{2\pi}$$

$$= \frac{1}{3}$$

Thus:

$$i_{B\,avg} = \frac{1}{T}\left[100 \cdot \frac{T}{3}\right]$$

$$= \underline{\underline{33.3\,mA}}$$

3.6

A	B	x	y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

$x = \underline{\underline{A\,B}}$ $\qquad$ $y = \underline{\underline{A + B}}$

CHAPTER 3 PROBLEMS

- x and y are the same for $A = B$
- x and y are opposite if $A \neq B$

3.7

$$\frac{5 - 0}{R} \leq 0.1\,mA$$

$$R \geq 5/0.1 = \underline{\underline{50\,k\Omega}}$$

3.8

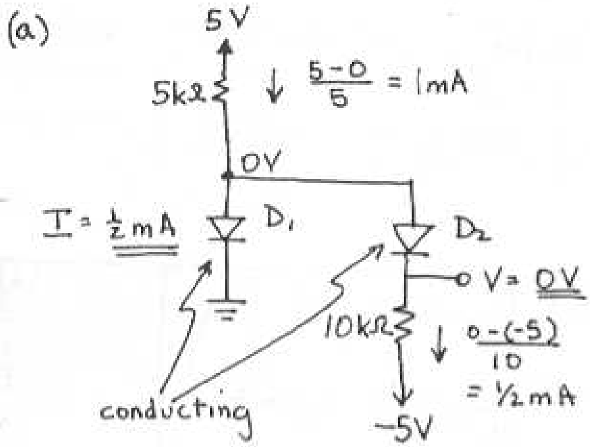
The maximum input current occurs when one input is low and the other two are high.

$$\frac{5 - 0}{R} \leq 0.1\,mA$$

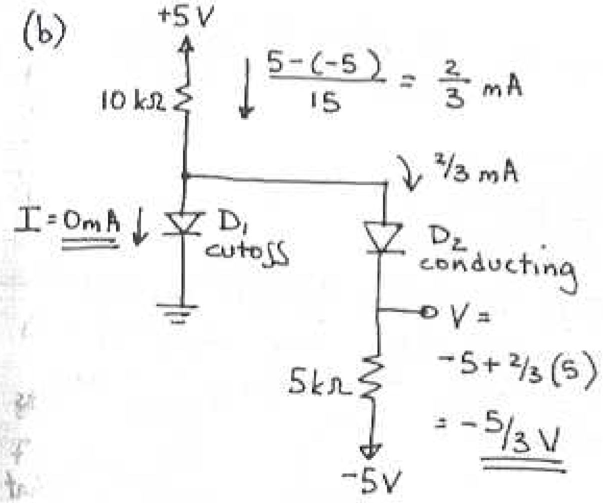
$$R \geq \underline{\underline{50\,k\Omega}}$$

3.9

(a)

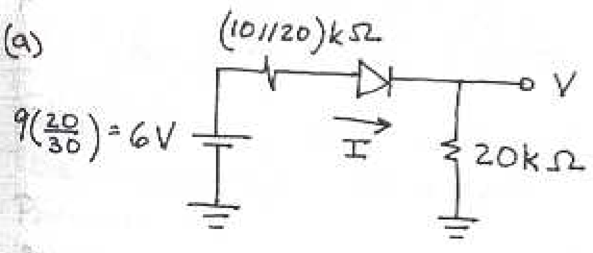


(b)



3.10

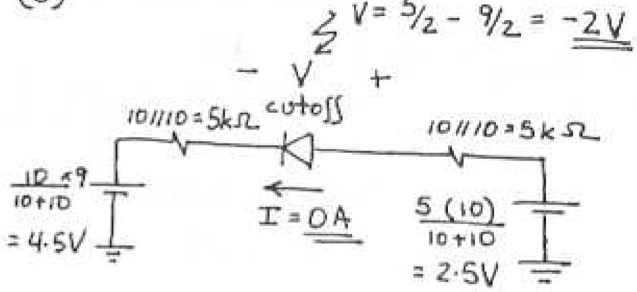
(a)



$$I = \frac{6}{(10 \| 20) + 20} = \underline{\underline{0.225\,\text{mA}}}$$

$$V = \frac{20}{(10 \| 20) + 20} \times 6 = \underline{\underline{4.5\,\text{V}}}$$

(b)



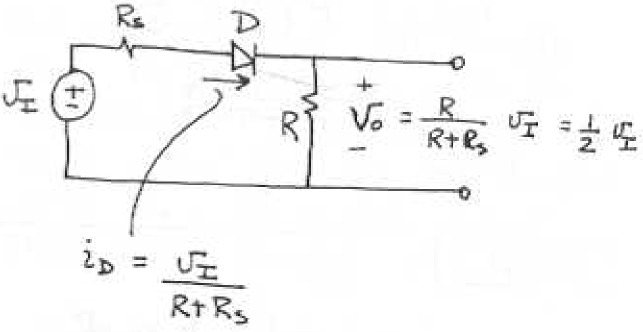
3.11

$$R \geq \frac{120\sqrt{2}}{50} \geq \underline{\underline{3.4\,\text{k}\Omega}}$$

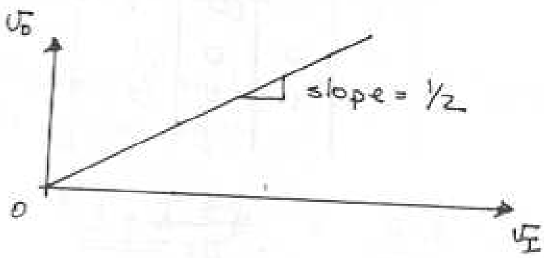
The largest reverse voltage appearing across the diode is equal to the peak input voltage

$$120\sqrt{2} = \underline{\underline{169.7\,\text{V}}}$$

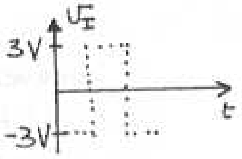
3.12

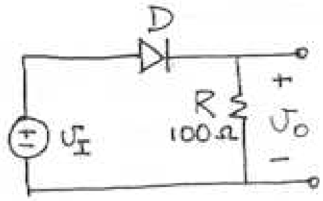


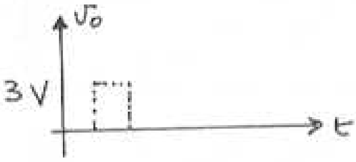
D starts to conduct when $v_I > 0$



3.13







$v_{O,\,peak} = 3V$

$$v_{O,avg} = \frac{1}{T}\int v_O\,dt$$

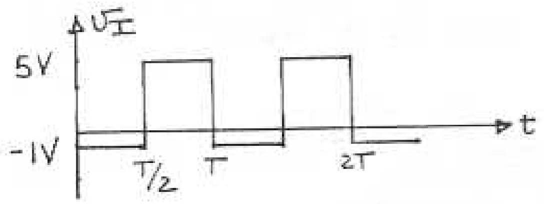
$$= \frac{1}{T}\left[3\,\frac{T}{2}\right] = \underline{\underline{3/2\ V}}$$

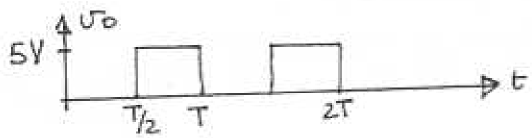
$i_{D,\,peak} = 3/100 = \underline{\underline{30\,mA}}$

$i_{D,\,avg} = \dfrac{3/2}{100} = \underline{\underline{15\ mA}}$

The maximum reverse diode voltage is $\underline{\underline{3\ V}}$

3.14





$v_{O,peak} = \underline{\underline{5V}}$

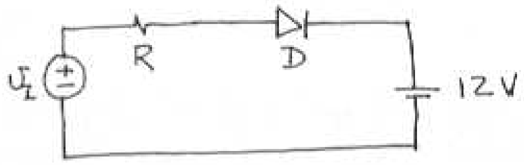
$v_{O,avg} = \underline{\underline{2.5V}}$

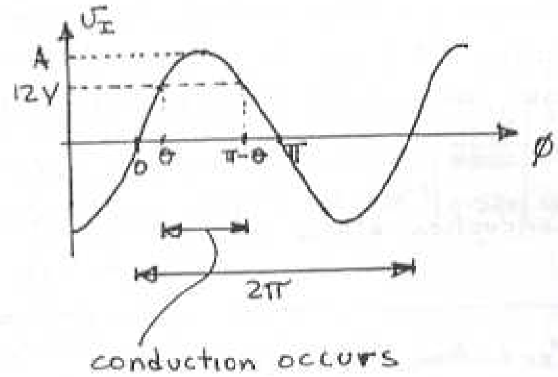
$i_{D,\,peak} = \dfrac{v_{O,\,peak}}{100} = \underline{\underline{50mA}}$

$i_{D,\,avg} = i_{D,\,peak}/2 = \underline{\underline{25mA}}$

maximum reverse voltage $= \underline{\underline{1V}}$

3.15





$v_I = A\sin\theta = 12$ ~ conduction across D occurs

For a conduction angle $(\pi - 2\theta)$ that is 20% of a cycle

$$\frac{\pi - 2\theta}{2\pi} = \frac{1}{5}$$

$$\theta = 0.3\pi$$

$$A = 12/\sin\theta = 14.83V$$

∴ Peak-to-peak sine wave voltage

$$= 2A = \underline{\underline{29.67V}}$$

Given the average diode current to be

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{A\sin\phi - 12}{R}\,d\phi = 100mA$$

$$\frac{1}{2\pi}\left[\frac{-14.83\cos\phi - 12\phi}{R}\right]_{\phi=0.3\pi}^{\phi=0.7\pi} = 0.1$$

$R = \underline{\underline{3.75\,\Omega}}$

Peak diode current $= \dfrac{A-12}{R} = \underline{\underline{0.75\text{A}}}$

Peak reverse voltage $= A+12 = \underline{\underline{26.83\text{V}}}$

For resistors specified to only one significant digit and peak-to-peak voltage to the nearest volt then choose $A = 15$ so the peak-to-peak sine wave voltage $= \underline{30\text{V}}$ and $R = \underline{\underline{3\,\Omega}}$

Conduction starts at $v_I = A\sin\theta = 12$

$15\sin\theta = 12$

$\theta = 0.93\text{ rad}$

Conduction stops at $\pi - \theta$

$\therefore$ fraction of cycle that current flows is $\dfrac{\pi - 2\theta}{2\pi} \times 100 = 20.5$

$\simeq \underline{\underline{20\%}}$

Average diode current =

$$\frac{1}{2\pi}\left[\frac{-15\cos\phi - 12\phi}{3}\right]_{\phi=0.93}^{2.21} = \underline{\underline{136\text{mA}}}$$

Peak diode current

$= \dfrac{15-12}{3} = \underline{\underline{1\text{A}}}$

Peak reverse voltage =

$A + 12 = \underline{\underline{27\text{V}}}$

3.16

V	RED	GREEN	
3V	ON	OFF	- D_1 conducts
0	OFF	OFF	- No current flows
-3V	OFF	ON	- D_2 conducts

3.17

$V_T = kT/q$ where $k = 1.38\times10^{-23}\,\text{J/K}$

$T = 273 + x\,°\text{C}$

$q = 1.60\times10^{-19}\,\text{C}$

x [°C]	V_T [mV]
-40	20
0	23.5
40	27
150	36.5

for $V_T = 25\text{mV}$

$T = \underline{16.8\,°\text{C}}$

3.18

$i = I_S e^{v/2\times0.025}$

$\therefore\ 1000\,I_S = I_S e^{v/0.05}$

$v = \underline{0.345\text{V}}$

at $v = 0.7\text{V}$

$i = I_S e^{0.7/0.05} = \underline{\underline{1.2\times10^6\,I_S}}$

3.19

$$i_1 = I_S e^{0.7/v_T} = 10^{-3}$$

$$i_2 = I_S e^{0.5/v_T}$$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5-0.7}{0.025}}$$

$$i_2 = \underline{\underline{0.335\ \mu A}}$$

3.20

$$i = I_S e^{v/nv_T} = I_S e^{0.7/0.025} = 5(10^{-3})$$

$$I_S = 5(10^{-3})e^{-0.7/0.025} = \underline{\underline{3.46\times10^{-15}\text{ A}}}$$

v	i
0.71 V	7.46 mA
0.8 V	273.21 mA
0.69 V	3.35 mA
0.6 V	91.65 μA

Let $i_1 = I_S e^{v_1/0.025}$

$$i_2 = 10 i_1 = I_S e^{v_2/0.025}$$

$$\frac{i_2}{i_1} = 10 = e^{\frac{v_2 - v_1}{0.025}}$$

$$\therefore\ \Delta v = v_2 - v_1 = \underline{\underline{57.56\text{ mV}}}$$

3.21

To calculate I_S use

$$I_S = I e^{-V/nV_T} = I e^{-V/n\times0.025}$$

To calculate the voltage at 1% of the measured current use

$i_2 = 0.01\, i_1$ so,

$$\frac{i_2}{i_1} = 0.01 = e^{\frac{V_2 - V_1}{nV_T}}$$

$$V_2 = V_1 + nV_T \ln 0.01$$

$$= V + n(0.025)\ln(0.01)$$

V [V]	I	I_S n=1 [A]	I_S n=2 [A]	V n=1 [V]	V n=2 [V
0.7	1 A	6.91×10^{-13}	8.32×10^{-7}	0.585	0.470
0.650	1 mA	5.11×10^{-15}	2.26×10^{-9}	0.535	0.420
0.650	10 μA	5.11×10^{-17}	2.26×10^{-11}	0.535	0.420
0.7	10 mA	6.91×10^{-15}	8.32×10^{-9}	0.584	0.470

3.22

Let $I_1 = I_S e^{V_1/nV_T}$ and

$$I_2 = I_S e^{V_2/nV_T} = I_1/10$$

Calculate n by :-

$$\frac{I_2}{I_1} = e^{\frac{V_2 - V_1}{nV_T}}$$

$$n = \frac{1}{V_T}\left[\frac{V_2 - V_1}{\ln I_2/I_1}\right] = \frac{1}{0.025}\left[\frac{V_2 - V_1}{\ln 0.1}\right.$$

Calculate I_S by :-

$$I_S = I_1 e^{-V_1/nV_T}$$

Calculate the diode voltage at $10I$ by :- $V_3 = nV_T \ln \frac{10 I_1}{I_S}$

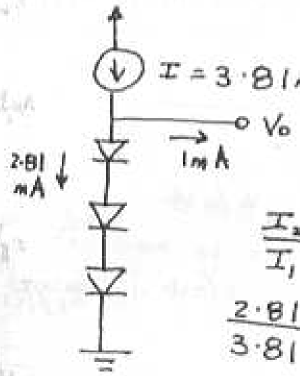
I	V_1 [V]	V_2 [V]	n	I_S [A]	V_3 [V]
10mA	0.7	0.6	1.737	10^{-9}	0.8
1mA	0.7	0.6	1.737	10^{-10}	0.8
10A	0.8	0.7	1.737	10^{-7}	0.9
1 mA	0.7	0.58	2.085	1.47×10^{-9}	0.82
10μA	0.7	0.64	1.042	2.15×10^{-17}	0.7

3.23

∴ The voltage across each diode is $V_0/3$

$$I = I_S e^{\frac{V_0/3}{nV_T}} = 10^{-14} e^{\frac{2/3}{0.025}}$$

$$= \underline{\underline{3.81\,mA}}$$

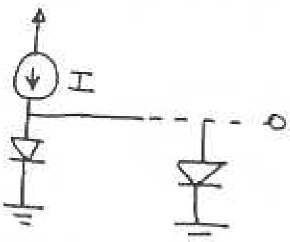


$$\frac{I_2}{I_1} = e^{\frac{(V_2 - V_1)/3}{0.025}}$$

$$\frac{2.81}{3.81} = e^{\frac{(V_2-2)/3}{0.025}}$$

$$\Delta V = V_2 - 2 = \underline{\underline{-22.8\,mV}}$$

3.24



With one diode the current through it is

$$I = I_S e^{V_1/nV_T}$$

With two diodes in parallel, the current splits between each diode so that the diodes each has half the current

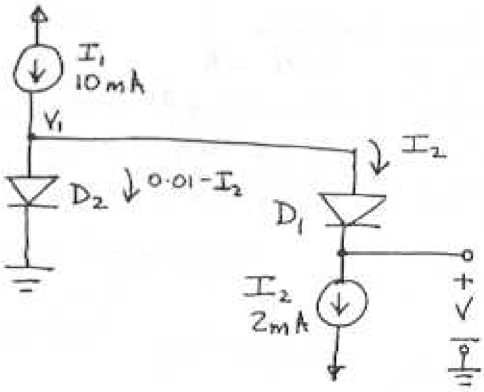
$$\frac{I}{2} = I_S e^{V_2/nV_T}$$

$$\therefore \frac{I/2}{I} = e^{\frac{V_2 - V_1}{nV_T}}$$

The change in voltage is

$$\Delta V = V_2 - V_1 = nV_T \ln\left(\tfrac{1}{2}\right) = \underline{-17.3\,mV}$$

3.25



The current through D_1 is

$$10 I_S e^{\frac{V_1 - V}{nV_T}} = I_2 \quad (A)$$

The current through D_2 is

$$I_S e^{\frac{V_1}{nV_T}} = 0.01 - I_2$$

$$I_S = (0.01 - I_2) e^{\frac{-V_1}{nV_T}} \quad (B)$$

(B) → (A)

$$10(0.01 - I_2) e^{\frac{-V}{nV_T}} = I_2$$

$$V = -V_T \ln\left(\frac{I_2}{10(0.01 - I_2)}\right)$$

$$= 0.025 \ln\left(\frac{2}{10(8)}\right) = \underline{\underline{92.2\,mV}}$$

For $V = 50\,mV$

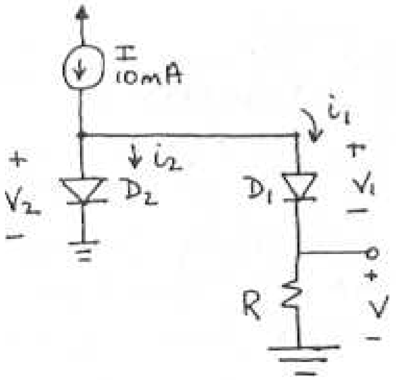
$$-V_T \ln\left(\frac{I_2}{10(10 - I_2)}\right) = 50 \times 10^{-3}$$

$$I_2 = 10(10 - I_2)e^{-2}$$

$$I_2(1 + 10e^{-2}) = 100e^{-2}$$

$$I_2 = \underline{\underline{5.75\,mA}}$$

3.26



Given for each diode

$$i = I_S e^{V/nV_T} \Rightarrow \begin{array}{ll} 10 \times 10^{-3} = I_S e^{0.7/n \times 0.025} & (1) \\ 100 \times 10^{-3} = I_S e^{0.8/n \times 0.025} & (2) \end{array}$$

(2)/(1) $\quad 10 = e^{0.1/n(0.025)}$

$$n = 1.737$$

$$V = V_2 - V_1 = nV_T \ln(i_2/i_1) = 80\,mV$$

$$1.737(25 \times 10^{-3}) \ln\left(\frac{0.01 - i_1}{i_1}\right) = 80$$

$$i_1 = 1.4\,mA$$

$$R = 80/i_1 = 80/1.4 = \underline{\underline{57.1\,\Omega}}$$

3.27

AT A CONSTANT TEMPERATURE, THE DIODE VOLTAGE DROP CHANGES WITH CURRENT ACCORDING TO

$$\Delta V = V_T \ln\left(\frac{I_2}{I_1}\right)$$

WHERE

$$V_T = \frac{kT}{q} = \frac{1.38 \times 10^{-23}(273 + \text{Temp. } ^\circ C)}{1.6 \times 10^{-19}}$$

THUS:

TEMP (°C)	0	50	75	100	−50
V_T (mV)	23.5	27.9	30	32.2	19.2

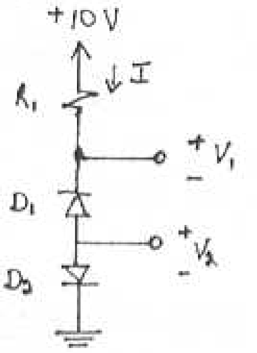
AT A CONSTANT CURRENT, THE DIODE VOLTAGE DROP CHANGES WITH TEMPERATURE ACCORDING TO

$$\Delta V = -2\,(mV) \times \text{Temperature change } (^\circ C)$$

THUS:

(a) 620 mV AT 10 μA AND 0°C
728 mV AT 1 mA AND 0°C
678 mV AT 1 mA AND 25°C

3.28



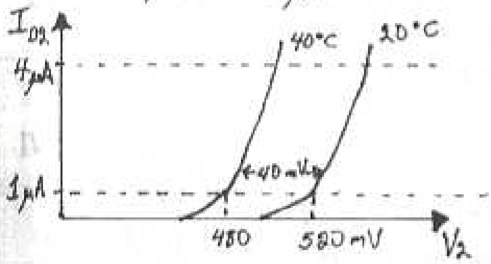
At 20°C:

$V_{R1} = V_2 = 520$ mV

$R_1 = 520$ kΩ

$I = \frac{520 \text{ mV}}{520 \text{ k}\Omega} = 1\,\mu A$

At 40°C, $I = 4\,\mu A$



$V_2 = 480 + 2.3 \times 1 \times 25 \log 4$
$= 514.6$ mV

$V_{R1} = 4\mu A \times 520 \text{ k}\Omega = \underline{\underline{2.08 V}}$

At 0°C, $I = \frac{1}{4}\,\mu A$

$V_2 = 560 - 2.3 \times 1 \times 25 \log 4$
$= \underline{\underline{525.4 \text{ mV}}}$

$V_{R1} = \frac{1}{4} \times 520 = \underline{\underline{0.13 V}}$

3.29

The voltage drop = 700 − 580 = 120 mV
Since the diode voltage decreases by approximately 2 mV for every 1°C increase in temperature, the junction temperature must have increased by

$\frac{120}{2} = \underline{60\,°C}$

Power being dissipated =
$580 \times 10^{-3} \times 15 = \underline{8.7 W}$

Thermal Resistance = temperature rise/watt
$= 60/8.7 = \underline{6.9\,°C/W}$

3.30

$i = I_S e^{v/nV_T}$

$10 = I_S e^{0.8/2(0.025)}$

$I_S = 1.12 \times 10^{-6}$ A

For current varying between $i_1 = 0.5$ mA to $i_2 = 1.5$ mA, the voltage varies from

$v_1 = 2(0.025) \ln\left(\frac{0.5 \times 10^{-3}}{1.12 \times 10^{-6}}\right) = \underline{\underline{0.305 V}}$

to:

$v_2 = 2(0.025) \ln\left(\frac{1.5 \times 10^{-3}}{1.12 \times 10^{-6}}\right) = \underline{\underline{0.360 V}}$

∴ the voltage decreases by approximately 2 mV for every 1°C increase in temperature, the voltage may vary by ∓50 mV for the ±25°C temperature variation.

3.31

$$i = I_S e^{v/nV_T}$$

$$\frac{I_{S2}}{I_{S1}} = \frac{1}{0.1\times10^{-3}} = 10^4$$

For identical currents

$$I_{S1} e^{v_1/nV_T} = I_{S2} e^{v_2/nV_T}$$

$$e^{\frac{v_1 - v_2}{nV_T}} = 10^4$$

$$v_1 - v_2 = nV_T \ln 10^4$$

$$= 25\times10^{-3} \ln 10^4$$

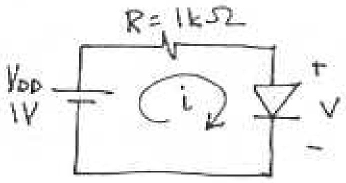
$$= +\underline{\underline{0.23\text{V}}}$$

I.E. THE VOLTAGE DIFFERENCE BETWEEN THE TWO DIODES IS +0.23 V INDEPENDENT OF THE CURRENT. HOWEVER, SINCE THE TWO CURRENTS CAN VARY BY A FACTOR OF 3 (0.5 mA TO 1.5 mA) THE DIFFERENCE VOLTAGE WILL BE:

$0.23\text{ V} \pm nV_T \ln 3 = 0.23\text{ V} \pm 2.75\text{ mV}$

SINCE TEMPERATURE CHANGE AFFECTS BOTH DIODES SIMILARLY THE DIFFERENCE VOLTAGE REMAINS CONSTANT.

3.32



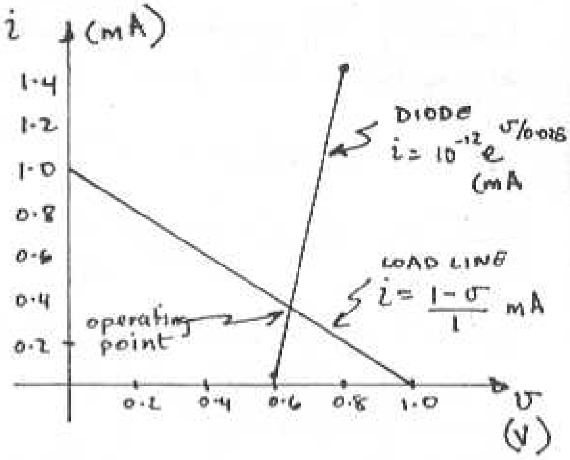
$$i = 10^{-15} e^{v/V_T}$$

where $n = 1$

$v = 0.7\text{V} \quad i = 1.45\text{ mA}$

$v = 0.6\text{V} \quad i = 0.026\text{mA}$

A sketch of the graphical construction to determine the operating point is shown below.



From the above sketch we see that the operating point must lie between $v = 0.6$ and 0.7 V and $i \approx 0.3$ to 0.4 mA. To find the point more accurately an enlarged graph is plotted.

For $i = 0.3\text{mA} = 10^{-12} e^{v/0.025}$

$\Rightarrow v = 660.7\text{ mV}$

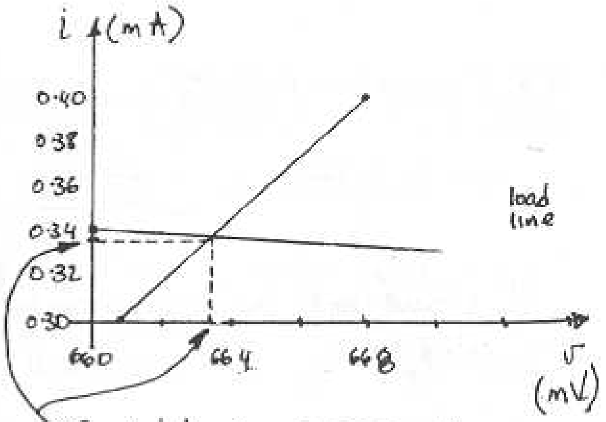
For $i = 0.4\text{mA} = 10^{-12} e^{v/0.025}$

$\Rightarrow v = 667.9\text{ mV}$

For the load line:

$v = 660\text{mV} \Rightarrow i = 0.34\text{mA}$

$v = 670\text{mV} \Rightarrow i = 0.33\text{mA}$



Graphical Point $\quad i = 0.337\text{mA}$

$v = 663.4\text{mV}$

CONT.

Comparing the graphical results to the exponential model gives:

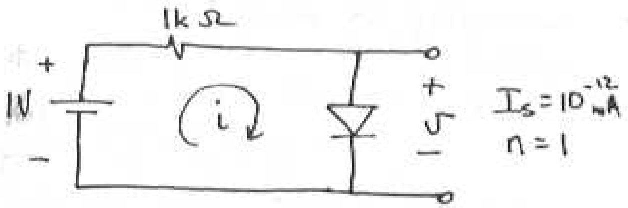
At $i = 0.337\,mA = 10^{-12} e^{v/0.025}$

$\Rightarrow v = 663.6\,mV$

which is only $(663.6 - 663.4) = \underline{\underline{0.2\,mV}}$ greater than the value found graphically!

3.33

Iterative Analysis:



#1 $v = 0.7V$ $\quad i = \frac{1-0.7}{1} = 0.3\,mA$

#2 $v = 0.25\ln\left(\frac{0.3}{10^{-12}}\right) = 0.6607V$

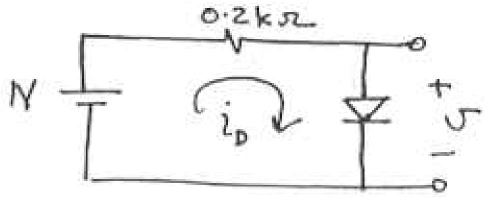
$i = \frac{1-0.6607}{1} = 0.3393\,mA$

#3 $v = 0.25\ln\left(\frac{0.3393}{10^{-12}}\right) = \underline{\underline{0.6638V}}$

$i = \frac{1-0.6638}{1} = \underline{\underline{0.3362}}$

$\therefore$ i did not change by much stop here.

3.34



(a) $i_D = \frac{1-0.7}{0.2} = \underline{\underline{1.5\,mA}}$

(b) Iterative Analysis given $v_D = 0.7V$ at $i_D = 1mA$

#1 $v = 0.7V$ $\quad i_D = \frac{1-0.7}{0.2} = 1.5\,mA$

#2 $\therefore\ i = I_S e^{v/nV_T}$ $\quad n = 2$

$$\frac{i_2}{i_1} = e^{\frac{v_2 - v_1}{0.05}}$$

thus $v_2 = v_1 + 0.05\ln i_2/i_1$

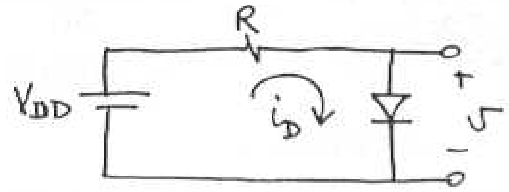
$\therefore$ for $i = 1.5\,mA$

$v = 0.7 + 0.05\ln\frac{1.5}{1}$ $\quad$ & $i_D = \frac{1-0.720}{0.2}$
$= 0.720V$ $\quad = 1.4\,mA$

#3
$v = 0.720 + 0.05\ln\left(\frac{1.4}{1.5}\right)$ & $i_D = \frac{1-0.716}{0.2}$
$= 0.716V$ $\quad = 1.42\,mA$

#4
$v = 0.716 + 0.05\ln\left(\frac{1.42}{1.4}\right)$ & $i_D = \underline{\underline{1.42\,mA}}$
$= \underline{\underline{0.716V}}$

3.35



Derivation of iterative equation

$$i_D = I_S e^{v/v_T}$$

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{v_2 - v_1}{n v_T}}$$

$$v_2 = v_1 + n v_T \ln\left(\frac{i_{D2}}{i_{D1}}\right)$$
$$= v_1 + \Delta V \log\left(\frac{i_{D2}}{i_{D1}}\right)$$

(a) $V = 0.7\,V$ $\quad i_D = \frac{10-0.7}{9.3} = 1\,mA$

(b) $v = 0.7V$ $\quad i_D = \frac{3-0.7}{2.3} = 1mA$

~ for both these cases the diode is rated at 1mA for 0.7V so stop.

(c) $V_{DD} = 2V \quad R = 2k\Omega$

#1 $v = 0.7\,V$ $\qquad i_D = \frac{2-0.7}{2} = 0.65mA$

#2 $v = 0.7 + 0.1 \log\left(\frac{0.650}{10}\right) = 0.581\,V$

$i_D = \frac{2-0.581}{2} = 0.709\ mA$

#3 $v = 0.581 + 0.1 \log\left(\frac{0.709}{0.650}\right)$

$= \underline{\underline{0.584V}}$

$i_D = \frac{2-0.584}{2} = \underline{\underline{0.708mA}}$

(d) $V_{DD} = 2V \qquad R = 2k\Omega$

#1 $V = 0.7V$ $\qquad i_D = \frac{2-0.7}{2}$

$= 0.650\,mA$

#2.

$V = 0.7 + 0.1\log\left(\frac{0.650}{1}\right)$ $\qquad i_D = \frac{2-0.681}{2}$

$= 0.681\,V$ $\qquad = 0.659\,mA$

#3

$V = 0.681 + 0.1\log\left(\frac{0.659}{0.650}\right)$ $\qquad i_D = \frac{2-0.682}{2}$

$= \underline{\underline{0.682V}}$ $\qquad = \underline{\underline{0.659mA}}$

(e) $V_{DD} = 1V \quad R = 0.3k\Omega$

#1 $v = 0.7V$ $\qquad i_D = \frac{1-0.7}{0.3} = 1mA$

#2

$v = 0.7 + 0.1 \log \frac{1}{10}$ $\qquad i_D = \frac{1-0.6}{0.3} = 1.333mA$

$= 0.6V$

#3

$v = 0.6 + 0.1\log \frac{1.333}{1}$ $\qquad i_D = \frac{1-0.612}{0.3} = 1.293\ mA$

$= 0.612V$

#4

$v = 0.612 + 0.1\log \frac{1.293}{1.333}$ $\qquad i_D = \frac{1-0.611}{0.3} = 1.297mA$

$= 0.611V$

#5

$v = 0.611 + 0.1 \log \frac{1.297}{1.293}$ $\qquad i_D = \underline{\underline{1.297mA}}$

$= \underline{\underline{0.611\,V}}$

(f) $V_{DD} = 1V \qquad R = 0.3k\Omega$

#1 $v = 0.7\,V$ $\qquad i_D = \frac{1-0.7}{0.3} = 1mA$

CONT.

#2

$v = 0.7 + 0.06 \log \frac{1}{10}$ $\qquad i_D = \frac{1-0.640}{0.3}$

$= 0.640\ V$ $\qquad = 1.2\ mA$

#3

$v = \quad + 0.06 \log \frac{1.2}{1}$ $\qquad i_D = \frac{1-0.645}{0.3}$

$= 0.645V$ $\qquad = 1.183\ mA$

#4

$v = \quad + 0.06 \log \frac{1.183}{1.2}$

$= \underline{\underline{0.645V}}$ $\qquad i_D = \underline{\underline{1.183mA}}$

(g) $V_{DD} = 1V \qquad R = 0.3k\Omega$

#1 $V = 0.7$ $\qquad i_D = \frac{1-0.7}{0.3} = 1mA$

#2

$v = 0.7 + 0.12 \log \frac{1}{10}$ $\qquad i_D = \frac{1-0.580}{0.3}$

$= 0.580V$ $\qquad = 1.381\ mA$

#3

$v = 0.580 + 0.12 \log \frac{1.381}{1}$ $\qquad i_D = \frac{1-0.697}{0.3}$

$= 0.697V$ $\qquad = 1.343\ mA$

#4

$v = 0.697 + 0.12 \log \frac{1.343}{1.381}$ $\qquad i_D = \frac{1-0.596}{0.3}$

$= 0.596\ V$ $\qquad = 1.347mA$

#5

$v = 0.596 + 0.12 \log \frac{1.347}{1.343}$

$= \underline{\underline{0.596V}}$ $\qquad i_D = \underline{\underline{1.347\ mA}}$

(h) $V_{DD} = 0.5V \qquad R = 30k\Omega$

#1 let $v = 0.4V$ $\qquad i_D = \frac{0.5-0.4}{30}$

$= 3.333\ \mu A$

#2

$v = 0.7 + 0.1 \log \frac{3.333 \times 10^{-3}}{10}$ $\qquad i_D = \frac{0.5-0.352}{30}$

$= 0.352V$ $\qquad = 4.933\ \mu A$

#3

$v = 0.352 + 0.1 \log \frac{4.933}{3.333}$ $\qquad i_D = \frac{0.5-0.369}{30}$

$= 0.369V$ $\qquad = 4.367\ \mu A$

#4

$v = 0.369 + 0.1 \log \frac{4.367}{4.933}$ $\qquad i_D = \frac{0.5-0.364}{30}$

$= 0.364V$ $\qquad = 4.533\ \mu A$

#5

$v = 0.364 + 0.1 \log \left(\frac{4.533}{4.367}\right)$ $\qquad i_D = \frac{0.5-0.366}{30}$

$= 0.366V$ $\qquad = 4.467\ \mu A$

#6

$v = 0.366 + 0.1 \log \frac{4.467}{4.533}$ $\qquad i_D = \frac{0.5-0.365}{30}$

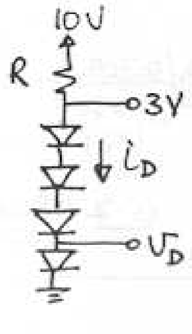
$= 0.365V$ $\qquad = 4.5\ \mu A$

#7

$v = 0.365 + 0.1 \log \frac{4.5}{4.467}$

$= \underline{\underline{0.365\ V}}$ $\qquad i_D = \underline{\underline{4.5\ \mu A}}$

$\boxed{3.36}$



$V_D = \frac{3}{4} = 0.75V$

$i_D = I_S e^{V_D/nV_T}$

$\frac{i_{D2}}{i_{D1}} = e^{\frac{V_{D2}-V_{D1}}{nV_T}}$

CONT.

$$\therefore i_D = i_{D2} = i_{D1}\, e^{\frac{v_{D2}-v_{D1}}{nV_T}}$$

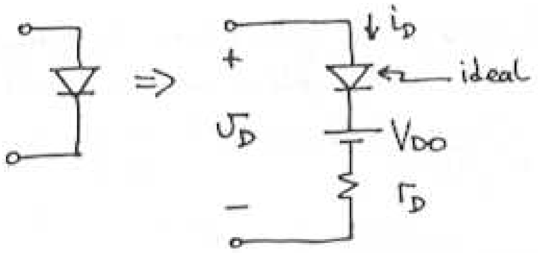
$$= 1 \times e^{\frac{0.75-0.7}{1\times 0.025}}$$

$$= 7.389 \text{ mA}$$

$$\therefore R = \frac{10-3}{I_D} = \frac{10-3}{7.389} = \underline{\underline{0.947 k\Omega}}$$

$\boxed{3.37}$

Piecewise linear model:



Given $n=2$, $V_D = 0.7V$ $i_D = 1mA$

The current throug the diode is given by:

$$i_D = \frac{v_D - V_{DO}}{r_D}$$ ~ need to find the parameters V_{DO} and r_D

Using the exponential model to find the diode voltage at 10mA

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{v_{D2}-v_{D1}}{nV_T}}$$

$$v_{D2} = nV_T \ln\left(\frac{i_{D2}}{i_{D1}}\right) = 0.05 \ln\left(\frac{10}{1}\right)$$

$$= 0.815V$$

Finding V_{DO} & r_D using the given facts:

$$i_D = \frac{0.7 - V_{DO}}{r_D} = 1mA \quad ①$$

$$i_D = \frac{0.815 - V_{DO}}{r_D} = 10mA \quad ②$$

$$②/① \Rightarrow \frac{0.815 - V_{DO}}{0.7 - V_{DO}} = 10$$

$$V_{DO} = \underline{0.687V}$$

$$r_D = \frac{0.7 - V_{DO}}{1} = \underline{\underline{12.8\,\Omega}}$$

Using the piecewise linear model

$$i_D = \frac{v_D - 0.687}{12.8} \Rightarrow v_D = 0.687 + 12.8\, i_D$$

Using the exponential model

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{v_D - 0.7}{0.05}} \Rightarrow v_D = 0.7 + 0.05 \ln\left(\frac{i_{D2}}{1}\right)$$

i_D (mA)	PIECEWISE LINEAR v_D (V)	EXPONENTIAL V_D (V)	Error (mV)
0.5	0.693	0.655	28
5	0.751	0.780	−29.5
14	0.866	0.832	34

$\boxed{3.38}$

Looking at the copy of FIG 3.12 below, we see at

$$i_D = 1mA \rightarrow v_D = 0.7V$$

$$i_D = 10mA \rightarrow v_D = 0.8V$$

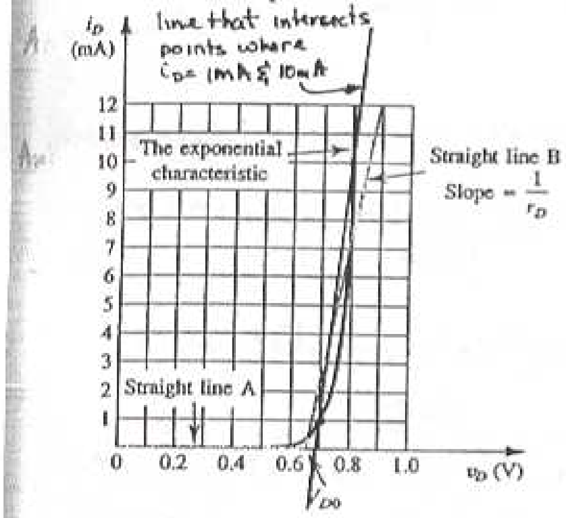
$$\therefore \text{slope} = \frac{1}{r_D} = \frac{10-1}{0.8-0.7} = 90 \frac{mA}{V}$$

$$\therefore r_D = \frac{1}{90\times 10^{-3}} = \underline{\underline{11.1\Omega}}$$

To find V_{DO}:

$$i_D = \frac{V_D - V_{DO}}{r_D}$$

$$10^{-3} = \frac{0.7 - V_{DO}}{11.1} \Rightarrow V_{DO} = \underline{0.689V}$$



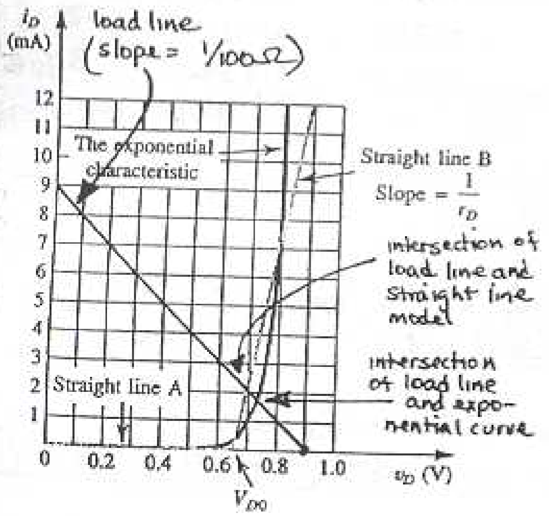
(a) The load line intersects the exponential model at:

$v_D = \underline{0.73V}$ $\quad i_D = \underline{1.7mA}$

(b) The load line intersects the straight-line model at

$v_D = \underline{0.7V}$ $\quad i_D = \underline{2mA}$

3.39



3.40

Calculating the parameters r_D & V_{DO} for the battery plus resistor model

$$i_D = I_s e^{v_D/nV_T} \qquad n=1$$

For $i_D = 0.1\, I_D$

$$v_{D2} = 0.7 + 0.025 \ln(0.1) = \underline{0.642V}$$

For $i_D = 10 I_D$

$$v_{D3} = 0.7 + 0.025 \ln(10) = \underline{0.758V}$$

Note that since the specifications for all of the diodes are given for 0.7V, the end voltages are the same as the voltage change for relative currents are independent to I_D & I_s.

$$\therefore \quad v_{D2} = V_{DO} + i_{D2} r_D \qquad (1)$$

$$v_{D3} = V_{DO} + i_{D3} r_D \qquad (2)$$

(2) → (1)

$$v_{D3} - v_{D2} = (i_{D3} - i_{D2}) r_D$$

CONT.

$$0.758 - 0.642 = (10 I_D - 0.1 I_D)\, r_D$$

$$0.166 = 9.9\, I_D r_D$$

$$r_D = \frac{0.0117}{I_D} \quad (3)$$

for (a) $I_D = 1mA$ $\quad r_D = \frac{0.0117}{1} = \underline{\underline{11.7\Omega}}$

(b) $I_D = 1A$ $\quad r_D = \frac{0.0117}{1} = \underline{\underline{0.0117\Omega}}$

(c) $I_D = 10\mu A$ $\quad r_D = \frac{0.0117}{10\mu A} = \underline{1.17k\Omega}$

(3) → (1)

$$0.642 = V_{DO} + 0.1 I_D \times \frac{0.0117}{I_D}$$

$$= V_{DO} + 0.00117$$

$V_{DO} = \underline{0.641V}$ ← same for all diodes

3.41

Since for a current of 10 mA, the diode voltage is $\underline{0.8V}$, this would be a suitable choice for the constant-voltage-drop model.

3.42

Constant Voltage drop Model:

Using $v_D = 0.7V \Rightarrow i_{D1} = \frac{V - 0.7}{R}$

Using $v_D = 0.6V \Rightarrow i_{D2} = \frac{V - 0.6}{R}$

For the difference in currents to vary by only 1% ⇒

$$i_{D2} = 1.01\, i_{D1}$$

$$V - 0.6 = 1.01\,(V - 0.7)$$

$$V = \underline{\underline{10V}}$$

For $V = 2V$ & $R = 1k\Omega$

At $V_D = 0.7V$ $\quad i_{D1} = \frac{2 - 0.7}{1} = 1.3mA$

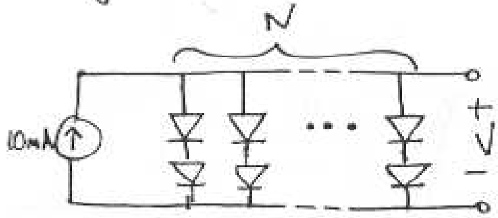
$V_D = 0.6V$ $\quad i_{D2} = \frac{2 - 0.6}{1} = 1.4mA$

$$\frac{i_{D2}}{i_{D1}} = \frac{1.4}{1.3} = 1.08$$

Thus the percentage difference is $\underline{8\%}$

3.43

Since $2V_D = 1.4V$ is close to the required 1.25V, use N parallel pairs of diodes to split the current evenly.



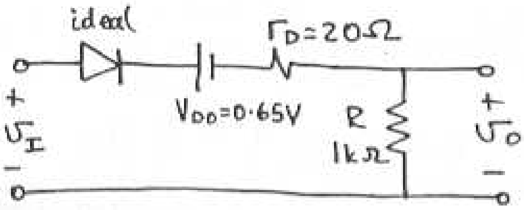
$$\therefore V = 2\left[0.7 + 0.1 \log \frac{10/N}{20}\right] = 1.25V$$

$N = 2.8$ ⇒ Use $\underline{\underline{3 \text{ sets of diodes}}}$

$$V = 2\left(0.7 + 0.1 \log \frac{10/3}{20}\right) = \underline{\underline{1.244V}}$$

3.44

Piecewise linear model in a half-wave rectifier.

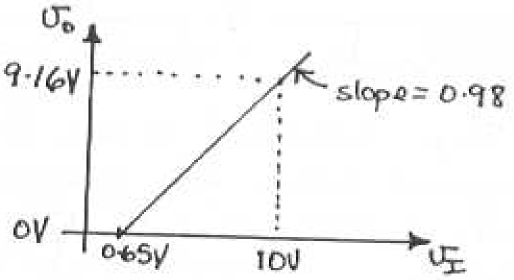


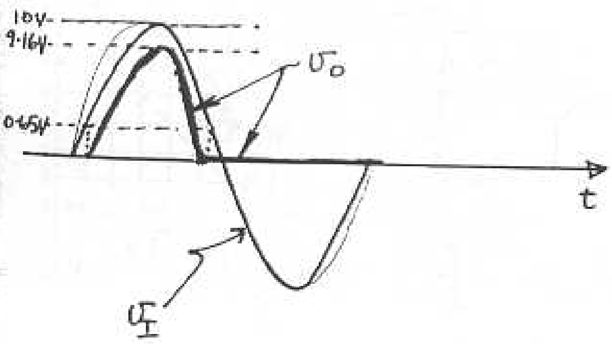
$$v_O = \frac{v_I - V_{D0}}{r_D + R} R \quad , \quad \text{for } v_I \gg V_{D0}$$

Thus, $v_O = 0.98(v_I - 0.65)$ for $v_I \geq 0.65$

$v_O = 0$ for $v_I < 0.65V$

Sketch of transfer characteristic :-

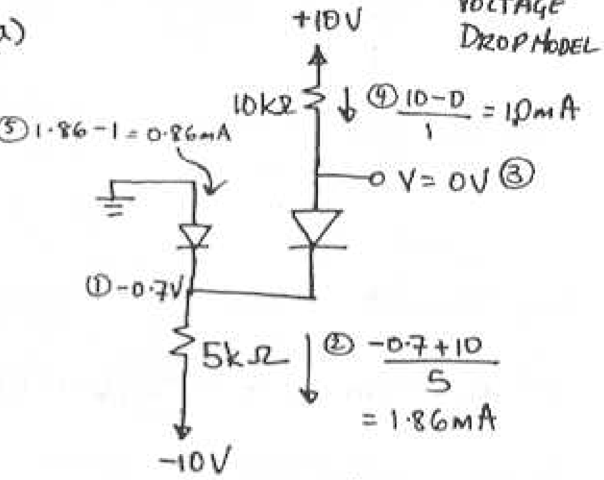




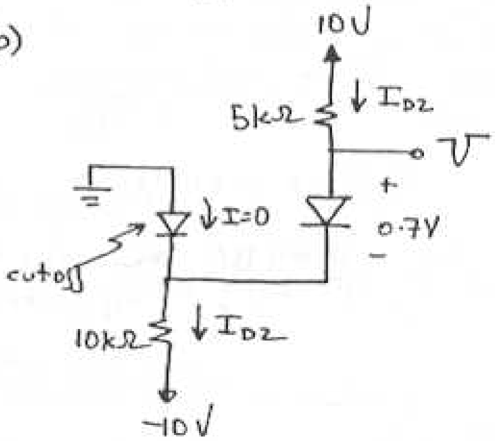
3.45

Refer to example 3-2 ~ CONSTANT VOLTAGE DROP MODEL

(a)



(b)

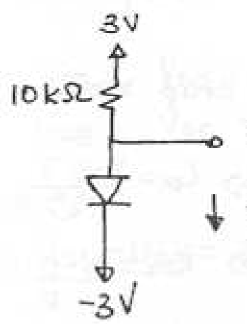


$$I_{D2} = \frac{10-(-10)-0.7}{15} = 1.29mA$$

$$V = -10 + 1.29(10) + 0.7 = \underline{3.6V}$$

3.46

(a)

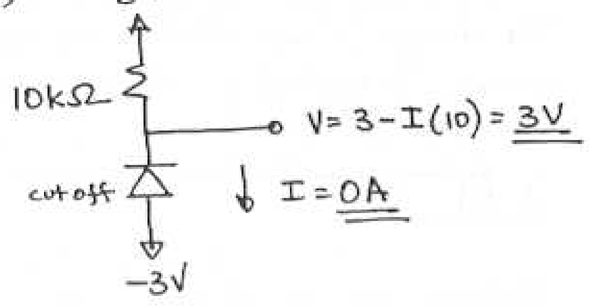


$V = -3 + 0.7 = \underline{-2.3V}$

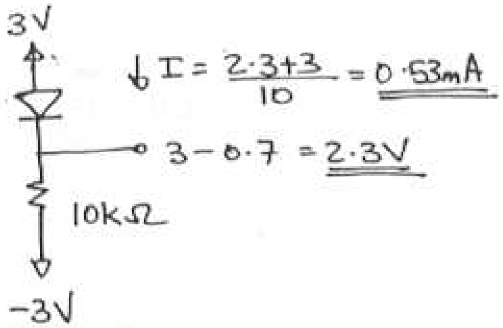
$$I = \frac{3+2.3}{10} = \underline{0.53mA}$$

CONT.

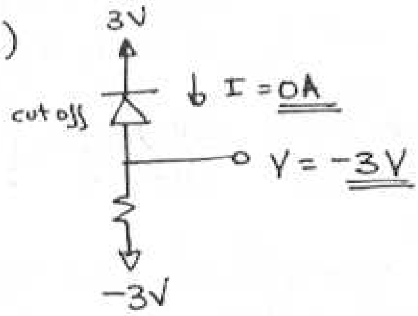
(b)



(c)

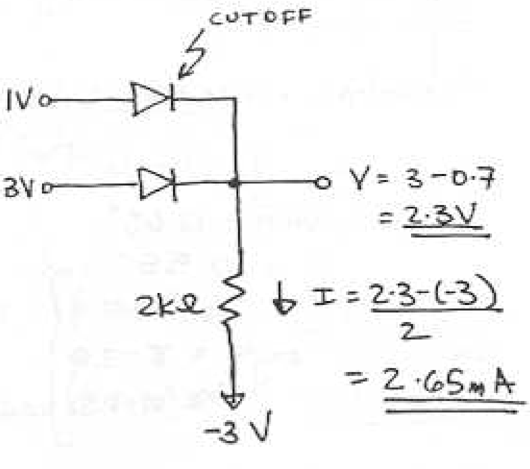


(d)

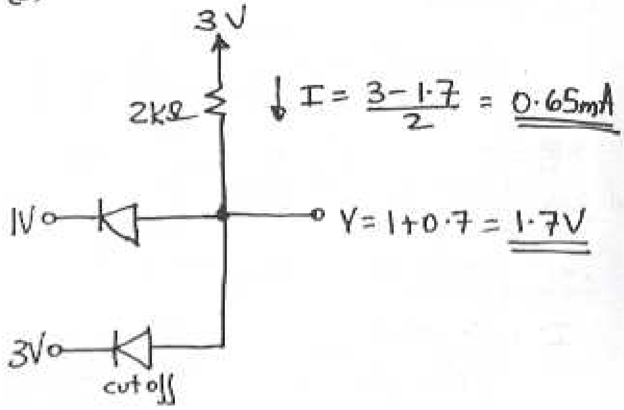


3.47

(a)

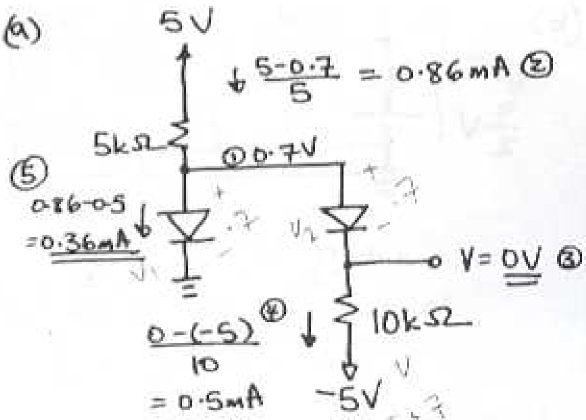


(b)

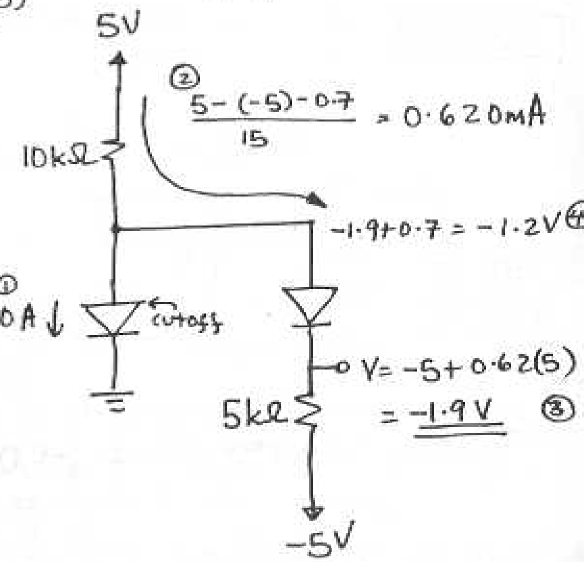


3.48

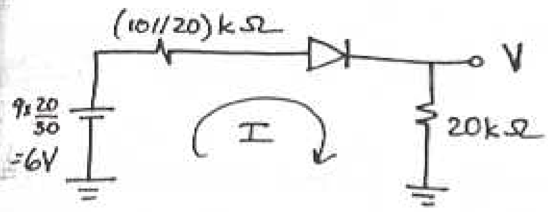
(a)



(b)



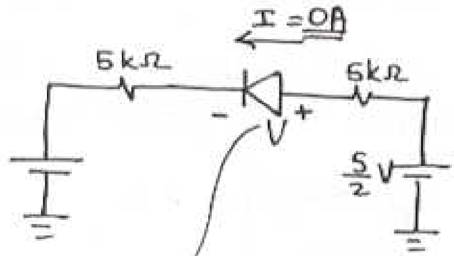
3.49



$$I = \frac{6-0.7}{(10//20)+20} = \underline{\underline{0.199\,mA}}$$

$$V = 20I = \underline{\underline{3.98V}}$$

(b)

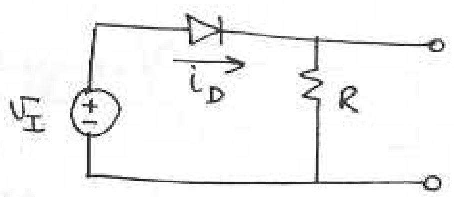


cutoff $\because \frac{5}{2} < \frac{9}{2}$

$\therefore I = \underline{\underline{0A}}$

$$V = 5/2 - 9/2 = \underline{\underline{-2V}}$$

3.50



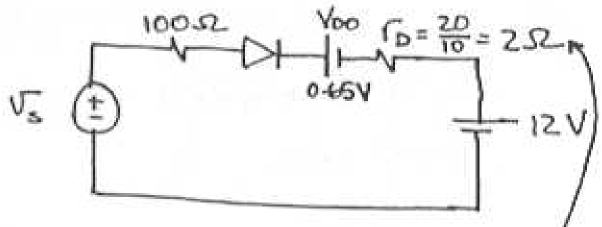
$$i_{D,peak} = \frac{v_{I,peak} - 0.7}{R} \leq 50$$

$$R \geq \frac{120\sqrt{2} - 0.7}{50} = \underline{\underline{3.38k\Omega}}$$

Reverse voltage $= 120\sqrt{2} = 169.7V$.
The design is essentially the same since the supply voltage >> 0.7V

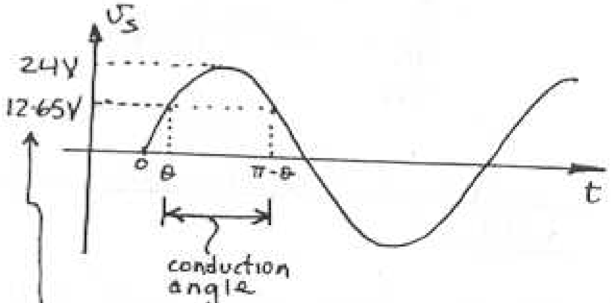
3.51

Battery plus resistance model



since the diode has 10x the area r_D is 1/10 as big.

$$i_D = I_s e^{v/nV_T} = \frac{V_s - V_{DO} - 12}{100+2}$$



Conduction starts when $v_s > 12 + V_{DO}$
$v_s > 12.65V$

$\therefore \quad 24 \sin\theta = 12.65°$
$\theta = 0.555$ rad

Conduction angle $= \pi - 2\theta$
$= 2.031$ rad.

Fraction of cycle for conduction $= \frac{2.031}{2\pi} = \underline{0.323}$

CONT.

$$i_{D,peak} = \frac{24-12.65}{100+2} = \underline{\underline{0.111\ A}}$$

Maximum reverse voltage occurs across the diode when v_s is at its negative peak and is equal to:

$$24 + 12 = \underline{\underline{36V}}$$

3.52

Using the exponential model

$$i_D = I_s\, e^{\Delta V/nV_T}$$

FOR A +10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/nV_T} = e^{0.01/n(0.025)}$$

$$= \begin{cases} 1.492 & \sim n=1 \\ 1.221 & \sim n=2 \end{cases}$$

$$\%\ \text{CHANGE} = \frac{i_{D2}-i_{D1}}{i_{D1}} \times 100$$

$$= \begin{cases} (1.492-1)\times 100 = \underline{\underline{+49.2\%}} & n=1 \\ (1.221-1)\times 100 = \underline{\underline{22.1\%}} & n=2 \end{cases}$$

FOR A −10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = 10^{-0.01/n(0.025)} = \begin{cases} 0.670 & n=1 \\ 0.819 & n=2 \end{cases}$$

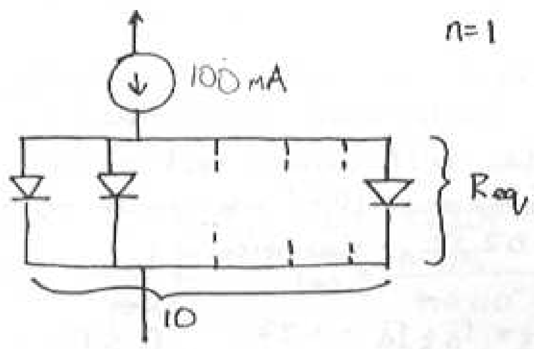
$$\%\ \text{CHANGE} = \begin{cases} (0.670-1)100 = \underline{\underline{-33\%}} & n=1 \\ (0.819-1)100 = \underline{\underline{-18\%}} & n=2 \end{cases}$$

For a current change limited to ±10%

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/n\times 0.025} = 0.9 \text{ to } 1.1$$

$$\Delta V = \begin{cases} \underline{\underline{-2.634mV \text{ to } 2.383mV}} & n=1 \\ \underline{\underline{-5.268mV \text{ to } 4.766mV}} & n=2 \end{cases}$$

3.53



n=1

Each diode has the current

$$i_D = \frac{0.1}{10} = 0.01A$$

Each diode has a small-signal resistance

$$r_d = \frac{nV_T}{I_D} = \frac{0.025}{0.01} = \underline{\underline{2.5\Omega}}$$

$$R_{eq} = r_d/10 = \underline{\underline{0.25\Omega}}$$

For one diode conducting 0.1 A

$$r_d = nV_T/0.1 = \frac{0.025}{0.1} = \underline{\underline{0.25\Omega}}$$

This is the same as Req. We can think of the parallel connection as equivalent to a single diode having 10 times the junction area of each diode. This large diode

is fed with 10x the current (or 0.1A) and this exhibits the same small-signal resistance as 10 parallel smaller diodes.

Now consider the series resistance of 0.2Ω to connect a diode. For the parallel combination above:

$$R_{eq} = \frac{1}{10}(0.2 + 2.5) = 0.27\Omega$$

To have an equivalent resistance, the single diode conducting all of the 0.1A would need a series resistance 10x as small or 0.02Ω. Specifically:

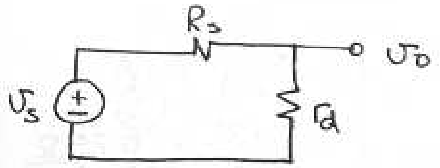
$$r_{out} = r_s + r_d = 0.27$$

$$= r_s + \frac{nV_T}{I_D} = 0.27$$

$$= r_s + 0.25 = 0.27$$

$$r_s = 0.27 - 0.25 = \underline{\underline{0.02\Omega}}$$

3.54



SMALL SIGNAL EQUIVALENT CIRCUIT

To find the small-signal response, v_o, open the dc current source I, and short the capacitors C_1 and C_2. Also replace the diode with its small signal resistance:

$$r_d = \frac{nV_T}{I} \qquad n=2$$

Now:

$$v_o = v_s \frac{r_d}{r_d + R_s}$$

$$= v_s \frac{nV_T/I}{\frac{nV_T}{I} + R_s} = \underline{\underline{v_s \frac{nV_T}{nV_T + IR_s}}}$$

Q.E.D.

$$v_o = 10\text{mV}\,\frac{0.05}{0.05 + 10^3 I}$$

$$= \begin{cases} 0.476\text{ mV} & \sim I = 1\text{mA} \\ 3.333\text{ mV} & \sim I = 0.1\text{mA} \\ 9.804\text{ mV} & \sim I = 1\mu\text{A} \end{cases}$$

For $v_o = \frac{1}{2}v_s = v_s \times \frac{0.05}{0.05 + 10^3 I}$

$$I = \underline{\underline{50\mu A}}$$

3.55

$R_s = 10k\Omega$, $n=1$, a 1mA diode

$$v_o/v_I = \frac{0.025}{0.025 + R_s I}$$

$$= \frac{0.025}{0.025 + 10^4 I} \quad (1)$$

For the current change limited to $\pm 10\%$ of I & using the exponential model we get

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/nV_T} = 0.9 \text{ to } 1.1$$

CONT.

$\Delta v = \underline{\underline{-2.63\,mV \text{ to } 2.38\,mV}}$

This is the amount the output will vary for a 10% change in diode current. Divide this by the specific gains given in the problem to find the limit on the input signal.

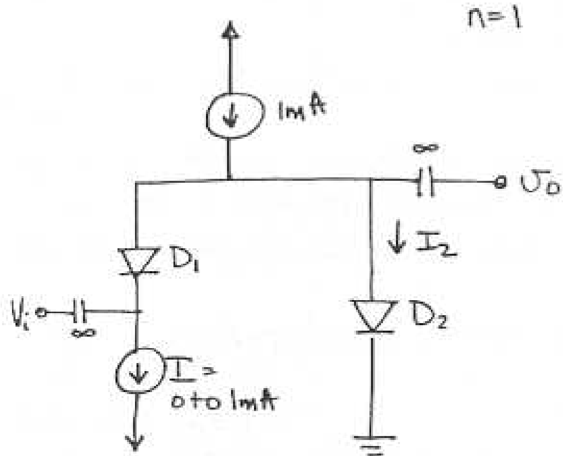
$$\Delta v_S = \frac{\Delta v_O}{v_O/v_I}$$

$$= \frac{-2.63\,mV}{\Delta v_O/v_I} \text{ to } \frac{2.38\,mV}{v_O/v_I} \quad ②$$

v_O/v_I	I use ① (mA)	v_S (using ②) (mV)
0.5	0.0025	5.26 to 4.76
0.1	0.0225	26.3 to 23.8
0.01	0.25	263 to 238
0.001	2.5	2630 to 2380

3.56

$n=1$



small signal model when D_1 & D_2 are conducting

(a) $I = 0\,\mu A$

D_1 - cutoff $\Rightarrow \frac{v_O}{v_I} = \underline{\underline{0\ V/V}}$

$I_2 = 1mA$

(b) $I = 1\,\mu A \qquad I_2 = 999\,\mu A$

$r_{d1} = \frac{nV_T}{I} = \frac{0.025}{I} = 25\Omega \qquad r_{d2} = \frac{0.025}{999\times10^{-6}} = 25.025\Omega$

$$\frac{v_O}{v_I} = \frac{r_{d2}}{r_{d1}+r_{d2}} = \underline{\underline{0.001\ \frac{V}{V}}}$$

(c) $I = 10\,\mu A \qquad I_2 = 990\,\mu A$

$r_{d1} = \frac{0.025}{10\times10^{-6}} = 2.5k\Omega \qquad r_{d2} = \frac{0.025}{990\times10^{-6}} = 25.25\Omega$

$v_O/v_I = \underline{\underline{0.01V/V}}$

(d) $I = 100\,\mu A \qquad I_2 = 900\,\mu A$

$r_{d1} = \frac{0.025}{100\times10^{-6}} = 250\Omega \qquad r_{d2} = \frac{0.025}{990\times10^{-6}} = 27.78\Omega$

$$\frac{v_O}{v_I} = \underline{\underline{0.1\ V/V}}$$

(e) $I = 500\,\mu A \qquad I_2 = 500\,\mu A$

$r_{d1} = r_{d2} = \frac{0.025}{500\times10^{-6}} = 50\Omega$

$$\frac{v_O}{v_I} = \underline{\underline{\frac{1}{2}\ V/V}}$$

(f) $I = 600\,\mu A \qquad I_2 = 400\,\mu A$

$r_{d1} = \frac{0.025}{600\times10^{-6}} = 41.67\Omega \qquad r_{d2} = \frac{0.025}{400\times10^{-6}} = 62.5\Omega$

56 CONT.

EN THE BIAS CURRENT IN EACH
DE IS $\geq 10\,\mu A$, THE DIODE RESISTANCE
BE $\leq 2.5\,k\Omega$. TO LIMIT THE
RENT SIGNAL TO A MAXIMUM OF 10%
BIAS, THE CURRENT SIGNAL MUST BE
μA. THUS, THE SIGNAL VOLTAGE ACROSS
"STARVED" DIODE WILL BE
mV WHICH IS APPROXIMATELY THE
UE TO WHICH THE INPUT SIGNAL
NG SHOULD BE LIMITED.

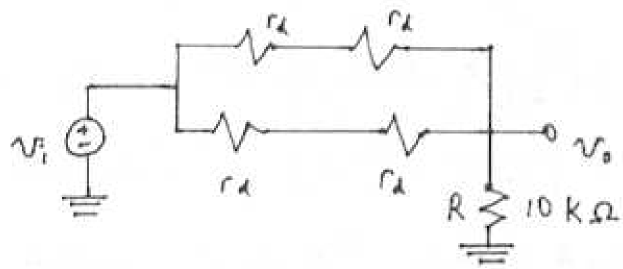
3.57

(a) $\frac{v_o}{v_i} = \frac{R}{R+(2r_d // 2r_d)}$

$= \frac{R}{R+r_d}$

WHERE $r_d = \frac{V_T}{I/2} = \frac{2V_T}{I}$

$= \frac{0.05\,V}{I}$



I (mA)	v_o/v_i (V/V)
0	0
10^{-3}	0.167
0.01	0.667
0.1	0.952
1.0	0.995
10	0.9995

(b) IF THE SIGNAL CURRENT IS TO BE LIMITED TO $\pm 10 I$, THE CHANGE IN DIODE VOLTAGE Δv_D CAN BE FOUND FROM

$$\frac{i_D}{I} = e^{\Delta v_D / nV_T} = 0.9 \text{ TO } 1.1$$

THUS, FOR $n=1$

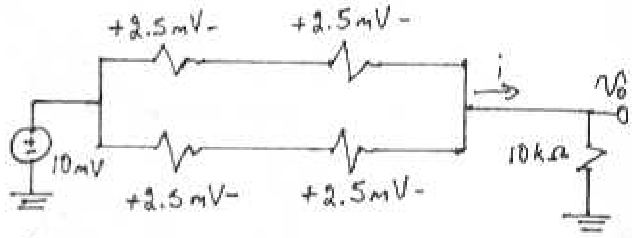
$\Delta v_D = -2.63\,mV$ TO $+2.38\,mV$

OR APPROXIMATELY $\pm 2.5\,mV$

CONT.

3.57 CONT.

(b CONT.) FOR THE DIODE CURRENT TO REMAIN WITHIN ±10% OF THEIR dc BIAS CURRENTS, THE SIGNAL VOLTAGE ACROSS EACH DIODE MUST BE LIMITED TO 2.5 mV. NOW, IF $v_{i\,PEAK} = 10\,mV$ WE CAN OBTAIN THE FOLLOWING SITUATION



WE SEE THAT $v_o = 5\,mV$ AND

$i = \frac{5\,mV}{10\,k\Omega} = 0.5\,\mu A$.

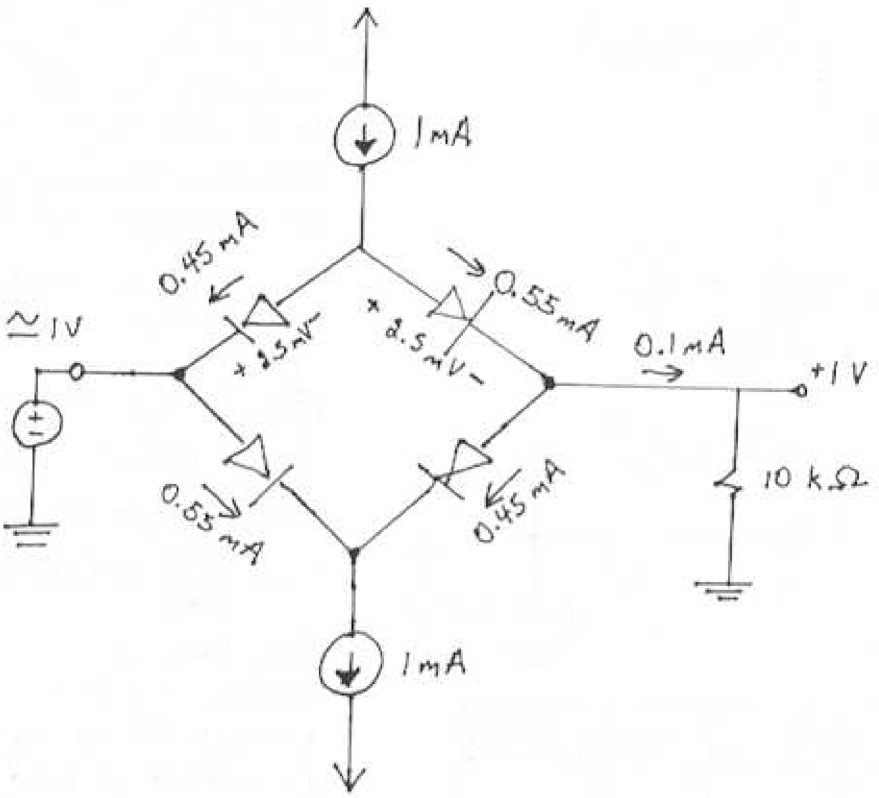
THUS, EACH DIODE IS CARRYING A CURRENT SIGNAL OF 0.25 mA. FOR THIS TO BE AT MOST 10% OF THE dc CURRENT, THE dc CURRENT IN EACH DIODE MUST BE AT LEAST 2.5 μA. IT FOLLOWS THAT THE MINIMUM VALUE OF I MUST BE $\underline{\underline{5\,\mu A}}$.

(c) FOR $I = 1\,mA$, $I_D = 0.5\,mA$, AND FOR MAXIMUM SIGNAL OF 10%, $I_D = 0.05\,mA$. THUS $i_D = 2i_d = 0.1\,mA$ AND THE CORRESPONDING MAXIMUM v_o IS $0.1\,mA \times 10\,k\Omega = 1\,V$.

THE CORRESPONDING PEAK INPUT CAN BE FOUND BY DIVIDING v_o BY THE TRANSMISSION FACTOR OF 0.995, THUS

$$v_{i\,MAX} = \frac{1\,V}{0.995\,V} = \underline{\underline{1.005\,V}}$$

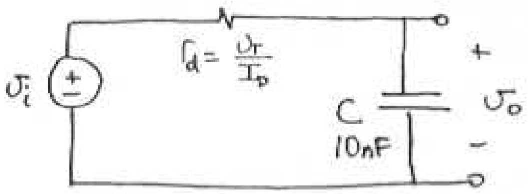
SEE FIGURE.
EACH DIODE HAS $r_d = 50\,\Omega$



3.58

Opening the current source we get the following small-signal circuit :

$(n=1)$



$$\frac{v_o}{v_i} = \frac{1/sC}{1/sC + r_d} = \frac{1}{1 + sCr_d}$$

$$\text{Phase Shift} = -\tan^{-1}\left(\frac{\omega C r_d}{1}\right)$$

$$= -\tan^{-1}\left(2\pi\, 10^5 \times 10\times10^{-9} \times 0.025/I\right)$$

For a phase shift of $-45°$ we have

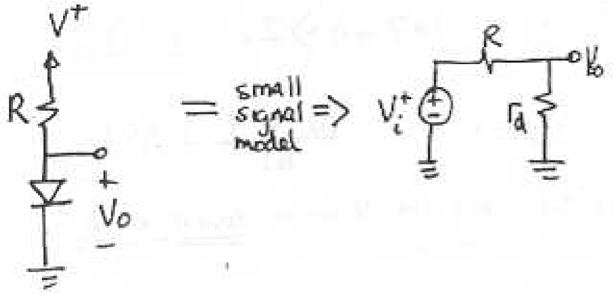
$$2\pi 10^5 \times 10(10^{-9}) \times \frac{0.025}{I} = 1$$

$$I = \underline{\underline{157\,\mu A}}$$

Range of phase shift for $I = 15.7\,\mu A$ to $1570\,\mu A$ is :

$$\underline{\underline{-84.3° \text{ to } -5.71°}}$$

3.59



CONT.

(a) $\frac{\Delta V_o}{\Delta V^+} = \frac{r_d}{r_d + R} = \frac{nV_T/I}{nV_T/I + R}$

$= \frac{nV_T}{nV_T + IR}$ where at No load $I = \frac{V^+ - 0.7}{R}$

$= \underline{\underline{\frac{nV_T}{nV_T + V^+ - 0.7}}}$ Q.E.D.

(b) For m diodes in series use

$I = \frac{V^+ - m \times 0.7}{R}$

Thus:

$\frac{\Delta V_o}{\Delta V^+} = \frac{m r_d}{m r_d + R} = \frac{m(nV_T)}{m(nV_T) + IR}$

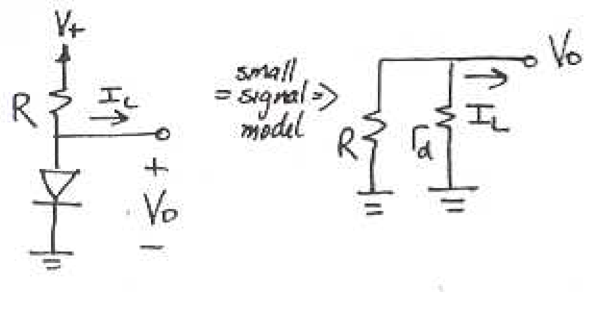
$= \underline{\underline{\frac{m(nV_T)}{m(nV_T) + V^+ - 0.7m}}}$

(c) Line Regulation for $V^+ = 10V$, $n = 2$

i) $m = 1$ $\frac{\Delta V_o}{\Delta V^+} = \underline{\underline{5.35 \text{ mV/V}}}$

ii) $m = 3$ $\frac{\Delta V_o}{\Delta V^+} = \underline{\underline{18.63 \text{ mV/V}}}$

$\boxed{3.60}$



$\Delta V_o = -I_L (R \| r_d)$

$\frac{\Delta V_o}{I_L} = \underline{\underline{-(R \| r_d)}}$ Q.E.D.

(b) Given at DC $I_D = \frac{V^+ - 0.7}{R}$

Also $r_d = \frac{nV_T}{I_D}$

We have:

$\frac{\Delta V_o}{I_L} = -\frac{1}{\frac{1}{R} + \frac{1}{r_d}}$

$= -\frac{1}{\frac{I_D}{V^+ - 0.7} + \frac{I_D}{nV_T}}$

$= -\frac{nV_T}{I_D} \frac{1}{1 + \frac{nV_T}{V^+ - 0.7}}$

$= \frac{-nV_T}{I_D} \frac{V^+ - 0.7}{V^+ - 0.7 + nV_T}$ Q.E.D.

for $\frac{\Delta V_o}{I_L} \leq 5 \frac{mV}{mA}$

$\frac{-2 \times 0.025}{I_D} \times \frac{10 - 0.7}{10 - 0.7 + 0.05} \leq \frac{5 \times 10^{-3}}{10^{-3}}$

$I_D \geq 9.947 mA \Rightarrow I_D = \underline{\underline{10 mA}}$

$R = \frac{V^+ - 0.7}{I_D} = \frac{10 - 0.7}{10} = \underline{\underline{930 \Omega}}$

Thus the diode should be a $\underline{\underline{10 mA \text{ diode}}}$.

(c) For m diodes

$I_D = \frac{V^+ - 0.7m}{R}$ & $r_d = \frac{m(nV_T)}{I_D}$

CONT.

$$\frac{\Delta V_o}{I_L} = \frac{-1}{\frac{1}{R} + \frac{1}{r_d}}$$

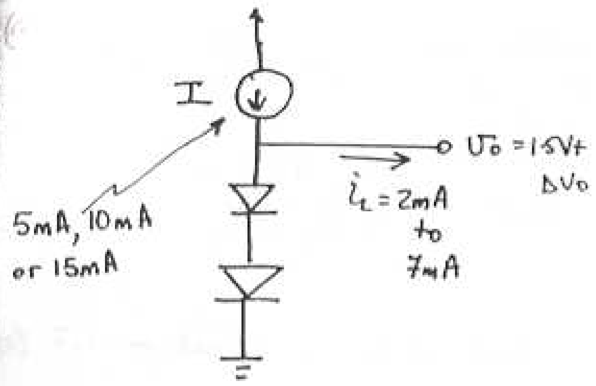
$$= \frac{-1}{\frac{I_D}{V^+ - 0.7m} + \frac{I_D}{mnV_T}}$$

$$= -\frac{mnV_T}{I_D} \frac{1}{\frac{mnV_T}{V^+ - 0.7m} + 1}$$

$$= -\frac{mnV_T}{I_D} \frac{V^+ - 0.7m}{V^+ - 0.7m + mnV_T}$$

3.61

3.62



For a load current of 2 to 7mA, I must be greater than 7 mA. Thus the 5 mA source would not do.

We are left to choose between the 10 and 15 mA sources. The 15mA source provides lower load regulation because the diodes will have more current flowing through them at all times. This is shown below:

Load Regulation if $I = 10mA$

use $\frac{i_{D2}}{i_{D1}} = e^{\Delta V / 2 \times n V_T}$ — 2 diodes

$\therefore e^{\Delta V / 0.05 \times 2} = 3/10$ to $8/10$

$\Delta V_o = -120mV$ to $-22.3mV$

$\therefore$ The peak to peak ripple is $-120 - (-22.3) \approx -100mV$

$$\text{Load Regulation} = \frac{\Delta V_O}{I_L} = \frac{-100}{5}$$

$$= -20 \ \frac{mV}{mA}$$

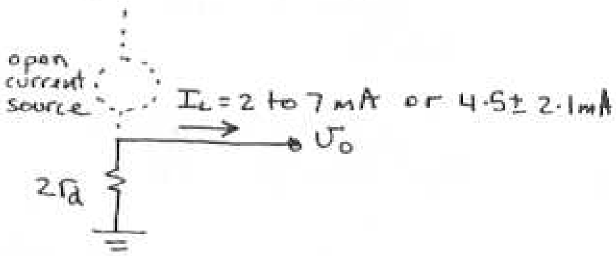
Load Regulation for $I = 15mA$. Here the current through the diodes change from 8 to 13 mA corresponding to

$$\Delta V_O = 0.1 \ln(8/13) = -49mV$$

$$\text{Load Regulation} = \frac{-49}{5} \approx -10 \frac{mV}{mA}.$$

The obvious disadvantage of using the 15mA supply is the requirement of higher current and higher power dissipation.

Alternate solution of Line Regulation using the small signal model



$$\text{Load Regulation} = \frac{\Delta V_O}{I_L} = -2r_d = \frac{-2nV_T}{I_D}$$

Where the bias current $I_D = 10 - 4.5$ for the 10mA source.

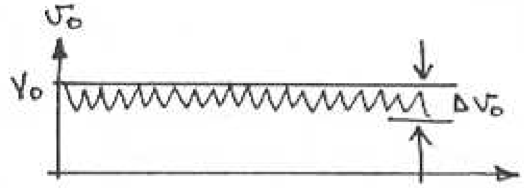
$$\Rightarrow \frac{\Delta V_O}{I_L} = \frac{-2 \times 2 \times 0.025}{10 - 4.5} = -18.2 \frac{mV}{mA}$$

For 15 mA source $I_D = 15 - 4.5$

$$\frac{\Delta V_O}{\Delta I_L} = \frac{-0.1}{15 - 4.5} = -9.5 \frac{mV}{mA}$$

CONT.

Sketch of output :-



$\boxed{3.63}$

3.64

(a) $V_Z = V_{Z0} + r_Z I_{ZT}$

$10 = 9.6 + r_Z \times 50\times10^{-3}$

$r_Z = \underline{\underline{8\,\Omega}}$

Power rating:

$V_Z = V_{Z0} + r_Z \times 2I_{ZT}$

$= 9.6 + 8\times100\times10^{-3}$

$= 10.4\,V$

$P = 10.4\times100\times10^{-3} = \underline{\underline{1.04\,W}}$

(b) $V_Z = V_{Z0} + r_Z I_{ZT}$

$9.1 = V_{Z0} + 30\times10\times10^{-3}$

$V_{Z0} = \underline{\underline{8.8\,V}}$

$V_Z = 8.8 + 30\times20\times10^{-3} = 9.4\,V$

$P = 9.4\times20\times10^{-3} = \underline{\underline{188\,mW}}$

(c) $6.8 = 6.6 + 2\times I_{ZT}$

$I_{ZT} = \underline{\underline{100\,mA}}$

$V_Z = 6.6 + 2\times200\times10^{-3} = 7\,V$

$P = 7\times200\times10^{-3} = \underline{\underline{1.4\,W}}$

(d) $18 = 17.2 + r_Z\times5\times10^{-3}$

$r_Z = \underline{\underline{160\,\Omega}}$

$V_Z = 17.2 + 160\times10\times10^{-3} = 18.8\,V$

$P = 18.8\times10\times10^{-3} = \underline{\underline{188\,mW}}$

(e) $7.5 = V_{Z0} + 1.5\times200\times10^{-3}$

$V_{Z0} = \underline{\underline{7.2\,V}}$

$V_Z = 7.2 + 1.5\times400\times10^{-3} = 7.8\,V$

$P = 7.8\times400\times10^{-3} = \underline{\underline{3.12\,W}}$

3.65

(a) Three 6.8 V zeners provide $3\times6.8 = 20.4\,V$ with $3\times10 = 30\,\Omega$ resistance. Neglecting R, we have

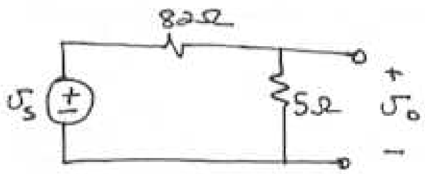
Load Regulation $= \underline{\underline{-30\,mV/mA}}$.

(b) For 5.1 V zeners we use 4 diodes to provide 20.4 V with $4\times30 = 120\,\Omega$ resistance.

Load regulation $= \underline{\underline{-120\,mV/mA}}$

3.66

Small signal model for line regulation:



$$\frac{\Delta v_o}{\Delta v_s} = \frac{5}{5+82}$$

$$\Delta v_o = \frac{5}{87}\times\Delta v_s$$

$$= \frac{5}{87}\times1.3$$

$$= \underline{\underline{74.7\,mV}}$$

$\boxed{3.67}$

$V_Z = V_{Z0} + r_Z I_{ZT}$

$9.1 = V_{Z0} + 5 \times 28 \times 10^{-3}$

$V_{Z0} = 8.96V$

$V_Z = V_{Z0} + 5I_Z = 8.96 \times 5I_Z$

FOR $I_Z = 10mA$ $\quad V_Z = \underline{\underline{9.01V}}$

FOR $I_Z = 100mA$ $\quad V_Z = \underline{\underline{9.46V}}$

3.68

$r_z = 30\,\Omega$

$I_{ZK} = 0.5\,mA$

$V_Z = 7.5\,V$

$I_Z = 12\,mA$

$7.5 = V_{Z0} + 12 \times 30 \times 10^{-3}$

$\Rightarrow V_{Z0} = 7.14\,V$

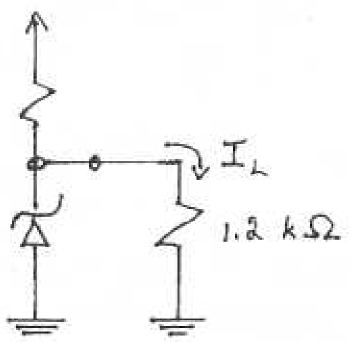
$I_L = \frac{7.5}{1.2} = 6.25\,mA$

SELECT $I = 10\,mA$

SO THAT $I_Z = 3.7\,mA$

WHICH IS $> I_{ZK}$

$R = \frac{10 - 7.5}{10} = \underline{\underline{250\,\Omega}}$



FOR $\Delta V^+ = \pm 1\,V$

$\Delta V_o = \pm 1 \times \frac{1.2 // 0.03}{0.250 + (1.2 // 0.03)}$

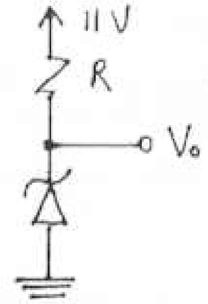
$= \pm\, 0.1\,V$

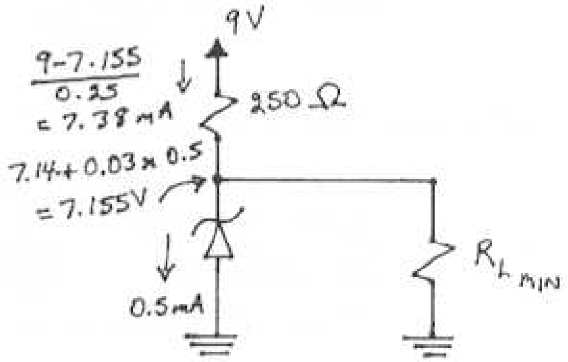
THUS $V_o = +7.4\,V$ TO $+7.6\,V$

WITH $V^+ = 11\,V$ AND $I_L = 0$

$V_o = V_{Z0} + \frac{11 - V_o}{0.25} \times 0.03$

$\Rightarrow V_o = \underline{\underline{7.55\,V}}$

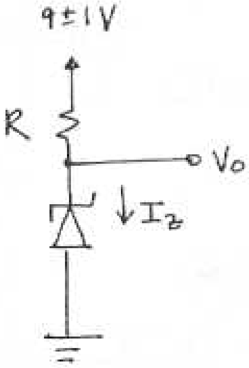




$R_{L_{MIN}} = \frac{7.155}{7.38 - 0.5}$

$= \underline{\underline{1.04\,k\Omega}}$

3.69



Given Parameters

$V_Z = 6.8V$, $r_z = 5\Omega$,
$I_Z = 20mA$

By knee

$I_{ZK} = 0.25mA$
$r_z = 750\Omega$

FIRST DESIGN – 9V supply can easily supply current.

Let $I_Z = 20mA$ ~ well above knee

$$\therefore R = \frac{9-0.68}{20} = \underline{\underline{110\Omega}}$$

$$\text{Line Regulation} = \frac{\Delta V_O}{\Delta V_S} = \frac{r_z}{r_z+R}$$

$$= \frac{5}{5+110}$$

$$= \underline{\underline{43.5\ \frac{mV}{V}}}$$

SECOND DESIGN ~ limited current from 9V supply

$I_Z = 0.25mA$

$V_Z = V_{ZK} \simeq V_{Z0}$ – calculate V_{Z0} from

$V_Z = V_{Z0} + r_z I_{ZT}$

$6.8 = V_{Z0} + 5 \times 0.02$

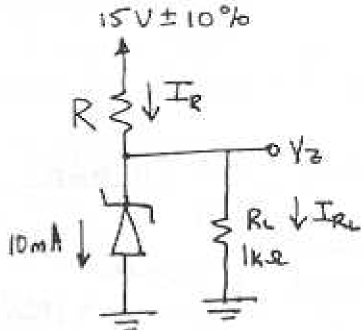
$V_{Z0} = 6.7V$

$$\therefore R = \frac{9-6.7}{0.25} = \underline{\underline{9.2k\Omega}}$$

$$\text{LINE REGULATION} = \frac{\Delta V_O}{\Delta V_S} = \frac{750}{750+9200}$$

$$= \underline{\underline{75.4\ \frac{mV}{V}}}$$

3.70



$V_Z = V_{Z0} + r_z I_Z$

$9.1 = V_{Z0} + 30(0.009)$

$V_{Z0} = \underline{\underline{8.83V}}$

CONT.

$V_Z = 8.83 + 30(0.01) = 9.13\text{ V}$

$I_{R_L} = 9.13/1\text{k}\Omega = 9.13\text{ mA}$

$I_R = 10 + 9.13 = \underline{\underline{19.13\text{ mA}}}$

$\therefore R = \dfrac{15-9.13}{19.13} = 306.8\ \Omega$

$\cong \underline{\underline{300\ \Omega}}$

$V_Z = 8.83 + 30\left(\dfrac{15-V_Z}{300} - \dfrac{V_Z}{1000}\right)$

$= 10.33 - V_Z/10 - 3/100 V_Z$

$V_Z = 9.14\text{V}$

$V_Z = 8.83 + 30\left(\dfrac{15 \pm 1.5 - V_Z}{300} - \dfrac{V_Z}{1000}\right)$

$= \dfrac{1}{1.13}\left[8.83 + 1.5 \pm 0.15\right] = 9.14 \pm 0.13\text{V}$

$\therefore$ $\underline{\pm 0.13\text{V}}$ variation in output voltage

Halfing the load current $\equiv R_L$ doubling

$V_Z = 8.83 + 30\left(\dfrac{15-V_Z}{300} - \dfrac{V_Z}{2000}\right)$

$= \dfrac{10.33}{1.115} = 9.26\text{V}$

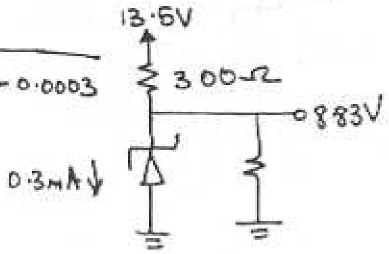
$\therefore 9.26 - 9.14 = 0.12\text{V}$ increase in output voltage.

At the edge of the breakdown region

$V_Z \cong V_{Z0} = 8.83\text{V} \quad I_{ZK} = 0.3\text{mA}$

$R_L = \dfrac{8.83}{\dfrac{13.5-8.83}{300} - 0.0003}$

$= \underline{\underline{578\ \Omega}}$



Lowest output voltage $= 8.83\text{V}$

Line Regulation $= \dfrac{r_z}{R + r_z} = \dfrac{30}{300+30}$

$= 90\ \dfrac{\text{mV}}{\text{V}}$

Load Regulation $= -(r_z \parallel R) = \underline{\underline{-29.1\ \dfrac{\text{mV}}{\text{mA}}}}$

$\boxed{3.71}$

(a) $V_{ZT} = V_{Z0} + r_z I_{ZT}$

$10 = V_{Z0} + 7(0.025)$

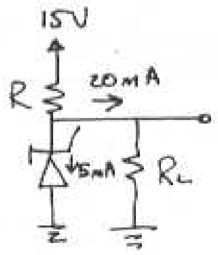
$\Rightarrow V_{Z0} = \underline{\underline{9.825\text{V}}}$

(b) The minimum zener current of 5mA occurs when $I_L = 20\text{mA}$ and V_S is at its minimum of $20(1-0.25) = 15\text{V}$. See the circuit below:

$R \leq \dfrac{15 - (V_{Z0} + r_Z I_Z)}{20+5}$

$\leq \dfrac{15 - (9.825 + 7(0.005))}{25}$

$\leq 205.6\ \Omega.$



$\therefore$ Use $R = \underline{\underline{205\ \Omega}}$

(c) Line Regulation $= \dfrac{7}{205+7} = \underline{\underline{33\ \dfrac{\text{mV}}{\text{V}}}}$

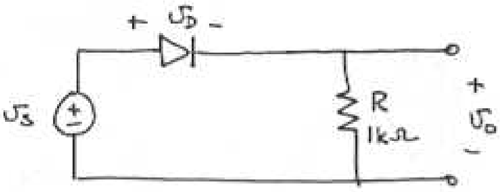
$\pm 25\%$ change in $V_S \equiv \pm 5\text{V}$

V_O changes by $\pm 5 \times 33 = \underline{\underline{\pm 165\text{ mV}}}$

corresponding to $\dfrac{\pm 165}{10} \times 100 = \underline{\underline{\pm 1.65\%}}$

CONT.

3.75



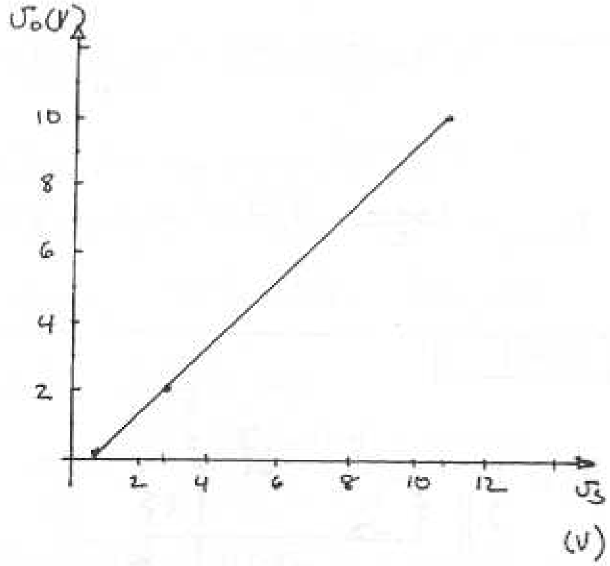
$$i_D = I_S e^{v_D/nV_T}$$

$$\frac{i_D}{1mA} = e^{\frac{v_D - 0.7}{nV_T}}$$

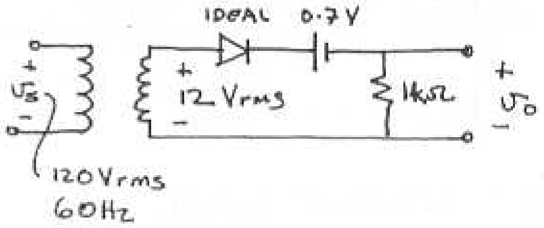
$$v_D - 0.7 = nV_T \ln\left(i_D/10^{-3}\right) = 0.1 \log\left(\frac{i_D}{10^{-3}}\right)$$

$$v_D = 0.7 + 0.1\log\left(\frac{v_O}{R}\right) \quad R \text{ in } k\Omega$$
$$= 0.7 + 0.1\log\left(\frac{v_O}{1}\right)$$

v_O (V)	v_D (V)	$v_S = v_D + v_O$ (V)
0.10	0.6	0.7
0.5	0.67	1.17
1	0.7	1.7
2	0.73	2.73
5	0.77	5.77
10	0.8	10.8



3.76



$$\hat{V}_o = 12\sqrt{2} - 0.7 = \underline{\underline{16.27V}}$$

Conduction begins at

$$v_S = 12\sqrt{2}\sin\theta = 0.7$$
$$\theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right)$$
$$= 0.0412 \text{ rad}$$

Conduction ends at $\pi - \theta$

$\therefore$ Conduction angle $= \pi - 2\theta = \underline{\underline{3.06 rad}}$

The diode conducts for

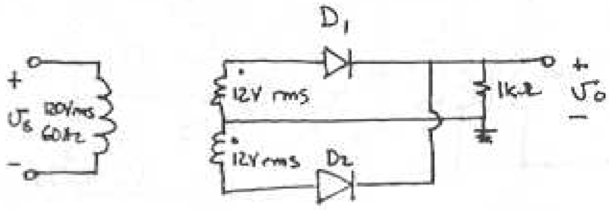
$$\frac{3.06}{2\pi} \times 100 = \underline{\underline{48.7\%}} \text{ of the cycle}$$

$$V_{o,avg} = \frac{1}{2\pi}\int_{\theta}^{\pi-\theta} 12\sqrt{2}\sin\phi - 0.7 \, d\phi$$

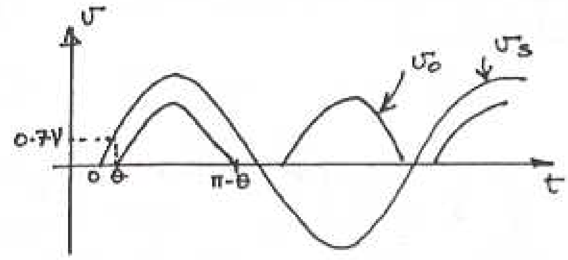
$$= \underline{\underline{5.06V}}$$

$$i_{D,avg} = \frac{v_{o,avg}}{R} = \underline{\underline{5.06mA}}$$

3.77



CONT.



$\hat{v}_O = 12\sqrt{2} - V_{DO} = \underline{\underline{16.27V}}$

Conduction starts at $\theta = \sin^{-1}\frac{0.7}{12\sqrt{2}}$
$= 0.0412$ rad

and ends at $\pi - \theta$. Conduction angle $= \pi - 2\theta = 3.06$ rad in each half cycle. Thus the fraction of a cycle for which one of the two diodes conduct $= \frac{2(3.06)}{2\pi} \times 100$

$= \underline{\underline{97.4\%}}$

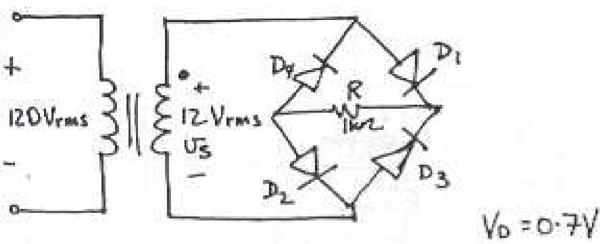
Note that during 97.4% of the cycle there will be conduction. However each of the two diodes conducts for only half the time, i.e. for 48.7% of the cycle.

$$v_{O,avg} = \frac{1}{\pi}\int_{\theta}^{\pi-\theta} 12\sqrt{2}\sin\phi - 0.7 \; d\phi$$

$= \underline{\underline{10.12V}}$

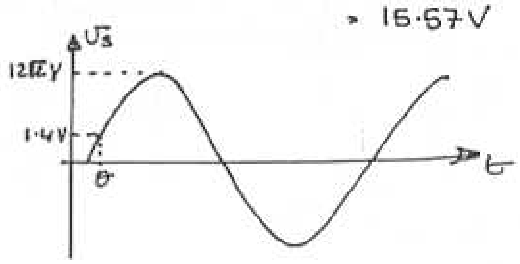
$i_{D,avg} = \frac{10.12}{1k\Omega} = \underline{\underline{10.12mA}}$

3.78



$V_D = 0.7V$

Peak voltage across $R = 12\sqrt{2} - 2V_D$
$= 12\sqrt{2} - 1.4$
$= 15.57V$



$\theta = \sin^{-1}\frac{1.4}{12\sqrt{2}} = 0.0826$ rad

Fraction of cycle that D_1 & D_2 conduct is $\frac{\pi - 2\theta}{2\pi} \times 100 = \underline{\underline{47.4\%}}$

Note D_3 & D_4 conduct in the other half cycle so that there is $2(47.4) = 94.8\%$ conduction interval.

$$v_{O,avg} = \frac{2}{2\pi}\int_{\theta}^{\pi-\theta} 12\sqrt{2}\sin\phi - 2V_D \; d\phi$$

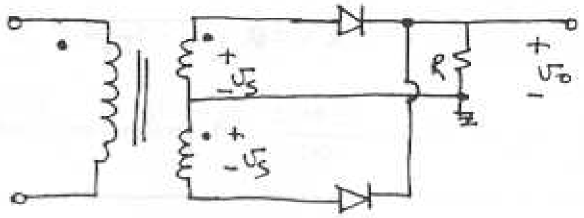
$$= \frac{1}{\pi}\Big[-12\sqrt{2}\cos\phi - 1.4\,\phi\Big]_{\theta}^{\pi-\theta}$$

$$= \frac{2(12\sqrt{2}\cos\theta)}{\pi} - \frac{1.4(\pi-2\theta)}{\pi}$$

$= \underline{9.44V}$

$i_{R,avg} = \frac{v_{O,avg}}{R} = \frac{9.44}{1} = 9.44$ mA

3.79



CONT.

For $V_{DO} \ll V_s$,

$V_{O,avg} \cong \frac{2}{\pi} V_s - V_{DO}$

(a) For $v_{O,avg} = 10V$

$10 = \frac{2}{\pi} V_s - 0.7$

$\Rightarrow V_s = 16.81V$

Line peak $= 120\sqrt{2}$

Thus,

turns ratio to each half of the secondary $= \frac{120\sqrt{2}}{16.81} = \underline{\underline{10.1:1}}$

OR $\underline{\underline{5.05:1}}$ centre tapped

(b) For $v_{O,avg} = 100V$

$V_s = \frac{\pi}{2}(100.7) = \underline{158.2V}$

Turns Ratio $= \frac{120\sqrt{2}}{158.2} = \underline{\underline{1.07:1}}$ to each half

OR $\underline{\underline{0.535:1}}$ centre tapped

3.80

Refer to Fig 3.27.

For $2V_{DO} \ll V_s$,

$v_{O,avg} = \frac{2}{\pi} V_s - 2V_{DO} = \frac{2}{\pi} V_s - 1.4$

(a) For $v_{O,avg} = 10V$

$V_s = \frac{\pi}{2} \times 11.4 = 17.91V$

Turns Ratio $= \frac{120\sqrt{2}}{17.91} = \underline{\underline{9.477 \text{ to } 1}}$

(b) For $v_{O,avg} = 100V$

$V_s = \frac{\pi}{2} \times 101.4 = 159.3V$

Turns Ratio $= \frac{120\sqrt{2}}{159.3} = \underline{\underline{1.065 \text{ to } 1}}$

3.81

$120\sqrt{2} \pm 10\% \; : \; 24\sqrt{2} \pm 10\%$

$\Rightarrow$ turns Ratio $= 5:1$

$V_s = \frac{24\sqrt{2}}{2} \pm 10\%$

$PIV = 2V_s|_{max} \; V_{DO}$

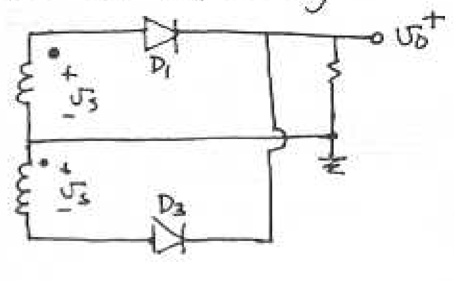
$= 2 \times \frac{24\sqrt{2}}{2} \times 1.1 - 0.7$

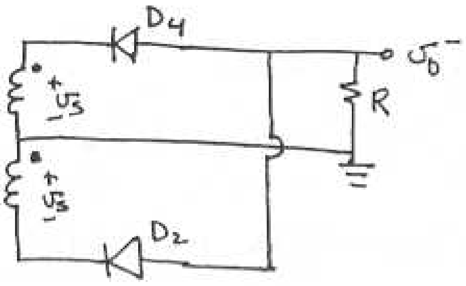
$= \underline{\underline{36.6V}}$

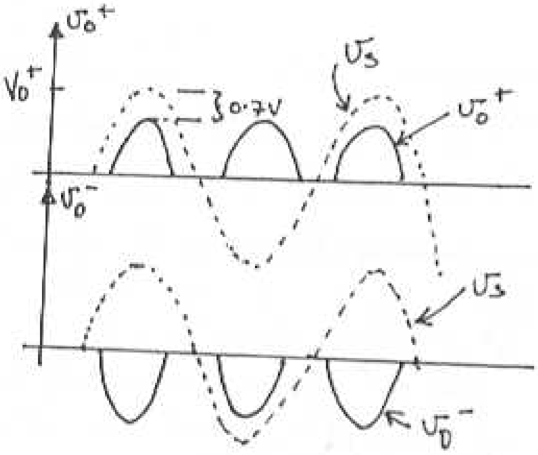
Using a factor of 1.5 for safety we select a diode having a PIV rating of $\underline{\underline{55V}}$

3.82

The circuit is a full wave rectifier with centre tapped secondary winding. The circuit can be analyzed by looking at v_O^+ and v_O^- separately.







$$v_{D,avg} = \frac{1}{2\pi}\int V_S \sin\phi - 0.7 \, d\phi = 15$$

$$= \frac{2V_S}{\pi} - 0.7 = 15 \qquad \text{assumed } V_S >> 0.7V$$

$$V_S = \frac{15+0.7}{2}\pi = 24.66V$$

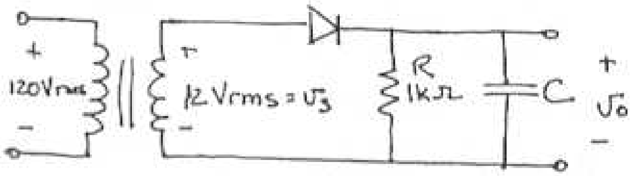
Thus voltage across secondary winding $= 2V_S = \underline{\underline{49.32V}}$

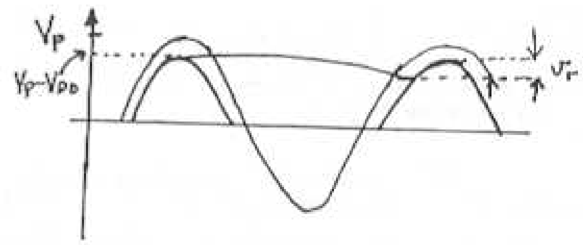
Looking at D_4

$$PIV = V_S - V_O^-$$
$$= V_S + (V_S - 0.7)$$
$$= 2V_S - 0.7$$
$$= \underline{\underline{48.6V}}$$

If choosing a diode, allow a safety margin of $1.5 PIV = \underline{\underline{73V}}$

3.83





(i) $V_r \cong (V_P - V_{DO})\frac{T}{CR}$ Eq. (3.28)

$$0.1(V_P - V_{DO}) = (V_P - V_{DO})\frac{T}{CR}$$

$$C = \frac{1}{0.1 \times 60 \times 10^3} = \underline{\underline{166.7\mu F}}$$

(ii) For $V_r = 0.01(V_P - V_{DO}) = (V_P - V_{DO})\frac{T}{CR}$

$$C = \underline{\underline{1667\ \mu F}}$$

(a)

(i) $$v_{O,avg} = V_P - V_{DO} - \tfrac{1}{2}V_r$$
$$= 12\sqrt{2} - 0.7 - \tfrac{1}{2}(12\sqrt{2} - 0.7)0.1$$
$$= (12\sqrt{2} - 0.7)\left(1 - \frac{0.1}{2}\right)$$
$$= \underline{\underline{15.5V}}$$

ii) $$v_{O,avg} = (12\sqrt{2} - 0.7)\left(1 - \frac{0.01}{2}\right)$$
$$= \underline{\underline{16.19V}}$$

CONT.

(b)

i) Using eq (3.30) we have the conduction angle =

$$\omega \Delta t \cong \sqrt{2V_r/(V_p - V_{DO})}$$

$$= \sqrt{\frac{2 \times 0.1 (V_p - 0.7)}{(V_p - 0.7)}}$$

$$= \sqrt{0.2}$$

$$= 0.447 \text{ rad}$$

$\therefore$ Fraction of cycle for conduction $= \frac{0.447}{2\pi} \times 100$

$$= \underline{\underline{7.1\%}}$$

ii) $\omega \Delta t \simeq \sqrt{2 \times 0.01 \frac{(V_p - 0.7)}{V_p - 0.7}} = 0.141 \text{ rad}$

Fraction of cycle $= \frac{0.141}{2\pi} \times 100 = \underline{\underline{2.25\%}}$

(c)(i) Use Eq (3.31)

$$i_{D,avg} = I_L \left(1 + \pi \sqrt{\frac{2(V_p - V_{DO})}{V_r}}\right)$$

$$= \frac{V_{o,avg}}{R} \left(1 + \pi \sqrt{\frac{2(V_p - V_{DO})}{0.1(V_p - V_{DO})}}\right)$$

$$= \frac{15.5}{10^3} \left(1 + \pi \sqrt{\frac{2}{0.1}}\right)$$

$$= \underline{\underline{233 \text{ mA}}}$$

ii) $i_{D,avg} = \frac{16.19}{10^3} \left(1 + \pi \sqrt{200}\right)$

$$= \underline{\underline{735 \text{ mA}}}$$

NB Text uses $I_L \cong V_p/R = \frac{V_p - V_{DO}}{R}$ but here we used $i_{L,avg} = \frac{V_p - V_{DO} - \frac{1}{2}V_r}{R}$ which is more accurate.

(d) i) $i_{D,peak} = I_L \left(1 + 2\pi \sqrt{\frac{2(V_p - V_{DO})}{V_r}}\right)$

$$= \frac{15.42}{10^3} \left(1 + 2\pi \sqrt{2/0.1}\right)$$

$$= \underline{\underline{449 \text{ mA}}}$$

ii) $i_{D,peak} = \frac{16.19}{10^3} \left(1 + 2\pi \sqrt{\frac{2}{0.01}}\right)$

$$= \underline{\underline{1455 \text{ mA}}}$$

3.84

i) $V_r = 0.1(V_p - V_{DO}) = \frac{(V_p - V_{DO})}{2fCR}$

the factor of 2 accounts for discharge occuring only half of the period $T/2 = \frac{1}{2f}$

$$C = \frac{1}{(2fR)0.1} = \frac{1}{2(60)\,10^3 \times 0.1} = \underline{\underline{83.3\,\mu F}}$$

ii) $C = \frac{1}{2(60)10^3\,0.01} = \underline{\underline{833\,\mu F}}$

(a) i) $V_o = V_p - V_{DO} - \frac{1}{2}V_r$

$$= (V_p - V_{DO})\left(1 - \frac{0.1}{2}\right)$$

$$= (16.27)\left(1 - \frac{0.1}{2}\right)$$

$$= \underline{\underline{15.5 V}}$$

ii) $V_o = (16.27)\left(1 - \frac{0.01}{2}\right) = \underline{\underline{16.19 V}}$

(b)

(i) Fraction of cycle $= \frac{2\omega\Delta t}{2\pi} \times 100$

$$= \frac{\sqrt{2V_r/(V_p - V_{DO})}}{\pi} \times 100$$

$$= \frac{1}{\pi}\sqrt{2(0.1)} \times 100 = \underline{\underline{14.2\%}}$$

CONT.

ii) Fraction of Cycle $= \frac{2\sqrt{2(0.01)}}{2\pi} \times 100$

$= \underline{\underline{4.5\%}}$

(c) Use eq (3.34)

i) $i_{D,avg} = I_L \left(1 + \pi\sqrt{\frac{V_P - V_{DO}}{2V_r}}\right)$

$= \frac{15.5}{1}\left(1 + \pi\sqrt{\frac{1}{2(0.1)}}\right) = \underline{\underline{124.4\,mA}}$

ii) $i_{D,avg} = \frac{16.19}{1}\left(1 + \pi\frac{1}{\sqrt{2(0.01)}}\right) = \underline{\underline{376\,mA}}$

(d) Use eq (3.35)

(i) $\hat{i}_D = I_L\left(1 + 2\pi\frac{1}{\sqrt{2(0.1)}}\right) = \underline{\underline{233\,mA}}$

ii) $\hat{i}_D = I_L\left(1 + 2\pi\frac{1}{\sqrt{0.02}}\right) = \underline{\underline{735\,mA}}$

3.85

i) $V_r = 0.1(V_P - V_{DO}\times 2) = \frac{V_P - 2V_{DO}}{2fCR}$

discharge occurs only over $\frac{1}{2}T = \frac{1}{2f}$

$C = \frac{(V_P - 2V_{DO})}{(V_P - 2V_{DO})}\frac{1}{2(0.1)fR} = \underline{\underline{83.3\,\mu F}}$

ii) $C = \frac{1}{2(0.01)fR} = \underline{\underline{833\,\mu F}}$

(b) i) Fraction of cycle $= \frac{2\omega\Delta t}{2\pi} \times 100$

$= \frac{\sqrt{2(0.1)}}{\pi} \times 100 = \underline{\underline{14.2\%}}$

ii) Fraction of cycle $= \frac{\sqrt{2(0.01)}}{\pi} \times 100 = \underline{\underline{4.5\%}}$

(c) i)

$i_{D,avg} = \frac{14.79}{1}\left(1 + \pi\sqrt{\frac{1}{0.2}}\right) = \underline{\underline{119\,mA}}$

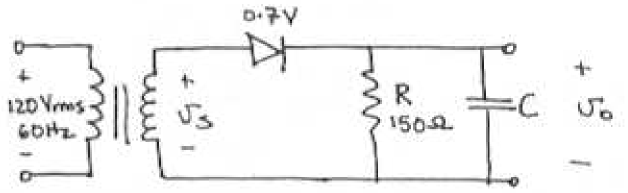
ii) $i_{D,avg} = \frac{15.49}{1}\left(1 + \pi/\sqrt{0.02}\right) = \underline{\underline{356\,mA}}$

(d)

(i) $\hat{i}_D = \frac{14.79}{1}\left(1 + 2\pi\sqrt{\frac{1}{0.2}}\right) = \underline{\underline{223\,mA}}$

(ii) $\hat{i}_D = \frac{15.49}{1}\left(1 + 2\pi\sqrt{\frac{1}{0.02}}\right) = \underline{\underline{704\,mA}}$

3.86



$v_{O,peak} = V_P - V_{DO} = 16$

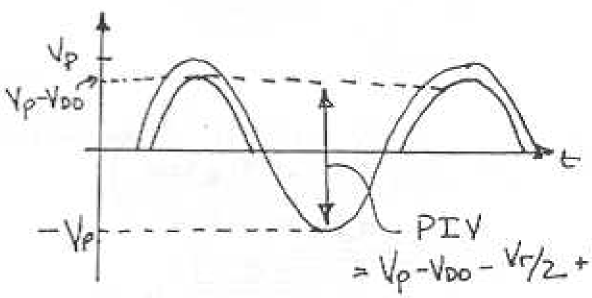
$V_P = 16.7\,V$

$V_{rms} = \frac{16.7}{\sqrt{2}} = 11.8$

(b) $V_r = (V_P - V_{DO})\frac{T}{CR}$ Eq (3.28)

$2 = \frac{16}{60 \times C \times 150}$

$C = 889\,\mu F$



PIV $= V_P - V_{DO} - V_r/2 + V_P$

$= V_{O,avg} + V_P$

$= 15 + 16.7$

$= 31.7\,V$

For a 50% safety margin PIV $= 1.5 \times 31.7$

$= \underline{\underline{47.6\,V}}$

CONT.

(d)

$$i_{D,avg} = I_L\left(1+\pi\sqrt{\frac{2(V_p - V_{DO})}{V_r}}\right)$$

using $I_L = \frac{v_{O,avg}}{R} = \frac{15}{R}$ we have

$$i_{D,avg} = \frac{15}{150}\left(1+\pi\sqrt{\frac{2(16)}{2}}\right)$$

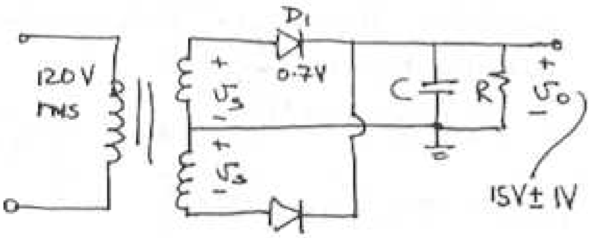
$$= \underline{\underline{1.36A}}$$

(e) $$i_{D,peak} = I_L\left(1+2\pi\sqrt{\frac{2(V_p - V_{DO})}{V_r}}\right)$$

$$= \frac{15}{150}\left(1+2\pi\sqrt{\frac{2(16)}{2}}\right)$$

$$= \underline{\underline{2.61\ A}}$$

3.87



(a) $\hat{v}_O = 16V$

$\therefore\ \hat{v}_S = 16 + V_{DO} = 16.7V$

RMS Voltage across secondary

$$= \frac{2\times 16.7}{\sqrt{2}}$$

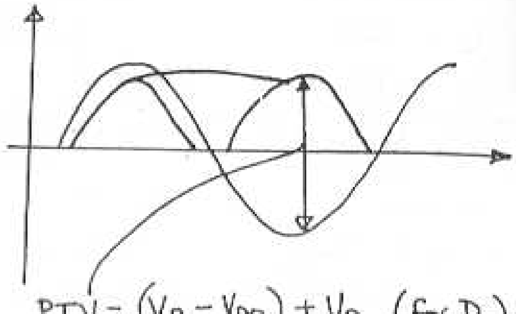
$$= \underline{\underline{23.6V}}$$

(b) Using Eq. (3.28)

$$V_r = \frac{V_p}{2fCR} = \frac{16}{2\times 60\times C\times 150} = 2$$

$$C = \underline{\underline{444.4\ \mu F}}$$

(c)



$$PIV = (V_p - V_{DO}) + V_p \quad (\text{for } D_1)$$

$$= 2(16.7) - 0.7$$

$$= \underline{\underline{32.7V}}$$

$\therefore$ Using a 50% margin

$$PIV = 1.5(32.7) = \underline{\underline{49V}}$$

(d) Using Eq (3.34)

$$i_{D,avg} = I_L\left(1+\pi\sqrt{V_p/2V_r}\right)$$

$$= \frac{15}{150}\left(1+\pi\sqrt{\frac{16}{2\times 2}}\right)$$

$$= \underline{\underline{0.73\ A}}$$

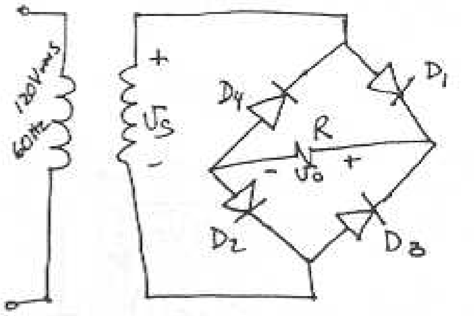
(e) Using Eq. (3.35)

$$i_{D,max} = I_L\left(1+2\pi\sqrt{V_p/2V_r}\right)$$

$$= \frac{15}{150}\left(1+2\pi\sqrt{\frac{16}{2\times 2}}\right)$$

$$= \underline{\underline{1.36A}}$$

3.88



$v_O = 15 \pm 1V$, $R = 150\Omega$

CONT.

$\hat{v}_o = 16V$

$\hat{v}_s = 16 + 2V_{DO} = 17.4V$

RMS Secondary Voltage $= \frac{17.4}{\sqrt{2}} = \underline{\underline{12.3V}}$

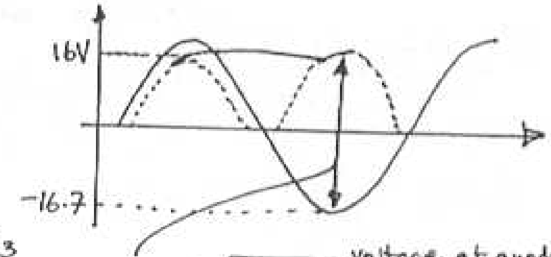
(b) $v_r = \dfrac{V_P}{2fCR}$

$$2 = \frac{16}{2 \times 60 \times C \times 150}$$

$C = \underline{\underline{444.4\,\mu F}}$

note: we got the same value for C as the full wave rectifier as discharge is over the same amt of time T/2 and the peak is the same – 16V.

c)



for D_3

$PIV = 16 - (-V_{DO})$ — Voltage at anode of $D_3 = 0 - V_{DO} = -V_{DO}$

$= 16 + 0.7$

$= 16.7V$

Allowing a 50% margin $= 16.7 \times 1.5$

$= \underline{25V}$

(d) Using 3.34

$$i_{D,avg} = I_L\left(1 + \pi\sqrt{V_P/2V_r}\right)$$

$$= \frac{15}{150}\left(1 + \pi\sqrt{16/2\times 2}\right)$$

$$= \underline{0.73A}$$

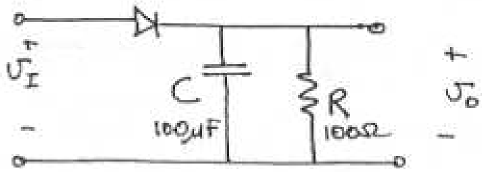
(e) Using (3.35)

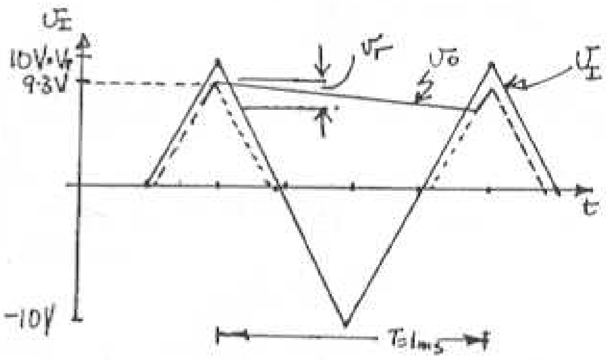
$$i_{D,max} = I_L\left(1 + 2\pi\sqrt{V_P/2V_r}\right)$$

$$= \frac{15}{150}\left(1 + 2\pi\sqrt{16/2\times 2}\right)$$

$$= \underline{1.36A}$$

3.89





During the diode's off interval, the capacitor discharges through the resistor R according to:

$$v_o = 9.3\,e^{-t/RC} \cong 9.3\left(1 - T/CR\right)$$

$$\therefore v_r = 9.3 - 9.3\left(1 - T/CR\right)$$

$$= 9.3T/CR$$

$$= \frac{9.3}{fCR} \qquad \text{NB this is Eq(3.38)}$$

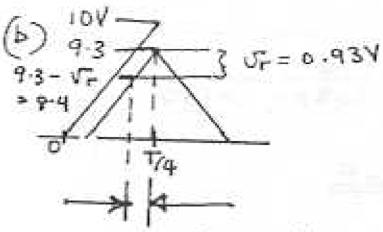
$$= 0.93V$$

$$v_{o,avg} = V_o - V_{DO} - \tfrac{1}{2}V_r$$

$$= 9.3 - \tfrac{1}{2}0.93$$

$$= \underline{8.84V}$$

(b)



$$\Delta t \Rightarrow \frac{10}{T/4} = \frac{0.93}{\Delta t}$$

$\Delta t = 0.02325T$

$= \underline{\underline{0.02325\ ms}}$

CONT.

(c) $\therefore$ Charge gained during conduction = Charge lost during discharge

$$i_{c,avg}\,\Delta t = C\,V_r$$

$$i_{c,avg} = \frac{CV_r}{\Delta t} = \frac{100\times10^{-6}\times 0.93}{0.02325\times10^{-3}}$$

$$= 4.0\text{ A}$$

$$i_{D,avg} \cong i_{c,avg} + \underbrace{i_{L,avg}}_{V_{O,avg}/R}$$

$$\cong 4.0 + \frac{8.84}{100} = \underline{4.09\text{A}}$$

(d) $$i_{c,max} = C\left.\frac{\delta V_O}{\partial t}\right|_{\text{at onset of conduction}}$$

$$= C\frac{\delta V_r}{\partial t}$$

$$= 100\times10^{-6}\times 40\times10^{3}$$

$$= \underline{4\text{A}}$$

$$i_{D,max} = i_{c,max} + i_{L,max}$$

$$= 4 + V_{O,max}/100$$

$$= 4 + 9.3/100$$

$$= 4.09\text{ A}.$$

Note that in this case $i_{D,avg} = i_{D,max}$ owing to the linear input (i_c is constant and i_L is approximately constant).

3.90

Refer to Fig P3.82 and let capacitor C be connected across each of the load resistors R. The two supplies, v_O^+ and v_O^- are identical. Each is a full-wave rectifier similar to that based on the centre-tapped-transformer circuit. For each supply, the dc output is 15 V and the ripple is 1V peak-to-peak. Thus $v_O = 15 \pm \frac{1}{2}$V. It follows that the peak value of v_S must be $15.5 + 0.7 = 16.2$V.

$\therefore$ Voltage across secondary $= 2(16.2) = 32.4$ V

RMS across secondary $= \frac{32.4}{\sqrt{2}} = \underline{22.9\text{ V}_{rms}}$

Turns Ratio $= \frac{120}{22.9} = 5.24:1$

Use Eq. (3.35) to find

$$i_{D,max} = I_L\left(1 + 2\pi\sqrt{V_P/2V_r}\right)$$

$$= 0.2\left(1 + 2\pi\sqrt{15.5/2}\right)$$

$$= \underline{3.70\text{A}}$$

$$V_r = \frac{V_P}{2fCR} = 1 \qquad \text{Eq. (3.28)}$$

DISCHARGE OCCURS OVER $T/2 = \frac{1}{2f}$

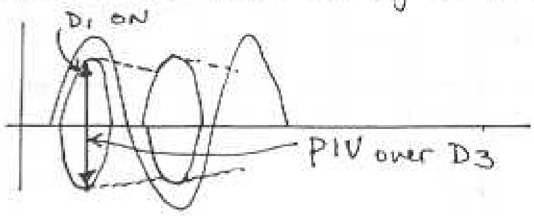
$$\Rightarrow C = \frac{15.5}{2\times60\times75}$$

where $200\text{mA} = \frac{15}{R}$

$R = \frac{15}{0.2} = 75\Omega$

$$= \underline{1722\,\mu\text{F}}$$

Consider D_3 when looking at PIV



CONT.

$PIV = \hat{v}_O + \hat{v}_S$

$= 15.5 + 16.2 = 31.7\,V.$

Allowing for 50% safety margin

$PIV = 1.5 \times 31.7 = \underline{47.6\,V}$

Use Eq (3.34) to find

$i_{D,avg} = I_L\left(1 + \pi\sqrt{V_P/2V_r}\right)$

$= 0.2\left(1 + \pi\sqrt{15.5/2}\right)$

$= \underline{1.95\,A}$

3.91

$v_O = v_I(1 + R/R)$

$= 2v_I$ when the diode is conducting.

(a) $v_I = +\underline{1V}$ $v_O = \underline{2V}$ $v_A = \underline{1.7V}$ $v_- = v_I = \underline{1V}$

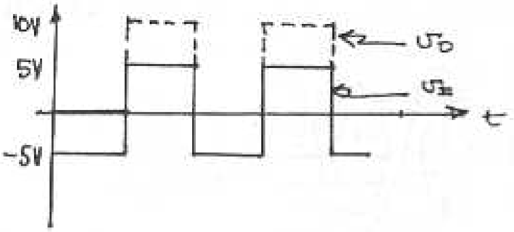
b) $v_I = \underline{2V}$ $v_O = \underline{4V}$ $v_A = \underline{4.7V}$ $v_- = \underline{2V}$

c) $v_I = -1V$ $v_A = \underline{-12V}$ ~ diode is cut off

$v_O = \underline{0V}$

$v_- = \underline{0V}$

d) $v_I = -2\,V$ $v_A = -\underline{12V}$ $v_O = \underline{0V}$ $v_- = \underline{0V}$



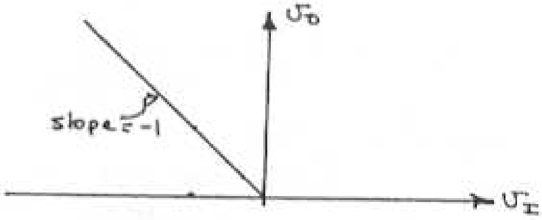
$v_{O,avg} = \underline{5V}$

3.92

$v_I > 0$ D_1 conducts D_2 cutoff

$v_I < 0$ D_1 cut off

D_2 conducts ~ $\frac{v_O}{v_I} = -1$



a) $v_I = 1V$ $v_O = \underline{0V}$

$v_A = \underline{-0.7V}$ - keeps D_2 off so no current flows through R

$\Rightarrow v_- = \underline{0V}$ - virtual gnd as fdbk is closed through D_1

(b) $v_I = 2V$

$v_O = \underline{0V}$

$v_A = -\underline{0.7V}$

$v_- = \underline{0V}$

(c) $v_I = -1V$

$v_O = \underline{1V}$

$v_A = \underline{1.7V}$

$v_- = 0\,V$ ~ virtual gnd as negative feedback is closed through R.

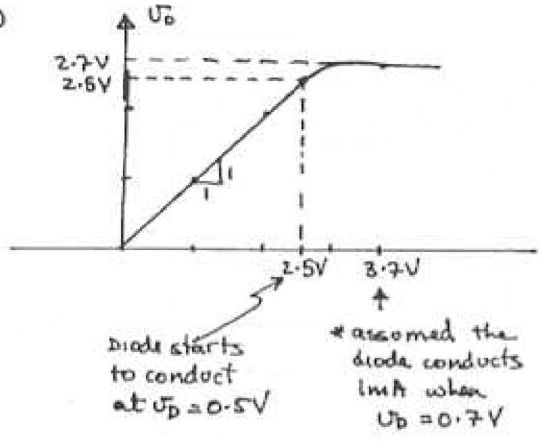
(d) $v_I = -2V \Rightarrow v_O = \underline{2V}$

$v_A = \underline{2.7V}$

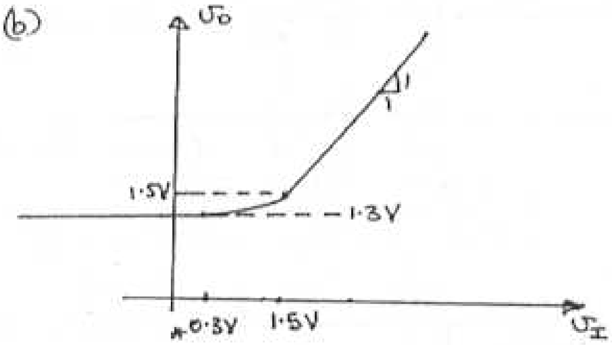
$v_- = \underline{0V}$

3.93

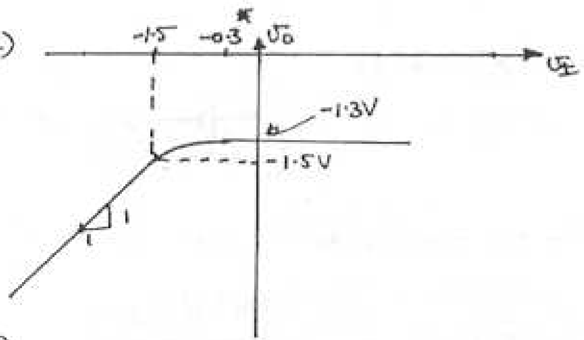
(a)



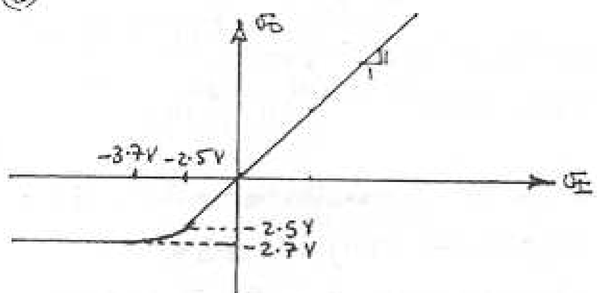
(b)



(c)



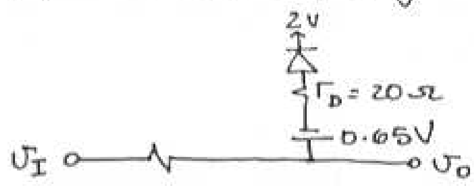
(d)

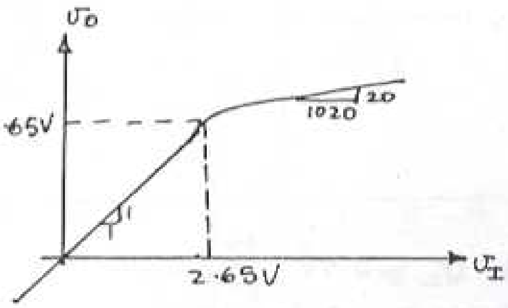


3.94

(a) $v_I < 2.65V$ ~ the diode is off and the circuit reduces to:

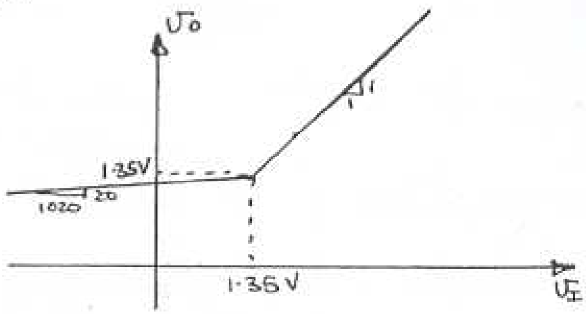
With the diode conducting we have:



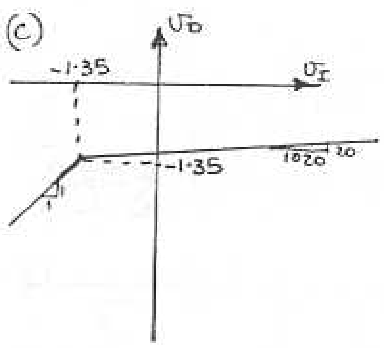


Similarily for (b), (c,) (d)

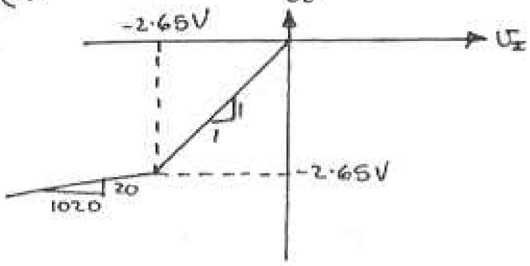
(b)



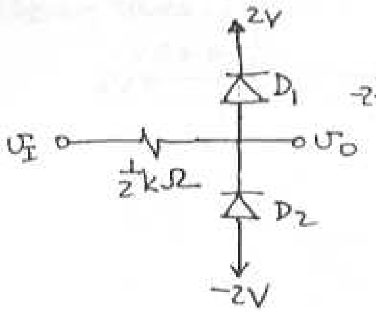
(c)



(d)



3.95



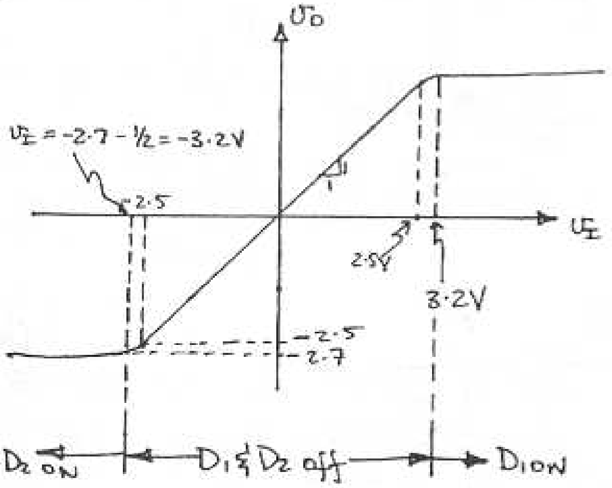
$-2.5 \leq v_I \leq 2.5$
~ both D_1 & D_2 off
~ $v_o = v_I$

For $v_I \geq 2.5V$ ~ D_1 on

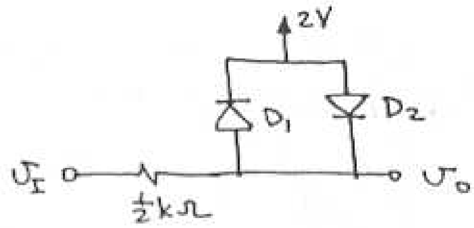
$v_{D1} = 0.7$ at $i_{D1} \geq 1mA$

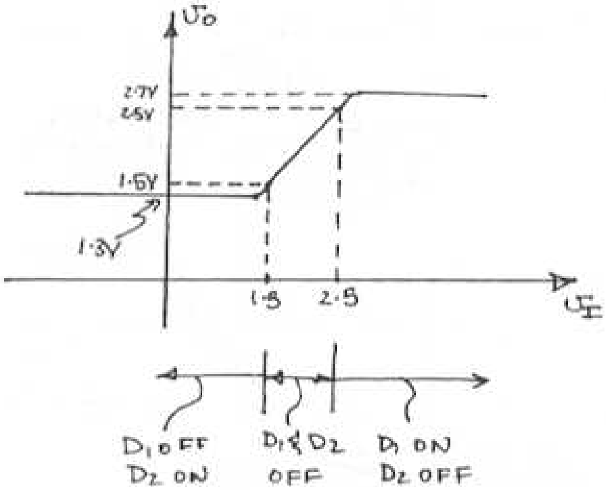
$v_O = 2.7V$ at $v_I = 2.7 + \frac{1}{2} \times 1$

$= \underline{\underline{3.2V}}$



3.96





3.97

For each diode

For the zener diode

$8.2 = V_{Z0} + 10 \times 10^{-3} \times 20$

$V_{Z0} = 8.0V$

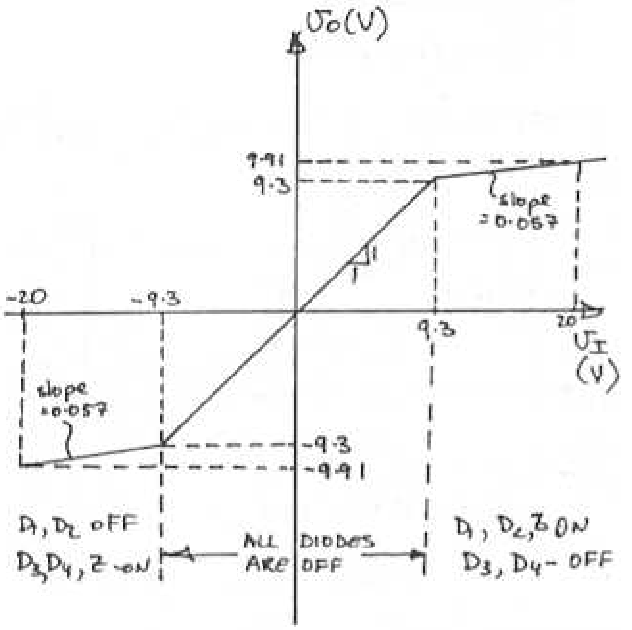
The limiter thresholds are

$\pm(2 \times 0.65 + 8.0) = \pm 9.3V$

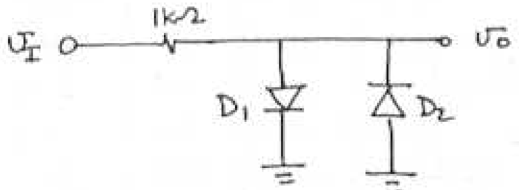
For $v_I > 9.3$ (as well as for $v_I < -9.3$)

$$\frac{\partial v_O}{\partial v_I} = \frac{r_{D1} + r_Z + r_{D2}}{1k\Omega + r_{D1} + r_Z + r_{D2}} = \frac{3(20)}{1k\Omega + 3(20)} = 0.057 \frac{V}{V}$$

CONT.



3.98



For D_1

Given $\frac{i_D}{1\text{mA}} = e^{\frac{v_O - 0.7}{nV_T}}$

$(v_O - 0.7) = nV_T \ln(i_D/1\text{mA})$

$= 0.1 \log\left(\frac{i_D}{10^{-3}}\right)$ ← ∴ can find v_O from i_D

$\therefore i_D = 10^{-3} \times 10^{\frac{v_O - 0.7}{0.1}}$

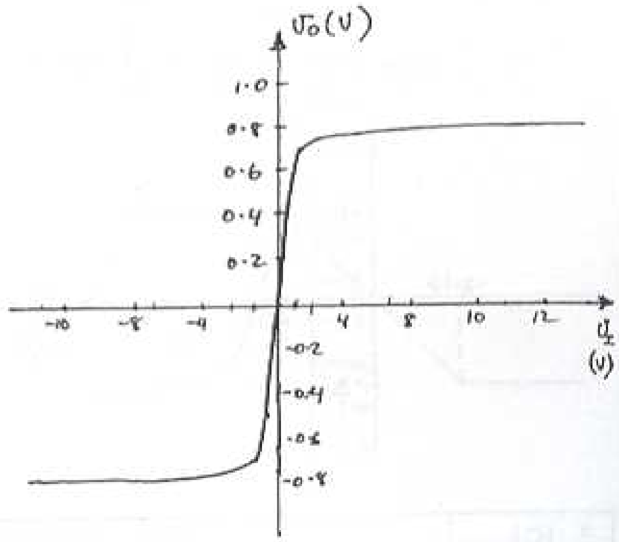
$= 10^{-3} \times 10^{10(v_O - 0.7)}$

& $v_I = v_O + i_D \times 10^3$

$= v_O + 10^{10(v_O - 0.7)}$

for D_2: $v_I = v_O - 10^{-10(v_O - 0.7)}$

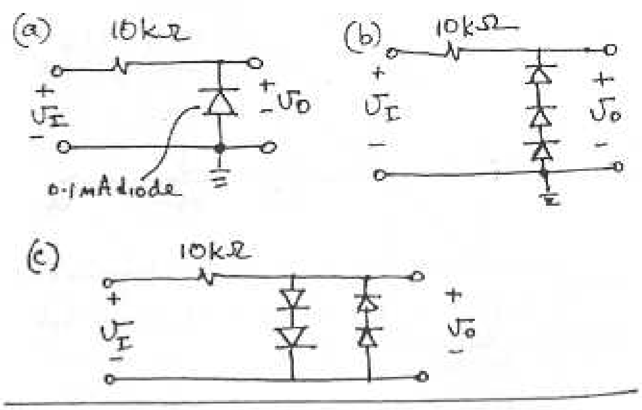
v_O (V)	v_I (V)	
0.5	0.510	D_1 ON
0.6	0.7	
0.7	1.7	
0.8	10.7	
0	0	
-0.5	-0.51	D_2 ON
-0.6	-0.7	
-0.7	-1.7	
-0.8	-10.7	



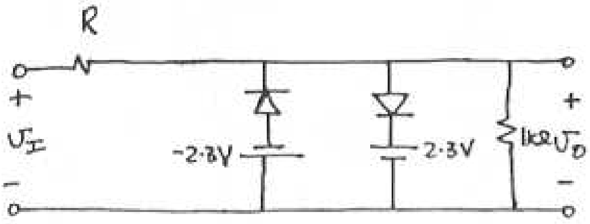
The limiter is fairly hard with a gain $K \cong 1$

$L_+ \cong \underline{\underline{0.8V}}$, $L_- \cong \underline{\underline{-0.8V}}$

3.99



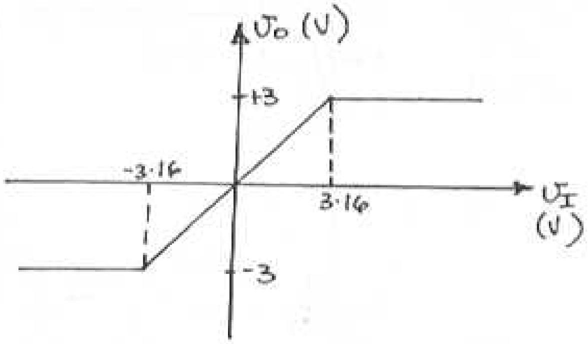
3.100



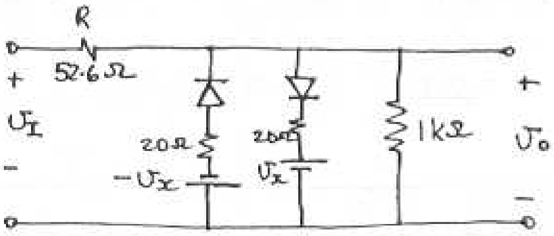
In the limiting region

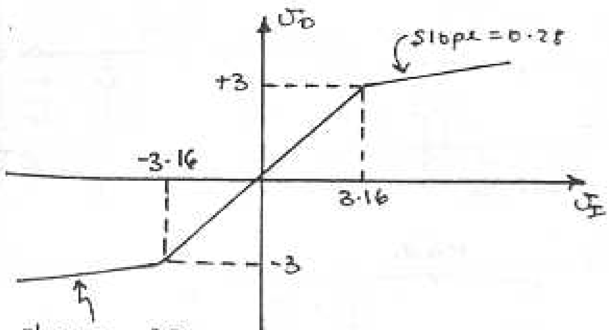
$$\frac{v_O}{v_I} = \frac{1000}{1000+R} \geq 0.95$$

$R \leq \underline{52.6\ \Omega}$



3.101

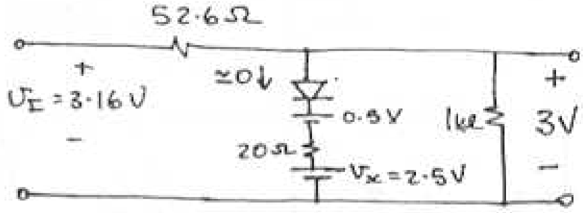




$$\text{slope} = \frac{20}{20+52.6} = 0.28$$

$$\frac{0.2}{10 \times 10^{-3}} = 20\ \Omega$$

At the verge of limiting in the positive direction we have :-



For $v_I = 10V$

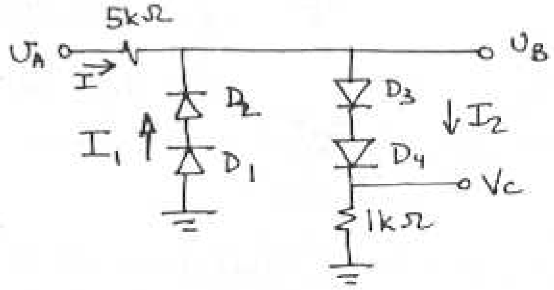
$$v_O \simeq 3 + 0.28(10 - 3.16)$$

$$= \underline{4.9V}$$

For $v_I = -10V$

$$v_O = \underline{-4.9V}$$

3.102



$$v_D = 0.7 + 0.1 \log (i_D / 0.1)$$

(a) For $v_A > 0$ $\quad D_1, D_2$ off $\Rightarrow I_1 = 0$

$$I = I_2 = v_C / 1k\Omega \qquad v_A = V_B + I_2 5$$

(b) For $v_I < 0$ $\quad D_3, D_4$ off $\Rightarrow v_C = 0$

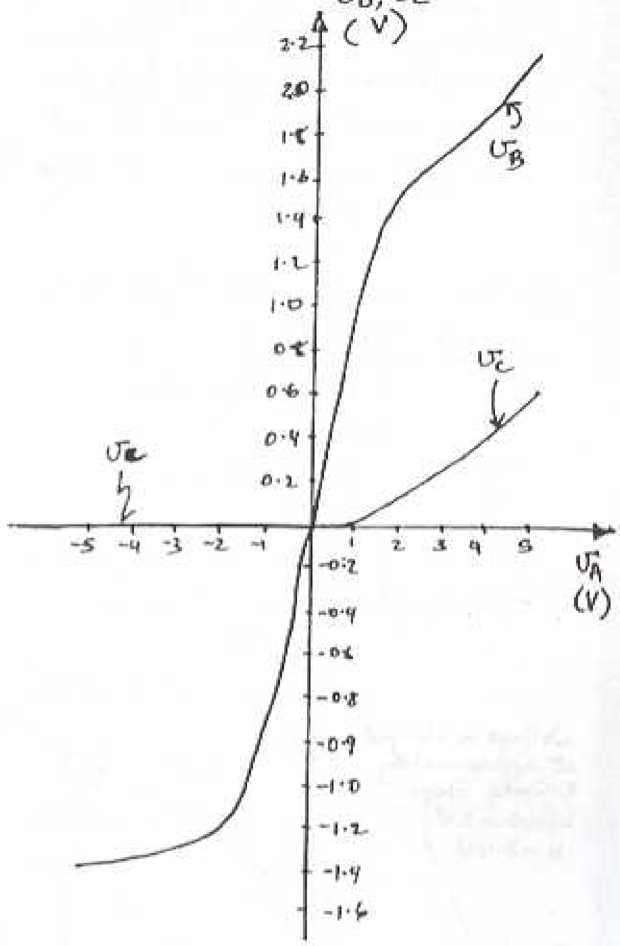
$$I = -I_1 \qquad v_B = -(v_{D1} + v_{D2})$$

$$v_A = -(V_B + 5I_1)$$

CONT.

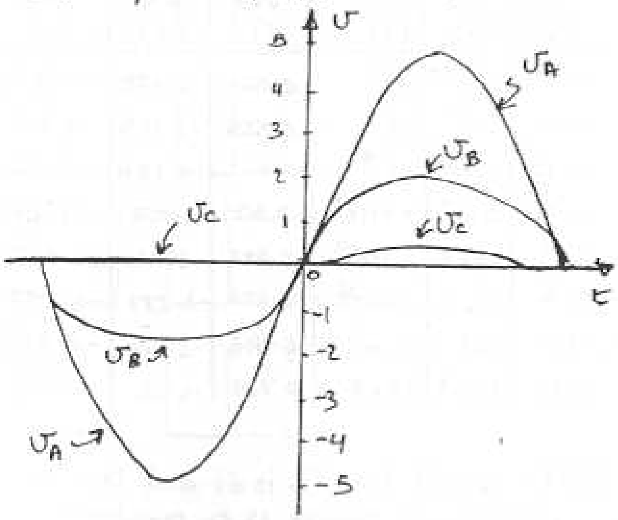
(a) List of points for $v_A > 0$

v_C (V)	I_2 (mA)	v_{D3}, v_{D4} (V)	$v_B = v_C + V_{D3} + V_{D4}$ (V)	v_A (V)
0.0001	0.0001	0.4	0.8	0.8
0.001	0.001	0.5	1.00	1.01
0.01	0.1	0.6	1.21	1.26
0.1	0.1	0.7	1.50	1.90
0.2	0.2	0.73	1.66	2.66
0.3	0.3	0.75	1.80	3.30
0.4	0.4	0.76	1.92	3.92
0.5	0.5	0.77	2.04	4.54
0.6	0.6	0.78	2.16	5.16

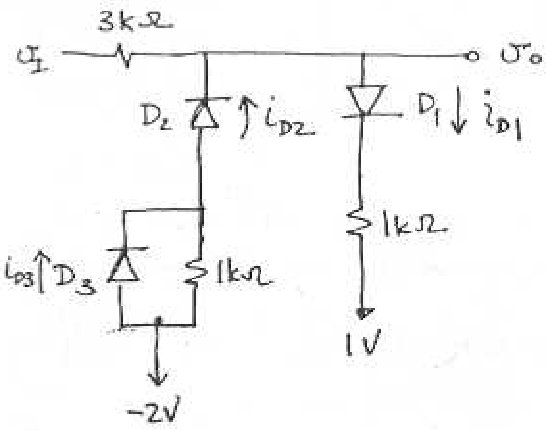


(b) List of Points for $v_A < 0$

I_1 (mA)	v_{D1}, v_{D2} (V)	v_B (V)	v_A (V)
0.0001	0.4	-0.80	-0.80
0.001	0.5	-1.00	-1.01
0.01	0.6	-1.20	-1.25
0.10	0.7	-1.40	-1.90
0.20	0.73	-1.46	-2.46
0.30	0.75	-1.50	-3.00
0.40	0.76	-1.52	-3.52
0.50	0.77	-1.54	-4.04
0.60	0.78	-1.56	-4.56
0.70	0.785	-1.57	-5.07



3.103



At currents $i_{D1} > 1mA$, $v_{D1} \simeq 0.7V$

Let $v_{D1} = 0.71V$ $\quad v_I > 5.7V$

CONT.

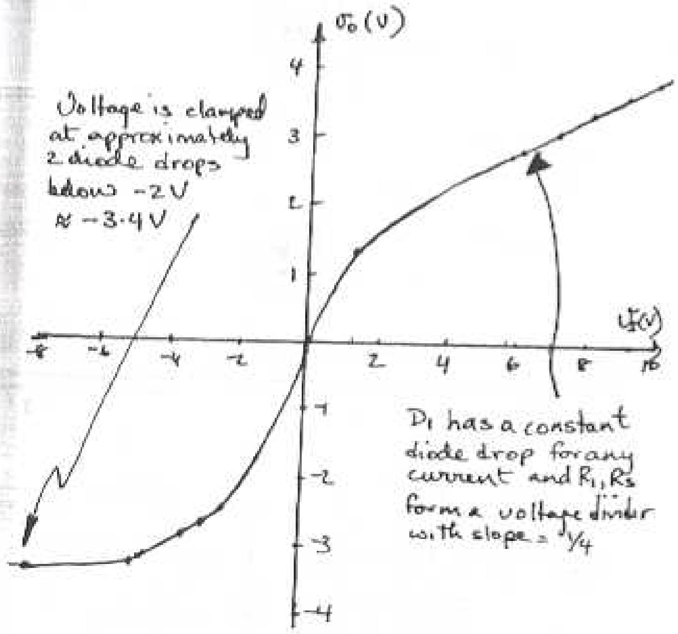
$$v_O = 1.71 + i_{D1} \times 1\,k\Omega$$

$$= 1.71 + \left(\frac{v_I - 1.71}{4}\right) \times 1$$

$$= \frac{v_I}{4} + 1.2825 \qquad \text{HB slope} = 1/4$$

$\therefore$ for $v_I > 5V$ slope $v_O/v_I \approx 1/4$

v_I (V)	v_O (V)
5.8	2.7325
6.0	2.7825
7.0	3.0325
8.0	3.2825
9.0	3.5325
10.0	3.7825



where points for $-8 \leq v_I \leq 6$ V are calculated as shown below:

$i_D = 1mA$ at $v_D = 0.7V \qquad n = 1$

$$i_D = I_S\, e^{0.7/0.025} = 10^{-3}$$

$$I_S = 6.914 \times 10^{-16}\ A.$$

$\therefore$ For Diodes use $\underline{i_D = 6.914 \times 10^{-16}\, e^{v_D/0.025}}$

D_1 conducting $\quad i_{D2} = 0$

i_{D1} (A)	v_{D1} (V)	v_O (V)	$v_I = (4k)i_{D1} + v_{D1} + 1$ (V)
10^{-10}	0.297	1.297	1.297
10^{-6}	0.527	1.528	1.5313
10^{-5}	0.584	1.595	1.625
10^{-4}	0.64	1.742	2.042
10^{-3}	0.70	2.7	5.7
0.2×10^{-2}	0.74	6.74	12.74
10^{-2}	0.758	11.75	41.75

even at small i_{D1} $v_O > 1V$, $v_O \approx v_I$ since $i_{D1} \approx 0$

For the D_2, D_3 arm conducting use the following equations:
Note $v_I < -2.5V$

Starting with a value for V_A (1) we have

$$V_{D3} = V_A + 2$$

$$i_{D3} = I_S\, e^{v_{D3}/0.025} \qquad (2)$$

$$i_{D2} = i_{D3} + \frac{V_A + 2}{1} \qquad (3)$$

$$v_{D2} = 0.025 \ln\left(\frac{i_{D2}}{6.914 \times 10^{-16}}\right) \qquad (4)$$

$$v_O = v_A - v_{D2} \qquad (5)$$

$$v_I = v_O - i_{D2} \times 3\,k\Omega \qquad (6)$$

(1) v_A (V)	(2) i_{D3} (A)	(3) i_{D2} (A)	(4) v_{D2} (V)	(5) v_O (V)	(6) v_I (V)
−2.001	7×10^{-16}	10^{-6}	0.527	−2.528	−2.531 (A)
−2.01	10^{-15}	10^{-5}	0.585	−2.595	−2.625
−2.10	3.8×10^{-14}	10^{-4}	0.642	−2.724	−3.024
−2.20	2×10^{-12}	0.2×10^{-3}	0.659	−2.859	−3.459
−2.5	33 μA	0.5×10^{-3}	0.682	−3.128	−4.628 (B)
−2.6	18 μA	0.6×10^{-3}	0.687	−3.287	−5.087
−2.7	1 mA	1.7×10^{-3}	0.713	−3.413	−8.516 (C)
−2.71	1.5 mA	2.2 mA	0.720	−3.43	−10

(A) for small i_{D2}, D_3 is off and D_2 is on
$\therefore$ i_2 flows through $1k\Omega$ resistor

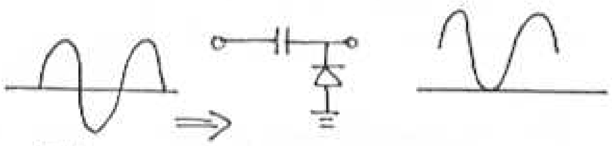
CONT.

(B) 0.5 V drop across D_3 causes D_3 to start to conduct

(C) $V_A = -2.7V$

The 0.7 voltage across D_3 clamps the voltage across R_3 so that D_3 control the current i_{D2}

3.104



10V rms

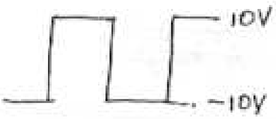
$10\sqrt{2}$ V, peak

Average (dc) value of output $= 10\sqrt{2}$

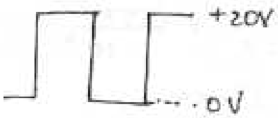
$= \underline{\underline{14.14V}}$

3.105

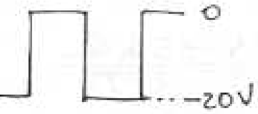
(a)



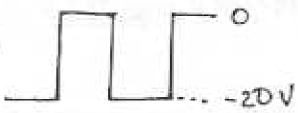
(b)



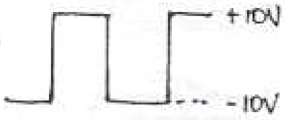
(c)



(d)



(e)

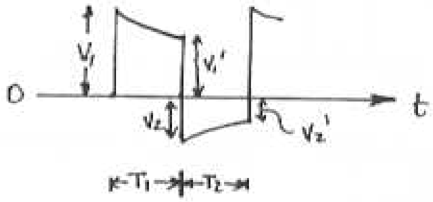


(f) Here there are two different time constants involved. To calculate the output levels we shall exagerate the discharge and charge waveforms.

During T_1 $v_O = V_1 e^{-t/RC}$

At $t = T_1 = T$ $v_O = V_1'$

$= V_1 e^{-T/RC}$



where for $T \ll CR$

$V_1' \cong V_1(1 - T/CR) = V_1(1-\alpha)$ where $\alpha \ll 1$

During the period T_2

$|v_O| = |V_2| e^{-t/CR/2}$

at the end of T_2 $t = T$, $v_O = V_2'$

where

$V_2' = |V_2| e^{-T/CR/2}$

$\cong |V_2|\left(1 - \frac{T}{CR/2}\right) = |V_2|(1-2\alpha)$

Now $V_1' + |V_2| = 20 \Rightarrow V_1 + |V_2| - \alpha V_1 = 20$ ①

and $|V_2'| + V_1 = 20 \Rightarrow V_1 + |V_2| - 2\alpha|V_2|$
$= 20$ ②

from ① & ② we find that

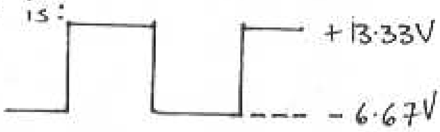
$V_1 = 2V_2$

Then using ① and neglecting αV_1 yields

$3|V_2| = 20 \Rightarrow |V_2| = \underline{\underline{6.67V}}$

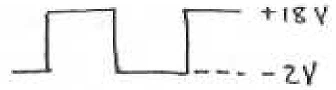
$V_1 = \underline{\underline{13.33V}}$

The result is:

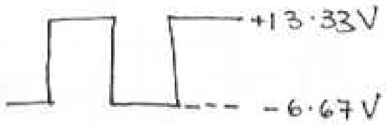


CONT.

(g)



(h) Using a method similar to that employed for case (f) above we obtain.



3.106

$$n_i^2 = BT^3 e^{-E_G/kT}$$

$B = 5.4\times10^{31}$

$E_G = 1.12\,eV$ for silicon

$k = 8.62\times10^{-5}\ \frac{eV}{K}$

Fraction of ionized atoms $= n_i / 5\times10^{22}$; $T = 273 + x°C$

T (K)	n_i ($\frac{\text{carriers}}{cm^3}$)	Fraction of ionized atoms.
203	2.68×10^{5}	5.37×10^{-18}
273	1.53×10^{9}	3.07×10^{-14}
293	8.65×10^{9}	1.73×10^{-13}
373	1.44×10^{12}	2.89×10^{-11}
398	4.75×10^{12}	9.51×10^{-11}

3.107

(a) intrinsic silicon

$$\rho = \frac{1}{q(p\mu_p + n\mu_n)}$$

$$= \frac{1}{1.6\times10^{-19}(1.5\times10^{10}\times480 + 1.5\times10^{10} + 1350)}$$

$$= 2.28\times10^{5}\ \Omega\cdot cm$$

$$R = \rho\, L/A = 2.28\times10^{5}\ \frac{10\times10^{-4}}{\underbrace{3.4\times10^{-4}}_{10^4/3}} = \underline{\underline{7.6\times10^{9}\ \Omega}}$$

(b)

$$\rho = \frac{1}{q(p_{no}\mu_p + n_{no}\mu_n)}$$

$$= \frac{1}{(1.6\times10^{-19})\left(\frac{(1.5\times10^{10})^2}{10^{16}}\times\underbrace{\frac{1200}{2.5}}_{\mu_p} + 10^{16}\times1200\right)}$$

$$= 0.521\ \Omega\cdot cm$$

$$R = \rho \times 10^4/3 = \underline{\underline{1.74\,k\Omega}}$$

(c) n doped $N_D = n_{no} = 10^{18}$

$n_{po} = n_i^2/n_{no}$

$$\rho = \frac{1}{1.6\times10^{-19}\left(\frac{(1.5\times10^{10})^2}{10^{18}}\times\frac{1200}{2.5} + 10^{18}\times1200\right)}$$

$$= 5.21\times10^{-3}\ \Omega\cdot cm$$

$$R = \rho\,\frac{10^4}{3} = \underline{\underline{17.4\ \Omega}}$$

(d) P doped $N_A = p_{po} = 10^{10}/cm^3$

$$\rho = \frac{1}{q(p_{po}\mu_p + n_{po}\mu_n)}$$

$$= \frac{1}{1.6\times10^{-19}\left(10^{10}\times\frac{1200}{2.5} + \frac{(1.5\times10^{10})^2}{10^{10}}\times1200\right)}$$

$$= 2.05\times10^{4}\ \Omega\cdot cm$$

$$R = \rho\,\frac{10^4}{3} = \underline{\underline{68.35\,k\Omega}}$$

(e) $R = 2.8\times10^4/3 = \underline{\underline{0.933\ \Omega}}$

3.108

$$p_{no} = \frac{n_i^2}{N_D} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^{4}\ /cm^3$$

CONT.

$$\frac{dP}{dx} = \frac{1000P_{no} - P_{no}}{W}$$

$$\cong \frac{2.25\times10^{-1}}{5\times10^{-4}}$$

$$J_P = -qD_P\frac{dP}{dx}$$

$$= -1.6\times10^{-19} \times 12 \times \frac{-2.25\times10^{-1}}{5\times10^{-4}}$$

$$= \underline{\underline{8.64\times10^{-8}\ A/cm^2}}$$

3.109

$$U_{drift,p} = \mu_p E$$

$$= 480\frac{cm^2}{V\cdot s} \times \frac{1}{10^{-3}}\frac{V}{cm}$$

$$= \underline{\underline{4.8\times10^5\ cm/s}}$$

$$U_{drift,n} = \mu_n E$$

$$= 1350\frac{cm^2}{V\cdot s} \times \frac{1}{10^{-3}}\frac{V}{cm}$$

$$= \underline{\underline{1.35\times10^6\ cm/s}}$$

3.110

$$J_{drift} = q(n\mu_n + p\mu_p)E \quad A/cm^2$$

$$I_{drift} = q(n\mu_n + p\mu_p)E\cdot A$$

$$= 1.6\times10^{-19}(10^5\cdot1350 + 10^{15}\cdot480)\times \frac{1}{10^{-3}}\times(5\times4\times10^{-8})$$

$$= \underline{\underline{15.36\ \mu A}}$$

3.111

$$E = \frac{1V}{10\times10^{-4}} = \frac{1}{10^{-3}}\frac{V}{cm}$$

$n_{no} = N_D \qquad P_{no} = \frac{n_i^2}{N_D}$ assume $P_{no} \ll n_{no}$

$$\therefore J_{drift} = q(\mu_n n + \mu_p P)E = \frac{10^{-3}}{10^{-8}}\frac{A}{cm^2}$$

$$\cong 1.6\times10^{-19}(1350N_D)\frac{1}{10^{-3}} = 10^5$$

$$N_D = 4.6\times10^{17}\ /cm^3$$

3.112

$P_{po} = N_A = 10^{16}\ /cm^3$ at all temperatures

At $25^\circ C \cong 300K$

$$n_i^2 = BT^3 e^{-E_G/kT}$$

$$= 5.4\times10^{31}(300)^3 e^{-1.12/8.62\times10^{-5}(300)}$$

$$= 2.26\times10^{20}$$

$$n_{po} = \frac{n_i^2}{N_A} = \underline{\underline{2.26\times10^4\ /cm^3}}$$

At $125^\circ C \cong 400K$

$$n_i^2 = 5.4\times10^{31}(400)^3 e^{1.12/8.62\times10^{-5}(400)}$$

$$= 2.7\times10^{25}$$

$$n_{po} = \frac{n_i^2}{N_A} = \underline{\underline{2.7\times10^9\ /cm^3}}$$

3.113

DOPING CONCENTRATION	μ_n $cm^2/V\cdot s$	μ_p $cm^2/V\cdot s$	D_n cm^2/s	D_p cm^2/s
INTRINSIC	1350	480	34	12
10^{16}	1100	400	28	10
10^{17}	700	260	18	6
10^{18}	360	150	9	4

where $D_n = V_T \mu_n = \underline{\underline{0.025\,\mu_n}}$

$D_p = V_T \mu_p = \underline{\underline{0.025\,\mu_p}}$

3.114

$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.025 \ln\left(\frac{10^{16}\,10^{16}}{10^{20}}\right)$

$= 1.27\,V$

$W_{dep} = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_0}$

$= \sqrt{\frac{2(11.7)\,8.85\times10^{-14}}{1.6\times10^{-19}}\left(\frac{1}{10^{16}} + \frac{1}{10^{16}}\right) 1.27}$

$= 57\times10^{-6}$ cm

$= \underline{\underline{0.57\,\mu m}}$

$\frac{x_n}{x_p} = \frac{N_A}{N_D} \Rightarrow x_n = x_p = \underline{\underline{0.28\,\mu m}}$

$q_J = q_N = q_P = q\frac{N_A N_D}{N_A + N_D} A\, W_{dep}$

$= \underline{\underline{45.6\times10^{-15}\,C}}$

$C_j = \frac{\epsilon_s A}{W_{dep}} = \frac{11.7(8.85\times10^{-14})\times100\times10^{-8}}{0.57\times10^{-4}}$

$= \underline{\underline{18.2\,fF}}$

3.115

$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.025 \ln\left(\frac{10^{16}\,10^{15}}{10^{20}}\right)$

$= 0.633\,V$

$W_{dep} = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$

$W_{dep} = \sqrt{\frac{2(11.7)(8.85\times10^{-14})}{1.6\times10^{-19}}\left(\frac{1}{10^{16}} + \frac{1}{10^{15}}\right)(0.633+5)}$

$= 2.83\times10^{-4}$ cm $= \underline{\underline{2.83\,\mu m}}$

$x_n = \frac{N_A}{N_D} x_p = 10\,x_p$

$\therefore\ x_p = \frac{2.83}{11} = \underline{\underline{0.26\,\mu m}}$

$x_n = \underline{\underline{2.57\,\mu m}}$

$q_J = q_N = q_P = q\frac{N_A N_D}{N_A + N_D} A\, W_{dep}$

$= 1.6\times10^{-19}\left(\frac{10^{16}\,10^{15}}{10^{16}+10^{15}}\right) 400\times10^{-8}\times 2.83\times10^{-4}$

$= \underline{\underline{1.65\times10^{-13}\,C}}$

$C_j = \frac{\epsilon_s A}{W_{dep}} = \frac{11.7(8.84\times10^{-14})\,400\times10^{-8}}{2.83\times10^{-4}}$

$= 14.6\times10^{-15} = \underline{\underline{14.6\,fF}}$

3.116

$q_J = q N x A = 1.6\times10^{-19}\times10^{16}\times0.1\times10^{-4}\times 100\times10^{-8}$

$= \underline{\underline{16\,fC}}$

3.117

$q_J = q\frac{N_A N_D}{N_A + N_D} A\, W_{dep}$

$= q\frac{N_A N_D}{N_A + N_D} A\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$

CONT.

$$\frac{dQ_J}{dV_R} = q\,\frac{\cancel{N_A N_D}}{\cancel{N_A+N_D}}\,A \times \frac{1}{q\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0+V_R)}} \times \frac{2\epsilon_s}{q}\cancel{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

$$= \frac{\epsilon_s A}{W_{dep}}$$

3.118

$$C_j = \frac{C_{j0}}{\left(1+V_R/V_0\right)^m}$$

$$V_R = 1\text{V} \quad C_j = \frac{0.6\text{pF}}{\left(1+1/0.75\right)^{1/3}} = \underline{0.45\text{pF}}$$

$$V_R = 10\text{V} \quad C_j = \frac{0.6\text{ pF}}{\left(1+10/0.75\right)^{1/3}} = \underline{0.25\text{pF}}$$

3.119

$V_Z = 10$

$$I_D = \frac{1}{2}\left(\frac{0.25}{10}\right) = \underline{12.5\text{mA}}$$

Since breakdown occurs only half the time, the average breakdown current can be twice the continuous value i.e. it can be 25 mA if the dissipation is limited to half the rated value or 50 mA if the dissipation is allowed to rise to the rated value.

3.120

$$I_p = Aq n_i^2\left(\frac{D_p}{L_p N_D}\right)\left(e^{V/V_T}-1\right)$$

$$I_n = Aq n_i^2\left(\frac{D_n}{L_n N_A}\right)\left(e^{V/V_T}-1\right)$$

$$\therefore \frac{I_p}{I_n} = \frac{D_p L_n N_A}{D_n L_p N_D} = \frac{10\times10\times10^{18}}{20\times5\times10^{16}} = \underline{100}$$

$$I = I_p + I_n = 100 I_n + I_n = 1\text{mA}$$

$$I_n = 1/101 = \underline{9.9\,\mu\text{A}}$$

$$I_p = 100(9.9) = \underline{990\,\mu\text{A}}$$

3.121

$$I_p = Aq n_i^2 \frac{D_p}{L_p N_D}\left(e^{V/V_T}-1\right)$$

$$I_n = Aq n_i^2 \frac{D_n}{L_n N_A}\left(e^{V/V_T}-1\right)$$

For p^+-n $\quad N_A >> N_D$

$$\therefore I \simeq I_p = Aq n_i^2 \frac{D_p}{L_p N_D}\left(e^{V/V_T}-1\right)$$

$$I_S \simeq Aq n_i^2 \frac{D_p}{L_p N_D} = Aq n_i^2\left(\frac{D_p}{\sqrt{D_p \tau_p}\,N_D}\right)$$

$$= 10^4(10^{-8})\,1.6\times10^{-19}\,\frac{(1.5\times10^{10})^2\quad 10}{\sqrt{10\times0.1\times10^{-6}}\times5\times10^{16}}$$

$$= \underline{0.72\times10^{-15}\text{ A}}$$

CONT.

$$I = I_s\left(e^{V/V_T}-1\right) = 0.2\times10^{-3}$$

$$0.72\times10^{-15}\left(e^{V/0.025}-1\right) = 0.2\times10^{-3}$$

$$V = \underline{0.684\,V}$$

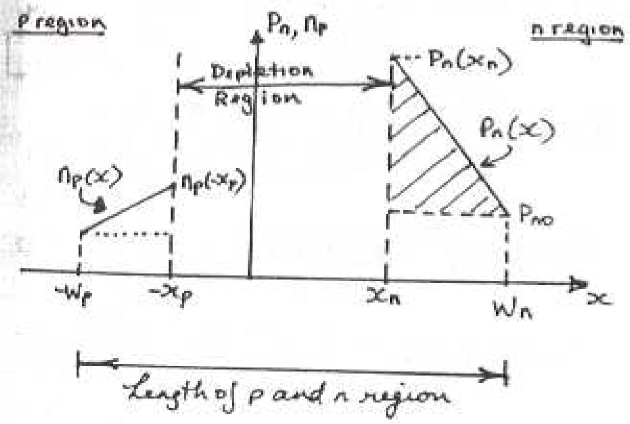
Excess minority charge

$$= Q_p + Q_n$$
$$= \tau_p I_p + \tau_n I_n \simeq \tau_p I_p$$
$$= (0.1\times10^{-6})(0.2\times10^{-3}) = \underline{\underline{2\times10^{-11}C}}$$

$$C_d = \frac{\tau_T I}{V_T} \simeq \frac{\tau_p I}{V_T}$$

$$= \frac{0.1\times10^{-6}\times0.2\times10^{-3}}{0.025} = \underline{\underline{800pF}}$$

3.122



(a) First consider I_p:

$$p_n(x_n) = p_{no}\,e^{V/V_T},$$
$$I_p = AJ_p, \quad \text{and}$$
$$p_{no} = \frac{n_i^2}{N_D}$$

Thus, $I_p = Aqn_i^2\dfrac{D_p}{(W_n-x_n)N_D}\left(e^{V/V_T}-1\right)$

& $I_n = Aqn_i^2\dfrac{D_n}{(W_p-x_p)N_A}\left(e^{V/V_T}-1\right)$

Thus: $I = I_p + I_n$

$$= Aqn_i^2\left[\frac{D_p}{(W_n-x_n)N_D}+\frac{D_N}{(W_p-x_p)N_A}\right]\times\left(e^{V/V_T}-1\right)$$ Q.E.D

The excess charge, Q_p, can be found by multiplying the area of the shaded triangle of the $p_n(x)$ distribution graph by Aq.

$$Q_p = Aq\times\frac{1}{2}\left[p_n(x_n)-p_{no}\right]\left[W_n-x_n\right]$$
$$= \frac{1}{2}Aq\,p_{no}\left(e^{V/V_T}-1\right)(W_n-x_n)$$
$$= \frac{1}{2}Aq\frac{n_i^2}{N_D}(W_n-x_n)\left(e^{V/V_T}-1\right)$$
$$= \frac{1}{2}\frac{(W_n-x_n)^2}{D_p}I_p$$
$$\simeq \frac{1}{2}\frac{W_n^2}{D_p}I_p \quad \text{for } W_n \gg x_n$$

Q.E.D.//

(c) $C_d = \dfrac{dQ}{dV} = \tau_T\dfrac{dI}{dV}$ But $I = I_s\left(e^{V/V_T}-1\right)$

$$\frac{dI}{dV} = \frac{I_s e^{V/V_T}}{V_T} \simeq I/V_T$$

$$\therefore C_d \simeq \frac{\tau_T I}{V_T}$$

(d) $C_d = \dfrac{1}{2}\dfrac{W_n^2}{10}\times\dfrac{10^{-3}}{0.025} = 8\times10^{-12}$

$$W_n = \underline{\underline{63.2\,\mu m}}$$

Chapter 4 - Problems

4.1

The capacitance per unit area is: $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$

$\epsilon_{ox} = 3.45 \times 10^{-11}$ F/m

$t_{ox} = 5nm \Rightarrow C_{ox} = \frac{3.45 \times 10^{-11}}{5 \times 10^{-9}} = 6.9\ fF/\mu m^2$

$t_{ox} = 40nm \Rightarrow C_{ox} = 0.86\ fF/\mu m^2$

For 1PF capacitance, we require an area A:

$A = \frac{10^{-12}}{6.9 \times 10^{-15}} = 145\ \mu m^2$ for $t_{ox} = 5nm$

$A = \frac{10^{-12}}{0.86 \times 10^{-15}} = 1163\ \mu m^2$ for $t_{ox} = 20nm$

For a square plate capacitor of 10PF:

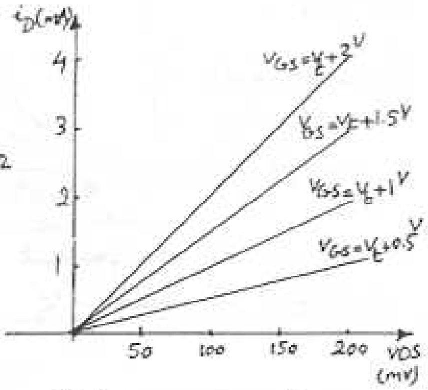
$A = 10 \times 145 = 1450\ \mu m^2$ or $38 \times 38\ \mu m^2$ square for $t_{ox} = 5^{nm}$

$A = 10 \times 1163 = 11630\ \mu m^2$ or $108 \times 108\ \mu m^2$ square for $t_{ox} = 20^{nm}$

4.2

Drain current is directly proportional to the width of the chanel. Therefore if width is 10 times greater, then i_D would be 10 times greater as well.

K = Constant of proportionality = $\frac{1}{0.5 \times 0.2} = 10\ mA/V^2$



$r_{DS} = \frac{i_D}{V_{DS}} = \frac{1}{0.2} = 5^{K\Omega}$ for $V_{ov} = 0.5^V$

$r_{DS} = 10^{K\Omega}$ for $V_{ov} = 1^V$

$r_{DS} = 15^{K\Omega}$ for $V_{ov} = 1.5^V$

$r_{DS} = 20K\Omega$ for $V_{ov} = 2^V$

$5^{K\Omega} \leq r_{DS} \leq 20K\Omega$ for $0.5^V \leq V_{ov} \leq 2^V$

4.3

eq. 4.6a : $i_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2$ $\quad K_n' = \mu_n C_{ox}$

for equal drain currents:

$\mu_n C_{ox} \frac{W_n}{L} = \mu_p C_{ox} \frac{W_p}{L} \Rightarrow \frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = \frac{1}{0.4} = \boxed{2.5}$

4.4

for small V_{DS}: $i_D \simeq K_n' \frac{W}{L} (V_{GS} - V_t) V_{DS}$

$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K_n' \frac{W}{L} (V_{GS} - V_t)} = \frac{1}{50 \times 10^{-6} \times 20 \times (5 - 0.8)}$

$r_{DS} = 238\Omega$ $\quad V_{DS} = r_{DS} \times i_D = 238mV$

For the same performance of a p-channel device: $\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L} \times 2.5 = 20 \times 2.5$

$\Rightarrow \frac{W_p}{L} = 50$

4.5

Eq. 4.5a: $i_D = K_n' \frac{W}{L} \left[(V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$ triode region

for small V_{DS}:

$i_D \simeq K_n' \frac{W}{L} (V_{GS} - V_t) V_{DS}$

$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K_n' \frac{W}{L} (V_{GS} - V_t)}$

for $r_{DS} = 1K\Omega$:

$K' = 100\ \mu A/V^2$: $1000 = \frac{1}{100 \times 10^{-6} \frac{W}{1} \times (5 - 0.8)}$

$\Rightarrow W = 2.4\mu m$

4.6

a) $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{15 \times 10^{-9}} = 2.3\ fF/\mu m^2$

$K_n' = \mu_n C_{ox} = 550 \times 10^{-4} \times 2.3 \times 10^{-3} = 126.5\ \mu A/V^2$

b) $i_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 100 = \frac{1}{2} \times 126.5 \times \frac{16}{0.8} (V_{GS} - 0.7)^2$

$V_{GS} - 0.7 = 0.28 \Rightarrow V_{OV} = 0.28V$

$V_{GS} = 0.98V$

$V_{DS min} = V_{GS} - V_t = 0.28V$

c) for small V_{DS}: (triode region) $i_D \simeq K_n' \frac{W}{L} V_{ov} \cdot V_{DS}$

Cont.

$$r_{DS} = \frac{v_{DS}}{i_D} = \frac{1}{k_n' \frac{W}{L} V_{OV}} = \frac{1}{126.5\times10^{-6}\times\frac{16}{0.8}V_{OV}} = 1000$$
$\Rightarrow V_{OV} = 0.4V$

$V_{GS} = V_{OV} + V_t = 0.4 + 0.7 = 1.1V$

4.7

$$k_n' = \mu_n C_{ox} = \mu_n \frac{\epsilon_{ox}}{t_{ox}} = 650\times10^{-4}\times\frac{3.45\times10^{-11}}{20\times10^{-9}} = 112.1\,\mu A/V^2$$

a) triode region: $v_{DS} < v_{GS} - V_t$

$$i_D = k_n' \frac{W}{L}\left[(v_{GS}-V_t)v_{DS} - \frac{1}{2}v_{DS}^2\right]$$
$$i_D = 112.1\times10^{-6}\times10\left[(5-0.8)\times1 - \frac{1}{2}\times1^2\right] = 4.15\,mA$$

b) edge of saturation region: $v_{DS} = v_{GS} - V_t$

$$i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS}-V_t)^2 = \frac{1}{2}\times112.1\times10^{-6}\times10\times(1.2)^2 = 0.8\,mA$$

c) triode region: $v_{DS} < v_{GS} - V_t$

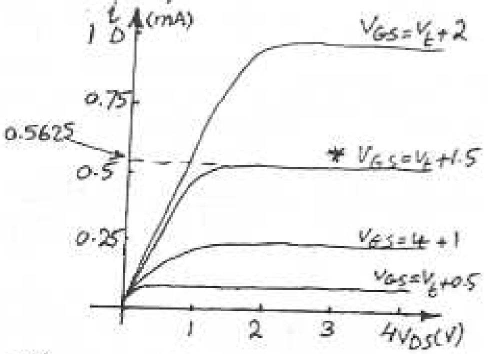
$$i_D = 112.1\times10^{-6}\times10\left[(5-0.8)\times0.2 - \frac{1}{2}\times0.2^2\right] = 0.92\,mA$$

d) Saturation region: $v_{DS} > v_{GS} - V_t$

$$i_D = \frac{1}{2}\times112.1\times10^{-6}\times10\times(5-0.8)^2 = 9.9\,mA$$

4.8

Refer to Fig. 4.11b, $i_D \propto \frac{W}{L}$ so if W is halved, then i_D would also be halved. The vertical axis, i_D, has to be divided by 2:



For $V_{OV} = 1.5^V$, by looking at the corresponding curve, $V_{GS} = V_t + 1.5$, we observe that: $i_D = 0.5625$ mA

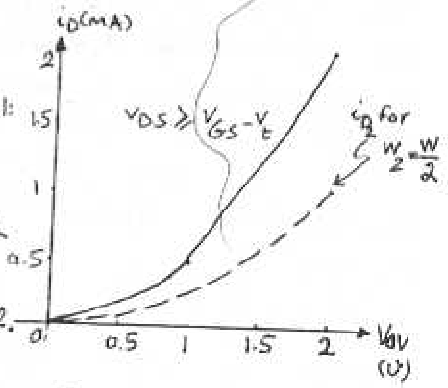
4.9

4.10

$i_D = \frac{1}{2}k_n' \frac{W}{L} V_{OV}^2$

assuming $k_n' \frac{W}{L} = 1$: $i_D = \frac{1}{2}V_{OV}^2$

For a device with half the width, i_D is divided by 2.



This graph is not dependent on V_t, while Fig. 4.12 is dependent on V_t.

4.11

Eq. 4.13: $r_{DS} = \left[k_n' \frac{W}{L}(V_{GS}-V_t)\right]^{-1}$ therefore:

$$\frac{r_{DS1}}{r_{DS2}} = \frac{V_{GS2}-V_t}{V_{GS1}-V_t} \Rightarrow \frac{1000}{200} = \frac{V_{GS2}-1}{1.5-1} \Rightarrow V_{GS2} = 3.5^V$$

Now for a device with twice the width:

Cont.

$$\frac{r_{DS1}}{r_{DS2}} = \frac{W_2(V_{GS2}-V_t)}{W_1(V_{GS2}-V_t)}$$

for $V_{GS}=1.5^V$ $\frac{r_{DS1}}{r_{DS2}} = 2 \Rightarrow r_{DS2} = \frac{1000}{2} = 500^{\Omega}$

for $V_{GS}=3.5^V$ $r_{DS2} = \frac{200}{2} = \underline{100\,\Omega}$

4.12

$i_D = \frac{1}{2}K_n'\frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 0.2\times10^{-3} = \frac{1}{2}\times0.1\times10^{-3}(V_{GS}-1)^2$

$V_{GS}-1=2 \Rightarrow V_{GS}=3V$

$V_{DSmin} = V_{GS}-V_t = 3-1 = 2V$

for $i_D=0.8mA$: $0.8 = \frac{1}{2}\times0.1(V_{GS}-1)^2$

$V_{GS}-1 = 4 \Rightarrow V_{GS}=5V$

$V_{DSmin} = V_{GS}-V_t = 5-1 = 4^V$

4.13

$V_{GS}=V_{DS}$ indicates operation in saturation mode; $i_D = \frac{1}{2}K_n'\frac{W}{L}(V_{GS}-V_t)^2$

$$\left.\begin{array}{l} 4 = \frac{1}{2}K_n'\frac{W}{L}(5-V_t)^2 \\ 1 = \frac{1}{2}K_n'\frac{W}{L}(3-V_t)^2 \end{array}\right\} \Rightarrow 4 = \frac{(5-V_t)^2}{(3-V_t)^2} \Rightarrow$$

$(5-V_t) = 2(3-V_t) \Rightarrow V_t = 1^V$, $K_n'\frac{W}{L} = 0.5\,mA/V^2$

4.14

$i_D = \frac{1}{2}K_n'\frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 0.8 = \frac{1}{2}\times50\times10^{-3}\frac{W}{L}(5-1)^2$

$\frac{W}{L} = 2 \Rightarrow W = 2\times2 = 4\mu m$

4.15

In triode region: $i_D = K_n'\frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$\frac{i_{D1}}{i_{D2}} = \frac{60\mu A}{160\mu A} = \frac{(2-V_t)0.1-0.01/2}{(4-V_t)0.1-0.01/2} \Rightarrow V_t = 0.75V$

if $K_n'=50\mu A/V^2$: $60 = 50\frac{W}{L}\left[(2-0.75)0.1 - \frac{0.01}{2}\right]$

$\frac{W}{L} = 10$

If $V_{GS}=3V$, $V_{DS}=0.15^V$ then $i_D = 50\times10\times\left[2.25\times0.15 - \frac{0.15^2}{2}\right]$

$i_D = 163.125\,\mu A$

If $V_{GS}=3V$, the channel reach pinchoff at $V_{DS}=V_{GS}-V_t = 3-0.75 = 2.25^V$ for which:

$i_D = \frac{1}{2}\times K_n'\frac{W}{L}(V_{GS}-V_t)^2 = \frac{1}{2}\times50\times10\times2.25^2 = 1.3^{mA}$

$\underline{i_D = 1.3mA}$

4.16

For the channel to remain continous:

$V_{DS} \leq V_{GS}-V_t \Rightarrow V_{DSmax} = 1.5-0.8 = 0.7^V$

4.17

Eq. 4.15 : $r_{DS} = \left[K_n'\frac{W}{L}V_{ov}\right]^{-1} = \frac{1}{50\times\frac{100}{5}(V_{GS}-1)}\,M\Omega$

$r_{DS} = \frac{1}{V_{GS}-1}\,K\Omega$

$V_{GS}=1.1V \Rightarrow r_{DS} = 10K\Omega$

$V_{GS}=11V \Rightarrow r_{DS} = 100\Omega$ $\Rightarrow 100^{\Omega} \leq r_{DS} \leq 10^{K\Omega}$

a) $r_{DS} \propto \frac{1}{W}$ so if W is halved, r_{DS} is doubled:

$200^{\Omega} \leq r_{DS} \leq 20K\Omega$

b) $r_{DS} \propto L$ so if L is halved, r_{DS} is also halved: $50^{\Omega} \leq r_{DS} \leq 5^{K\Omega}$

c) $r_{DS} \propto \frac{L}{W}$ so if both W and L are halved, $\frac{W}{L}$ stays unchanged and so does r_{DS}.

$100^{\Omega} \leq r_{DS} \leq 10K\Omega$

4.18

According to eq. 4.17 : $V_{GD} \leq V_t$ for saturation region. Since V_{GD} is zero in Fig. P4.18, then the device is always in saturation region:

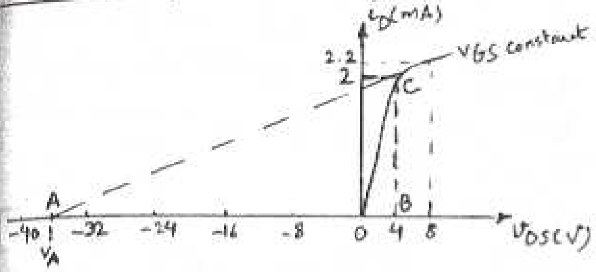
$i_D = i = \frac{1}{2}K_n'\frac{W}{L}(V_{GS}-V_t)^2$

if we replace V_{GS} by v and $K' = \frac{1}{2}K_n'$

$i = K'\frac{W}{L}(V-|V_t|)^2$

$r = \left(\frac{\partial i}{\partial v}\right)^{-1} = \frac{1}{2K'\frac{W}{L}(V-|V_t|)} = \frac{1}{K_n\frac{W}{L}V_{ov}}$ where $v = |v_t| + v_{ov}$

4.19



$r_o = \frac{\Delta V_{DS}}{\Delta i_D}\Big|_{V_{GS}\,\text{Const}} = \frac{8-4}{2.2-2} = 20\,\text{K}\Omega$

To calculate V_A, consider the ABC triangle:

$V_A + 4 = 2\text{mA} \times r_o = 2 \times 20 = 40^V \Rightarrow V_A = 36V$

$\lambda = \frac{1}{V_A} = 0.028\ V^{-1}$

4.20

Eq. 4.26, $r_o = \frac{V_A}{i_D} = \frac{50}{i_D}$ $\quad 0.1^{mA} \leq i_D \leq 1^{mA} \Rightarrow 50^{K\Omega} \leq r_o \leq 500^{K\Omega}$

$r_o = \frac{\Delta V_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{r_o}$

$i_D = 0.1\,\text{mA} \quad \Delta i_D = 2\,\mu\text{A} \quad \frac{\Delta i_D}{i_D} = 2\%$

$i_D = 1\,\text{mA} \quad \Delta i_D = 20\,\mu\text{A} \quad \frac{\Delta i_D}{i_D} = 2\%$

4.21

$V_A = V_A' L$ where V_A' is completely process dependent. Also, $r_o = \frac{V_A}{i_D}$. therefore to achieve desired r_o (which is 4 times larger) we should increase L. $L = 4 \times 2 = 8\,\mu\text{m}$

In order to keep I_D unchanged, $\frac{W}{L}$ ratio has to stay unchanged. There fore:

$W = 4 \times 10 = 40\,\mu\text{m}$. (So $\frac{W}{L}$ is kept at 5)

$V_A = r_o i_D = 0.5\text{M} \times 100\mu = 50V$ (For standard)

$V_A = 4 \times 0.5\text{M} \times 100 = 200V$ (For new device)

4.22

$\lambda = 0.02\ V^{-1} \Rightarrow V_A = 50V$ for $L = 1\,\mu\text{m}$

$V_A = V_A' L \Rightarrow V_A' = 50V$

For $L = 3\,\mu\text{m}$: $V_A = 50 \times 3 = 150V$

$r_o = \frac{V_A}{I_D} = \frac{150}{0.08} = 1875\,\text{K}\Omega$

$r_o = \frac{\Delta V_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta V_{DS}}{r_o} = \frac{5-1}{1875} = 2.13\,\mu\text{A}$

For V_{DS} raised from 1^V to 5^V, i_D increases from $80\,\mu\text{A}$ to $82.13\,\mu\text{A}$.

$\frac{\Delta i_D}{i_D} = 2.7\%$ change in i_D.

In order to reduce $\frac{\Delta i_D}{i_D}$ by a factor of 2, Δi_D has to be halved, or equivalently r_o has to be doubled. In order to double r_o, V_A has to be doubled and this can be done by doubling the length. $L = 2 \times 3 = 6\,\mu\text{m}$

4.23

$V_A = V_A' L = 20 \times 1.6 = 32V \quad , \lambda = \frac{1}{V_A} = 0.031\ V^{-1}$

$V_{DS} = 2^V > (V_{GS} - V_t) = 0.5V \Rightarrow$ device in saturation region

$i_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 130 \times \frac{16}{1.6}\ 0.5^2 = 162.5\,\mu\text{A}$

$r_o = \frac{V_A}{i_D} = \frac{32}{162.5\mu} = 197\,\text{K}\Omega$

$r_o = \frac{\Delta V_{DS}}{\Delta i_D} \Rightarrow 197\,\text{K}\Omega = \frac{1}{\Delta i_D} \Rightarrow \Delta i_D = 5.1\,\mu\text{A}$

4.24

MOS	1	2	3	4
$\lambda\ (V^{-1})$	0.02	0.01	0.1	0.005
V_A (V)	50	100	10	200
I_D (mA)	5	3.33	0.1	0.2
r_o (KΩ)	10	30	100	1000

$r_o = \frac{V_A}{I_D}$, $\lambda = \frac{1}{V_A}$

4.25

$V_A = \frac{1}{\lambda} = 100V \quad V_A = V_A' L$ if L is doubled, so is V_A.

$V_A = 2 \times 100 = 200V$, $\lambda = \frac{1}{200} = 0.005\ V^{-1}$

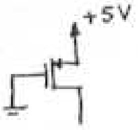
a) if V_{GS} is fixed, then i_D is halved, as a result of doubling L. $I_D \propto \frac{1}{L} \Rightarrow I_D = \frac{1}{2} = 0.5\text{mA}$

Cont.

$r_o = \frac{V_A}{I_D} = \frac{200}{0.5} = 400\,k\Omega$

b) if I_D is fixed: $r_o = \frac{V_A}{I_D} = \frac{200}{1} = 200\,k\Omega$

4.26



$V_{GS} = -5V$

To operate in saturation:

$V_{DS} \le V_{GS} - V_t$ or $V_{DS} \le -5-(-1.5)$ or $V_{DS} \le -3.5$

a) $V_D = 4^V \Rightarrow V_{DS} = -1V > -3.5^V \Rightarrow$ triode region

$i_D = K_p' \frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$ eq. 4.29

$i_D = 80\left[(-5-(-1.5))\times(-1) - \frac{1}{2}\times 1\right] = 0.24\,mA$

b) $V_D = 1.5^V \Rightarrow V_{DS} = -3.5^V = V_{GS}-V_t \Rightarrow$ edge of saturation

$i_D = \frac{1}{2}K_p'\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$ eq. 4.32

$i_D = \frac{80}{2}(3.5)^2(1+0.02\times 3.5) = 0.52\,mA$

c) $V_D = 0 \Rightarrow V_{DS} = -5^V < -3.5 \Rightarrow$ saturation

$i_D = \frac{1}{2}\times 80\times(3.5)^2(1+0.02\times 5) = 0.54\,mA.$

d) $V_D = -5V \Rightarrow V_{DS} = -10^V < -3.5 \Rightarrow$ saturation

$i_D = \frac{1}{2}\times 80(3.5)^2(1+0.02\times 10) = 0.59\,mA$

4.27

$V_{GS} = -3V$ $\quad V_{SG} = 3V$ $\quad V_t = -1V$

$V_{DS} = -4V$ $\quad V_{SD} = 4V$ $\quad V_A = -50^V$ $\quad \lambda = -0.02\,V^{-1}$

$i_D = \frac{1}{2}K_p'\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$

$3 = \frac{1}{2}K_p'\frac{W}{L}(-3+1)^2(1+0.02\times 4) = 2.16\,K_p'\frac{W}{L}$

$K_p'\frac{W}{L} = 1.39\,mA/V^2$

4.28

Eq. 4.34: $\gamma = \frac{\sqrt{2qN_A\epsilon_s}}{C_{ox}} \Rightarrow N_A = \frac{\gamma^2 C_{ox}^2}{2q\epsilon_s}$

$t_{ox} = 20nm$ $\quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45\times 10^{-11}}{20\times 10^{-9}} = 1.73\,fF/\mu m^2$

$N_A = \frac{0.5^2\times 1.73^2\times 10^{-6}}{2\times 1.6\times 10^{-19}\times 11.7\times 8.854\times 10^{-14}\times 10^{2}} = 2.3\times 10^{22}/m^3$

$N_A = 2.3\times 10^{16}\,/cm^3$

if $t_{ox} = 100nm$ $\quad \frac{\gamma_1}{\gamma_2} = \frac{C_{ox2}}{C_{ox1}} = \frac{t_{ox1}}{t_{ox2}} \Rightarrow \frac{0.5}{\gamma_2} = \frac{20}{100}$

$\Rightarrow \gamma_2 = 2.5\,V^{1/2}$

If γ is kept at $0.5\,V^{1/2}$, then when t_{ox} is changed:

$\frac{\sqrt{N_{A1}}}{C_{ox1}} = \frac{\sqrt{N_{A2}}}{C_{ox2}} \Rightarrow \frac{\sqrt{N_{A1}}}{\sqrt{N_{A2}}} = \frac{C_{ox1}}{C_{ox2}} = \frac{t_{ox2}}{t_{ox1}} = \frac{100}{20} = 5$

$\sqrt{N_{A2}} = \frac{\sqrt{N_{A1}}}{5} \Rightarrow N_{A2} = \frac{N_{A1}}{25} = 9.2\times 10^{14}\,/cm^3$

4.29

Eq. 4.33 $\quad V_t = V_{t_0} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$

$V_{SB} = 0^V \Rightarrow V_t = V_{t_0} = 1V$

$V_{SB} = 4^V \Rightarrow V_t = 1 + 0.5\left[\sqrt{0.6+4} - \sqrt{0.6}\right] = 1.69^V$

$1^V \le V_t \le 1.69^V$

If t_{ox} is increased by a factor of 4:

from Eq. 4.34 $\gamma \propto \frac{1}{C_{ox}}$ or $\gamma \propto t_{ox}$ or $\frac{\gamma_1}{\gamma_2} = \frac{t_{ox1}}{t_{ox2}}$

$\Rightarrow \gamma_2 = 4\times 0.5 = 2V^{1/2}$

$V_t = 1 + 2(\sqrt{0.6+4} - \sqrt{0.6}) = 3.74V$ for $V_{SB} = 2^V$

$V_t = 1^V$ for $V_{SB} = 0$

Therefore for $\gamma = 2$: $1 \le V_t \le 3.74^V$

4.30

$|V_{SB}| = 3^V$ $\quad$ Eq. 4.33 $V_t = V_{t_0} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$

$V_t = -1 - 0.5\left[\sqrt{0.6+3} - \sqrt{0.6}\right] = -1.56V$

4.31

a) $i_D = \frac{1}{2}K_n'\frac{W}{L}(V_{GS}-V_t)^2$

$\frac{\partial i_D}{\partial T} = \frac{1}{2}\frac{W}{L}\frac{\partial K_n'}{\partial T}(V_{GS}-V_t)^2 - K_n'\frac{W}{L}(V_{GS}-V_t)\frac{\partial V_t}{\partial T}$

$\frac{\partial i_D/i_D}{\partial T} = \frac{\partial K_n'/K_n'}{\partial T} - 2\frac{\partial V_t}{\partial T}\frac{1}{V_{GS}-V_t}$

Cont.

b) $\frac{\partial V_t}{\partial T} = -2mV/^\circ C$ For $\frac{\partial i_D/i_D}{\partial T} = -0.2\% = -0.002/^\circ C$

$-0.002 = \frac{\partial k_n'/k_n'}{\partial T} - 2\times(-2^{mV}) \frac{1}{5-1} \Rightarrow$

$\frac{\partial k_n'/k_n'}{\partial T} = -0.003/^\circ C = -0.3\%/^\circ C$

4.32

Case	Transistor	V_S	V_G	V_D	I_D	type	mode	$\mu C_{ox}\frac{W}{L}$	V_t
a	1	0	2	5	100	N	Sat.	200	1
	1	0	3	5	400	N	Sat.	200	1
b	2	5	3	.45	50	P	Sat.	400	-1.5
	2	5	2	.05	450	P	Sat.	400	-1.5
c	3	5	3	4	200	P	Sat.	400	-1
	3	5	2	0	800	P	Sat.	400	-1
d	4	-2	0	0	72	N	Sat.	100	0.8
	4	-4	0	-3	270	N	Triode	100	0.8

Case a) transistor 1: $V_{GS} = 2^V$ $V_{DS} = 5^V$ $I_D = 100\mu A$

This must be an NMOS operating in Saturation.

$I_D = 100 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2-V_t)^2$

when $V_{GS}=3V$: $400 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-V_t)^2 \Rightarrow 4 = \frac{(3-V_t)^2}{(2-V_t)^2}$

$\Rightarrow 2(2-V_t) = 3-V_t \Rightarrow V_t = 1^V,\ \mu_n C_{ox}\frac{W}{L} = 200\frac{\mu A}{V^2}$

Case b) transistor 2: $V_{GS} = 3-5 = -2^V$ $V_{DS} = -9.5^V$

Therefore $V_{DS} < V_{GS} - V_t$ regardless of value of V_t, and the device operates in saturation. (PMOS)

$I_D = 50 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-2-V_t)^2$

$450 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-3-V_t)^2 \Rightarrow 9 = \frac{(3+V_t)^2}{(2+V_t)^2}$

$\Rightarrow 3(2+V_t) = 3+V_t \Rightarrow V_t = -1.5V,\ \mu_p C_{ox}\frac{W}{L} = 400\frac{\mu A}{V^2}$

Case c) transistor 3: $V_{GS} = -2^V$ $V_{DS} = -1^V \Rightarrow$ PMOS

This device can be either in saturation or triode region. First, we assume saturation region:

$I_D = 200 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-2-V_t)^2$

$800 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-3-V_t)^2 \Rightarrow 4 = \frac{(3+V_t)^2}{(2+V_t)^2}$

$\Rightarrow 2(2+V_t) = 3+V_t \Rightarrow V_t = -1V,\ \mu_p C_{ox}\frac{W}{L} = 400\frac{\mu A}{V^2}$

So our assumption was right:

$(V_{DS} = -1^V) \leq (V_{GS} - V_t) = -2+1 = -1^V$

edge of saturation

Case d) transistor 4: $V_{GS} = 2^V$ $V_{DS} = 2^V \Rightarrow$ NMOS

So $V_{DS} > V_{GS} - V_t \Rightarrow$ saturation region

① $I_D = 72 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2-V_t)^2$

But for $V_{GS} = 4^V, V_{DS} = 1^V$ and considering that $V_t < 2V$, then the device is in triode region:

$270 = \mu_n C_{ox}\frac{W}{L}\left[(4-V_t)\times 1 - \frac{1}{2}\times 1\right]$

② $270 = \mu_n C_{ox}\frac{W}{L}(3.5-V_t)$

①,② $\Rightarrow V_t = 0.8V$ $\mu_n C_{ox}\frac{W}{L} = 100\mu A/V^2$

4.33

a) $I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 2 = \frac{1}{2}k_n'\frac{W}{L}(3-1)^2$

$\Rightarrow k_n'\frac{W}{L} = 1\,mA/V^2$

$V_1 = V_{DS} = 3V$

b) $V_2 = V_S = V_D - V_{DS} = 1-3 = -2^V$

c) $V_3 = V_S = V_D - V_{DS} = 0-(-3) = 3^V$

d) $V_4 = V_D = V_S + V_{DS} = 5+(-3) = 2^V$

In order to calculate R_{max} that can be inserted in series with the drain, V_{DS} has to be equal to $V_{GS} - V_t$, so that the device is operating on the edge of saturation: $|V_{DS}| = 3-1 = 2^V$. Note that since i_D is the same, V_{GS} stays the same.

a) $R_{D\,max} = \frac{3-2}{2^{mA}} = 0.5K\Omega$

b) $V_2 = -2^V \Rightarrow V_D = -2+2 = 0 \Rightarrow R_{D\,max} = \frac{1}{2} = 0.5^{K\Omega}$

Note that V_2 is fixed through $V_{GS} = 3^V$.

c) $V_{GS} = -3V \Rightarrow V_S = V_3 = 3^V$. Now for V_{DS} to be $-2V$, V_D has to be 1^V.

$R_{DMax} = \frac{1^V}{2^{mA}} = 0.5k\Omega$

d) $V_{GS} = -3V \Rightarrow V_G = V_4 = 2^V$. Adding the resistor between V_4 and drain means that V_D has to be $5-2 = 3^V$ and this leaves 1^V voltage drop on the resistor $\therefore R_{Dmax} = \frac{1}{2} = 0.5k\Omega$

In order to calculate the largest Resistor added to the gates, note that since the gate doesn't draw any current, the value of the resistor is immaterial.

Cont.

Now we calculate R_{smax}, assuming that the voltage drop across the current source is at least 2^V:

a) $V_1 = 8^V$ then $V_{GS} = 3^V \Rightarrow V_S = 8-3 = 5^V$

$R_{SMax} = \frac{5}{2} = 2.5K\Omega$

b) $V_2 = -9+2 = -7^V$, $V_S = 1-|V_{GS}| = -2^V$

$R_{Smax} = \frac{-2-(-7)}{2} = 2.5K\Omega$

c) $V_3 = 10-2 = 8^V$, $V_S = 0+|V_{GS}| = 3^V$

$R_{Smax} = \frac{8-3}{2} = 2.5K\Omega$

d) $V_4 = -5+2 = -3^V$; $V_S = -3+|V_{GS}| = 0^V$

$R_{Smax} = \frac{5}{2} = 2.5K\Omega$

4.34

$I_D = \frac{V_{DD}-V_D}{R_D} \Rightarrow \frac{5-0}{R_D} = 1^{mA} \Rightarrow R_D = 5K\Omega$

$V_D = V_G \Rightarrow$ saturation

therefore: $i_D = \frac{1}{2}K_n' \frac{W}{L}(V_{GS}-V_t)^2$

$1 = \frac{1}{2} \times 60 \times 10^{-3} \times \frac{100}{3} (V_{GS}-1)^2$

$\Rightarrow V_{GS} = 2V \quad \Rightarrow V_S = -2^V$

$R_S = \frac{-2-(-5)}{1} = 3K\Omega$

4.35

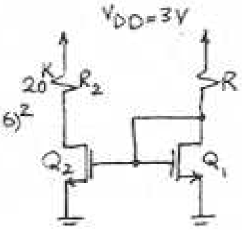
a) $i_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_t)^2$

$i_{D_1} = 0.2^m = \frac{1}{2} \times 200 \times 10^{-6} \times \frac{8}{0.8}(V_{GS_1}-0.6)^2$

$V_{GS_1} - 0.6 = \sqrt{0.2}$

$V_{GS_1} = 1.05V$

$R = \frac{3-1.05}{0.2} = 9.75K\Omega$



b) For Q_2 to conduct 0.5mA:

$R_2 = \frac{3-1}{0.5} = 4K\Omega$

Since the gates are connected together, both trasistors have the same V_{GS} and hence same i_D. In order to conduct 0.5mA or multiply i_D by 5: $\frac{W_2}{W_1} = 5 \Rightarrow W_2 = 5 \times 8$

$W_2 = 40\mu m$

4.36

Refer to Fig. P4.36.

$R = \frac{3.5}{0.115} = 3.04K\Omega$

$0.115_{mA} = \frac{1}{2} \times 60 \times 10^{-3} \times \frac{W}{0.8}(-1.5-(-0.7))^2 \Rightarrow W = 4.8\mu m$

4.37

Refer to Fig. P4.37

$V_{GS1} = 1.5V$, $120\mu A = \frac{1}{2} \times 120 \times \frac{W_1}{1}(1.5-1)^2$

$\Rightarrow W_1 = 8\mu m$

$V_{GS2} = 3.5-1.5 = 2^V$, $120 = \frac{1}{2} \times 120 \frac{W_2}{1}(2-1)^2$

$\Rightarrow W_2 = 2\mu m$

$R = \frac{5-3.5}{0.120} = 12.5K\Omega$

4.38

Refer to Fig. P4.38.

$V_{GS1} = 1.5^V \quad 120\mu A = \frac{1}{2} \times 120 \times \frac{W_1}{1}(1.5-1)^2 \Rightarrow W_1 = 8\mu m$

$V_{GS2} = 2^V \quad 120\mu A = \frac{1}{2} \times 120 \frac{W_2}{1}(2-1)^2 \Rightarrow W_2 = 2\mu m$

$V_{GS3} = 1.5^V \quad W_3 = 8\mu m$

4.39

Refer to Fig. 4.23a. $V_{GS} = 5-6I_D \Rightarrow$

$I_D = \frac{1}{2} \times 2 \times (5-6I_D-2)^2 \Rightarrow 36I_D^2 - 37I_D + 9 = 0$

$\Rightarrow I_D = 0.395^{mA}$ or $0.633\,mA$

$I_D = 0.633mA$ is not acceptable, because it results in $V_{GS} = 1.2^V$ which is lower than $V_t = 2.^V$

Therefore $I_D = 0.395mA \cong 0.4mA$

$V_D = 10-6I_D = 7.6^V$

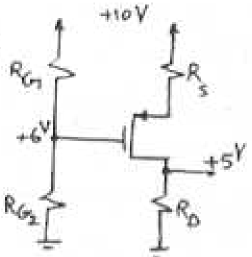
These results show that although V_t and $K_n'\frac{W}{L}$ are doubled, I_D is only decreased by 20% and V_D is only increased by 8.5%. Therefore the conclusion is

Cont.

that the circuit is tolerant to changes in device parameters.

4.40

$V_G = 6^V \Rightarrow R_{G_1} = 4^{M\Omega}$, $R_{G_2} = 6^{M\Omega}$



$I_D = 0.5mA \Rightarrow R_D = \frac{5}{0.5} = 10K\Omega$

$I_D = 0.5 = \frac{1}{2}\times 1 \times (V_{GS}+1.5)^2$

$V_{GS} = -2.5V$ or $-0.5V$

($V_{GS} = -0.5V$ is rejected, because it is less than V_t)

$V_{GS} = -2.5V \Rightarrow V_S = 8.5^V \Rightarrow R_S = \frac{10-8.5}{0.5} = 3K\Omega$

4.41

Refer to Fig. P4.41.

$V_I = V_{GS} = 5V$, $V_o = V_{DS} = 0.05V$

$r_{DS} = 50\Omega = \frac{V_{DS}}{I_D} \Rightarrow I_D = \frac{0.05}{50} = 0.001^A = 1mA$

$R = \frac{V_{DD}-V_o}{I_D} = \frac{5-0.05}{1} = 4.95K\Omega$

$V_{DS} < V_{GS} - V_t \Rightarrow$ triode region

$I_D = k_n' \frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$1 = 100\times10^{-3}\frac{W}{L}\left[(5-1)\times0.05 - \frac{0.05^2}{2}\right] \Rightarrow \frac{W}{L} = 50$

4.42

Refer to Fig. P4.42.

In circuit a: $V_2 = 10 - 4\times2 = 2^V$

Assume saturation: $I_D = 2 = \frac{1}{2}\times1\times(V_{GS}-2)^2 \Rightarrow V_{GS} = 4^V$

$\Rightarrow V_1 = -4^V$, $V_{DS} = 6V > V_{GS} - V_t$ So our assumption was correct.

In circuit b: $I_D = 1 = \frac{1}{2}\times1\times(V_{GS}-2)^2 \Rightarrow V_{GS} = 3.41^V$

$V_3 = 3.41^V$

Circuit c: $I_D = 2mA \Rightarrow V_{GS} = -4^V \Rightarrow V_5 = 4^V = V_4$

$V_5 = -10\times2.5\times2 = -5^V$

In circuit d: $I_D = 2mA \Rightarrow V_{GS} = -4^V \Rightarrow V_6 = 6^V$

$\Rightarrow V_7 = V_6 - 4 = 2^V$

If we replace the current source with a resistor in each of those circuits:

in circuit a:

$R = \frac{-4-(-10)}{2} \cong 3.01^{K\Omega}$ (by looking at the table for 1% resistors)

Now recalculate I_D: $I_D = \frac{1}{2}\times1\times(V_{GS}-V_t)^2$

$V_{GS} - V_t = 0-(-10+3.01I_D) - 2 = 8 - 3.01I_D$ } $\Rightarrow$

$2I_D = (8-3.01I_D)^2 \Rightarrow I_D = 1.99mA \Rightarrow V_2 = 2.04^V$

$V_1 = -4.01^V$

in circuit b:

$R = \frac{10-3.41}{1} = 6.59^K \cong 6.65^{K\Omega}$

Then $V_{GS} = 10 - 6.65I$

$I = \frac{1}{2}\times1(10-6.65I-2)^2 \Rightarrow I = 0.99mA$

$V_3 = 10 - 6.65\times0.99 = 3.41^V$

in circuit c:

$R = \frac{10-4}{2} \cong 3.01K\Omega$, $V_{GS} = -(10-3.01I)$

$I = \frac{1}{2}\times1\times(-10+3.01I+2)^2 \Rightarrow I_D = 1.99mA$

$V_4 = 10 - 3.01\times1.99 = 4.01V$

$V_5 = -10 + 2.5K\times1.99 = -5.03V$

in circuit d:

$R = \frac{2}{2} = 1K$ so V_7 is still 2^V

4.43

a) $V_{GS} = -V_1$. $10\mu A = \frac{1}{2}\times0.4\times10^3(V_{GS}-1)^2 \Rightarrow$

$V_{GS} = 1.22^V \Rightarrow V_1 = -1.22^V$

b) $100\mu A = \frac{1}{2}\times0.4\times10^3\times(V_{GS}-1)^2 \Rightarrow V_{GS} = 1.71^V$, $V_2 = -1.71^V$

c) $1 = \frac{1}{2}\times0.4\times(V_{GS}-1)^2 \Rightarrow V_{GS} = 3.23^V \Rightarrow V_3 = -3.23V$

d) $10 = \frac{1}{2}\times0.4\times10^3(V_{GS}-1)^2 \Rightarrow V_{GS} = 1.22^V \Rightarrow V_4 = 1.22V$

e) $1 = \frac{1}{2}\times0.4(V_{GS}-V_t)^2 \Rightarrow V_{GS} = 3.24^V \Rightarrow V_5 = 3.24V$

f) $I = \frac{1}{2}\times0.4\times(5-100I-1)^2 \Rightarrow I = 0.045mA, 0.036mA$

$V_6 = 5 - 100\times0.036 = 1.4^V$

g) $I = \frac{1}{2} \times 0.4 \times (5 - 1 \times I - 1)^2 \Rightarrow I = 1.38\,mA$

$V_7 = 5 - 1.38 \times 1 = \underline{3.62^V}$

h) $I = \frac{1}{2} \times 0.4 \times (5 - 100 - I)^2 \Rightarrow I = \cancel{0.045^{mA}}, 0.036^{mA}$

$V_8 = -5 + 100 \times 0.036 = \underline{-1.4^V}$

Note that $I = 0.045^{mA}$ in circuits h and f is not acceptable, because it results in $V_{GS} < V_t$ that is not physically possible.

4.44

Refer to Fig. P4.44.

a) $V_{GS2} = -V_2$, $I = \frac{V_2 - (-5)}{1K} = \frac{1}{2} \times 2 \times (-V_2 - 1)^2$

$\Rightarrow V_2 + 5 = V_2^2 + 2V_2 + 1 \Rightarrow V_2^2 + V_2 - 4 = 0 \Rightarrow \cancel{V_2 = 1.55^V}$, $V_2 = -2.56^V$

$V_2 = 1.55^V$ is not acceptable because it results in $V_{GS} < 0$ that is not possible for an NMOS. Therefore $\underline{V_2 = -2.56V}$

$i_{D1} = i_{D2} \Rightarrow \frac{V_2 - (-5)}{1K} = \frac{1}{2} \times 2 (5 - V_1 - 1)^2 \Rightarrow$

$2.44 = (4 - V_1)^2 \Rightarrow 4 - V_1 = \pm 1.56^V \Rightarrow V_1 = 2.44^V$, $V_1 = 5.56^V$ X

The second answer results in $V_{GS} = 5 - 5.56 < 0$ which is not acceptable. Therefore $\underline{V_1 = 2.44^V}$

b) $\frac{10 - V_3}{1K} = \frac{V_5}{1K} = i_D \Rightarrow 10 - V_3 = V_5$ ①

$i_{D_1} = \frac{V_5}{1K} = \frac{1}{2} \times 2 \times (V_3 - V_4 - 1)^2 \Rightarrow V_5 = (V_3 - V_4 - 1)^2$ ②

$i_{D_2} = V_5/_{1K} = \frac{1}{2} \times 2 \times (V_4 - V_5 - 1)^2 \Rightarrow V_5 = (V_4 - V_5 - 1)^2$ ③

②, ③ $\Rightarrow V_3 - V_4 - 1 = V_4 - V_5 - 1 \Rightarrow V_5 = 2V_4 - V_3$ ④

①, ④ $\Rightarrow 2V_4 - V_3 = 10 - V_3 \Rightarrow \underline{V_4 = 5^V}$

③ $\Rightarrow V_5 = (4 - V_5)^2 \Rightarrow V_5^2 - 9V_5 + 16 = 0 \Rightarrow V_5 = 6.55^V$ X, $V_5 = 2.45^V$

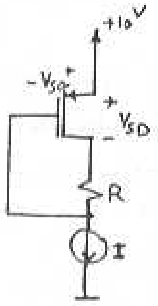
$V_5 = 6.55$ results in $i_D = 6.55\,mA$, $V_3 = 4.45^V$ and this is not physicly possible. So $\underline{V_5 = 2.45^V}$

$V_3 = 10 - 2.45 = \underline{7.55\,V}$

4.45

The PMOS transistor operates in saturation region if $V_{SD} \geq V_{SG} - |V_t|$

or $V_{SD} \geq V_{SG} - 1$



Also, $V_{SD} + IR = V_{SG} \Rightarrow V_{SD} = V_{SG} - IR$

$\Rightarrow IR \leq |V_t|$ For PMOS to be in saturation.

a) $R = 0 \Rightarrow IR = 0 < |V_t|$

saturation:

$I = 100 = \frac{1}{2} \times 8 \times 25 \times (V_{SG} - |V_t|)^2$

$V_{SG} - 1 = \pm 1 \Rightarrow \underline{V_{SG} = 2^V = V_{SD}}$

b) $R = 10K\Omega \Rightarrow IR = 10 \times 0.1 = 1^V \Rightarrow$ saturation

$\underline{V_{SG} = 2^V} \Rightarrow V_{SD} = 2 - 1 = \underline{1^V}$

c) $R = 30K\Omega \Rightarrow IR = 30 \times 0.1 = 3^V \Rightarrow$ triode region

$100 = 8 \times 25 \left[(V_{SG} - |V_t|) V_{SD} - \frac{1}{2} V_{SD}^2 \right]$

$0.5 = \left[(V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2} (V_{SG} - 3)^2 \right]$

$0.5 = 0.5 V_{SG}^2 - V_{SG} - 1.5 \Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$

$\underline{V_{SG} = 3.24^V}$, -1.2^V X

$\underline{V_{SD} = 3.24 - 3 = 0.24V}$

d) $R = 100K\Omega \Rightarrow IR = 100 \times 0.1 = 10^V \Rightarrow$ triode region

$100 = 8 \times 25 \left[(V_{SG} - 1)(V_{SG} - 10) - \frac{1}{2} (V_{SG} - 10)^2 \right]$

$0.5 = 0.5 V_{SG}^2 - V_{SG} - 40 \Rightarrow V_{SG}^2 - 2V_{SG} - 81 = 0$

$\underline{V_{SG} = 10.1^V} \Rightarrow \underline{V_{SD} = 0.1^V}$

e) $V_{SD} = V_{SG}$ when $R = 0$

f) $V_{SD} = V_{SG}/2$ For the device in saturation, $V_{SG} = 2^V$ (for $I = 100\mu A$), therefore $V_{SD} = \frac{V_{SG}}{2} = 1^V$ implies: $IR = 1^V \Rightarrow \underline{R = 10K\Omega}$

g) $V_{SD} = \frac{V_{SG}}{10}$ In saturation: $V_{SG} = 2^V$, $V_{SD} = 0.2^V$

$\Rightarrow V_{SD} < V_{SG} - |V_t| \Rightarrow$ therefore the device can't be in saturation and it is in triode region: $100 = \frac{1}{2} \times 8 \times 25 \left[(V_{SG} - 1)\left(\frac{V_{SG}}{10}\right) - \left(\frac{V_{SG}}{10}\right)^2 \frac{1}{2} \right]$

$0.5 = \frac{V_{SG}^2}{10} - \frac{V_{SG}^2}{200} - 0.1 V_{SG} \Rightarrow 19 V_{SG}^2 - 20 V_{GS} - 100 = 0$

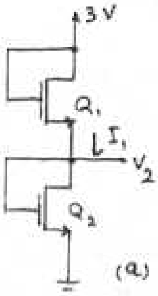
$\Rightarrow \underline{V_{SG} = 2.88^V}$, -1.82^V X

$V_{SD} = \frac{2.88}{10} = 0.29^V \Rightarrow IR = 2.88 - 0.29 = 2.59^V$

$\Rightarrow R = 25.9^{K\Omega}$

4.46

a) Q_2, Q_1 operating in Saturation: $i_{D1} = i_{D2}$



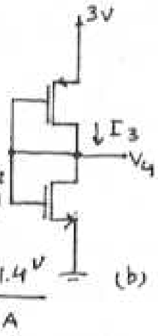
$\Rightarrow V_{GS1} = V_{GS2}$

$3V = V_{GS1} + V_{GS2} \Rightarrow V_{GS1} = V_{GS2} = 1.5^V$

$\underline{V_2 = 1.5^V}$

$I_1 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.5-1)^2 = \underline{7.5 \mu A}$

b) Both transistors have $V_D = V_G$ and therefore they are operating in Saturation: $i_{D1} = i_{D2}$



$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_4 - 1)^2 = \frac{1}{2} \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (3 - V_4 - 1)^2$

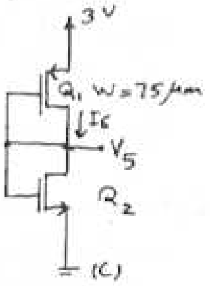
$2.5 (V_4 - 1)^2 = (2 - V_4)^2$

$1.58 (V_4 - 1) = (2 - V_4) \Rightarrow V_4 = \underline{1.39\, V \simeq 1.4^V}$

$I_3 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.39-1)^2 = \underline{4.6 \mu A}$

c) $\frac{W_1}{L_1} = \frac{75}{10} = 7.5$

$\frac{W_2}{L_2} = \frac{30}{10} = 3 \qquad \frac{\frac{W_1}{L_1}}{\frac{W_2}{L_2}} = 2.5$



$i_{D1} = i_{D2}$

Since $\mu_n C_{ox} \frac{W_2}{L_2} = \mu_p C_{ox} \frac{W_1}{L_1}$

$\Rightarrow V_{GS1} = V_{GS2} = \frac{3}{2} = 1.5V = V_5$

$I_6 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.5-1)^2 = \underline{7.5 \mu A}$

4.47

Refer to Fig. P4.47.

Since $V_{G1} = V_{D1}$ then Q_1 is in saturation. We assume that Q_2 is also in saturation, then because $i_{D1} = i_{D2}$, V_{GS1} would be equal to V_{GS2}.

$V_{GS1} = V_{GS2} = \frac{5}{2} = 2.5V$

$I_1 = \frac{1}{2} \times 50 \times \frac{10}{1} (2.5-1)^2 = 562.5 \mu A$

$V_{GS3} = V_{GS1} = 2.5V$. Since Q_3 and Q_4 have the same drain current, then $V_{GS3} = V_{GS4} = 2.5^V$

This is based on the assumption that Q_3 & Q_4 are saturated:

$V_{GS3} = V_{GS1} \Rightarrow I_2 = I_{GS3} = I_{GS1} = \underline{562.5 \mu A}$

$\underline{V_2 = 5 - 2.5 = 2.5V}$

Now if Q_3 and Q_4 have $W = 100 \mu m$ then:

$I_2 = \frac{1}{2} \times 50 \times \frac{100}{1} (2.5-1)^2 = 5.625\, mA$ or

$\frac{I_{Q3}}{I_{Q1}} = \frac{W_3}{W_1} = \frac{100}{10} \Rightarrow I_{Q3} = 10 \times 562.5 \mu A = 5.625 mA$

4.48

a) $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 20$

$V_{GS1} = V_{GS2} = -V_3$

Assuming that the transistors are in saturation, since k'_n and $\frac{W}{L}$ are the same for both Q_2 and Q_1 we can write: $i_{D1} = i_{D2} = \frac{200}{2} = 100 \mu A$

$100 \mu A = \frac{1}{2} \times 100 \times 20 \times (V_{GS} - 1)^2 \Rightarrow 0.1 = (V_{GS} - 1)^2 \Rightarrow$

$V_{GS} = 1.32^V = -V_3 \Rightarrow \underline{V_3 = -1.32V}$

$\underline{V_1 = 5 - 40 \times 100 = 1V} \qquad \underline{V_2 = 1V}$

b) $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 \times 1.5 = 20$ assume Q_1, Q_2 are saturated:

$V_{GS1} = V_{GS2} = -V_3 \Rightarrow \frac{i_{D1}}{i_{D2}} = \frac{(W/L)_1}{(W/L)_2} = 1.5$

$\left.\begin{array}{l} i_{D1} = 1.5 i_{D2} \\ i_{D1} + i_{D2} = 0.2 mA \end{array}\right\} \Rightarrow i_{D1} = 120 \mu A, \; i_{D2} = 80 \mu A$

$i_{D1} = 120 = \frac{1}{2} \times 100 \times 20 \times (V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 1.35^V$

$\Rightarrow \underline{V_3 = -1.35^V}$

$V_1 = 5 - 40 \times 0.12 = \underline{0.2^V} \quad , V_2 = 5 - 40 \times 0.8 = \underline{1.8^V}$

$V_{DS1} = 0.2 + 1.35 = 1.55V \quad > V_{GS} - 1$

$V_{DS2} = 1.8 + 1.35 = 3.15V \quad > V_{GS} - 1$

This Confirms that both transistors are indeed saturated and our assumption was correct.

4.49

a)

Point A : $V_{IA} = V_t = 1^V$, $V_{OA} = V_{DD} = 5^V$

For $V_i < V_t$, the transistor is not on, $V_{GS} < V_t$.

Point A is when $V_{GS} = V_t$ and the transistor turns on. As V_i increases, the i_D increases and v_o decreases. v_o decreases to the point that it is below V_I by V_t volts. At this point, B, the MOSFET enters the triode region: $V_{OB} = V_{IB} - V_t$ or $V_{DS} = V_{GS} - V_t$. So at point B: $I = \frac{V_{DD} - V_{OB}}{R}$

$I = \frac{V_{DD} - (V_{GS} - V_t)}{R} = \frac{1}{2} \times k'_n \frac{W}{L} (V_{GS} - V_t)^2$

$\frac{5 - V_{GS} + 1}{24} = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2 \Rightarrow 12 V_{GS}^2 - 23 V_{GS} + 6 = 0$

Cont.

$V_{GS} = 1.61V \Rightarrow V_I = 1.61^V \quad V_O = 1.61 - 1 = 0.61^V$

Point B: $V_{OB} = 0.61^V \quad V_{IB} = 1.61V$

b) $I_Q = \frac{1}{2} \times 1 \times 0.5^2 = 0.125mA$

$V_{OQ} = 5 - 24 \times 0.125 = 2^V$

$V_{IQ} = V_{GS} = V_{ov} + V_t = 0.5 + 1 = 1.5^V$

Now to calculate the incremental gain A_v at this bias point, From equation 4.41, we have: $A_v = -2\, V_{RD}/V_{ov} = \frac{-2(V_{DD}-V_{OQ})}{V_{ov}}$

$$A_v = \frac{-2(5-2)}{0.5} = -12 V/V$$

C) $V_{IQ} = 1.5^V$, $V_t = 1^V$, $V_{IB} = 1.61^V$. Thus the largest amplitude of a sine wave that can be applied to the input while the transistor remain in saturation is: $1.61 - 1.5 = 0.11V$

The amplitude of the output voltage signal that results is approximately equal to $V_{OQ} - V_{OB} = 2 - 0.61 = 1.39^V$. The gain implied by this amplitudes is:

$gain = \frac{-1.39}{0.11} = 12.64 V/V$

This gain is 5.3% different from the incremental gain calculated in part(b). This difference is due to the fact that the segment of the voltage transfer curve considered here is not perfectly linear.

4.50

$V_{DS} = 1^V = V_{OQ} \Rightarrow I_D = \frac{V_{DD} - V_{OQ}}{R} = \frac{10-1}{18} = 0.5mA$

$I_D = 0.5 = \frac{1}{2} \times 1 \times V_{ov}^2 \Rightarrow V_{ov} = \sqrt{2I_D} = 1V$

$V_{ov} = V_{GS} - V_t \Rightarrow V_{GS} = 2^V$

$A_v = \frac{-2\, V_{RD}}{V_{ov}} = -\frac{2 \times 18 \times 0.5}{1} = -18 V/V$

$v_o^+ = V_{DD} - V_{OQ} = 10 - 1 = 9^V$

$V_o^- = V_{OQ} - V_{OB}$

In order to find V_{OB} where $V_{DS} = V_{GS} - V_t$ or

$V_{OB} = V_{IB} - V_t \quad : \quad v_O = v_{DD} - R_D I_D$

$v_{OB} = 10 - 18 \times \frac{1}{2} \times 1\, V_{OB}^2 \Rightarrow 9V_{OB}^2 + V_{OB} - 10 = 0 \Rightarrow V_{OB} = 1^V$

$V_o^- = V_{OQ} - V_{OB} = 1 - 1 = 0$

Using the same formulas we can calculate the rest of the values for $V_{DS} = 2, 3, \ldots, 10^V$

V_{DS} (V)	I_D (mA)	V_{OV} (V)	V_{GS} (V)	A_V (V/V)	v_o^+ (V)	v_o^- (V)
1	0.5	1	2	18	9	0
2	0.44	0.94	1.94	17.02	8	1
3	0.39	0.88	1.88	15.9	7	2
4	0.33	0.81	1.81	14.8	6	3
5	0.28	0.75	1.75	13.3	5	4
6	0.22	0.66	1.66	12.1	4	5
7	0.17	0.58	1.58	10.3	3	6
8	0.11	0.47	1.47	8.5	2	7
9	0.06	0.35	1.35	5.7	1	8
10	0	0	$\leq 1^V$	0	0	0

4.51

$R_D = 20k\Omega$, $V_{RD} = 2^V \Rightarrow I_D = 0.1mA$

$A_v = -\frac{2\, V_{RD}}{V_{ov}} \Rightarrow -10 = -\frac{2 \times 2}{V_{ov}} \Rightarrow V_{ov} = 0.4^V$

$V_{GS} = 1.2V \Rightarrow V_t = 1.2 - 0.4 = 0.8V$

$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 \Rightarrow 0.1 = \frac{1}{2} \times 50 \times 10^{-3} \frac{W}{L}\, 0.4^2$

$\Rightarrow \frac{W}{L} = 25$

4.52

Eq. 4.41 : $A_v = -\frac{2(V_{DD} - V_{OQ})}{V_{ov}}$

a) $A_{vmax} = -\frac{2(V_{DD} - 0)}{V_{ovmin}} = -\frac{2 \times 5}{0.2} = 50 V/V$

b)

c)

4.53

From Figure 4.26(c) we have: $\hat{v}_o = V_{DS} - V_{OB}$

Assuming linear operation around the bias point:

$$A_v = -\frac{\hat{v}_o}{V_{IB} - V_{IQ}} = -\frac{\hat{v}_o}{V_{OB} + V_t - V_{IQ}} = -\frac{\hat{v}_o}{V_{OB} + V_t - (V_{ov} + V_t)}$$

$$A_v = -\frac{\hat{v}_o}{V_{OB} - V_{ov}} = -\frac{\hat{v}_o}{V_{OB} - V_{DS} + V_{DS} - V_{ov}} = -\frac{\hat{v}_o}{-\hat{v}_o + V_{DS} - V_{ov}}$$

$$-A_v \hat{v}_o + A_v (V_{DS} - V_{ov}) = -\hat{v}_o \Longrightarrow \hat{v}_o (1 - A_v) = -A_v (V_{DS} - V_{ov})$$

$$\hat{v}_o = \frac{A_v (V_{DS} - V_{ov})}{A_v - 1} \Longrightarrow \hat{v}_o = \frac{V_{DS} - V_{ov}}{1 - \frac{1}{A_v}}$$

V_{DS} (V)	A_v (V/V)	$\hat{v}_o$ (V)	$\hat{v}_i$ (V)
1	-16	0.47	0.029
1.5	-14	0.93	0.066
2	-12	1.38	0.12
2.5	-10	1.81	0.18

$$I_D = \frac{1}{2} \times 1 \times (0.5)^2 = 0.125\,mA$$

$$R_D = \frac{V_{DD} - V_{DS}}{I_D} = \frac{5-1}{0.125} = 32\,k\Omega$$

4.54

$$i_{D_1} = i_{D_2} \Rightarrow \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_t)^2 = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_2 (V_{GS2} - V_t)^2$$

Note that $V_{GS1} = V_I$

$V_{GS2} = V_{DD} - V_O$

$$\Rightarrow \left(\frac{W}{L}\right)_1 (V_I - V_t)^2 = \left(\frac{W}{L}\right)_2 (V_{DD} - V_O - V_t)^2$$

$$\sqrt{\left(\frac{W}{L}\right)_1 / \left(\frac{W}{L}\right)_2}\ (v_I - v_t) = v_{DD} - v_O - v_t$$

$$v_O = v_{DD} - v_t + \sqrt{\frac{(W/L)_1}{(W/L)_2}}\, v_t - \sqrt{\frac{(W/L)_1}{(W/L)_2}}\, v_I$$

If $\left(\frac{W}{L}\right)_1 = \frac{50}{0.5} = 100$ $\quad \left(\frac{W}{L}\right)_2 = \frac{5}{0.5} = 10$

then:

$$A_v = \frac{\partial v_O}{\partial v_I} = -\sqrt{\frac{100}{10}} = -3.16\ V/V$$

4.55

$$I_D = 2\,mA = \frac{1}{2} \times 80 \times 10^{-3} \times \frac{240}{6} \times (V_{GS} - 1.2)^2 \Rightarrow$$

$V_{GS} = 2.32\,V$ $\qquad$ Refer to Fig. 4.30c

$$R_D I_D = \frac{15}{3} = 5^V \Rightarrow R_D = \frac{5}{2^{mA}} = 2.5\,k\Omega$$

$$R_S I_D = 5^V \Rightarrow R_S = \frac{5}{2} = 2.5\,k\Omega$$

$$V_G = 5 + V_{GS} = 7.32\,V$$

$$\frac{15}{R_{G1} + R_{G2}} \times R_{G2} = 7.32 \qquad R_{G1} = 22\,M\Omega \Rightarrow R_{G2} = 20.97\,M\Omega$$

$V_{DS} = 5^V$

at the edge of saturation $V_{DS} = V_{GS} - V_t$ or

$V_{DS} = 2.32 - 1.2 = 1.12^V$. So V_{DS} is $5 - 1.12 = 3.88^V$

away from the edge of saturation.

4.56

Refer to Fig. 4.30e

$$I_D = 2\,mA = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow 2 = \frac{1}{2} \times 50 \times 10^{-3} \times \frac{200}{4} V_{OV}^2$$

$V_{OV} = 1.26\,V$

$V_{DS} = V_{OV}$ edge of triode

Midway of cutoff ($V_{DS} = V_{DD}$) and beginning of triode operation ($V_{DS} = V_{ov}$) is when $V_{DS} = \frac{30 + 1.26}{2}$

$V_{DS} = 15.63\,V$

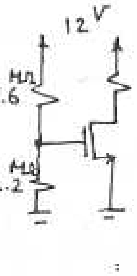
$V_{GS} = 2.32\,V \Rightarrow V_S = -2.32\,V \Rightarrow R_S = \frac{-2.32 + 15}{2}$

$\underline{R_S = 6.34\,k\Omega}$

$V_D = V_S + V_{DS} = -2.32 + 15.63 = 13.31^V \Rightarrow R_D = \frac{15 - 13.31}{2}$

$\underline{R_D = 0.85\,k\Omega}$

4.57



$$V_G = 12 \times \frac{2.2}{2.2 + 5.6} = 3.4^V$$

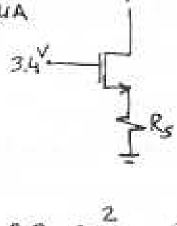
$k_n' \frac{W}{L} = 220$ to $380\ \mu A/V^2$

$V_t = 1.3$ to $2.4\,V$

$$I_D = \frac{1}{2} \times k_n' \frac{W}{L} (3.4 - V_t)^2$$

$$I_{Dmin} = \frac{1}{2} \times 220\,(3.4 - 2.4)^2 = 110\,\mu A$$

$$I_{Dmax} = \frac{1}{2} \times 380\,(3.4 - 1.3)^2 = 838\,\mu A$$



to limit I_{Dmax} to $150\,\mu A$:

$$150 = \frac{1}{2} \times 380\,(3.4 - 0.15 R_S - 1.3)^2$$

$R_S = 8.1\,k\Omega$

Select $\underline{R_S = 8.2\,k\Omega}$

$$I_{Dmax} = \frac{1}{2} \times 380 \times (3.4 - I_{Dmax} \times 8.2 - 1.3)^2$$

$I_{Dmax} = 0.15\,mA$ or $0.\cancel{34}\,mA$

The second answer results in negative V_{GS}

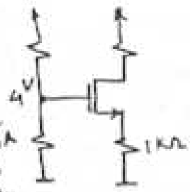
Cont.

and therefore it is not acceptable.

$I_{Dmin} = \frac{1}{2} \times 0.22 \times (3.4 - 8.2 I_{Dmin} - 2.4)^2$

$\underline{I_{Dmin} = 0.04 mA}$

4.58



$V_t = 2^V$, $K_n' \frac{W}{L} = 2 mA/V^2$

$I_D = \frac{1}{2} \times 2 \times (4 - I_D \times 1 - 2)^2$

$I_D = 4 + I_D^2 - 4I_D \Rightarrow I_D = 1^{mA}, \cancel{4^{mA}}$

$I_D = 4^{mA}$ results in $V_{GS} = 0$ which is not acceptable, therefore $I_D = 1mA$.

For $K' \frac{W}{L}$ 50% larger, i.e. $K' \frac{W}{L} = 3 mA/V^2$

$I_D = \frac{1}{2} \times 3 (4 - I_D - 2)^2 \Rightarrow \underline{I_D = 1.13 mA}$

I_D increases by 13%.

4.59

Refer to Fig. 4.30.c

$V_{GS} = 5 - 2 = 3V$, $I_D = \frac{V_S}{R_S} = \frac{2}{1} = 2mA$

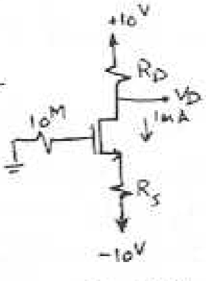
$I_D = 2 = \frac{1}{2} \times 2 \times (3 - V_t)^2 \Rightarrow 1.41 = 3 - V_t \Rightarrow V_t = 1.59^V$

For a device with $V_t = 1.59 - 0.5 = 1.09^V$:

$I_D = \frac{1}{2} \times 2 \times (5 - I_D \times 1 - 1.09)^2 \Rightarrow \underline{I_D = 2.37^{MA}}$

$\underline{V_S = 2.37V}$

4.60



To maximize gain, we design for the lowest possible V_D consistant with allowing a 2V p-p signal swing. $V_{Dmin} = V_D - 1$

$V_{Dmin} = V_G - V_t = 0 - 2$

$V_D - 1 = -2 \Rightarrow V_D = -1V \Rightarrow R_D = \frac{10 - (-1)}{1 mA} = 11 K\Omega$

$I_D = \frac{1}{2} \times 2 [0 - (-10 + 1 \times R_S) - 2]^2 = 1 \Rightarrow 1 = (8 - R_S)^2$

$\underline{R_S = 7 K\Omega}$

4.61

For p-channel MOSFET to be 1V from the edge of saturation: $V_{DS} = V_{GS} - V_t - 1$. Since $V_D = 3V$ and $I_D = 1mA$: $R_D = \frac{3}{1} = 3K\Omega$, $R_1 + R_2 = \frac{10V}{10\mu A} = 1M\Omega$

a) $|V_t| = 1V$, $K_p' \frac{W}{L} = 0.5 mA/V^2$

$I_D = 0.5 \times \frac{1}{2} \times (V_{GS} + 1)^2 = 1 \Rightarrow V_{GS} = -3V$

$V_{DS} = -3 + 1 - 1 = -3V$

$V_S = 6V$, $V_G = 3V$ $R_S = \frac{10 - 6}{1} = 4K\Omega$

$\frac{R_2}{R_1 + R_2} = \frac{3}{10} \Rightarrow R_1 = 0.7M\Omega$, $R_2 = 0.3 M\Omega$

b) $|V_t| = 2V$, $K_p' \frac{W}{L} = 1.25 mA/V^2$

$1 = \frac{1}{2} \times 1.25 (V_{GS} + 2)^2 \Rightarrow V_{GS} = -3.26^V$ or -0.74^V

the second answer is not acceptable $|V_{GS}| < |V_t|$

$\underline{V_{GS} = -3.26V}$

$V_{DS} = -3.26 + 2 - 1 = -2.26V$

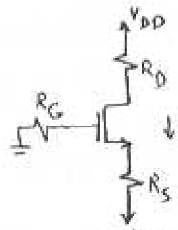
$V_S = 3 + 2.26 = 5.26 V$

$V_G = 2V$

$R_S = \frac{10 - 5.26}{1} = 4.74 K\Omega$, $R_D = 3K\Omega$

$\frac{R_2}{R_1 + R_2} = \frac{2}{10} \Rightarrow R_2 = 0.2 M\Omega$ $R_1 = 0.8 M\Omega$

4.62



$K = \frac{1}{2} K' \frac{W}{L}$

a) $I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2$

$I_D = K (0 + V_{SS} - R_S I_D - V_t)^2$

$\frac{\partial I_D}{\partial K} = (V_{SS} - R_S I_D - V_t)^2 + 2K (V_{SS} - R_S I_D - V_t)(-R_S) \frac{\partial I_D}{\partial K}$

$\frac{\partial I_D}{\partial K} = \frac{I_D}{K} - 2R_S \sqrt{\frac{I_D}{K}} \, K \frac{\partial I_D}{\partial K}$

$\frac{\partial I_D}{\partial K} (1 + 2\sqrt{K I_D} R_S) = \frac{I_D}{K} \Rightarrow S_K^{I_D} = \frac{\partial I_D}{\partial K} \frac{K}{I_D} = \frac{1}{1 + 2\sqrt{K I_D} R_S}$

b) $K = 100 \, \mu A/V^2$, $\frac{\Delta K}{K} = \pm 10\%$, $V_t = 1V$, $I_D = 100 \mu A$

$\frac{\Delta I_D}{I_D} = \pm 1\%$

$S_K^{I_D} \simeq \frac{\partial I_D / I_D}{\partial K / K} = \frac{1}{10} = 0.1 = \frac{1}{1 + 2\sqrt{100 \times 10^{-3} \times 100 \times 10^{-3}} R_S}$

$\underline{\Rightarrow R_S = 45 K\Omega}$

Cont.

$100 = 100\,(V_{GS}-1)^2 \Rightarrow V_{GS} = 2V$

$V_{GS} = V_{SS} - I_D R_S \Rightarrow 2 = V_{SS} - 0.1\times 45 \Rightarrow \underline{V_{SS} = 6.5^V}$

c) For $V_{SS} = 5V$:

$R_S = \dfrac{-V_{GS}+V_{SS}}{I_D} = \dfrac{-2+5}{0.1} = 30k\Omega$

$S_K^{I_D} = \dfrac{1}{1+2\sqrt{0.1\times0.1}\times 30} = \dfrac{1}{1+2\times0.1\times30} = 0.14$

Therefore for $\dfrac{\Delta K}{K} = \pm 10\%$, $\dfrac{\Delta I_D}{I_D} = \pm 1.4\%$

4.63

Both cases are in saturation region, because $V_{DG} \geq V_t$.

$V_D = 10 - 5\times 1 = 5^V$

a) $1 = \frac{1}{2}\times 0.5\times(V_{GS}-1)^2 \Rightarrow V_{GS} = 3V$, $V_S = -3V$

$V_{DS} = 8V$

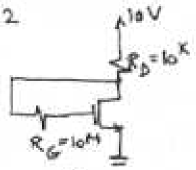
b) $1 = \frac{1}{2}\times 1.25\times(V_{GS}-2)^2 \Rightarrow V_{GS} = 3.3V$, $V_S = -3.3^V$

$V_{DS} = 8.3V$

4.64

$V_D = V_G = V_{GS}$ Refer to Fig. 4.32

to operate in saturation:



$V_{DS} \geq V_{GS} - V_t \Rightarrow V_{DG} \geq -V_t$

$V_{DG} = 0$

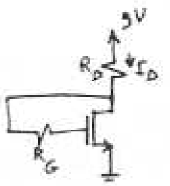
a) $\dfrac{10 - V_D}{10} = \frac{1}{2}\times 0.5\times(V_D-1)^2 \Rightarrow V_D = 2.7^V$

$V_G = 2.7^V$

b) $\dfrac{10-V_D}{10} = \frac{1}{2}\times 1.25\,(V_D-2)^2 \Rightarrow V_D = 3.05V$

$V_G = 3.05V$

4.65



For $I_D = 0.2mA$:

$0.2 = \frac{1}{2}\times 0.4\times(V_{GS}-1)^2$

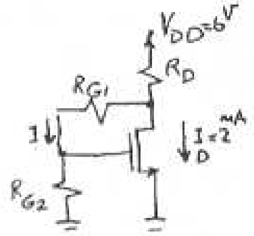
$V_{GS} = 2V$, $V_D = V_G = V_{GS} = 2V$

$R_D = \dfrac{9-2}{0.2} = 35K\Omega$

Select $R_D = 36K\Omega \Rightarrow \dfrac{9-V_D}{R_D} = \frac{1}{2}\times 0.4(V_D-1)^2$

$\dfrac{9-V_D}{36} = 0.2(V_D-1)^2 \Rightarrow V_D = 2V$, $I_D = 0.21mA$.

4.66



$I_D = 2 = \frac{1}{2}\times 3.2\times(V_{GS}-1.2)^2$

$V_{GS} - 1.2 = 1.12 \Rightarrow V_{GS} = 2.32^V$

$V_G = 2.32^V$

$V_{DSmin} = V_{GS} - V_t = 1.12^V$

$V_{DS} = V_{DSmin} + 2 = 3.12^V$

$R_{G_2} = 22M\Omega \Rightarrow I = \dfrac{2.32}{22} = 0.11\mu A$

$R_{G_1} = \dfrac{3.12-2.32}{0.11} = \underline{7.58M\Omega}$

$R_D = \dfrac{6-3.12}{2+0.11\times10^{-3}} = \underline{1.44K\Omega}$

4.67

Eq. 4.57 implies:

$i_D = I_D + K_n'\frac{W}{L}(V_{GS}-V_t)v_{gs}\sin\omega t + \frac{1}{2}K_n'\frac{W}{L}v_{gs}^2\sin^2\omega t$

$i_D = I_D + K_n'\frac{W}{L}(V_{GS}-V_t)v_{gs}\sin\omega t + \frac{1}{2}K_n'\frac{W}{L}v_{gs}^2\left(\frac{1-\cos 2\omega t}{2}\right)$

$i_D = I_D + K_n'\frac{W}{L}(V_{GS}-V_t)v_{gs}\sin\omega t + \frac{1}{4}K_n'\frac{W}{L}v_{gs}^2 - \frac{v_{gs}^2}{4}K_n'\frac{W}{L}\cos 2\omega t$

Second Harmonic Distortion $= \dfrac{\frac{1}{4}K_n' W/L\, v_{gs}^2}{K_n' W/L\,(V_{GS}-V_t)V_{gs}}\times 100$

$= \dfrac{1}{4}\,\dfrac{V_{gs}}{V_{GS}-V_t}\times 100$

Second Harmonic Distortion $= \underline{\dfrac{1}{4}\,\dfrac{V_{gs}}{V_{ov}}\times 100}$

For $V_{gs} = 10mV$ $\quad \dfrac{1}{4}\times\dfrac{10\times10^{-3}}{V_{ov}}\times 100 \leq 1$

$\Rightarrow V_{ov} \geq 0.25V \Rightarrow V_{ov\,min} = \underline{0.25V}$

4.68

$I_D = \frac{1}{2}K_n'\frac{W}{L}V_{ov}^2 \Rightarrow I_D = \frac{1}{2}\times 2\times 1^2 = 1mA$

$i_D = \frac{1}{2}\times 2\times(1+0.1)^2 = 1.21mA \quad (v_{gs} = 0.1V)$

$i_D = 1.21 - 1 = 0.21mA$

If $v_{gs} = -0.1^V \Rightarrow i_D = \frac{1}{2}\times 2(1-0.1)^2 = 0.81mA$

$i_D = 0.81 - 1 = -0.19mA$

For positive increment: $g_m = \dfrac{\Delta i_D}{\Delta v_{gs}} = \dfrac{0.21}{0.1} = 2.1\ mA/V$

For negative increment: $g_m = \dfrac{0.19}{0.1} = 1.9mA/V$

An estimate of $g_m = \dfrac{2.1+1.9}{2} = 2mA/V$

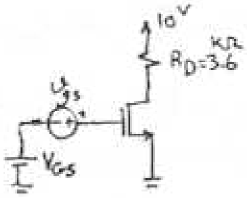
Eq. 4.62 $g_m = K_n'\frac{W}{L}V_{ov} = 2\times 1 = \underline{2mA/V}$ (same as estimate!)

4.69

a) $I_D = \frac{1}{2} \times 1 \times (4-2)^2 = \underline{2mA}$

$V_D = V_{DD} - R_D I_D = 10 - 2 \times 3.6$

$\underline{V_D = 2.8V}$



b) $g_m = k'_n \frac{W}{L} V_{ov} = 1 \times (4-2) = \underline{2mA/V}$

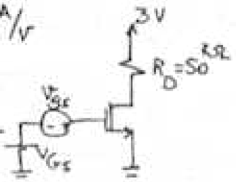
c) $A_v = \frac{v_d}{v_{gs}} = -g_m R_D = -2 \times 3.6 = \underline{-7.2 V/V}$

d) $r_o \simeq \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 2} = \underline{50k\Omega}$

$A_v = \frac{v_d}{v_{gs}} = -g_m (R_D \| r_o) = -2(3.6\|50) = \underline{-6.7 V/V}$

4.70

$g_m R_D = 5 \Rightarrow g_m = \frac{5}{50} = 0.1 mA/V$



For 0.5V output signal and a gain of $5 V/V$, $v_{gs} = \frac{0.5}{5} = 0.1V$

So we can write $V_{DS} - 0.5 \geq V_{GS} + 0.1 - V_t$

or $V_{DS} \geq V_{GS} + 0.6 - 0.8 \Rightarrow \underline{V_{DS} \geq V_{GS} - 0.2}$

Also, from the other side: $V_{DS} + 0.5 \leq V_{DD}$

or $V_{DS} \leq 3 - 0.5 \Rightarrow \underline{V_{DS} \leq 2.5V}$

We design the circuit for lowest possible V_{DS} that guarantees the device operation in saturation: $V_{DS} = V_{GS} - 0.2$

$V_{DS} = V_{DD} - R_D I_D \Rightarrow V_{GS} - 0.2 = 3 - 50 \times I_D$

$\Rightarrow \underline{I_D = \frac{3.2 - V_{GS}}{50}}$

Also, from eq. 4.71: $g_m = \frac{2 I_D}{V_{GS} - V_t} = 0.1$

$0.1 = \frac{2}{V_{GS} - 0.8} \times \frac{3.2 - V_{GS}}{50}$

$\Rightarrow V_{GS} = 1.49V$, $I_D = \underline{0.034mA}$

$V_{DS} = 1.49 - 0.2 = 1.29V$ $\quad V_{ov} = 1.49 - 0.8 = \underline{0.69V}$

$\frac{W}{L} = \frac{I_D}{\frac{1}{2} k'_n V_{ov}^2} = \frac{0.034 \times 10^3}{\frac{1}{2} \times 100 \times 0.69^2} = 1.43$

$\underline{\frac{W}{L} = 1.43}$

4.71

$$\left.\begin{array}{l} A_v = -g_m R_D , \\ g_m = \frac{2 I_D}{V_{ov}} \quad \text{eq. 4.71} \end{array}\right\} \Rightarrow A_v = -\frac{2 R_D I_D}{V_{ov}} = -\frac{2(V_{DD} - V_D)}{V_{ov}} \quad (1)$$

Minimum v_{DS} for edge of saturation:

$v_{DS} \geq v_{GS} - V_t$ or $v_{DSmin} = v_{GSmax} - V_t$

$V_{DS} - |A_v| \hat{v}_i = V_{GS} + \hat{v}_i - V_t$

IF we replace A_v with (1):

$V_D - \frac{2(V_{DD} - V_D)}{V_{ov}} \hat{v}_i = V_{ov} + \hat{v}_i$

$\Rightarrow V_D \left(1 + \frac{2\hat{v}_i}{V_{ov}}\right) = V_{ov} + \hat{v}_i + \frac{2V_{DD}}{V_{ov}} \hat{v}_i$

$$V_D = \frac{V_{ov} + \hat{v}_i + 2V_{DD}(\hat{v}_i / V_{ov})}{1 + 2(\hat{v}_i / V_{ov})}$$

$V_{DD} = 3V$, $\hat{v}_i = 20mV$ $\quad m = 10 = \frac{V_{ov}}{\hat{v}_i} \Rightarrow \underline{V_{ov} = 0.2V}$

$V_D = \frac{0.2 + 0.02 + 2 \times 3 \times 10^{-1}}{1 + 2 \times 0.1} = \underline{0.68V}$

$A_v = -\frac{2(3 - 0.68)}{0.2} = \underline{-23.2 V/V}$

IF $I_D = 100\mu A = 0.1mA$:

$A_v = -\frac{2 R_D I_D}{V_{ov}} \Rightarrow 23.2 = \frac{2 \times R_D \times 0.1}{0.2} \Rightarrow$

$\underline{R_D = 23.2k\Omega}$

$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{ov}^2 \Rightarrow 0.1 = \frac{1}{2} \times 100 \times 10^{-3} \frac{W}{L} 0.2^2$

$\Rightarrow \underline{\frac{W}{L} = 50}$

4.72

$k'_n = \mu_n C_{ox} = 500 \times 10^8 \times 0.4 \times 10^{-15} = 20\mu A/V^2$, $k'_p = 10\mu A/V^2$

Case	Type	I_D (mA)	$\|V_{GS}\|$ (V)	$\|V_t\|$ (V)	V_{ov} (V)	W (μm)	L (μm)	$\frac{W}{L}$	$k'\frac{W}{L}$ (mA/V^2)	g_m (mA/V)
a	N	1	3	2	1	100	1	100	2	2
b	N	1	1.2	0.7	0.5	50	$\frac{1}{8}$	400	8	4
c	N	10	?	?	2	250	1	250	5	10
d	N	0.5	?	?	0.5	?	?	200	4	2
e	N	0.1	?	?	1.41	10	2	5	0.1	0.14
f	N	0.1	1.8	0.8	1	40	4	10	0.2	0.2
g	P	1	?	?	2	?	?	25	0.25	1
h	P	1	3	1	2	?	?	50	0.5	1
i	P	10	?	?	1	4000	2	2000	20	20

Cont.

Case	Type	I_D	V_{GS}	V_t	V_{ov}	W	L	$\frac{W}{L}$	$\frac{k'W}{L}$	g_m
j	P	10	?	?	4	?	?	125	1.25	5
K	P	0.05	?	?	1	30	3	10	0.1	0.1
L	P	0.1	?	?	5	?	?	0.8	$\frac{8}{1000}$	0.04

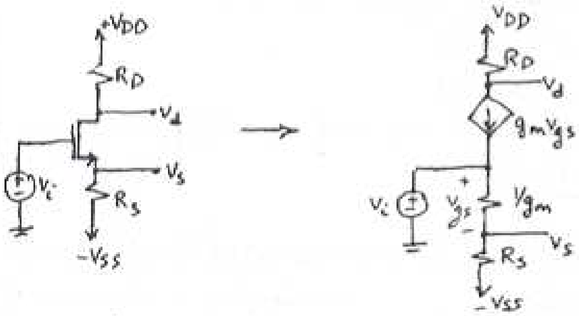
4.73

Eq. 4.70 : $g_m = \sqrt{2k_n' \frac{W}{L} I_D} \Rightarrow \frac{W}{L} = \frac{g_m^2}{2k_n' I_D}$

$\frac{W}{L} = \frac{1}{2 \times 50 \times 10^{-3} \times 0.5} \Rightarrow W = 20\,\mu m$

$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2 \times 0.5}{1} = 1 \Rightarrow V_{GS} = 1 + V_t = 1.7^V$

4.74



$\frac{v_s}{v_i} = \frac{R_S}{R_S + \frac{1}{g_m}} = \frac{R_S g_m}{R_S g_m + 1}$

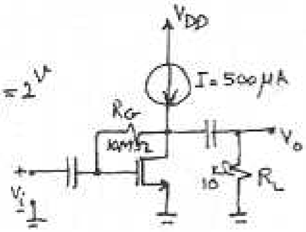
$\frac{v_d}{v_i} = \frac{-g_m v_{gs} R_D}{v_i} = -g_m R_D \frac{1/g_m}{1/g_m + R_S} = \frac{-g_m R_D}{1 + g_m R_S}$

4.75

$r_o \simeq \frac{V_A}{I_D} = \frac{50}{0.5} = 100\,k\Omega$

$g_m = \frac{2 I_D}{V_{GS} - V_t}$, $V_{GS} = V_{DS} = 2^V$

$g_m = \frac{2 \times 0.5}{2 - 0.9} = 0.91\,mA/V$



$\frac{v_o}{v_i} = -g_m (r_o \| R_L) = -0.91 (100^k \| 10^k) = -8.3\,V/V$

For I = 1 mA or twice the current:

$\frac{I_{D1}}{I_{D2}} = \frac{(V_{GS} - V_t)^2}{(V_{GS_2} - V_t)^2} \Rightarrow V_{GS2} = V_t + \sqrt{2}\,(V_{GS1} - V_t)$

$V_{GS2} = 0.9 + \sqrt{2}\,(2 - 0.9) = 2.5^V$

$\frac{g_{m1}}{g_{m2}} = \sqrt{\frac{I_{D1}}{I_{D2}}} \Rightarrow g_{m2} = \sqrt{2}\, g_{m1} = 1.3\,mA/V$

$\frac{r_{o1}}{r_{o2}} = \frac{I_{D2}}{I_{D1}} \Rightarrow r_{o2} = \frac{100}{2} = 50\,k\Omega$

$A_v = -1.3 \times (50k \| 10k) = -10.8\,V/V$

4.76

NMOS: $g_m = \sqrt{2 k_n' \frac{W}{L} I_D} = \sqrt{2 \times 90 \times 10^{-3} \times \frac{20}{2} \times 0.1} = 0.42\,\frac{mA}{V}$

$r_o = \frac{|V_A|}{I_D} = \frac{8 \times 2}{0.1} = 160\,k\Omega$

$\chi = \frac{\gamma}{2\sqrt{2\phi_f + |V_{SB}|}} = \frac{0.5}{2\sqrt{2 \times 0.34 + 1}} = 0.2$

$g_{mb} = \chi g_m = 0.2 \times 0.42 = 0.084\,mA/V$

$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2 \times 0.1}{0.42} = 0.48\,V$

PMOS: $g_m = \sqrt{2 \times 30 \times 10^{-3} \times \frac{20}{2} \times 0.1} = 0.24\,mA/V$

$r_o = \frac{|V_A|}{I_D} = \frac{12 \times 2}{0.1} = 240\,k\Omega$

$\chi = 0.2 \Rightarrow g_{mb} = 0.2 \times 0.24 = 0.048\,mA/V$

$V_{ov} = \frac{2 \times 0.1}{0.24} = 0.83\,V$

4.77

Refer to Fig. P4.77.

a) $V_G = 15 \times \frac{5}{10+5} = 5^V$ $\quad V_S = 3 \times I_D \Rightarrow V_{GS} = 5 - 3I_D$

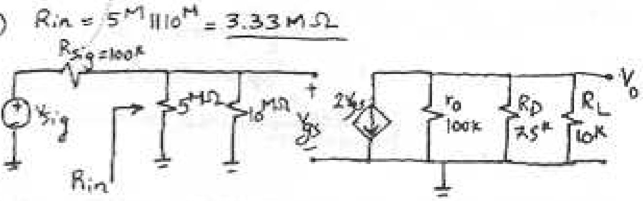
$I_D = \frac{1}{2} \times k' \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow I_D = \frac{1}{2} \times 2 \times (5 - 3I_D - 1)^2$

$\Rightarrow 16 - 25 I + 9 I^2 = 0 \Rightarrow I = 1\,mA$

$V_{GS} = 2\,V \quad V_D = 15 - 7.5 = 7.5^V$

b) $g_m = \frac{2 I_D}{V_{ov}} = \frac{2 \times 1}{2 - 1} = 2\,mA/V \quad r_o = \frac{V_A}{I_D} = 100\,k\Omega$

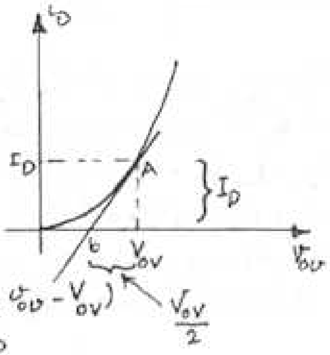
c) $R_{in} = 5^M \| 10^M = 3.33\,M\Omega$



d) $\frac{V_{gs}}{V_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} = \frac{3.33}{0.1 + 3.33} = 0.97\,V/V$

$\frac{V_o}{V_{gs}} = 2 \times (r_o \| R_D \| R_L) = 2 \times 4.1^K = 8.2\,V/V \quad \frac{V_o}{V_{sig}} = 7.95\,V/V$

4.78



$i_D = \frac{1}{2} k_n' \frac{W}{L} v_{ov}^2$

The equation for the tangent line passing from point A (V_{ov}, I_D) can be written as:

$$(i_D - I_D) = \left.\frac{\partial i_D}{\partial v_{ov}}\right|_{\substack{i=I_D \\ v=V_{ov}}} (v_{ov} - V_{ov})$$

$$\left.\frac{\partial i_D}{\partial v_{ov}}\right|_{i_D=I_D} = k_n' \frac{W}{L} V_{ov}$$

$$\Rightarrow i_D - I_D = k_n' \frac{W}{L} V_{ov} (v_{ov} - V_{ov})$$

$$i_D - \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = k_n' \frac{W}{L} V_{ov} (v_{ov} - V_{ov})$$

The tangent intersects the v_{ov} axis at $i_D = 0$

$$0 - \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = k_n' \frac{W}{L} V_{ov} (v_{ov} - V_{ov})$$

$$-\frac{V_{ov}}{2} = v_{ov} - V_{ov} \Rightarrow v_{ov} = \frac{V_{ov}}{2}$$

The slope of the tangent is g_m and it is equal to: $g_m = \frac{\Delta i_D}{\Delta v_{ov}} = \frac{I_D}{V_{ov} - \frac{V_{ov}}{2}} = \frac{I_D}{\frac{V_{ov}}{2}} = \frac{2I_D}{V_{ov}}$

4.79

For this common-source amplifier we have:

$g_m = 2\frac{mA}{V}$, $r_o = 50K\Omega$, $R_D = 10K\Omega$

$R_G = 10M\Omega$, $R_{sig} = 0.5M\Omega$ and $R_L = 20K\Omega$

From equation 4.82 we have:

$$G_v = -\frac{R_G}{R_G + R_{sig}} g_m (r_o \| R_D \| R_L) = -\frac{10 \times 2}{10 + 0.5} (50^K \| 10^K \| 20^K)$$

$$G_v = -11.2\ V/V$$

4.80

a) $A_{v_o} = -2\frac{(V_{DD} - V_D)}{V_{ov}} = -2\frac{(10 - 2.5)}{1} = -15 V/V$

b) If V_{ov} is halved ($V_{ov} = 0.5$) then I_D is divided by 4, i.e. $I_D = \frac{0.5}{4} = 0.125\,mA$

Since V_D is kept unchanged at 2.5^V then:

$R_D = \frac{10 - 2.5}{0.125} = 60K\Omega$, $g_m = \frac{2I_D}{V_{ov}} = 0.5\frac{mA}{V}$

$r_o = \frac{V_A}{I_D} \Rightarrow r_o = 4 \times r_{o1} = 4 \times \frac{75}{0.5} = 600K\Omega$

$A_{v_o} = -15 \times 2 = -30 V/V$ (without r_o)

c) If we take r_o into account:

$A_{v_o} = -g_m (r_o \| R_D) = -0.5 (600^K \| 60^K) = -27.3 V/V$

$R_{out} = R_D \| r_o = 600^K \| 60^K = 54.5K\Omega$

d) $R_{in} = R_G = 4.7M\Omega$

$R_o = R_{out} = 54.5K\Omega$

$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{v_o} \frac{R_L}{R_L + R_o} = \frac{4.7}{4.7 + 0.1} \times 27.3 \times \frac{15}{15 + 54.5}$

$G_v = 5.77 V/V$

e) As we can see by reducing V_{ov} to half of its value or equivalently multiplying drain current by 4, A_{v_o} is almost doubled, while R_{out} is multiplied by 4.

As a result G_v which is proportional to both A_{v_o} and $\frac{1}{R_{out}}$ is only slightly reduced.

(G_v was $-7 V/V$ before and it is $5.8 V/V$ now)

4.81

For an NMOS common-gate amplifier with $g_m = 5\,mA/V$, $R_D = 5K\Omega$, $R_L = 2K\Omega$, $R_{sig} = 200\Omega$ we have:

$R_{in} = \frac{1}{g_m} = \frac{1}{5} = 0.2K\Omega = 200\Omega$

From Eq. 4.96b we know that the overall voltage gain of this amplifier is:

$$G_v = \frac{g_m (R_D \| R_L)}{1 + g_m R_{sig}} = \frac{5(5K \| 2K)}{1 + 5 \times 0.2} = 3.57 V/V$$

If we increase the bias current by a factor of 4, while maintaining the other parameter constant (assuming linear operation), we have:

$g_m = \sqrt{2 k_n' \frac{W}{L} I_D} \Rightarrow \frac{g_{m2}}{g_{m1}} = \sqrt{\frac{I_{D2}}{I_{D1}}} = \sqrt{4} = 2$

$g_m = 2 \times 5 = 10\ mA/V$

$R_{in} = \frac{1}{g_m} = 0.1\ K\Omega = 100\Omega$

$G_v = g_m \frac{R_D \| R_L}{1 + g_m R_{sig}} = 10 \frac{(5^K \| 2^K)}{1 + 10 \times 0.2} = 4.76 V/V$

4.82

Refer to Eq. 4.90: $G_v = \frac{R_G}{R_G+R_{sig}} \frac{g_m(R_D \| R_L)}{1+g_m R_s}$

$G_{v1} = -16\ V/V$ reduced by factor of 4:

$\frac{G_{v2}}{G_{v1}} = \frac{1}{1+g_m R_s} \Rightarrow \frac{1}{4} = \frac{1}{1+2R_s} \Rightarrow R_s = 1.5K\Omega$

4.83

Eq. 4.90: $G_v = \frac{R_G}{R_G+R_{sig}} \frac{g_m(R_D \| R_L)}{1+g_m R_s}$

$G_v = -10\ V/V$ for $R_s = 1K\Omega$ and $G_v = -20\ V/V$ for $R_s = 0$

$\frac{G_{v1}}{G_{v2}} = \frac{1+g_m R_{s2}}{1+g_m R_{s1}} \Rightarrow \frac{10}{20} = \frac{1+g_m \times 0}{1+g_m \times 1} \Rightarrow 1+g_m = 2$

$g_m = 1\ mA/V$

In order to have $G_v = -8\ V/V$:

$\frac{G_{v1}}{G_{v3}} = \frac{20}{8} = \frac{1+g_m R_{s3}}{1+0} \Rightarrow 1+R_{s3} = 2.5 \Rightarrow R_{s3} = 1.5^{K\Omega}$

4.84

$A_{vo} = 0.98\ V/V$ and $A_v = \frac{0.98}{2} = 0.49\ V/V$ for $R_L = 500^\Omega$

Eq. 4.102a: $A_v \simeq \frac{R_L}{R_L+\frac{1}{g_m}} \Rightarrow 0.49 = \frac{0.5}{0.5+\frac{1}{g_m}} \Rightarrow$

$g_m = 1.92\ mA/V$

Eq. 4.103: $A_{vo} = \frac{r_o}{r_o+\frac{1}{g_m}} \Rightarrow 0.98 = \frac{r_o}{r_o+0.52}$

$\Rightarrow r_o = 25.5\ k\Omega$

4.85

We have $g_m = 5\ mA/V$ and $r_o = 20K\Omega$. From Eq. 4.103 we know: $A_{vo} = \frac{r_o}{r_o+\frac{1}{g_m}} = \frac{20}{20+\frac{1}{5}} = 0.99\ V/V$

$A_{vo} = 0.99\ V/V$

From Eq. 4.105: $R_{out} = \frac{1}{g_m} \| r_o = \frac{1}{5}^K \| 20^K = 198\Omega$

$R_{out} \simeq 200\Omega$

With $R_L = 1K\Omega$ we have:

$A_v = \frac{R_L \| r_o}{(R_L \| r_o)+\frac{1}{g_m}} = \frac{1^K \| 20^K}{(1^K \| 20^K)+\frac{1}{5}}$

$\Rightarrow A_v = 0.83\ V/V$

4.86

$R_{i2} = \frac{1}{g_{m2}} = 50\Omega \Rightarrow g_{m2} = \frac{1}{0.05} = 20\ mA/V$

if Q_1 is biased the same as Q_2, then $g_{m1} = g_{m2}$

$i_{D1} = g_{m1} v_i = 20 \times 5^{mV} = 100\mu A = 0.1mA$

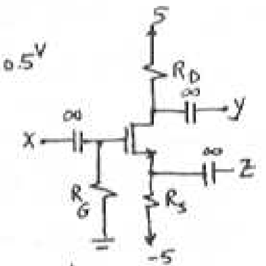
$v_{D1} = i_{D1} \times 50^\Omega = 0.5^V$

In order to have $v_{d2} = v_o = 1^V$:

$v_o = i_D R_D \Rightarrow 1 = 0.1 \times R_D \Rightarrow R_D = 10K\Omega$

4.87

a) $I_D = 0.1 = \frac{1}{2} \times 0.8 \times V_{ov}^2 \Rightarrow V_{ov} = 0.5^V$



$\Rightarrow V_{GS} = 0.5+1 = 1.5^V$

$V_G = 0 \Rightarrow V_S = -1.5V$

$R_s = \frac{-1.5-(-5)}{0.1} = 35K\Omega$

$V_{DS} = 5 - R_D \times 0.1$

Largest possible R_D is achieved for V_{DSmin}

$V_{DS} \geq V_{GS} - V_t \Rightarrow V_{DSmin} = V_{ov} \Rightarrow V_{DS} - 1 = V_{ov}$

$\Rightarrow V_{DS} = 1+0.5 = 1.5V \Rightarrow R_D = \frac{5-1.5}{0.1} = 35K\Omega$

$R_G = 10M\Omega$.

b) $g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 0.1}{0.5} = 0.4\ mA/V$

$r_o = \frac{V_A}{I_D} = \frac{40}{0.1} = 400K\Omega$

c) If Z is grounded then the circuit becomes a common-source configuration. The voltage gain according to Eq. 4.82:

$G_v = -\frac{R_G}{R_G+R_{sig}} g_m (r_o \| R_D \| R_L)$

$G_v = \frac{10M}{10^M+1^M} \times 0.4 \times (400^K \| 35^K \| 40^K) = 6.5\ V/V$

$G_v = 6.5\ V/V$

d) If y is grounded, then the circuit becomes a source follower configuration.

Eq. 4.103: $A_{vo} = \frac{r_o}{r_o+\frac{1}{g_m}} = \frac{400}{400+\frac{1}{0.4}} = 0.99\ V/V$

$R_{out} = \frac{1}{g_m} \| r_o = \frac{1}{0.4} \| 400$

$R_{out} = 2.48K\Omega$

Cont.

e) If x is grounded, the circuit becomes a common-gate configuration.

$R_{in} = \frac{1}{g_m} \| R_s = 35k \| \frac{1}{0.4} = 2.33k\Omega$

Eq. 4.98: $i_i = i_{sig} \frac{R_{sig}}{R_{sig} + R_{in}} \Rightarrow$

$$i_i = 10\mu A \frac{100^k}{100^k + 2.33^k} = 9.77\mu A$$

$v_y = R_D \times i_i = 35 \times 9.77\mu A = \underline{0.34V}$

4.88

a) Fig. P4.88a is a source follower:

Eq. 4.103: $A_{v_o} = \frac{r_o}{r_o + \frac{1}{g_m}}$; $r_o \gg \frac{1}{g_m} \Rightarrow \underline{A_{v_o} \simeq 1V/V}$

$R_{out} = \frac{1}{g_m} = \frac{1}{5} = \underline{0.2k\Omega}$

b) Fig. P4.88b is a common-gate configuration:

$R_{in} = \frac{1}{g_m} = \frac{1}{5} = 0.2k\Omega$

Eq. 4.94: $A_v = g_m (R_D \| R_L) = 5(5^k \| 2^k) = \underline{7.1V/V}$

c) If we connect both stages together, then:

for the first stage: $A_v = A_{v_o} \frac{R_L}{R_L + R_{out}}$

where R_L is in fact R_{in} of the second stage.

Therefore: $A_{v_1} = 1 \times \frac{0.2k}{0.2 + 0.2} = 0.5V/V$

For the second stage: $A_{v_2} = 7.1V/V$

Overall gain $A_v = A_{v_1} A_{v_2} = 7.1 \times 0.5 = 3.55V/V$

4.89

4.90

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-11}}{8\times10^{-9}} = 4.3\times10^{-3}\,F/_{m^2} = 4.3\ fF/_{\mu m^2}$

$K'_n = \mu_n C_{ox} = 450\times10^{-4}\times4.3\times10^{-3} = 193.5\,\mu A/_{V^2}$

$I_D = 100\,\mu A = \frac{1}{2}\times193.5\times\frac{20}{1}V_{ov}^2 \Rightarrow \underline{V_{ov} = 0.23V}$

$V_{DS} = 1.5^V > V_{ov} \Rightarrow$ Saturation

$g_m = \frac{2I_D}{V_{ov}} = \underline{880\,\mu A/V}$, $r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05\times0.1} = \underline{200^{k\Omega}}$

$\chi = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} = \frac{0.5}{2\sqrt{0.65+1}} = 0.19$

$g_{mb} = \chi g_m = \underline{167.2\,\mu A/V}$

$C_{ov} = W L_{ov} C_{ox} = 20\times0.05\times4.3 = 4.3fF$

$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov} = \frac{2}{3}\times20\times1\times4.3 + 4.3 = \underline{61.6fF}$

$C_{gd} = C_{ov} = \underline{4.3\ fF}$

$C_{sb} = \frac{C_{sbo}}{\sqrt{1+\frac{V_{SB}}{V_o}}} = \frac{15}{\sqrt{1+\frac{1}{0.7}}} = \underline{9.6\ fF}$

$C_{db} = \frac{C_{dbo}}{\sqrt{1+\frac{V_{dB}}{V_o}}} = \frac{15}{\sqrt{1+\frac{(1+1.5)}{0.7}}} = \underline{7\ fF}$

4.91

$g_m = \frac{2I_D}{V_{ov}} = \frac{2\times0.1}{0.25} = 0.8\,mA/_V$

$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{0.8\times10^{-3}}{2\pi(20+5)\times10^{-15}} = 5.1\,GHz$

4.92

$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$, $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$

Also $C_{gs} \simeq \frac{2}{3}WLC_{ox}$, if $C_{gs} >> C_{gd}$ then we can ignore C_{gd}. If we replace for g_m and C_{gs} in the f_T formula, we have:

$$f_T = \frac{\sqrt{2\mu_n C_{ox} W/L\, I_D}}{2\pi\times\frac{2}{3}WLC_{ox}} = \frac{1.5}{\pi L}\sqrt{\frac{\mu_n I_D}{2C_{ox}WL}}$$

Therefore we can see that the higher the current I_D, then the higher is the f_T. Also the frequency is reverse proportional to the size of the device, i.e. higher frequencies are achievable for smaller devices.

4.93

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} \quad (1)$$

For $C_{gs} >> C_{gd}$ and the overlap capacitance of C_{gs} negligibly small: $C_{gs} \simeq \frac{2}{3}WLC_{ox}$

Also $g_m = \frac{2I_D}{V_{ov}} = K'_n\frac{W}{L}V_{ov}$

If we substitude g_m and C_{gs} in (1) from the above formulas: $f_T = K'_n\frac{W}{L}V_{ov}\frac{1}{2\pi\times\frac{2}{3}WLC_{ox}}$

$\Rightarrow f_T = \frac{3\mu_n V_{ov}}{4\pi L^2}$

Therefore, for a given device f_T is proportional to V_{ov}. $f_T \propto V_{ov}$

For $L = 1\,\mu m$, $V_{ov} = 0.25$:

$f_T = \frac{3\times450\times10^{-4}\times0.25}{4\times\pi\times1\times10^{-12}} = \underline{2.7\ GHz}$

For $V_{ov} = 0.5V$: $\frac{f_{T1}}{f_{T2}} = \frac{V_{ov1}}{V_{ov2}} \Rightarrow f_{T_2} = 2.7\times\frac{0.5}{0.25}$

$\underline{f_T = 5.4\,GHz}$

4.94

$A_M = -27\,V/_V$, $C_{gs} = 0.3pF$, $C_{gd} = 0.1pF$

Common-Source configuration.

Eq. 4.127 : $C_{in} = C_{gs} + C_{gd}(1+g_m R'_L)$.

Also $A_M = -g_m R'_L$, therefore:

$C_{in} = 0.3 + 0.1\times(1+27) = 3.1pF$

Now to find the range of R_{sig} that results in 3-db frequencies over 10MHZ, we use eq. 4.132: $f_H = \frac{1}{2\pi C_{in}R'_{sig}}$

If we neglect R_G effect then $R'_{sig} \simeq R_{sig}$.

$f_H \geq 10^{MHZ} \Rightarrow \frac{1}{2\pi\times3.1\times10^{-12}\times R_{sig}} \geq 10^{MHZ}$

$\Rightarrow \underline{R_{sig} \leq 5.1^{k\Omega}}$

4.95

$R_{sig} = 100^{k\Omega}$, $R_{in} = 100k\Omega$, $C_{gs} = 1pF$, $C_{gd} = 0.2pF$

Cont.

$$A_M = -\frac{R_G}{R_G+R_{sig}} g_m (r_o \| R_D \| R_L) \quad \text{(Eq. 4.119)}$$

Also $R_{in} = 100K\Omega = R_G$

$$A_M = \frac{-100}{100+100} 3(50^K \| 8^K \| 10^K) = \underline{-6.1 \, V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad \text{(Eq. 4.132)}$$

$R'_{sig} = R_{sig} \| R_G = 100\|100 = 50^{K\Omega}$

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$R'_L = r_o \| R_D \| R_L = 4.1K\Omega$

$C_{in} = 1 + 0.2(1 + 3\times 4.1) = 3.66pF$

Now we can calculate f_H:

$$f_H = \frac{1}{2\pi \times 3.66\times10^{-12} \times 50\times10^3} = \underline{870^{KHz}}$$

In order to double f_H, we have to either decrease C_{in} (by reducing R_{out}) or reduce R'_{sig} by reducing R_{in}.

If we reduce $R_{out} = R_D \| r_o$:

$$\frac{f_{H2}}{f_{H1}} = \frac{C_{in1}}{C_{in2}} \Rightarrow 2 = \frac{3.66\,pF}{1 + 0.2(1+3\times R'_L)}$$

$\Rightarrow R'_L = 1.27K\Omega \qquad R'_L = R_{out} \| R_L = R_{out} \| 10^k$

$\Rightarrow \underline{R_{out} = 1.45K\Omega}$

Therefore in order to double f_H to $870\times2 = 1.74^{MHz}$, we have to reduce $R_{out} = r_o \| R_D$ to $1.45^{K\Omega}$ or equivalently reducing R_D to $1.5^{K\Omega}$. The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R'_{L2}}{R'_{L1}} \Rightarrow A_{M2} = -6.1 \times \frac{1.27}{4.1} = -1.9 V/V$$

Gain is almost reduced by a Factor of $\underline{3}$.

If we reduce $R_{in} = R_G$:

$$\frac{f_{H2}}{f_{H1}} = \frac{R'_{sig1}}{R'_{sig2}} \Rightarrow 2 = \frac{50^K}{R'_{sig2}} \Rightarrow R'_{sig2} = 25^{K\Omega}$$

$\Rightarrow 25K\Omega = 100K \| R_G \Rightarrow \underline{R_G = 33^{K\Omega} = R_{in}}$

Therefore in order to double f_H, R_{in} is reduced by a factor of 3, from $100k\Omega$ to $33k\Omega$.

The new mid band gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R_{G2}}{R_{G1}} \cdot \frac{R_{G1}+R_{sig}}{R_{G2}+R_{sig}} \Rightarrow A_{M2} = -6.1 \times \frac{1}{3} \times \frac{100+100}{33+100}$$

$\underline{A_{M2} = -3.06 \, V/V}$

Gain is almost reduced by a factor of $\underline{2}$.

4.96

$R_{in} = 2M\Omega$, $g_m = 4mA/V$, $r_o = 100k\Omega$, $R_D = 10k\Omega$

$C_{gs} = 2pF$, $C_{gd} = 0.5pF$, $R_{sig} = 500k\Omega$, $R_L = 10K\Omega$

a) using Eq. 4.119 and noting that $R_G = R_{in}$, we have:

$$A_M = -\frac{R_G}{R_G+R_{sig}} g_m (r_o \| R_D \| R_L) = -\frac{2\times4}{2+0.5}(100^K \| 10^K \| 10^K)$$

$\underline{A_M = -15.2 \, V/V}$

b) Eq. 4.132: $f_H = \dfrac{1}{2\pi C_{in} R'_{sig}}$ where

$C_{in} = C_{gs} + C_{gd}(1 + g_m(r_o \| R_D \| R_L))$, $R'_{sig} = R_{sig} \| R_G$

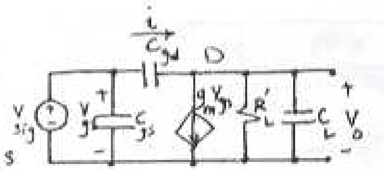
$C_{in} = 2 + 0.5(1 + 4\times(100^K \| 10^K \| 10^K)) = \underline{12.02pF}$

$R'_{sig} = 0.5^M \| 2^{M\Omega} = 0.4M\Omega = 400k\Omega$

$$f_H = \frac{1}{2\pi \times 12.02\times10^{-12} \times 400\times10^3} = \underline{33.1^{KHz}}$$

4.97

If we write KCL at node D:



$$i = g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s$$

then: $V_{sig} = i \times \dfrac{1}{C_{gd}s} + v_o$

$$V_{sig} = (g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s)\frac{1}{C_{gd}s} + v_o, \quad V_{sig} = V_{gs}$$

$$v_{sig}\left(1 - \frac{g_m}{C_{gd}s}\right) = v_o\left(1 + \frac{1}{R'_L C_{gd}s} + \frac{C_L s}{C_{gd}s}\right)$$

$$\frac{v_o}{v_{sig}} = -g_m R'_L \frac{(1 - s(C_{gd}/g_m))}{R'_L C_{gd}s + 1 + R'_L C_L s}$$

$$\frac{v_o}{v_{sig}} = -g_m R'_L \frac{1 - s\, C_{gd}/g_m}{1 + s(C_L + C_{gd})R'_L}$$

$$\text{If } (g_m/C_{gd}) \gg \omega \Rightarrow \frac{v_o}{v_{sig}} = \frac{-g_m R'_L}{1 + s(C_L + C_{gd})R'_L}$$

For $C_{gd} = 0.5pF$, $C_L = 2pF$, $g_m = 4mA/V$, $R'_L = 5k\Omega$

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + s/\omega_H} \Rightarrow \begin{cases} A_M = -g_m R'_L = -4\times5 = -20 V/V \\ f_H = \dfrac{1}{2\pi(C_L + C_{gd})R'_L} \end{cases}$$

$$\Rightarrow f_H = \frac{10^{12}}{2\pi \times (2+0.5) \times 5\times10^3}$$

$\underline{f_H = 12.7MHz}$

$g_m/C_{gd} = \dfrac{4}{0.5} = 8^{GRad/s} \gg \omega_H$

4.98

Eq. 4.134 $\quad \omega_{p_1} = \dfrac{1}{C_{c_1}(R_G + R_{sig})}$

For $f_p = 10\text{Hz}$:

$$2\pi \times 10 = \frac{1}{C_{c_1}(1+1)\times 10^6} \Rightarrow C_{c_1} = 7.96\text{nF}$$

To ensure that f_p is not exceeding 10Hz, C_{c_1} has to be greater than 7.96nF or: $C_{c_1} = 8\text{nF}$. If $C_{c_1} = 8\text{nF} \Rightarrow f_p = 9.95\text{Hz}$

To lower f_p, R_G has to be increased. For the lowest possible f_p, the largest available R_G has to be used, which in this case is $10 \times 1 = 10\text{M}\Omega$.

To calculate the new f_p for $R_G = 10^{\text{M}\Omega}$:

$$\frac{f_{p_2}}{f_{p_1}} = \frac{R_{G_1} + R_{sig}}{R_{G_2} + R_{sig}} \Rightarrow f_{p_2} = 9.95\,\frac{1+1}{10+1} = 1.81^{\text{Hz}}$$

f_p is reduced by a factor of 5.5.

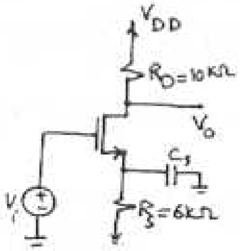
4.99

$I_D = 1\text{mA}$, $g_m = 1\text{mA/V}$

Using eq. 4.89 we have:

$$A_M = \frac{-g_m R_D}{1 + g_m R_S} = -\frac{1\times 10}{1+1\times 6}$$

$A_M = 1.43\,\text{V/V}$



$$f_L = \frac{1}{2\pi\left(\frac{1}{g_m} \| R_S\right) C_S} = 10\text{Hz}$$

$$C_S = \frac{1}{2\pi \times 10 (1^{\text{k}} \| 6^{\text{k}})} = 18.57\,\mu\text{F}$$

4.100

$$f_{C_{C_2}} = \frac{1}{2\pi C_{C_2}(R_L + R_D \| r_o)} \leq 10^{\text{Hz}}$$

$$\Rightarrow C_{C_2} \geq \frac{1}{10 \times 2\pi \times (10^{\text{K}} + 15^{\text{K}} \| 150^{\text{K}})} \Rightarrow C_{C_2} \geq 0.67\,\mu\text{F}$$

$$\Rightarrow C_{C_2} = 0.7\,\mu\text{F} \Rightarrow f_{C_{C_2}} = 9.62\text{Hz}$$

If I_D is doubled with both r_o and R_D halved:

$$f_{C_{C_2}} = \frac{1}{2\pi \times 0.7^{\mu}\left(10^{\text{K}} + \frac{15^{\text{K}}}{2} \| \frac{150^{\text{K}}}{2}\right)} = 13.5^{\text{Hz}}$$

For higher-power designs, where I_D is increased and consequently r_o and R_D are reduced. For smallest r_o and R_D where $r_o \| R_D \ll R_L$, R_L becomes dominant in determining the corner frequency:

$$f_{C_{C_2}\max} = \frac{1}{2\pi C_{C_2}(R_L)} = 22.75\text{Hz}$$

4.101

Refer to Fig. P4.101: $g_m = 1\text{mA/V}$

$$A_M = -\frac{R_G}{R_G + R_{sig}}\, g_m (R_D \| R_L) \quad \text{where } R_G = 10^{\text{M}} \| 47^{\text{M}}$$

$R_G = 8.25^{\text{M}\Omega}$

$$A_M = -\frac{8.25}{8.25+0.1} \times 1 \times (4.7^{\text{K}} \| 10^{\text{K}}) = -3.16\,\text{V/V}$$

$$f_{P_1} = \frac{1}{2\pi C_{C_1}(R_G + R_{sig})} \quad \text{(Eq. 4.134)}$$

$$f_{P_1} = \frac{1}{2\pi \times 0.01\times 10^{-6} \times (8.25+0.1)\times 10^6} = 1.9^{\text{Hz}}$$

$$f_{P_2} = \frac{1}{2\pi C_S (R_S \| \frac{1}{g_m})} = \frac{1}{2\pi \times 10 \times 10^{-6} \times (2^{\text{K}} \| \frac{1^{\text{K}}}{1})} = 23.9^{\text{Hz}}$$

$$f_{P_3} = \frac{1}{2\pi C_{C_2}(R_D + R_L)} = \frac{1}{2\pi \times 0.1\times 10^{-6} \times (4.7+10)\times 10^3} = 108.3^{\text{Hz}}$$

$f_L \simeq 108.3\text{Hz}$

4.102

If $g_m = 1\text{mA/V}$ and $r_o = 100\text{k}\Omega$:

$$A_M = -\frac{R_G}{R_G + R_{sig}}\, g_m (r_o \| R_D \| R_L) \quad \text{where } R_G = 10^{\text{M}} \| 47^{\text{M}}$$

$R_G = 8.25^{\text{M}\Omega}$

$$A_M = -\frac{8.25}{8.25+0.1}\, 1\,(100^{\text{K}} \| 4.7^{\text{K}} \| 10^{\text{K}}) = -3.06\,\text{V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad \text{where } R'_{sig} = R_{sig} \| R_G = 0.1^{\text{M}} \| 8.25^{\text{M}\Omega}$$

$R'_{sig} = 0.1^{\text{M}\Omega}$

$$C_{in} = C_{gs} + C_{gd}(1 + g_m (r_o \| R_D \| R_L))$$

$$C_{in} = 1 + 0.2(1 + 1(100^{\text{K}} \| 4.7^{\text{K}} \| 10^{\text{K}})) = 1.82\text{pF}$$

$$f_H = \frac{1}{2\pi \times 1.82\times 10^{-12} \times 0.1\times 10^6} = 875\text{KHz}$$

4.103

$$A_M = -\frac{R_G}{R_G+R_{sig}} g_m (r_o \| R_D \| R_L) = -\frac{2\times3}{2+0.5}(20^k \| 10^k)$$
$(r_o=\infty)$

$A_M = -16 V/V$

Using equations 4.134, 4.136 and 4.138 we have:

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G+R_{sig})} \Rightarrow 3^{Hz} = \frac{1}{2\pi \times C_{C1}(2+0.5)\times10^6}$$

$\rightarrow C_{C1} = 21.2\,nF$

$$f_{P2} = \frac{g_m}{2\pi C_S} \rightarrow 50^{Hz} = \frac{3\times10^{-3}}{2\pi\times C_S} \Rightarrow C_S = 9.6\mu F$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D+R_L)} \Rightarrow 10^{Hz} = \frac{1}{2\pi C_{C2}(10^k+20^k)}$$

$\rightarrow C_{C2} = 0.5\mu F$

$f_L = 50^{Hz}$

4.104

Refer to Fig. P4.104.

$$f_{P1} = \frac{1}{2\pi C_{C1} R_{in}} = 1^{Hz} \quad \text{(a decade lower than } 10^{Hz}\text{)}$$

$$f_{P2} = \frac{1}{2\pi C_{C2}(R_L + R_D \| R_o)} = 10^{Hz}$$

$$f_{P1} = 1^{Hz} = \frac{1}{2\pi C_{C1}\times 2.33\times10^6} \Rightarrow C_{C1} = 68.3\,nF$$

$$f_{P2} = 10^{Hz} = \frac{1}{2\pi C_{C2}(10^k + 10^k \| 47^k)} \Rightarrow C_{C2} = 873\,nF$$

4.105

Note that $K_n' \frac{W_n}{L} = K_p' \frac{W_p}{L}$ (matched)

a) when V_o is low: (eq. 4.140)

$$r_{DSN} = \frac{1}{K_n'(\frac{W}{L})_n(V_{DD}-V_{tn})} = \frac{1}{120\times10^{-6}\times\frac{1.2}{0.8}\times(3-0.7)}$$

$r_{DSN} = 2.4\,K\Omega$

when V_o is high (V_{OH}): (eq. 4.141)

$$r_{DSP} = \frac{1}{K_p'(\frac{W}{L})_p(V_{DD}-|V_{tp}|)} = \frac{1}{60\times10^{-6}\times\frac{2.4}{0.8}\times(3-0.7)}$$

$r_{DSP} = 2.4\,K\Omega$

b) $I_{max} = K_n'(\frac{W}{L})_n (V_{DD}-V_{tn})\times 0.1$

$I_{max} = 120\times10^{-6}\times\frac{1.2}{0.8}\times(3-0.7)\times0.1 = 41.4\mu A$

c) Eq. 4.148: $V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$

$V_{IH} = \frac{1}{8}(5\times3 - 2\times0.7) = 1.7V$

Eq. 4.149: $V_{IL} = \frac{1}{8}(3V_{DD}+2V_t) = 1.3^V$

$N_{MH} = V_{OH} - V_{IH} = 3 - 1.7 = 1.3V$

$N_{ML} = V_{IL} - V_{OL} = 1.3^V - 0 = 1.3^V$

4.106

From the formula given in Excercise 4.44 we have: $V_{th} = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{1+r}$

where $r = \sqrt{\frac{K_p'(\frac{W}{L})_p}{K_n'(\frac{W}{L})_n}}$

We have $V_{tn} = |V_{tp}| = 0.7V$, $V_{DD} = 3^V$, $K_n' = 120\mu A/V^2$, $K_p' = 60\mu A/V^2$. Thus: for $(\frac{W}{L})_p = (\frac{W}{L})_n$ we have:

$r = \sqrt{\frac{60}{120}} = 0.71$

$$V_{th} = \frac{0.71(3-0.7)+0.7}{1+0.71} = 1.36^V$$

For $(\frac{W}{L})_p = 2(\frac{W}{L})_n$ (the matched case)

we have: $r = \sqrt{\frac{60}{120}\times2} = 1$

$$V_{th} = \frac{1\times(3-0.7)+0.7}{1+1} = 1.5^V$$

For $(\frac{W}{L})_p = 4(\frac{W}{L})_n$ we have: $r = \sqrt{\frac{60\times4}{120}} = 1.41$

$$V_{th} = \frac{1.41\times(3-0.7)+0.7}{1+1.41} = 1.64^V$$

The results are summarized in the following table:

$(\frac{W}{L})_p = (\frac{W}{L})_n$	$V_{th} = 1.36^V$
$(\frac{W}{L})_p = 2(\frac{W}{L})_n$	$V_{th} = 1.5^V$
$(\frac{W}{L})_p = 4(\frac{W}{L})_n$	$V_{th} = 1.64^V$

4.107

equal sizes NMOS and PMOS, but $k'_n = 2k'_p$

$V_t = 0.7^V$

For V_{IH}: Q_N in triode and Q_p in saturation

$$k'_n\left(\frac{W}{L}\right)_n\left[(v_I - V_t)v_O - \frac{1}{2}v_O^2\right] = \frac{1}{2}k'_p\left(\frac{W}{L}\right)_p(V_{DD} - v_I - V_t)^2$$

$$4(v_I - V_t)v_O - 2v_O^2 = (V_{DD} - v_I - V_t)^2 \quad ①$$

Differentiating both sides relative to v_I results in:

$$\frac{\partial}{\partial v_I}: 4(v_I - V_t)\frac{\partial v_O}{\partial v_I} + 4v_O - 4v_O\frac{\partial v_O}{\partial v_I} = 2(V_{DD} - v_I - V_t)(-1)$$

substitude the values together with $v_I = V_{IH}$, $\frac{\partial v_O}{\partial v_I} = -1$:

$$4(V_{IH} - 0.7)(-1) + 4v_O + 4v_O = 2(V_{IH} - 3 + 0.7)$$

$$V_{IH} = \frac{8v_O + 7.4}{6} = 1.33v_O + 1.23 \quad ②$$

From ①: $4(V_{IH} - 0.7)v_O - 2v_O^2 = (3 - V_{IH} - 0.7)^2$

③ $4(V_{IH} - 0.7)v_O - 2v_O^2 = (2.3 - V_{IH})^2$

Solving ② and ③: $1.55v_O^2 + 4.97v_O - 1.14 = 0$

$\Rightarrow v_O = 0.22V$

$V_{IH} = 1.52V$

For V_{IL}: Q_N is in saturation and Q_p in triode

$$\frac{1}{2}k'_n\left(\frac{W}{L}\right)_n(v_I - V_t)^2 = k'_p\left(\frac{W}{L}\right)_p\left[(V_{DD} - v_I - V_t)(V_{DD} - v_O) - \frac{1}{2}(V_{DD} - v_O)^2\right]$$

$$(v_I - 0.7)^2 = (3 - v_I - 0.7)(3 - v_O) - \frac{1}{2}(3 - v_O)^2 \quad ①$$

$$(v_I - 0.7)^2 = (2.3 - v_I)(3 - v_O) - \frac{1}{2}(3 - v_O)^2$$

$$\frac{\partial}{\partial v_I} \Rightarrow 2(v_I - 0.7) = (2.3 - v_I)\left(-\frac{\partial v_O}{\partial v_I}\right) - (3 - v_O) + (3 - v_O)\frac{\partial v_O}{\partial v_I}$$

$v_I = V_{IL}$ and $\frac{\partial v_O}{\partial v_I} = -1$

$$2V_{IL} - 1.4 = 2.3 - V_{IL} - 3 + v_O - 3 + v_O$$

$$V_{IL} = \frac{2}{3}v_O - 1.15$$

From ①: $(V_{IL} - 0.7)^2 = (2.3 - V_{IL})(3 - v_O) - \frac{1}{2}(3 - v_O)^2$

$(0.66v_O - 1.85)^2 = (3.45 - 0.66v_O)(3 - v_O) - \frac{1}{2}(3 - v_O)^2$

$v_O = 2.96V$

$V_{IL} = 0.81V$

$NM_H = 3 - 1.52 = 1.48^V$, $NM_L = 0.81 - 0 = 0.81^V$

4.108

Matched MOSFETs with $V_t = 1^V$, $V_{DD} = 10^V$.

$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$ Eq. 4.149

$V_{IL} = \frac{1}{8}(3\times10 + 2\times1) = 4^V$

$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$ Eq. 4.148

$V_{IH} = \frac{1}{8}(5\times10 - 2\times1) = 6^V$

$NM_H = V_{OH} - V_{IH} = 10 - 6 = 4^V$

$NM_L = V_{IL} - V_{OL} = 4 - 0 = 4^V$

For $V_{DD} = 15^V$

$V_{IL} = \frac{1}{8}(3\times15 + 2\times1) = 5.875V$

$V_{IH} = \frac{1}{8}(5\times15 - 2\times1) = 9.125V$

$NM_H = 15 - 9.125 = 5.875V$

$NM_L = 5.875 - 0 = 5.875V$

4.109

$V_t = 0.5V$

$I_{max} = k'_n\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{tn})\times0.5 - \frac{1}{2}\times0.5^2\right]$

$I_{max} = 20\times20[(10 - 0.5)\times0.5 - 0.125]$

$I_{max} = 185\mu A$

$V_t = 1.5V \Rightarrow I_{max} = 165\mu A$

$V_t = 2^V \Rightarrow I_{max} = 155\mu A$

4.110

$i_{Dn} = k'_n\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_t)v_O - \frac{1}{2}v_O^2\right]$

$i_{Dnmax} = k'_n\left(\frac{W}{L}\right)_n\left[(V_{DD} - 0.2V_{DD})\times0.1V_{DD} - \frac{1}{2}\times0.01V_{DD}^2\right]$

$i_{Dnmax} = k'_n\left(\frac{W}{L}\right)_n \times 0.075V_{DD}^2$.

If $V_{DD} = 3V$, $k'_n = 120\mu A/V^2$, $L_n = 0.8\mu m$, $i_{Dmax} = 1mA$

$i_{Dnmax} = 1 = 120\times10^{-3}\times\frac{W_n}{0.8}\times0.075\times9$

$W_n = 9.9 \simeq 10\mu m$

4.111

we use the I_{peak} equation given in Table 4.6:

$$I_{peak} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n \left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$I_{peak} = \frac{1}{2} \times 120 \times \frac{1.2}{0.8} \left(\frac{3}{2} - 0.7\right)^2$$

$$\underline{I_{peak} = 57.6\,\mu A}$$

4.112

Eq. 4.156: $t_{PHL} = \dfrac{2C}{k_n'\left(\frac{W}{L}\right)_n (V_{DD}-V_t)}\left[\dfrac{V_t}{V_{DD}-V_t} + \dfrac{1}{2}\ln\left(\dfrac{3V_{DD}-4V_t}{V_{DD}}\right)\right]$

$$\Rightarrow t_{PHL} = \frac{2 \times 0.05 \times 10^{-12}}{120 \times 10^{-6} \times \frac{1.2}{0.8} \times (3-0.7)}\left[\frac{0.7}{3-0.7} + \frac{1}{2}\ln\left(\frac{3\times 3 - 4\times 0.7}{3}\right)\right]$$

$$\underline{t_{PHL} = 0.16\,ns}$$

If we use the estimation formula of Eq. 4.157:

$$t_{PHL} = \frac{1.6C}{k_n'\left(\frac{W}{L}\right)_n V_{DD}} = \frac{1.6 \times 10^{-12} \times 0.05}{120 \times 10^{-6} \times \frac{1.2}{0.8} \times 3} = \underline{0.15\,ns}$$

The value obtained using the estimation formula is 6% lower. Eq. 4.157 is for $V_t \simeq 0.2 V_{DD}$, but in our case $V_t = 0.7 \simeq 0.23 V_{DD}$.

4.113

We use the estimation formulas for t_{PHL} and t_{PLH} provided in Table 4.6:

$$t_{PLH} = \frac{1.6C}{k_p'\left(\frac{W}{L}\right)_p V_{DD}} = \frac{1.6 \times 0.05 \times 10^{-12}}{60 \times 10^{-6} \times \left(\frac{W}{L}\right)_p \times 3} = \frac{444 \times 10^{-12}}{\left(\frac{W}{L}\right)_p} \leq 60^{ps}$$

$$\Rightarrow \frac{444}{\left(\frac{W}{L}\right)_p} \leq 60 \Rightarrow \left(\frac{W}{L}\right)_p \geq 7.41 \Rightarrow W_p \geq 5.93\,\mu m$$

$$\left(\frac{W}{L}\right)_p = 2\left(\frac{W}{L}\right)_n \Rightarrow W_p = 2W_n \Rightarrow W_n \geq 2.97\,\mu m$$

We choose $\underline{W_p = 6\,\mu m}$ and $\underline{W_n = 3\,\mu m}$

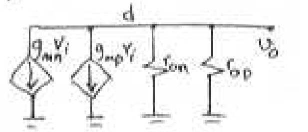
4.114

At $V_I = V_O = \frac{V_{DD}}{2}$ both Q_N and Q_P are operating in saturation with I_D given by:

$$I_D = \frac{1}{2} k_n'\left(\frac{W}{L}\right)_n \left(\frac{V_{DD}}{2} - V_t\right)^2$$

Thus each device has g_m given by $g_m = \dfrac{2I_D}{V_{GS}-V_t}$ or $g_m = \dfrac{2I_D}{V_{DD}/2 - V_t}$ and r_o given by $r_o = \dfrac{|V_A|}{I_D}$ where we have assumed matched devices: $g_{mn} = g_{mp} = g_m$, $r_{op} = r_{on} = r_o$.

From the small signal equivalent circuit:



$$A_v = \frac{v_o}{v_i}$$

$$A_v = -(g_{mn}+g_{mp})(r_{on}\|r_{op})$$

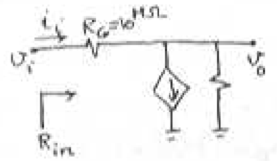
$$A_v = -2g_m \frac{r_o}{2} = -g_m r_o$$

Therefore:

$$A_v = -\frac{2|V_A|}{\frac{V_{DD}}{2} - V_t}$$

b) $V_I = V_O = \frac{V_{DD}}{2} = 1.5^V$

$$A_v = -\frac{2 \times 50}{\frac{3}{2} - 0.7} = -\frac{100}{0.8} = \underline{-125^V/_V}$$



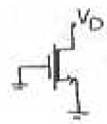
$$R_{in} = \frac{v_i}{i_i} = \frac{v_i}{\frac{v_i - v_o}{R_G}} = \frac{R_G}{1-A_v}$$

$$R_{in} = \frac{10 \times 10^3}{1+125} = 79.4\,K\Omega$$

$$\underline{R_{in} \simeq 80\,K\Omega}$$

4.115

we have $k_n'\frac{W}{L} = 2\,mA/_{V^2}$, $V_t = -3V$



$V_{GS} = 0 > V_t \Rightarrow$ device is on

$V_{GS} - V_t = 3^V$

a) $V_D = 0.1V$ $V_{DS} = 0.1 < V_{GS} - V_t \Rightarrow$ triode region

$$i_D = k_n' \frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$i_D = 2 \times \left[3 \times 0.1 - \frac{1}{2}0.1^2\right] = \underline{0.59\,mA}$$

b) $V_D = 1^V \Rightarrow V_{DS} = 1^V < V_{GS} - V_t \Rightarrow$ triode region

$$i_D = 2 \times \left[3 \times 1 - \frac{1}{2} \times 1\right] = \underline{5\,mA}$$

c) $V_D = 3^V \Rightarrow V_{DS} = 3^V = V_{GS} - V_t \Rightarrow$ edge of saturation

$$i_D = \frac{1}{2} k_n' \frac{W}{L}(V_{GS}-V_t)^2 = \frac{1}{2} \times 2 \times 3^2 = \underline{9\,mA}$$

d) $V_D = 5^V$, $V_{DS} = 5^V > V_{GS} - V_t \Rightarrow$ saturation

$$i_D = \frac{1}{2} \times 2 \times 3^2 = \underline{9\,mA}$$

4.116

$V_{GS}=0 \quad V_t=-2V \quad V_{GS}-V_t=2V$

* $V_{DS}=1V$: $V_{DS}<V_{GS}-V_t \Rightarrow$ triode

$i_D=k_n'\frac{W}{L}\left[(V_{GS}-V_t)V_{DS}-\frac{1}{2}V_{DS}^2\right]=200\left[2\times1-\frac{1}{2}\times1\right]$

$\underline{i_D=300\mu A}$

If W is doubled, with L the same: $\frac{i_{D2}}{i_{D1}}=\frac{W_2}{W_1}=2$

$\Rightarrow i_{D2}=2\times300=\underline{600\mu A}$

If W is doubled and L is also doubled, then $\frac{W}{L}$ remains the same and therefore $\underline{i_D=300\mu A}$

* If $V_{DS}=2V$ then $V_{DS}=V_{GS}-V_t \Rightarrow$ edge of saturation

$i_D=\frac{1}{2}k_n'\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})=\frac{1}{2}\times200\times2^2\times(1+0.02\times2)$

$\underline{i_D=416\mu A}$

If W is doubled and L is the same, then:

$i_D=2\times416=\underline{832\mu A}$

If W is doubled and L is also doubled, then $\frac{W}{L}$ stays the same but $V_A=V_A'L$ is doubled or equivalently λ is halved, thus:

$i_D=\frac{1}{2}\times200\times2^2\times(1+0.01\times2)=\underline{408\mu A}$

* If $V_{DS}=3V$, then $V_{DS}>V_{GS}-V_t \Rightarrow$ saturation

$i_D=\frac{1}{2}\times200\times2^2\times(1+0.02\times3)=\underline{424\mu A}$

if W is doubled and L is the same: $i_D=2\times424\Rightarrow$

$\underline{i_D=848\mu A}$

if W is doubled and L is also doubled, then $\frac{W}{L}$ stays unchanged, V_A is doubled, λ is halved:

$\frac{i_{D2}}{i_{D1}}=\frac{(1+\lambda_2V_{DS})}{(1+\lambda_1V_{DS})}\Rightarrow i_{D2}=424\times\frac{(1+0.01\times3)}{(1+0.02\times3)}=\underline{412\mu A}$

* if $V_{DS}=10V$, then $V_{DS}>V_{GS}-V_t \Rightarrow$ saturation

$i_D=\frac{1}{2}\times200\times2^2\times(1+0.02\times10)=480\mu A$

if W is doubled and L is the same: $i_D=\underline{960\mu A}$

if W is doubled and L is doubled: $i_D=480\,\frac{(1+0.01\times10)}{1+0.02\times10}$

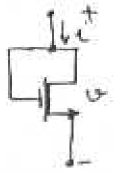
$\underline{i_D=440\mu A}$

4.117

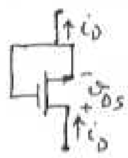
$V_{GS}=V_{DS}=v$. for $v\geq V_t$ the device will be conducting and since $V_{DS}=v<v-V_t$, the device will be operating in the triode region and thus

$i=k_n'\frac{W}{L}\left[(v-V_t)v-\frac{1}{2}v^2\right]$

$i=\frac{1}{2}k_n'\frac{W}{L}(v^2-2V_tv) \quad v\geq V_t$



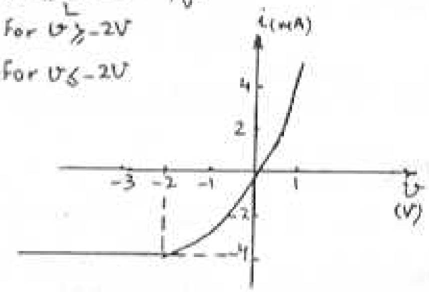
For $v\leq V_t$, the source and drain exchange roles and the MOSFET operates with $v_{GS}=0$ and $v_{DS}=-v\geq-V_t$ which implies saturation region operation with $i_D=\frac{1}{2}k_n'\frac{W}{L}V_t^2$ (for $v\leq V_t$)



For $V_t=-2V$ and $k_n'\frac{W}{L}=2mA/V^2$

$i=v^2+4v \quad$ for $v\geq-2V$

$i=-4\ mA \quad$ for $v\leq-2V$



4.118

$k_n'\frac{W}{L}=4mA/V^2$, $V_t=-2V$

Since $V_{DG}\leq|V_t|$, the MOSFET will be operating in the triode region:

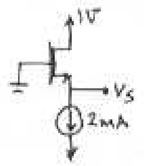
$i_D=k_n'\frac{W}{L}\left[(V_{GS}-V_t)V_{DS}-\frac{1}{2}V_{DS}^2\right]$

$2=4\left[(-V_S+2)(1-V_S)-\frac{1}{2}(1-V_S)^2\right]$

$2=+4V_S^2+8-12V_S-2V_S^2-2+4V_S$

$2V_S^2-8V_S+4=0\Rightarrow V_S^2-4V_S+2=0$

$V_S=3.4V$ or $\underline{0.59V}$



The first answer is not physically meaningful, for it results in $V_{GS}=-3.4<V_t$ and that implies cutoff. Therefore: $\underline{V_S=0.59V}$

4.119

$i_D=\frac{1}{2}k_n'\frac{W}{L}\left[V_{GS}-V_t\right]^2$, $\lambda=0$

① $\left.\begin{array}{l}1=\frac{k_n'}{2}\frac{W}{L}(-1-V_t)^2\\9=\frac{k_n'}{2}\frac{W}{L}(1-V_t)^2\end{array}\right\}\Rightarrow 9=\frac{(1-V_t)^2}{(1+V_t)^2}\Rightarrow\pm3=\frac{1-V_t}{1+V_t}$

$\Rightarrow V_t=-0.5V, -2V$

Cont.

$V_t=-0.5V$ is not acceptable, for it results in $V_{GS}=-1V<V_t$. Therefore $V_t=-2V$.

If we replace this in ①:

$1=\frac{1}{2}k_n'\frac{W}{L}(-1+2)^2 \Rightarrow k_n'\frac{W}{L}=2\,mA/V^2$

$I_{DSS}=\frac{1}{2}k_n'\frac{W}{L}V_t^2=\frac{1}{2}\times2\times2^2=4\,mA$

4.120

$I_{DSS}=\frac{1}{2}k_n'\frac{W}{L}V_t^2 \rightarrow 4=\frac{1}{2}k_n'\frac{W}{L}\times2^2 \rightarrow k_n'\frac{W}{L}=2\,mA/V^2$

Refer to Fig. P4.120

Consider Q_1: $V_{GS1}=-I_DR_1$

$I_{D1}=\frac{1}{2}k_n'\frac{W}{L}(V_{GS1}-V_t)^2 \Rightarrow 1=\frac{1}{2}\times2\times(V_{GS}+2)^2 \Rightarrow$

$V_{GS}=-1V$

$R_1=\frac{1V}{1mA}=1k\Omega$, $R_1=R_2=1k\Omega$

$V_B=6V \rightarrow R_3=\frac{10-6}{1mA}=4k\Omega$

$V_C=V_A-V_{GS2}-I_{D2}R_2=0-V_{GS1}-I_{D2}R_2=0-(-1)-1\times1$

$V_C=0V$ (for $V_A=0$)

Now if $V_A=\pm1V$: $v_C=V_A-V_{GS1}-R_2I_2=V_A$

$\rightarrow v_C=\pm1V$

Since $v_C=v_A$, the source follower has zero offset. Q_2 enters the triode region when:

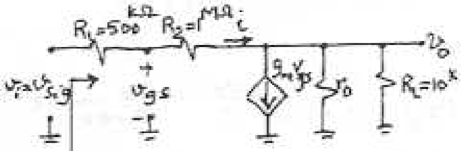
$v_B=V_A+|V_t| \rightarrow V_A=6-2=4V$

Q_1 enters the triode region when $v_C=-10+2$

$v_C=-8V$ which corresponds to $V_A=-8V$

4.121

a) $v_{sig}=v_i$



If we write KCL at the output node: R_{in}

$$\frac{v_i-v_o}{R_1+R_2}=g_mv_{gs}+\frac{v_o}{r_o}+\frac{v_o}{R_L} \quad ①$$

Also note that $v_{gs}=v_i+R_1\frac{v_o-v_i}{R_1+R_2}$

$$v_{gs}=\frac{R_2}{R_1+R_2}v_i+\frac{R_1}{R_1+R_2}v_o$$

By substituting for v_{gs} in ①:

$$\frac{v_i-v_o}{R_1+R_2}=g_m\frac{R_2}{R_1+R_2}v_i+\frac{g_mR_1}{R_1+R_2}v_o+\frac{v_o}{R_L}+\frac{v_o}{r_o}$$

$$v_i\left(\frac{1-g_mR_2}{R_1+R_2}\right)=v_o\left(\frac{1+g_mR_1}{R_1+R_2}+\frac{1}{r_o}+\frac{1}{R_L}\right)$$

$$\frac{v_o}{v_i}=\frac{1-g_mR_2}{1+g_mR_1+\frac{(R_1+R_2)}{r_o\|R_L}}$$

$g_m=1\,mA/V$, $r_o=100k\Omega$, $R_1=0.5M\Omega$, $R_2=1M\Omega$

$R_L=10k\Omega$

$$\frac{v_o}{v_i}=\frac{1-1\times1000}{1+1\times500+\frac{1500}{10k\|100k}}=-1.5\,V/V$$

Now we calculate $R_{in}=\frac{v_i}{i_i}$:

$v_i=(R_1+R_2)i_i+(i_i-g_mv_{gs})(r_o\|R_L)$

$v_{gs}=v_i-R_1i_i$

$v_i=(R_1+R_2)i_i+(i_i-g_mv_i+g_mR_1i_i)(r_o\|R_L)$

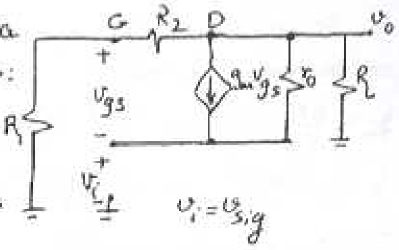
$$\frac{v_i}{i_i}=R_{in}=\frac{R_1+R_2+(1+g_mR_1)(r_o\|R_L)}{1+g_m(r_o\|R_L)}$$

$$R_{in}=\frac{500+1000+(1+1\times500)\times(10k\|100k)}{1+1(10k\|100k)}$$

$R_{in}=600k\Omega$

b) If we write a KCL for node G:

$$① \ \frac{v_{gs}+v_i}{R_1}+g_mv_{gs}+\frac{v_o-v_i}{r_o}+\frac{v_o}{R_L}=0$$



$v_i=v_{sig}$

Also: $v_{gs}=v_o\frac{R_1}{R_1+R_2}-v_i$

substitute for v_{gs} in ①:

$$\frac{v_o}{R_1+R_2}-\frac{v_i}{R_1}+\frac{v_i}{R_1}+g_m\frac{R_1}{R_1+R_2}v_o-g_mv_i+\frac{v_o}{r_o}-\frac{v_i}{r_o}+\frac{v_o}{R_L}=0$$

$$v_o\left(\frac{1+g_mR_1}{R_1+R_2}+\frac{1}{r_o}+\frac{1}{R_L}\right)=v_i\left(+g_m+\frac{1}{r_o}\right)$$

$$\frac{v_o}{v_i}=\frac{g_m+\frac{1}{r_o}}{\frac{g_mR_1+1}{R_1+R_2}+\frac{1}{r_o}+\frac{1}{R_L}}$$

$$\frac{v_o}{v_i}=\frac{1+\frac{1}{100}}{\frac{1\times500+1}{500+1000}+\frac{1}{100}+\frac{1}{10}}=2.27\,V/V$$

Now we find $R_i=\frac{v_i}{i_i}$:

Cont.

$$i_i = \frac{v_o}{R_1+R_2} + \frac{v_o}{R_L}$$ (Note that (R_1+R_2) is parallel to R_L)

$$i_i = v_o\left(\frac{1}{R_1+R_2} + \frac{1}{R_L}\right)$$

$$\frac{i_i}{v_i} = \frac{v_o}{v_i}\left(\frac{1}{R_1+R_2} + \frac{1}{R_L}\right)$$

$$R_i = \frac{v_i}{i_i} = \frac{1}{\frac{v_o}{v_i}\left(\frac{1}{R_1+R_2} + \frac{1}{R_L}\right)}$$

$$R_i = \left(\frac{g_m R_1+1}{R_1+R_2} + \frac{1}{r_o} + \frac{1}{R_L}\right) \Big/ \left(g_m + \frac{1}{r_o}\right)\left(\frac{1}{R_1+R_2} + \frac{1}{R_L}\right)$$

$$R_i = \left(\frac{1\times 500+1}{500+1000} + \frac{1}{100} + \frac{1}{10}\right) \Big/ \left(1+\frac{1}{100}\right)\left(\frac{1}{500+1000} + \frac{1}{10}\right)$$

$$R_i = 4.37\,k\Omega$$

4.122

For DC-bias analysis purposes, both circuits are basically the same.

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2\times 0.1}{1} = 0.2^V$$

$$I_D = \frac{1}{2}k_n' \frac{W}{L} V_{ov}^2 \Rightarrow k_n'\frac{W}{L} = \frac{2\times 0.1}{0.2^2} = 5\,mA/V^2$$

$$r_o = \frac{V_A}{I_D} \Rightarrow V_A = 100\times 0.1 = 10^V$$

In order to find the required value for V_{DD}:

$$V_{ov} = V_{GS} - V_t \Rightarrow V_{GS} = 0.2+0.6 = 0.8V$$

$$V_{GS} = R_1 I_{R_1} \Rightarrow I_{R_1} = \frac{0.8}{500k} = 1.6\,\mu A$$

$$V_{DS} = (R_1+R_2) I_{R_1} = (1+0.5)\times 1.6 = 2.4V$$

$$V_{DD} = V_{DS} + (I_{R_1} + I_D)R_L = 2.4 + (0.0016+0.1)\times 10^k$$

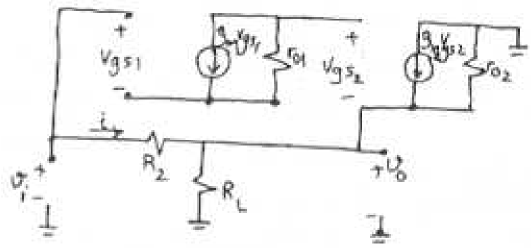
$$V_{DD} = 3.416V$$

4-123

$V_{R_1} = V_{GS} = 0.8V = R_1 I_{R_1}$, $I_{R_1} = 0.01 I_{D2} = 0.01mA$

$$\Rightarrow 0.8 = R_1\times 0.01 \Rightarrow R_1 = 80k\Omega$$

$$(R_1+R_2)\times 0.01 = 2^V \Rightarrow R_2 = 120k\Omega$$



$$v_{gs1} = v_i$$

$$v_i = R_2 i + v_o \quad (1)$$

$$v_{gs2} = -g_{m1}v_{gs1}r_{o1} - v_o$$

$$i = \frac{v_o}{R_L} - g_{m2}v_{gs2} + \frac{v_o}{r_{o2}}$$

substitute i in (1):

$$v_i = \frac{R_2}{R_L}v_o + g_{m2}R_2 g_{m1} r_{o1} v_i + g_{m2}R_2 v_o + \frac{v_o}{r_{o2}}R_2$$

$$\frac{v_o}{v_i} = \frac{1 - g_{m2}R_2 g_{m1} r_{o1}}{\frac{R_2}{R_L} + g_{m2}R_2 + \frac{R_2}{r_{o2}}}$$

We calculate the parameters:

$$g_{m1} = \frac{2I_D}{V_{ov}} = \frac{2\times 0.01}{0.8-0.6} = 0.1\,mA/V$$

$$g_{m2} = \frac{2\times 1}{0.2} = 10\,mA/V$$

$$r_{o1} = \frac{V_A}{I_D} = \frac{20}{0.01} = 2000\,k\Omega = 2M\Omega$$

$$r_{o2} = \frac{20}{1} = 20k\Omega$$

$$\frac{v_o}{v_i} = \frac{1 - 10\times 120\times 0.1\times 2000}{\frac{120}{80} + 10\times 120 + \frac{120}{20}} = -198.8\,V/V$$

We can write: $V_i = V_o + R_2 i_i \Rightarrow 1 = \frac{v_o}{v_i} + R_2\frac{i_i}{v_i}$

$$\Rightarrow R_{in} = \frac{V_i}{i_i} = \frac{R_2}{1-\frac{v_o}{v_i}}$$

$$R_{in} = \frac{120^k}{1+198.8} = 0.6\,k\Omega$$

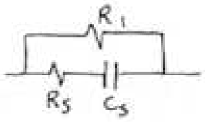
$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_1+R_{in}}\frac{v_o}{v_i} = \frac{0.6^k}{80+0.6}\times(-198.8) = -1.48\,V/V$$

This two stage amplifier consisting of a CS stage followed by a source follower stage with a relatively high gain $\left(\frac{v_o}{v_i}\right)$ acts similar to an op-amp in an inverting configuration. Also note that $\frac{v_o}{v_{sig}} \simeq \frac{-R_2}{R_1} = -1.5\,V/V$

Cont.

In order to increase the gain magnitude to $5\,V/V$, we need to reduce the effective source resistance R_1. To do so, the following configuration can be used:

To calculate R_S: $R_1' = R_1 \,||\, R_S$
(midband)



$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_1' + R_{in}} \cdot \frac{v_o}{v_i}$$

$$-5 = \frac{0.6}{R_1' + 0.6} \times (-198.8) \Rightarrow R_1' = 23.3\,k\Omega$$

$$R_1' = 23.3^k = 80^k \,||\, R_S \Rightarrow \underline{R_S \simeq 33\,k\Omega}$$

4.124

$$I_1 = 10\,\mu A = I_{D1} = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 (0.8 - 0.6)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 2.5$$

$$I_{D2} = 1\,mA = \frac{1}{2} \times \frac{200}{1000} \times \left(\frac{W}{L}\right)_2 \times (0.8 - 0.6)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 250$$

CHAPTER 5 PROBLEMS

5.1

Case	Mode
1	active
2	saturation
3	active
4	saturation
5	inverted active mode
6	active
7	cut-off
8	cut-off

5.2

Using eq. (5.4)

$$I_S = \frac{A_E q D_n n_i^2}{N_A W}$$

$$= \frac{10\times10\times10^{-8}\times1.6\times10^{-19}\times21.3\times1.5^2\times10^{20}}{10^{17}\times1\times10^{-4}}$$

$$= \underline{\underline{7.7\times10^{-17}\,A}}$$

$$\beta = \frac{1}{\dfrac{D_p N_A W}{D_n N_D L_p} + \dfrac{1}{2}\dfrac{W^2}{D_n \tau_b}}$$

$$= \frac{1}{\dfrac{D_p N_A W}{D_n N_D L_p} + \dfrac{1}{2}\dfrac{W^2 \cancel{D_n}}{\cancel{D_n} L_n^2}} \quad Eq\ (5.12)$$

$$= \frac{1}{\dfrac{1.7\times10^{17} W}{21.3\times10^{19}\times0.6\times10^{-4}} + \dfrac{1}{2}\dfrac{W^2}{19^2\times10^{-8}}}$$

(a) $W = 10^{-4}$ cm $I_S = \underline{7.7\times10^{-17} A}$ $\beta = \underline{368}$

(b) $W = 2\times10^{-4}$ cm $I_S = \underline{3.8\times10^{-17} A}$ $\beta = \underline{122}$

(c) $W = 5\times10^{-4}$ cm $I_S = \underline{1.5\times10^{-17} A}$ $\beta = \underline{24.2}$

5.3

$$i_C = I_S e^{v_{BE}/V_T}$$

For Device #1

$$0.2\times10^{-3} = I_{S1} e^{0.72/0.025}$$

$$I_{S1} = \underline{\underline{6.214\times10^{-17}\,A}}$$

For Device #2

$$12\times10^{-3} = I_{S2} e^{0.72/0.025}$$

$$I_{S2} = \underline{\underline{3.728\times10^{-15}\,A}}$$

Since $I_S \propto A$, the relative junction areas is:

$$\frac{A_2}{A_1} = \frac{I_{S2}}{I_{S1}} = \frac{i_{C2}}{i_{C1}} = \frac{12}{0.2} = \underline{\underline{60}}$$

5.4

$$i_C = \beta i_B$$

$$400 = \beta\times7.5$$

$$\beta = \frac{400}{7.5} = \underline{\underline{53.3}}$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{53.3}{54.3} = \underline{\underline{0.982}}$$

5.5

Use $\beta = \alpha/(1-\alpha)$

α	β
0.5	1
0.8	4
0.9	9
0.95	19

α	β
0.99	99
0.995	199
0.999	999

5.6

Use $\alpha = \dfrac{\beta}{\beta+1}$

β	α	β	α
1	0.5	100	0.9907
2	0.6667	200	0.9950
10	0.9091	1000	0.9990
20	0.9524	2000	0.9995

5.7

(a) $I_C = \beta I_B$

$10^{-3} = \beta \times 50 \times 10^{-6}$

$\beta = \underline{\underline{20}}$

$\alpha = \dfrac{\beta}{\beta+1} = \dfrac{20}{21} = \underline{\underline{0.9524}}$

$I_C = \alpha I_E$

$I_E = \dfrac{I_C}{\alpha} = \dfrac{10^{-3}}{0.9524} = \underline{\underline{1.05 mA}}$

$I_C = I_S e^{V_{BE}/V_T}$

$I_S = I_C e^{-V_{BE}/V_T}$

$= 10^{-3} e^{-0.69/0.025}$

$= \underline{\underline{1.03 \times 10^{-15} A}}$

(b) $I_C = \alpha I_E$

$10^{-3} = \alpha \times 1.07 \times 10^{-3} \Rightarrow \alpha = \underline{\underline{0.9346}}$

$\beta = \dfrac{\alpha}{1-\alpha} = \underline{\underline{14.286}}$

$I_B = I_C/\beta = \dfrac{10^{-3}}{14.286} = \underline{\underline{69.998\ \mu A}}$

$I_C = I_S e^{v_{BE}/V_T}$

$I_S = 10^{-3} e^{-0.69/0.025} = \underline{\underline{1.03 \times 10^{-15}}}$

(c) $I_C = \alpha I_E = \beta I_B$

$\dfrac{\beta}{\beta+1} I_E = \beta I_B$

$I_E = (\beta+1) I_B$

$0.137 \times 10^{-3} = (\beta+1) 7 \times 10^{-6} \Rightarrow \beta = \underline{\underline{18.571}}$

$\alpha = \dfrac{\beta}{\beta+1} = \underline{\underline{0.9489}}$

$I_C = \beta I_B = 18.571 \times 7 \times 10^{-6} = \underline{\underline{0.130 mA}}$

$I_C = I_S e^{V_{BE}/V_T}$

$I_S = 0.130 \times 10^{-3} e^{-0.58/0.025} = \underline{\underline{10.922 \times 10^{-15}}}$

(d) $I_E = I_C + I_B = 10.1 + 0.120 = \underline{\underline{10.22 mA}}$

$I_C = \alpha I_E$

$\alpha = I_C/I_E = \dfrac{10.1}{10.22} = \underline{\underline{0.9883}}$

$\beta = I_C/I_B = 10.1/0.120 = \underline{\underline{84.167}}$

$I_C = 10.1 \times 10^{-3} = I_S e^{0.78/0.025}$

$I_S = \underline{\underline{284.66 \times 10^{-18} A}}$

(e) $I_E = I_C + I_B$

$I_C = I_E - I_B = 75 - 1.05$

$= \underline{\underline{73.95 mA}}$

$\alpha = I_C/I_E = 73.95/75 = \underline{\underline{0.986}}$

$\beta = I_C/I_B = 73.95/1.05 = \underline{\underline{70.429}}$

$I_C = 73.95 \times 10^{-3} = I_S e^{0.82/0.025}$

$I_S = \underline{\underline{4.208 \times 10^{-16} A}}$

5.8

$i_C = I_S e^{v_{BE}}$

$10 \times 10^{-3} = I_S e^{0.76/0.025} \Rightarrow I_S = 6.273 \times 10^{-16} A$

For $v_{BE} = 0.7V \Rightarrow i_C = 6.273 \times 10^{-16} e^{0.7/0.025}$

$= \underline{\underline{0.907 mA}}$

For $i_C = 10 \mu A \Rightarrow 10 \times 10^{-6} = 6.273 \times 10^{-16} e^{v_{BE}/0.025}$

$\therefore\ v_{BE} = \underline{\underline{0.587 V}}$

Alternate way - without calculating I_s

For $V_{BE} = 0.7V$

$$\frac{i_C}{10mA} = e^{\frac{0.7-0.76}{0.025}}$$

$$\therefore i_C = \underline{\underline{0.907mA}}$$

For $i_C = 10\mu A$

$$\frac{10\times10^{-6}}{10\times10^{-3}} = e^{\frac{V_{BE}-0.76}{0.025}}$$

$$V_{BE} = \underline{\underline{0.587V}}$$

5.9

$$\beta = \frac{\alpha}{1-\alpha}, \text{ for } \alpha \to \alpha + \Delta\alpha$$
$$\beta \to \beta + \Delta\beta$$

$$\beta + \Delta\beta = \frac{\alpha + \Delta\alpha}{1-\alpha-\Delta\alpha} \quad \text{solve for } \Delta\beta$$

$$\Delta\beta = \frac{\alpha+\Delta\alpha}{1-\alpha-\Delta\alpha} - \frac{\alpha}{1-\alpha}$$

$$= \frac{\alpha}{1-\alpha} \cdot \frac{1+\Delta\alpha/\alpha}{1-\frac{\Delta\alpha}{1-\alpha}} - \frac{\alpha}{1-\alpha}$$

$$= \beta \times \frac{\Delta\alpha/\alpha + \Delta\alpha/1-\alpha}{1-\frac{\Delta\alpha}{1-\alpha}}$$

Thus,

$$\frac{\Delta\beta}{\beta} = \frac{\Delta\alpha}{\alpha} \cdot \frac{1+\frac{\alpha}{1-\alpha}}{1-\frac{\Delta\alpha}{1-\alpha}}$$

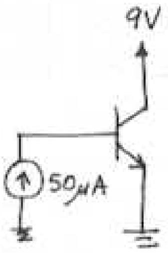
$$= \frac{\Delta\alpha}{\alpha} \frac{1-\alpha+\alpha}{1-\alpha-\Delta\alpha}$$

$$= \frac{\Delta\alpha}{\alpha} \frac{1}{1-\alpha-\Delta\alpha}$$

for small $\Delta\alpha$, $\frac{1}{1-\alpha-\Delta\alpha} \simeq \beta$

$$\therefore \frac{\Delta\beta}{\beta} \simeq \underline{\underline{\left(\frac{\Delta\alpha}{\alpha}\right)\beta}} \qquad \text{Q.E.D.}$$

5.10



$\beta = 60$ to 300

$I_C = \beta I_B$ ranges from

$= 60 \times 50\mu A$ to $300 \times 50\mu A$

$= \underline{\underline{3mA \text{ to } 15mA}}$

$I_E = I_C + I_B$ ranges from

$= \underline{\underline{3.05mA \text{ to } 15.05mA}}$

Max Power $= 9 \times I_{Cmax} = 9 \times 15$

$= \underline{\underline{135mW}}$

5.11

$$i_C = I_S e^{V_{BE}/V_T}$$

$$10\times10^{-3} = I_S e^{0.7/0.025}$$

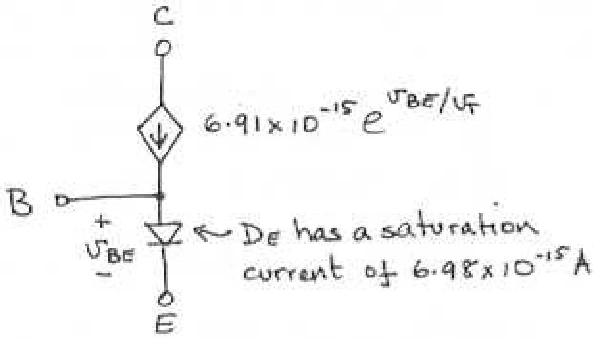
$$\Rightarrow I_S = 6.91\times10^{-15} A$$

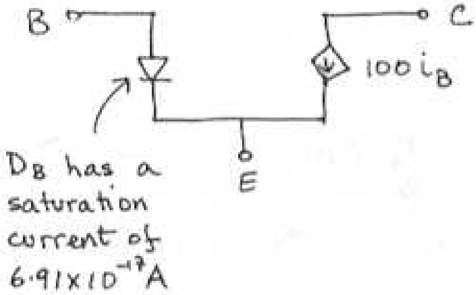
$$\alpha = \frac{i_C}{i_C+i_B} = \frac{10}{10+0.1} = 0.990$$

$$\beta = \frac{i_C}{i_B} = \frac{10}{0.1} = 100$$

$$\frac{I_S}{\alpha} = 6.98\times10^{-15} A$$

$$\frac{I_S}{\beta} = 6.91\times10^{-17} A$$

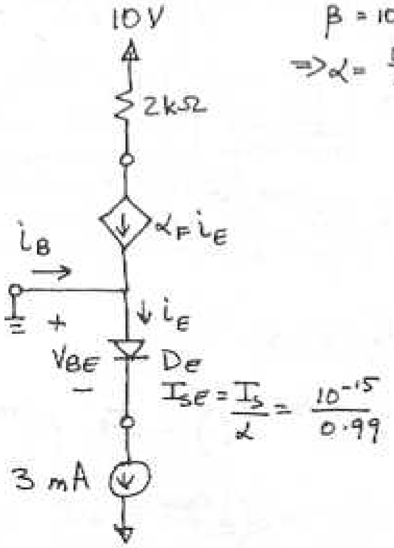




5.12

$\beta = 100,\ I_s = 10^{-15}$

$\Rightarrow \alpha = \frac{100}{101} = 0.99$



$i_E = I_{SE}\, e^{V_{BE}/V_T}$

$3\times10^{-3} = \frac{10^{-15}}{0.99} e^{V_{BE}/0.025}$

$V_{BE} = 0.718\text{V}$

$V_E = 0 - 0.718 = \underline{\underline{-0.718\text{V}}}$

$i_c = \alpha i_E = 0.99 \times 3\text{mA}$

$V_c = 10 - 2 i_c = 10 - 2\times0.99\times3$

$= \underline{\underline{4.06\text{ V}}}$

5.13

$\beta_F = 100 \quad \alpha_R = 0.01 \quad I_s = 10^{-15}\text{A}$

$\Rightarrow \alpha_F = 0.99 \quad \beta_R = 0.111$

(a) Forward Active Mode Operation

$I_B = 10\mu\text{A} \quad V_{CB} = 1\text{V} \quad V_{BE}, I_c, I_E =$

Using the Ebers Moll model we have from eq. (5.28)

$$i_B = \frac{I_s}{\beta_F}\left(e^{V_{BE}/V_T} - 1\right) + \frac{I_s}{\beta_R}\left(e^{V_{BC}/V_T} - 1\right)$$

NB. NEGATIVE IN FORWARD ACTIVE MODE

$$10\times10^{-6} = \frac{10^{-15}}{100}\left(e^{V_{BE}/0.025} - 1\right) + \underbrace{\frac{10^{-15}}{0.111}\left(e^{\frac{-1}{0.025}} - 1\right)}_{9.11\times10^{-16}\ \approx 0}$$

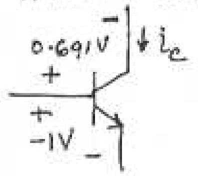
$= \underline{\underline{0.691\text{V}}}$

$I_c = I_s e^{V_{BE}/V_T} = 10^{-15} e^{\frac{0.691}{0.025}}$

$= \underline{\underline{1\text{mA}}}$ ← NB $I_c = \beta I_B = 100 I_B$

$I_E = I_c + I_B = \underline{1.01\text{mA}}$

(b) Reverse Active Mode Operation



Eq (5.28)

this term dominates in reverse active mode

$$i_B = \frac{I_s}{\beta_F}\left(e^{V_{BE}/V_T} - 1\right) + \overbrace{\frac{I_s}{\beta_R}\left(e^{V_{BC}/V_T} - 1\right)}$$

$$= \underbrace{\frac{10^{-15}}{100}\left(e^{\frac{-1}{0.025}} - 1\right)}_{\approx 0} + \frac{10^{-15}}{0.111}\left(e^{\frac{0.691}{0.025}} - 1\right)$$

$$= -10^{-17} + 0.00909$$

$$= \underline{\underline{9.08\,mA}}$$

Eq (5.27)

$$i_C = I_s\left(e^{V_{BE}/V_T} - 1\right) - \frac{I_s}{\alpha_R}\left(e^{V_{BC}/V_T} - 1\right)$$

$$= \underbrace{10^{-15}\left(e^{-1/0.025} - 1\right)}_{\approx 0} - \frac{10^{-15}}{0.1}\left(e^{\frac{0.691}{0.025}} - 1\right)$$

$$= -10^{-15} - 0.01009$$

$$= \underline{\underline{-10.09\,mA}}$$

$$i_E = i_C + i_B = \underline{-1.01\,mA}$$

Check with Eq (5.26) - Ebers Moll

$$i_E = \underbrace{\frac{I_s}{\alpha_F}\left(e^{-1/0.025} - 1\right)}_{\approx 0} - I_s\left(e^{\frac{0.691}{0.025}} - 1\right)$$

$$= \underline{\underline{-1.01\,mA}}$$

$$\alpha_F I_{SE} = \alpha_R I_{SC} = I_S$$

Since $I_S \propto$ Area

$$\frac{I_{SC}}{I_{SE}} = \frac{\alpha_F}{\alpha_R} = 10$$

$$\alpha_R = \frac{\alpha_F}{10} = \frac{1}{10}, \quad \beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{1}{9}$$

Using Eq (5.26)

$$i_E = \frac{I_s}{\alpha_F}\left(e^{V_{BE}/V_T} - 1\right) - I_s\left(e^{V_{BC}/V_T} - 1\right)$$

$= \underline{\underline{0A}}$ - Since $V_{BE} = V_{BC}$

Using Eq (5.28)

$$i_B = \frac{I_s}{\beta_F}\left(e^{V_{BE}/V_T} - 1\right) + \frac{I_s}{\beta_R}\left(e^{V_{BC}/V_T} - 1\right)$$

$$= I_s\left(\underbrace{\frac{1}{\beta_F}}_{\approx 0} + \frac{1}{\beta_R}\right)\left(e^{V_{BE}/V_T} - 1\right)$$

$$\therefore I_s\left(e^{V_{BE}/V_T} - 1\right) = \beta_R i_B = \frac{10^{-3}}{9}$$

Using Eq (5.27)

$$i_C = I_s\left(e^{V_{BE}/V_T} - 1\right) - \frac{I_s}{1/10}\left(e^{V_{BC}/V_T} - 1\right)$$

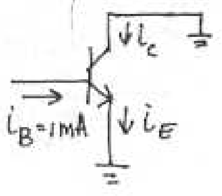
$$= I_s(1-10)\left(e^{V_{BE}/V_T} - 1\right)$$

$$= -9\times 10^{-3}/9 = \underline{\underline{-1\,\mu A}}$$

5.14

$\alpha_F \cong 1$

$\therefore \beta_F = \infty$



5.15

Using the complete Ebers Moll equations we have:

from Eq (5.26)

$$i_E = \frac{I_S}{\alpha_F}\left(e^{V_{BE}/V_T}-1\right) - I_S\left(e^{V_{BC}/V_T}-1\right)$$

$$\Rightarrow I_S\left(e^{V_{BE}/V_T}-1\right) = \alpha_F\left(i_E + I_S\left(e^{\frac{V_{BC}}{V_T}}-1\right)\right)$$

substitute this expression in Eq (5.27):

$$i_C = I_S\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \frac{I_S}{\alpha_R}\left(e^{V_{BC}/V_T}-1\right) \quad (A)$$

$$= \alpha_F\left(i_E + I_S\left(e^{\frac{V_{BC}}{V_T}}-1\right)\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$$

$$= \alpha_F i_E + I_S\left(\alpha_F e^{V_{BC}/V_T} - \alpha_F - \frac{e^{V_{BC}/V_T}}{\alpha_R} + \frac{1}{\alpha_R}\right)$$

SEE FIG 5.9

$i_E = I_E$ since the npn transistor is fed with a constant current

$$= \underset{\substack{\approx \\ 1}}{\alpha_F} I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)e^{\frac{V_{BC}}{V_T}} + \underbrace{I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)}$$

- neglect as $I_S \ll 1$
- this is the same as neglecting terms not involving exponents

Thus,

$$i_C = I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)e^{V_{BC}/V_T} \quad (B)$$

Q.E.D.

(b) $I_S = 10^{-15}$ $\alpha_F \cong 1$ $\alpha_R = 0.1$

$\beta_F = \infty$

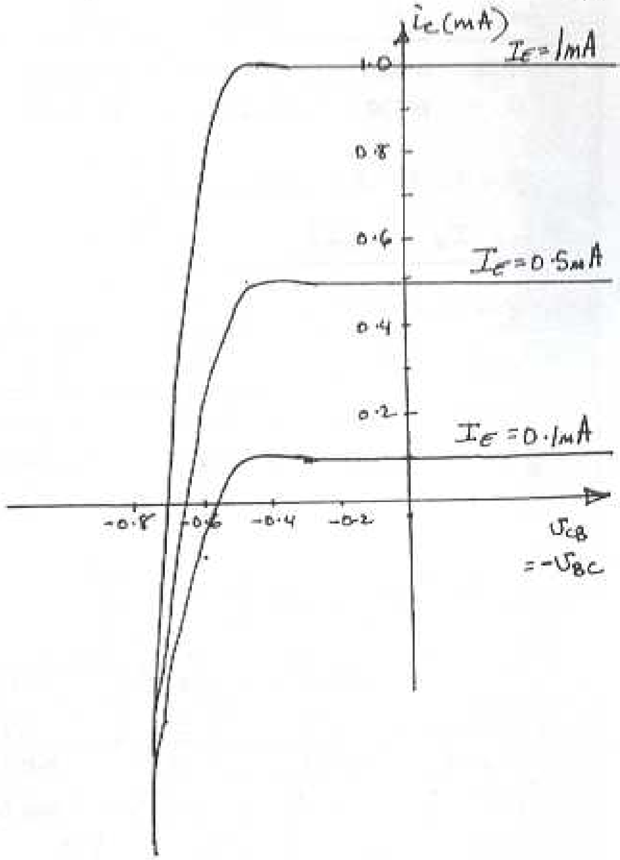
from (B)

$$i_C = I_E - 9\times10^{-15}e^{V_{BC}/0.025} \quad (C)$$

This expression is used to create the following tables and hence the curves of i_C vs V_{BC}.

V_{BC} (V)	$I_E = 0.1$ mA, i_C (mA)	$i_E = 0.5$ mA, i_C (mA)	$i_E = 1$ mA, i_C (mA)
0.7	-12.92	-12.52	-12.02
0.6	-0.14	0.26	0.762
0.5	+0.1	0.5	1
0.45	+0.1	0.5	1
0.4	0.1	0.5	1
0.35	0.1	0.5	1
0.3	0.1	0.5	1
0	0.1	0.5	1
-1	0.1	0.5	1
-2	0.1	0.5	1

V_{BC} is getting smaller so less current is subtracted from I_E. $\therefore i_C \to I_E$



For a particular I_E and i_C, v_{BC} can be calculated using (C)

$$i_C = I_E - 9\times10^{-15} e^{v_{BC}/0.025} \quad (1)$$

Having calculated v_{BC}, v_{BE} can be calculated using (A)

$$i_C = 10^{-15}\left(e^{\frac{v_{BE}}{V_T}}-1\right) - 10^{-14}\left(e^{\frac{v_{BC}}{V_T}}-1\right) \quad (2)$$

Now v_{CE} can be calculated with

$$v_{CE} = -v_{BC} + v_{BE} \quad (3)$$

Thus,

for $I_E = 0.1\,mA$

i_C (mA)	v_{BC}(1) (V)	v_{BE} (2) (V)	v_{CE} (3) (V)
$0.5I_E$	0.561	0.644	0.083
0	0.578	0.635	0.057

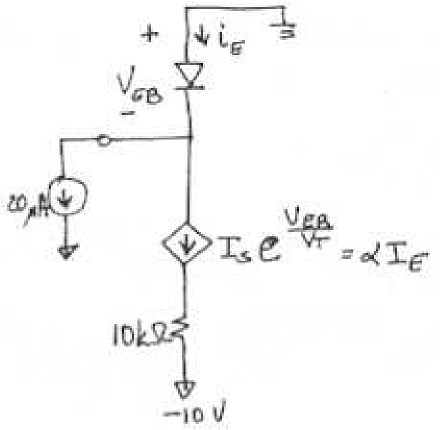
For $I_E = 0.5\,mA$

i_C (mA)	v_{BC} (1) (V)	v_{BE}(2) (V)	v_{CE} (3) (V)
$0.5I_E$	0.601	0.674	0.073
0	0.618	0.675	0.057

for $I_E = 1\,mA$

i_C (mA)	v_{BC}(1) (V)	v_{BE} (2) (V)	v_{CE} (3) (V)
$0.5I_E = \frac{1}{2}$	0.618	0.691	0.073
0	0.638	0.696	0.058

5.16



$\beta = 40$

$\alpha_F = \frac{40}{41}$

$I_S = 10^{-13}$

$$i_E = \frac{I_S}{\alpha} e^{V_{EB}/V_T} = I_S e^{V_{EB}/V_T} + 0.02\times10^{-3}$$

$$I_S e^{V_{EB}/V_T}\left(\frac{1}{\alpha}-1\right) = 0.02\times10^{-3}$$

$$10^{-13} e^{V_{EB}/0.025}\left(\frac{41}{40}-1\right) = 0.02\times10^{-3}$$

$$V_{EB} = 0.570V \Rightarrow V_B = \underline{\underline{-0.670V}}$$

$$i_E = \frac{I_S}{\alpha} e^{V_{EB}/V_T} = \frac{10^{-13}}{40} e^{\frac{0.57}{0.025}}$$

$$= \underline{0.82\,mA}$$

$$i_C = \alpha i_E \Rightarrow V_C = -10 + \alpha i_E \times 10$$

$$= -10 + \frac{40}{41}\times0.82\times10$$

$$= \underline{-2V}$$

5.17

$\therefore i_C = I_S e^{V_{EB}/V_T}$

Use $\frac{i_C}{1A} = e^{\frac{V_{EB}-0.8}{0.025}}$

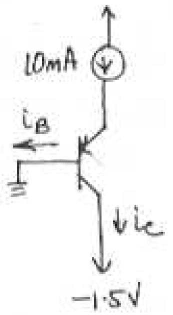
to calculate V_{EB} for a particular i_C

CONT.

for $i_C = 10\,mA$ $\quad v_{EB} = 0.685\,V$

$i_C = 5\,A$ $\quad v_{EB} = 0.840\,V$

5.18



$\beta = 10$

$i_C = \alpha i_E = \frac{10}{11} \times 10 = \underline{9.09\,mA}$

$i_B = i_E - i_C = \underline{0.91\,mA}$

$i_C = I_S e^{V_{EB}/V_T}$

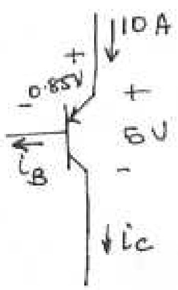
$9.09 \times 10^{-3} = 10^{-16} e^{V_{EB}/0.025}$

$V_E = V_{EB} = \underline{0.803\,V}$

For $\beta = 1000$

$i_C = \frac{\beta}{\beta+1} i_E = \frac{1000}{1001} \times 10 = \underline{9.99\,mA}$

5.19



for $\beta = 15$

$i_E = (\beta + 1) i_B$

$10 = (\beta + 1) i_B$

$i_B = \frac{10}{16} = \underline{0.625\,A}$

Calculating I_{S1}

$i_C = \frac{\beta}{\beta+1} i_E = I_{S1} e^{v_{EB}/V_T}$

$\frac{15}{16} \times 10 = I_{S1} e^{0.85/0.025}$

$I_{S1} = 1.608 \times 10^{-14}\,A$

Compare this to

$I_{S2} = i_C e^{-v_{EB}/V_T}$

$= 10^{-3} e^{-0.7/0.025}$

$= 6.914 \times 10^{-16}$

$\therefore$ $I_S \propto$ Area

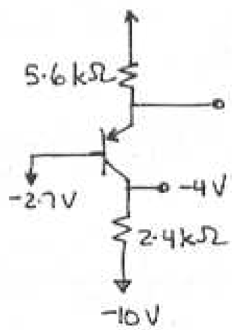
$\frac{\text{Area 1}}{\text{Area 2}} = \frac{I_{S1}}{I_{S2}} = \frac{1.608 \times 10^{-14}}{6.914 \times 10^{-16}}$

$= \underline{23.3 \text{ times larger}}$

5.20

(a) $I_1 = \frac{10.7 - 0.7}{10} = \underline{1\,mA}$

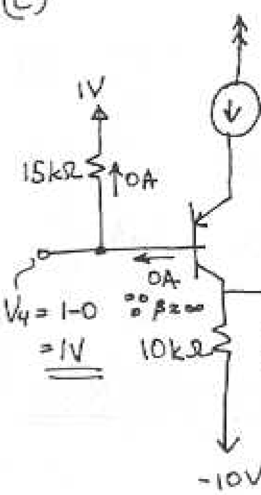
(b)



$V_2 = 12 - 5.6 \times 2.5 = \underline{-2V}$

$\frac{-4 + 10}{2.4} = 2.5\,mA$

(c)



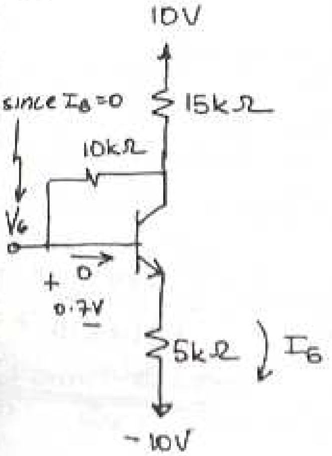
$I_3 = \underline{1\,mA}$ $\quad \therefore \alpha \approx 1$

$V_4 = 1 - 0 = \underline{1V}$

$\frac{0 - (-10)}{10} = 1\,mA$

CONT

(d)



$I_e = I_c$

$$\frac{V_6 - 0.7 + 10}{5} = \frac{10 - V_6}{15}$$

$$15V_6 + 139.5 = 50 - 5V_6$$

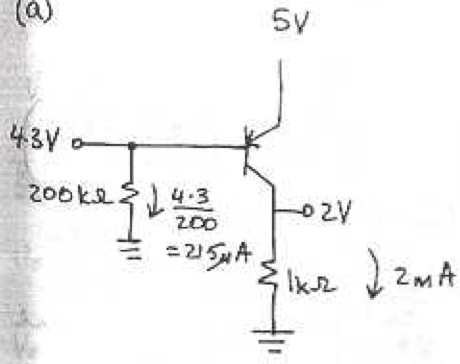
$$V_6 = \underline{\underline{-4.475V}}$$

$$I_6 = \frac{V_6 - 0.7 + 10}{5}$$

$$= \frac{-4.475 - 0.7 + 10}{5} = \underline{\underline{0.965mA}}$$

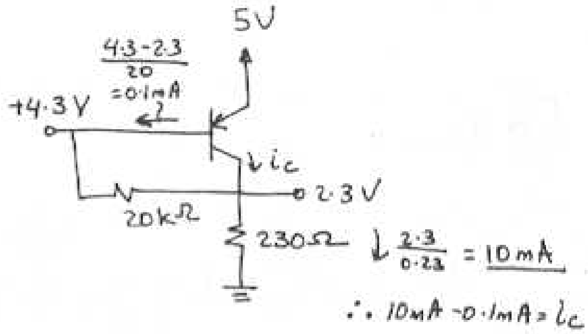
5.21

(a)



$$\frac{i_c}{i_B} = \beta = \frac{2}{0.0215} = \underline{\underline{93.0}}$$

(b)



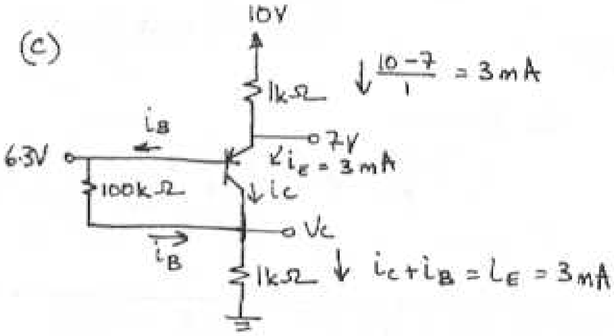
$\therefore 10mA - 0.1mA = i_c$

$9.9mA = i_c$

$$\frac{i_c}{i_B} = \frac{9.9}{0.1} = \beta$$

$$\beta = \underline{\underline{99}}$$

(c)



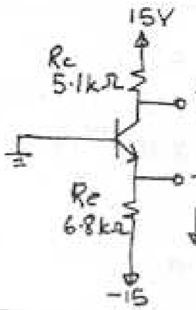
$$V_c = 3 \times 1 = 3V$$

$$i_B = \frac{6.3 - 3}{100} = \frac{3.3}{100}$$

$$\beta + 1 = \frac{i_E}{i_B} = \frac{3 \times 100}{3.3}$$

$$\beta = \frac{300}{3.3} - 1 = \underline{\underline{89.9}}$$

5.22



$\alpha = 1$

$$V_C = 15 - 5.1 \times i_E = 15 - 5.1 \times 2.1$$

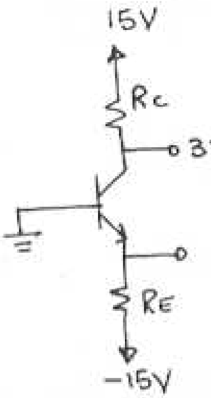
$$= \underline{\underline{4.3V}}$$

$-0.7V$

$$i_E = i_C = \frac{-0.7 + 15}{6.8} = \underline{\underline{2.1mA}}$$

5.23

$I_c = 5mA,\ V_c = 3V$



$R_c = \dfrac{15-3}{5}$

$= \underline{\underline{2.4k\Omega}}$

~ assume $V_E = -0.7V$

$R_E = \dfrac{-0.7-(-15)}{5/\left(\frac{100}{101}\right)}$

$= \underline{\underline{2.83\,k\Omega}}$

5.24

(a) $V_B = \underline{0V}$

$V_E = V_B - 0.7 = \underline{-0.7V}$

$I_E = \dfrac{-0.7+3}{2.2} = \underline{1.05mA}$

$I_c = \dfrac{30}{31} I_E = \underline{1.02mA}$

$V_c = 3 - 1.02 \times 2.2 = \underline{0.756V}$

$I_B = \dfrac{I_c}{\beta} = \dfrac{1.02}{30} = \underline{\underline{0.034mA}}$

(b) $V_B = \underline{0V}$

$V_E = V_B + 0.7 = \underline{0.7V}$

$I_E = \dfrac{3-V_E}{1} = \dfrac{3-0.7}{1} = \underline{2.3mA}$

$I_c = \alpha I_E = \dfrac{30}{31} \times 2.3 = \underline{\underline{2.23mA}}$

$V_c = -3 + 1 \times I_c = -3 + 2.23$

$= \underline{-0.77V}$

$I_B = \dfrac{I_c}{\beta} = \dfrac{2.23}{30} = \underline{\underline{0.0743mA}}$

(c) $V_B = \underline{3V}$

$V_E = V_B + 0.7 = \underline{3.7V}$

$I_E = \dfrac{9-V_E}{1.1} = \dfrac{9-3.7}{1.1} = \underline{\underline{4.82mA}}$

$I_c = \alpha I_e = \dfrac{30}{31} \times 4.82 = \underline{\underline{4.66mA}}$

$V_c = I_c \times 0.56 = \underline{2.62V}$

$I_B = \dfrac{I_c}{\beta} = \dfrac{4.66}{30} = \underline{\underline{0.155mA}}$

(d) $V_B = \underline{3V}$

$V_E = 3 - 0.7 = \underline{2.3V}$

$I_E = V_E/0.47 = 2.3/0.47 = \underline{4.89mA}$

$I_c = \alpha I_E = \dfrac{30}{31} \times 4.89 = 4.73mA$

$V_c = 9 - 1 \times I_c = 9 - 4.73 = \underline{4.22V}$

$I_B = I_c/\beta = \dfrac{4.73}{30} = \underline{0.158mA}$

5.25

Instead of v_{BE} being independent of the current line $v_{BE} = 0.7V$ only at $I_c = 1mA$. If I_c is different v_{BE} will change.

(a) $V_E = -V_{BE}$

$I_c = \alpha I_E = \dfrac{30}{31}\left(\dfrac{-V_{BE}+3}{2.2}\right)$

$= \dfrac{-30V_{BE}+90}{31(2.2)}$

$\therefore \dfrac{I_c}{1mA} = e^{\frac{V_{BE}-0.7}{0.025}}$ we have

$\dfrac{-30V_{BE}+90}{31(2.2)} \Big/ 1 = e^{\frac{V_{BE}-0.7}{0.025}}$

CONT.

$$V_{BE} = 0.025 \ln\left(\frac{90-30V_{BE}}{31\times 2.2}\right) + 0.7$$

Iteration #1

$V_{BE} = 0.7 \rightarrow V_{BE} = 0.7\,0029$
$\simeq 0.7V$

$\therefore$ All the values calculated in 5.24(a) remain the same. This makes sense as I_c in 5.24(a) was 1.02mA which has a rated V_{BE} voltage of 0.7V. Specifically we have:

$V_B = \underline{0V}$	$I_B = \underline{0.034mA}$
$V_E = \underline{-0.7V}$	$I_E = \underline{1.05mA}$
$V_c = \underline{0.756}V$	$I_c = \underline{1.02mA}$.

(b) Continuing to use the iterative process on relative currents $\frac{I_c}{1mA} = e^{\frac{V_{BE}-0.7}{0.025}}$ we have:

$$\frac{I_c}{1} = \frac{\alpha I_E}{1} = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$\frac{30}{31}\left(\frac{3-V_{EB}}{1}\right) = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$V_{EB} = 0.7 + 0.025\ln\left(\frac{90-30V_{EB}}{31}\right)$$

By Iteration

$V_{EB} = 0.7V \rightarrow 0.72V \rightarrow 0.72V$

$V_E = \underline{\underline{0.72V}}$ $\quad I_E = \frac{3-0.72}{1} = \underline{\underline{2.28mA}}$

$I_c = \frac{30}{31} I_E = \underline{\underline{2.21mA}}$

$V_c = -3 + 2.21 = \underline{\underline{-0.79V}}$

$V_B = 0 \quad I_B = I_c/\beta = \frac{2.21}{30} = \underline{\underline{0.074mA}}$

(c) $I_c = \alpha I_E = e^{\frac{V_{EB}-0.7}{0.025}}$

$$\frac{30}{31}\left(\frac{9-(3+V_{EB})}{1}\right) = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$V_{EB} = 0.7 + 0.025\ln\left[\frac{180-30V_{EB}}{31}\right]$$

By Iteration

$V_{EB} = 0.7 \rightarrow 0.741 \rightarrow 0.741$

$\therefore V_E = \underline{\underline{3.741V}} \quad I_E = \frac{9-3.741}{1.1} = \underline{\underline{4.78mA}}$

$I_c = \frac{30}{31} I_E = \underline{\underline{4.63mA}} \quad V_c = 0.56 I_c = \underline{\underline{2.59V}}$

$I_B = \frac{I_c}{30} = \underline{\underline{0.154mA}} \quad V_B = \underline{\underline{3V}}$

(d) $I_c = \alpha I_E = e^{\frac{V_{BE}-0.7}{0.025}}$

$$\frac{30}{31}\left(\frac{3-V_{BE}}{0.47}\right) = e^{\frac{V_{BE}-0.7}{0.025}}$$

$$V_{EB} = 0.7 + \ln\left[\frac{90-30V_{BE}}{31\times 0.47}\right]$$

By Iteration:

$V_{BE} = 0.7 \rightarrow 0.739 \rightarrow 0.738 \rightarrow 0.738$

$\therefore V_E = 3 - 0.738 = \underline{2.26V}$

$I_E = V_E/0.47 = \underline{4.81mA}$

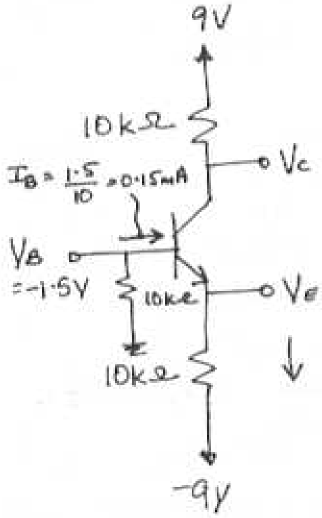
CONT.

$I_C = \frac{30}{31} I_E = \underline{\underline{4.65 mA}}$ $V_C = 9 - I_C \times 1 = \underline{\underline{4.35 V}}$

$I_B = \frac{I_C}{30} = \underline{\underline{0.155 mA}}$ $V_B = \underline{\underline{3V}}$

5.26

FIRST WITH FINITE β



$V_E = -1.5 - 0.7 = \underline{\underline{-2.2V}}$

$\downarrow I_E = \frac{-2.2+9}{10} = 0.68 mA$

$$\frac{I_E}{I_B} = \beta + 1$$

$$\frac{0.68}{0.15} = \beta + 1 \Rightarrow \beta = \underline{\underline{3.53}}$$

$$\therefore \alpha = \frac{\beta}{\beta+1} = \underline{\underline{0.779}}$$

$$V_C = 9 - 10(\alpha I_E)$$
$$= 9 - 10(0.779 \times 0.68)$$
$$= \underline{\underline{3.7V}}$$

SECOND WITH INFINITE β

$\beta = \infty$ $V_{BE} = 0.7V \Rightarrow V_E = \underline{\underline{-2.2V}}$

$\alpha = 1$

$I_E = 0.68 mA$

$I_B = 0$ $V_B = \underline{\underline{0}}$

$V_C = 9 - 10 I_E = 9 - 6.8 = \underline{\underline{2.2V}}$

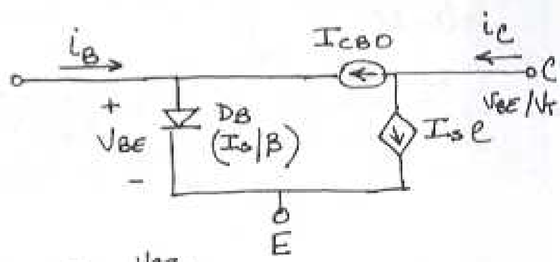
5.27

I_{CBO} doubles for every 10°C rise in temperature.

Thus if $I_{CBO} = 20nA$ at 25°C

At 85°C $I_{CBO} = 2^{\frac{85-25}{10}} \times 20$
$= \underline{\underline{1280 nA}}$

5.28



$$i_B = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} - I_{CBO} \quad (1)$$

$$i_C = I_S e^{\frac{V_{BE}}{V_T}} + I_{CBO} \quad (2)$$

$$i_E = I_S \left(1 + \frac{1}{\beta}\right) e^{V_{BE}/V_T} \quad (3)$$

for β opencircuited, $i_B = 0$ and (1) gives

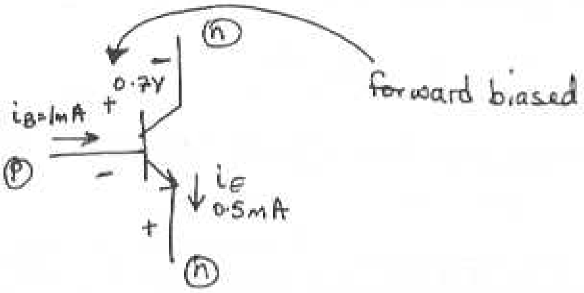
$$\frac{I_S}{\beta} e^{V_{BE}/V_T} = I_{CBO} \Rightarrow e^{V_{BE}/V_T} = \frac{\beta I_{CBO}}{I_S}$$

SUBSTITUTE INTO (2) & (3) $\Rightarrow$

$$i_C = (\beta + 1) I_{CBO}$$

$$i_E = (\beta + 1) I_{CBO}$$

5.29



Reverse Active Mode - see Table 5.1

Use Ebers Moll Model:

Eq (5.26)

$$i_E = \frac{I_S}{\alpha_F}\overbrace{\left(e^{V_{BE}/0.7} - 1\right)}^{\text{NEGLECT}} - I_S\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$$

I_S IS SMALL

NEGATIVE AND THUS VERY SMALL

Eq (5.28)

$$i_B = \underbrace{\frac{I_S}{\beta_F}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)}_{\text{NEGLECT THIS TERM}} + \frac{I_S}{\beta_R}\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$$

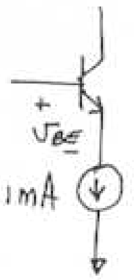
∴ Neglecting the first terms in the expressions for i_E and i_B we have

$$i_E = -I_S\left(e^{\frac{V_{BC}}{V_T}} - 1\right) = 0.5\times10^{-3}$$

$$i_B = \frac{I_S}{\beta_R}\left(e^{\frac{V_{BC}}{V_T}} - 1\right) = 10^{-3}$$

$$\left|\frac{i_E}{i_B}\right| = \underline{\underline{\beta_R = 1/2}} \qquad \underline{\underline{\alpha_R = \frac{\beta_R}{\beta_R+1} = \frac{1}{3}}}$$

5.30



V_{BE} changes by -2 mV/°C

GIVEN V_{BE} = 0.69V at 25°C

Thus,

at 0°C $V_{BE} = 0.69 - 2\times10^{-3}(-25)$

$= \underline{\underline{0.74V}}$

At 100°C $V_{BE} = 0.69 - 2\times10^{-3}(75)$

$= \underline{\underline{0.54V}}$

5.31

GIVEN $\left.\begin{array}{l} i_E = 0.5mA \\ V_{EB} = 0.692V \end{array}\right\}$ AT 20°C

(a) The junction temperature rises to 50°C

$V_{EB} = 0.692 - 2\times10^{-3}(50-20)$

$= \underline{\underline{0.632V}}$

(b) The Base-Emitter Voltage is fixed V_{EB} = 0.7V at ALL TEMPERATURES

At 20°C ~ i_E = 0.5mA at V_{EB} = 0.692V

Thus for V_{EB} = 0.7V we have

$$\frac{i_E}{0.5\times10^{-3}} = e^{\frac{0.7-0.692}{0.025}}$$

$i_E = \underline{\underline{0.689mA}}$

CONT.

Now if $T = 50°C$ & $V_{EB} = 0.7V$

from (a) we see that at 50°C,

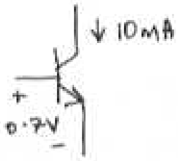
$I_E = 0.5mA, \quad V_{EB} = 0.632V$

Therefore for $V_{EB} = 0.7V$

$$\frac{i_E}{0.5\times10^{-3}} = e^{\frac{0.7-0.632}{0.025}}$$

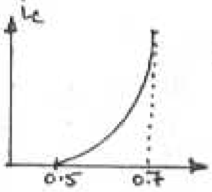
$$i_E = \underline{\underline{7.59mA}}$$

5.32



$$\frac{i_C}{10mA} = e^{\frac{v_{BE}-0.7}{0.025}}$$

$$= e^{\frac{0.5-0.7}{0.025}}$$



$$i_C = \underline{3.35\,\mu A}$$

Notice the current drops significantly at $v_{BE} = 0.5V$ See Fig. 5.16

5.33

v_{BE} changes by $-2mV/°C$ <u>FOR A PARTICULAR CURRENT</u>. Given that at 25°C $v_{BE} = 0.7V$ and $i_C = 10mA$

Thus

@ −25°C $v_{BE} = 0.7 - 2\times10^{-3}(-50)$
$= \underline{0.8V}$ and $i_C = 10mA$

@ 125°C $v_{BE} = 0.7 - 2\times10^{-3}(100)$
$= \underline{0.5V}$ and $i_C = 10mA$

5.34

Using the complete Ebers Moll model in eqs. (5.26) and (5.27) and neglecting terms not containing exponentials the i_C dependence on v_{BC} with fixed $i_E = I_E$ can be derived :-

$$i_C = I_S\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$$

$$\simeq I_S e^{V_{BE}/V_T} - \frac{I_S}{\alpha_R} e^{V_{BC}/V_T} \quad (1)$$

$$i_E = \frac{I_S}{\alpha_F}\left(e^{V_{BE}/V_T}-1\right) - I_S\left(e^{V_{BC}/V_T}-1\right)$$

$$\simeq \frac{I_S}{\alpha_F} e^{V_{BE}/V_T} - I_S e^{v_{BC}/V_T}$$

Noting the emitter current as being fixed at I_E we get

$$I_S e^{v_{BE}/V_T} = \alpha_F I_E + \alpha_F I_S e^{v_{BC}/V_T}$$

Substituting this into (1) we get:

$$i_C = \alpha_F I_E + \alpha_F I_S e^{V_{BC}/V_T} - \frac{I_S}{\alpha_R} e^{V_{BC}/V_T}$$

$$= \underline{\underline{\alpha_F I_E + I_S\left(\alpha_F - 1/\alpha_R\right) e^{V_{BC}/V_T}}}$$

$\equiv$ Eq. (5.35)
Q.E.D.

5.35

See details in problem 5.15(b).

5.36

For the saturated transistor in Fig P.5.36

$V_{BC} > 0 \Rightarrow e^{V_{BC}/V_T} \gg 1$

$V_{BE} > 0 \Rightarrow e^{V_{BE}/V_T} \gg 1$

Therefore only exponential terms in the complete Ebers Moll model are kept. Specifically in Eq(5.27) we have

$$i_C = I_S e^{V_{BE}/V_T} - \frac{I_S}{\alpha_R} e^{V_{BC}/V_T}$$

and in Eq (5.26) we have

$$i_E = \frac{I_S}{\underbrace{\alpha_F}_{\approx 1}} e^{V_{BE}/V_T} - I_S e^{V_{BC}/V_T}$$

Noting that $V_{BE} = V_{BC} + V_{CE,sat}$ we get:

$$\frac{i_C}{i_E} = \frac{I_S e^{\frac{V_{BC}+V_{CEsat}}{V_T}} - \frac{I_S}{\alpha_R} e^{\frac{V_{BC}}{V_T}}}{I_S e^{\frac{V_{BC}+V_{CE,sat}}{V_T}} - I_S e^{\frac{V_{BC}}{V_T}}}$$

$$= \frac{e^{V_{CEsat}/V_T} - \frac{1}{\alpha_R}}{e^{\frac{V_{CE,sat}}{V_T}} - 1}$$

$$\left(e^{\frac{V_{CEsat}}{V_T}} - 1\right)\frac{I_{Csat}}{I_E} = e^{\frac{V_{CEsat}}{V_T}} - \frac{1}{\alpha_R}$$

$$e^{\frac{V_{CE,sat}}{V_T}}\left[1 - \frac{I_{Csat}}{I_E}\right] = \frac{1}{\alpha_R} - \frac{I_{Csat}}{I_E}$$

$$V_{CE,sat} = V_T \ln\left[\frac{1/\alpha_R - I_{Csat}/I_E}{1 - \frac{I_{Csat}}{I_E}}\right]$$ Q.E.D.

Given $\alpha_R = 0.1$ then

$$V_{CE,sat} = 0.026 \ln\left[\frac{10 - \frac{I_{Csat}}{I_E}}{1 - I_{Csat}/I_E}\right]$$

I_{Csat}/I_E	$V_{CE,sat}$ (V)
0.9	0.113
0.5	0.0736
0.1	0.06
0	0.058

5.37

Eq(5.36):

$$i_C = I_S e^{\frac{V_{BE}}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right) \leftarrow I_S = 10^{-15}\text{A}, \; V_A = 100\text{V}$$

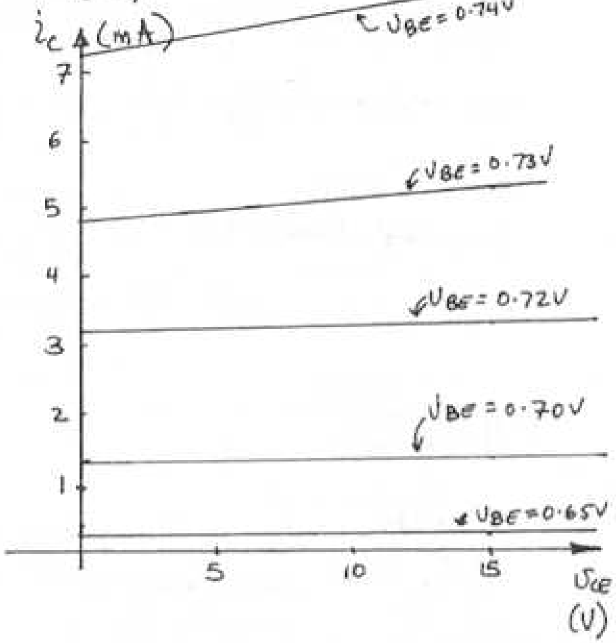
$$= 10^{-15} e^{V_{BE}/0.025}\left(1 + \frac{V_{CE}}{100}\right)$$

V_{BE} = / V_{CE} (V)	0.65V i_C (mA)	0.70V i_C (mA)	0.72V i_C (mA)	0.73V i_C (mA)	0.74V i_C (mA)
0.2	0.1961	1.4491	3.2251	4.8113	7.1777
0.8	0.1973	1.4578	3.2445	4.8402	7.2207
1.0	0.1977	1.4607	3.2509	4.8498	7.2350
4.0	0.2036	1.5041	3.3475	4.9938	7.4499
10	0.2153	1.5909	3.5406	5.2819	7.8797
12	0.2192	1.6198	3.6409	5.3780	8.0230
15	0.2936	1.6632	3.7015	5.5220	8.2379

To find the intercept of the straight-line characteristics on the i_C axis, we substitute $V_{CE} = 0$ and evaluate $i_C = 10^{-15} e^{V_{BE}/0.025}$ A for the given value of V_{BE}. The slope of each straight line is equal to this value divided by 100V (V_A). Thus we obtain

CONT.

v_{BE} (V)	0.65	0.70	0.72	0.73	0.74
Intercept (mA)	0.2	1.45	3.22	4.80	7.16
Slope (mA/V)	0.002	0.015	0.032	0.048	0.072



5.38

$r_o = \frac{1}{3\times10^{-5}} = \underline{\underline{33.3k\Omega}}$

$V_A = r_o I_c = 33.3\times10^{3} \times 3\times10^{-3}$

$= \underline{\underline{100V}}$

$r_o = \frac{V_A}{I_c} = \frac{100}{30} = \underline{\underline{3.3k\Omega}}$

5.39

$r_o = V_A / I_c = 200/I_c$

@ $I_c = 1mA$ $\quad r_o = \underline{\underline{200k\Omega}}$

@ $I_c = 100\mu A$ $\quad r_o = \frac{200}{0.1} = \underline{\underline{2.0M\Omega}}$

5.40

$v_{BE} = 0.72V$ - $i_c = 1.8mA$ $\quad v_{CE} = 2V$

$i_c = 2.4mA$ $\quad v_{CE} = 14V$

$r_o = \frac{\Delta v_{CE}}{\Delta i_c} = \frac{14-2}{2.4-1.8} = 20k\Omega$

Near saturation $v_{CE} = 0.3V$

$\therefore \frac{\Delta v_{CE}}{\Delta i_c} = \frac{0.3-2}{i_c - 1.8} = 20$

$i_c = \underline{\underline{1.72mA}}$

calculating v_{CE} for $i_c = 2.0mA$

$\frac{\Delta v_{CE}}{\Delta i_c} = r_o$

$\frac{v_{CE}-2}{2-1.8} = 20 \Rightarrow v_{CE} = \underline{\underline{6V}}$

Take the ratio of currents to fi[nd] the Early Voltage (with Eq 5.36)

$$\frac{2.4}{1.8} = \underbrace{e^{\frac{v_{BE}-v_{BE}}{V_T}}}_{=1}\left(\frac{1+14/V_A}{1+2/V_A}\right)$$

$2.4 + \frac{4.8}{V_A} = 1.8 + \frac{25.2}{V_A}$

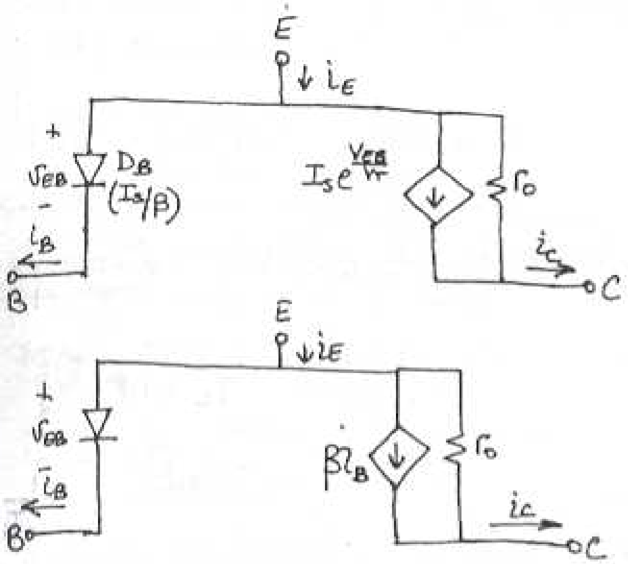
$V_A = \underline{\underline{34V}}$

$r_o = \frac{V_A}{I_c'}$ where I_c' is the curre[nt] near saturation $\Leftrightarrow$ active boundary. As calculated above $I_c' = 1.72$

$= \frac{34}{1.72}$

$= \underline{\underline{19.8k\Omega}}$ $\leftarrow$ $\approx$ to the above calculation of 20kΩ

5.41



5.42

Large signal or DC β:

$$h_{FE} = \frac{i_C}{i_B} = \frac{1.2\,mA}{8\,\mu A} = \underline{150}$$

Small signal $\quad h_{fe} = \frac{0.1\,mA}{0.8\,\mu A} = \underline{125}$

$$r_o = \frac{V_A}{I_C} = \frac{100\,V}{1.2\,mA} = 83.3\,k\Omega$$

$$\Delta i_C = h_{fe}\,\Delta i_B + \frac{\Delta v_{CE}}{r_o}$$

$$= 125 \times 2\,\mu A + 2/83.3\,k\Omega = 0.274\,mA$$

$$\therefore i_C = 1.2\,mA + \Delta i_C = \underline{\underline{1.474\,mA}}$$

5.43

	-55°C	25°C	125°C
$I_C = 100\,\mu A$ $\beta \Rightarrow$	100	187	300
$I_C = 10\,mA$ $\beta \Rightarrow$	78	178	322

For $I_C = 100\,\mu A$

Temp coef below 25°C $= \frac{187-100}{25+55} = \underline{\underline{1.09/°C}}$

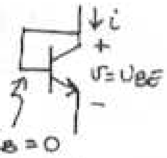
Temp coef above 25°C $= \frac{300-187}{125-25} = \underline{\underline{1.13/°C}}$

For $I_C = 10\,mA$

Temp coef below 25°C $= \frac{178-78}{25+55} = \underline{\underline{1.25/°C}}$

Temp. coef above 25°C $= \frac{322-178}{125-25} = \underline{\underline{1.44/°C}}$

5.44



$\therefore$ active mode operation

Use Eq. (5.27) & (5.28) but neglect $(e^{v_{BC}/V_T} - 1) = 0$ terms as $v_{BC} = 0$.

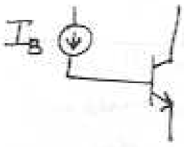
$$i = i_C + i_B$$

$$= I_S(e^{v_{BE}/V_T} - 1) + \frac{I_S}{\beta_F}(e^{v_{BE}/V_T} - 1)$$

$$= I_S \underbrace{(e^{v_{BE}/V_T} - 1)}_{\approx e^{v/V_T}} (1 + 1/\beta_F) \quad \frac{\beta_F + 1}{\beta_F} = \frac{1}{\alpha_F} \approx 1$$

$$= \underline{\underline{I_S\, e^{v/V_T}}} \qquad Q.E.D.$$

5.45

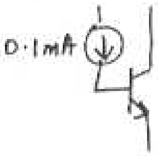


$$v_{CEsat} = v_{CEoff} + \underbrace{I_{Csat}}_{0 \because \text{ collector is open}} R_{Csat}$$

$$= V_T \ln\left(\frac{1}{\alpha_R}\right)$$

$$= 0.025 \ln \frac{1}{0.2} = \underline{\underline{40.2\,mV}}$$

5.46



$\beta_{forced} = 20$

$\beta_F = 50, \ \alpha_F = 0.98$

$\beta_R = 0.2, \ \alpha_R = 0.167.$

Eq (5.48)

$$R_{CEsat} = \frac{1}{10\beta_F I_B} = \frac{1}{10\times 50\times 0.1} = \underline{\underline{20\,\Omega}}$$

Eq (5.49)

$$v_{CEsat} = V_T \ln\left[\frac{1+\frac{\beta_{forced}+1}{\beta_R}}{1-\beta_{forced}/\beta_F}\right]$$

$$= 0.025\ln\left[\frac{1+21/0.2}{1-20/50}\right] = \underline{\underline{0.187V}}$$

5.47

Using Eq (5.47) which gives the collector current for a transistor in saturation and letting $a = \frac{i_C}{\beta_F I_B}$

we have:

$$i_C = \beta_F I_B\left(\frac{e^{V_{CE}/V_T} - 1/\alpha_R}{e^{V_{CE}/V_T}+\frac{\beta_F}{\beta_R}}\right)$$

solve for v_{CE} & take derivative

$$\left.\frac{\partial v_{CE}}{\partial i_C}\right|_{i_B=I_B,\ i_C=I_{Csat}} = R_{CEsat}$$

$$\therefore \ \frac{i_C}{\beta_F I_B}\left(e^{v_{CE}/V_T}+\frac{\beta_F}{\beta_R}\right) = e^{V_{CE}/V_T}-\frac{1}{\alpha_R}$$

$$e^{V_{CE}/V_T}\left(\frac{i_C}{\beta_F I_B}-1\right) = -\frac{\beta_F}{\beta_R}\frac{i_C}{\beta_F I_B}-\frac{1}{\alpha_R}$$

$$e^{\frac{V_{CE}}{V_T}}(a-1) = -\frac{\beta_F}{\beta_R}a - \frac{1}{\alpha_R}$$

$$v_{CE} = V_T\ln\left[\frac{a\frac{\beta_F}{\beta_R}+\frac{1}{\alpha_R}}{1-a}\right]$$

Now:

$$R_{CEsat} = \left.\frac{\partial V_{CE}}{\partial i_C}\right|_{i_C=I_{Csat}}$$

$$= \frac{\partial v_{CE}}{\partial a}\frac{\partial a}{\partial i_C} = \frac{\partial v_{CE}}{\partial a}\times\frac{1}{\beta_F I_B}$$

$$= V_T\left[\frac{1-a}{\frac{a\beta_F}{\beta_R}+\frac{1}{\alpha_R}}\times\left(\frac{\beta_F/\beta_R}{1-a}+\frac{a\beta_F/\beta_R+\frac{1}{\alpha_R}}{(1-a)^2}\right)\right]\frac{1}{\beta_F I_B}$$

$$= \frac{V_T}{\beta_F I_B}\left[\underbrace{\frac{\beta_F/\beta_R}{a\beta_F/\beta_R+\frac{1}{\alpha_R}}}_{\approx\frac{1}{a}} + \frac{1}{1-a}\right] \quad \because \ \frac{\beta_F}{\beta_R}a \gg \frac{1}{\alpha_R}$$

$$\cong \frac{V_T}{\beta_F I_B}\left[\frac{1-a+a}{a(1-a)}\right]$$

But at $i_C=I_{Csat}$, $a=\frac{I_{Csat}}{\beta_F I_B}=x$

$$= \underline{\underline{\frac{V_T}{\beta_F I_B}\times\frac{1}{x(1-x)}}}$$

Q.E.D.

Using this expression to find R_{CEsat} at $\beta_{forced}=\beta_F/2 \Rightarrow x = \frac{\beta_{forced}}{\beta_F}=\frac{1}{2}$

$$R_{CEsat} = \frac{V_T}{\beta_F I_B}\times\frac{1}{\frac{1}{2}(1-\frac{1}{2})}$$

$$= \frac{4\times 0.025}{\beta_F I_B}$$

$$= \frac{1}{10\beta_F I_B} \quad \longleftarrow \text{same as eq (5.48)}$$

5.48

$\beta_F = 70 \rightarrow \alpha_F = 70/71$

$\beta_R = 0.7 \rightarrow \alpha_R = \frac{0.7}{1.7}$

$I_B = 2mA$

For $i_c = 3mA$

$\beta_{forced} = \frac{3}{2} = 1.5$

Note the small β_{forced} means we are deeper in saturation. Hence Eq. (5.48) which approximates the slope at $i_c = \frac{\beta_F I_B}{2}$ is not useful here!

Thus we have to use the expression in Problem 5.47

$$R_{CEsat} = \frac{V_T}{\beta_F I_B}\left[\frac{1}{\frac{\beta_{forced}}{\beta_F}\left(1 - \frac{\beta_{forced}}{\beta_F}\right)}\right]$$

$$= \frac{0.025}{70 \times 2 \times 10^{-3}}\left[\frac{1}{\frac{1.5}{70}\left(1 - \frac{1.5}{70}\right)}\right]$$

$$= \underline{\underline{8.516\ \Omega}}$$

Using Eq (5.49) to calculate V_{CEsat}

$$V_{CEsat} = V_T \ln\left[\frac{1 + \frac{\beta_{forced}+1}{\beta_R}}{1 - \beta_{forced}/\beta_F}\right]$$

$$= 0.025 \ln\left[\frac{1 + \frac{2.5}{0.7}}{1 - 1.5/70}\right]$$

$$= 38.5\,mV$$

$\therefore$ $V_{CEsat} = V_{CEoff} + R_{CEsat} I_{Csat}$

$38.5 = V_{CEoff} + 8.516 \times 3 \times 10^{-3}$

$V_{CEoff} = \underline{\underline{12.95\,mV}}$

Now for $i_c = 0.3\,mA$

$$\beta_{forced} = \frac{0.3}{2} = 0.15$$

$$R_{CE,sat} = \frac{V_T}{\beta_F I_B}\left[\frac{1}{\frac{\beta_{forced}}{\beta_F}\left(1 - \frac{\beta_{forced}}{\beta_F}\right)}\right]$$

$$= \frac{0.025}{70 \times 2 \times 10^{-3}}\left[\frac{1}{\frac{0.15}{70}\left(1 - \frac{0.15}{70}\right)}\right]$$

$$= 83.51\ \Omega$$

$$V_{CEsat} = 0.025 \ln\left[\frac{1 + \frac{\beta_{forced}+1}{\beta_R}}{1 - \beta_{forced}/\beta_F}\right]$$

$$= 0.025 \ln\left[\frac{1 + \frac{1.15}{0.7}}{1 - 0.15/70}\right]$$

$$= 24.35\,mV$$

Thus $V_{CE,off}$ can be calculated from

$V_{CE,sat} = V_{CEoff} + R_{CEsat} I_{Csat}$

$V_{CEoff} = V_{CEsat} - R_{CEsat} I_{Csat}$

$= \quad 83.51 \times 0.3 \times 10^{-3}$

$= \underline{\underline{-0.7\,mV}}$

5.49

$\beta_F = 150 \rightarrow \alpha_F = \frac{150}{151}$

$\alpha_F I_{SE} = \alpha_R I_{SC} = I_S$

$\frac{A_{CBJ}}{A_{EBJ}} \propto \frac{I_{SC}}{I_{SE}} = \frac{\alpha_F}{\alpha_R} = 10$

$\therefore \alpha_R = \frac{\alpha_F}{10} = 0.0993$

$\beta_R = \frac{\alpha_R}{1-\alpha_R} = 0.110$

Calculate V_{CEsat} with

$$V_{CEsat} = V_T \ln\left[\frac{1+\frac{\beta_{forced}+1}{\beta_R}}{1-\beta_{forced}/\beta_F}\right]$$

$$= 0.025 \ln\left[\frac{1+\frac{\beta_{forced}+1}{0.110}}{1-\beta_{forced}/\beta_F}\right]$$

$\frac{\beta_{forced}}{\beta_F}$	β_{forced}	V_{CEsat} (V)
0.99	148.5	0.296
0.95	142.5	0.254
0.9	135	0.236
0.5	75	0.180
0.1	15	0.127
0.01	1.5	0.079
0	0	0.058

As β_{forced} is decreased, V_{CEsat} decreases and the BJT is drawn further into saturation.

5.50

$V_{BE} = 0.72\text{V}$ at $i_C = 0.6$ mA

(a)

$\beta_F = 150 \rightarrow \alpha_F = \frac{150}{151} = \underline{\underline{0.993}}$

$\frac{A_{CBJ}}{A_{EBJ}} \propto \frac{I_{SC}}{I_{SE}} = \frac{\alpha_F}{\alpha_R} = 20$

$\Rightarrow \alpha_R = \frac{\alpha_F}{20} = \frac{0.993}{20} = \underline{\underline{0.0497}}$

$\therefore \beta_R = \underline{\underline{0.0523}}$

(b) $i_C = 5$mA ~ assuming active mode

Using the Ebers Moll model in active mode – Eq (5.32)

$$i_C = I_S e^{V_{BE}/V_T} + I_S\left(\frac{1}{\alpha_R} - 1\right)$$

$$0.6\times10^{-3} = I_S\left(e^{0.72/0.025} + \frac{1}{0.0497} - 1\right)$$

$$I_S = 1.86\times10^{-16}$$

Therefore at $i_C = 5$mA we have

$$5\times10^{-3} = 1.86\times10^{-16}\left(e^{V_{BE}/0.025} + \frac{1}{0.0497} - 1\right)$$

$V_{BE} = \underline{0.773\text{V}}$

Use Eq (5.33) to calculate i_B

$$i_B = \frac{I_S}{\beta_F} e^{V_{BE}/V_T} - I_S\left(\frac{1}{\beta_F} + \frac{1}{\beta_R}\right)$$

$$= \frac{1.86\times10^{-16}}{150} e^{\frac{0.773}{0.025}} - 1.86\times10^{-16}\left(\frac{1}{150} + \frac{1}{0.0523}\right)$$

$= \underline{\underline{33.2\mu A}}$

CONT.

This value for i_B makes sense as $\frac{i_C}{\beta_F} = \frac{5}{150} = 33.3\mu A$.

It is a little less due to the reverse leakage currents.

(c) The base current is doubled

$$\left.\begin{array}{l} i_B = 66.4\mu A \\ i_C = 5mA \end{array}\right\} \beta_{forced} = \frac{i_C}{i_B} = \frac{I_{Csat}}{i_B}$$

$$= \frac{5}{0.0664}$$

$$= \underline{\underline{75.3}}$$

Note β_{forced} is not small so we are not deep in saturation. Hence we can use Eq (5.48) to calculate R_{CEsat}:

$$R_{CEsat} = \frac{1}{10\beta_F I_B} = \frac{1}{10 \times 150 \times 66.4\mu A}$$

$$= \underline{\underline{10.04\,\Omega}}$$

checking this result with the expression from PROB. 5.47 we get the same result:

$$R_{CEsat} = \frac{V_T}{\beta_F I_B}\left(\frac{1}{\frac{\beta_{forced}}{\beta_F}\left(1 - \frac{\beta_{forced}}{\beta_F}\right)}\right)$$

$$= \frac{0.025}{150 \times 66.4 \times 10^{-6}}\left(\frac{1}{\frac{75.3}{150}\left(1 - \frac{75.3}{150}\right)}\right)$$

$$= 10.04\,\Omega$$

From Eq (5.49)

$$|V_{CEsat}| = V_T \ln\left[\frac{1 + \frac{\beta_{forced}+1}{\beta_R}}{1 - \beta_{forced}/\beta_F}\right]$$

$$|V_{CEsat}| = 0.025 \ln\left[\frac{1 + 76.3/0.0523}{1 - 75.3/150}\right]$$

$$= \underline{\underline{0.1996\,V}}$$

Use the complete Ebers Moll equations to calculate v_{BE} and v_{BC}.

Note that $v_{CE} = v_{CB} + v_{BE}$

$$= -v_{BC} + v_{BE}$$

$$\therefore v_{BE} = v_{CEsat} + v_{BC}$$

Eq (5.27):

$$i_C = I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

$$\frac{i_C}{I_S} = e^{\frac{V_{CEsat} + v_{BC}}{V_T}} - 1 - \frac{e^{v_{BC}/V_T}}{\alpha_R} + \frac{1}{\alpha_R}$$

$$\frac{5 \times 10^{-3}}{1.86 \times 10^{-16}} = e^{\frac{0.1996}{0.025}} e^{\frac{v_{BC}}{V_T}} - \frac{e^{v_{BC}/V_T}}{\alpha_R} - 1 + \frac{1}{\alpha_R}$$

$$v_{BC} = 0.025 \ln\left[\frac{\frac{5 \times 10^{-3}}{1.86 \times 10^{-16}} + 1 - \frac{1}{0.0497}}{e^{\frac{0.1996}{0.025}} - \frac{1}{0.0497}}\right]$$

$$= \underline{\underline{0.5736}}$$

Now

$$v_{BE} = v_{CEsat} + v_{BC}$$

$$= 0.1996 + 0.5736$$

$$= \underline{\underline{0.7732\,V}}$$

5.51

Eq (5.49)

$$V_{CEsat} = V_T \ln\left[\frac{1 + \frac{\beta_{forced}+1}{\beta_R}}{1 - \beta_{forced}/\beta_F}\right]$$

For E grounded and C open: $\beta_{forced} = 0$

$$\therefore\ 60 = 25 \ln\left[\frac{1 + 1/\beta_R}{1-0}\right]$$

$$\ln\left(1 + \frac{1}{\beta_R}\right) = \frac{60}{25} = 2.4$$

$$\beta_R = \underline{\underline{0.01}}$$

For C grounded and E open

$\beta_{forced} = 0 \quad \beta_R^* = \beta_F \quad \beta_F^* = \beta_R$

$$\therefore\ 1 = 25 \ln\left[\frac{1 + \frac{0+1}{\beta_F}}{1-0}\right]$$

$$\ln\left(1 + \frac{1}{\beta_F}\right) = \frac{1}{25}$$

$$\beta_F = \underline{\underline{24.5}}$$

Check:

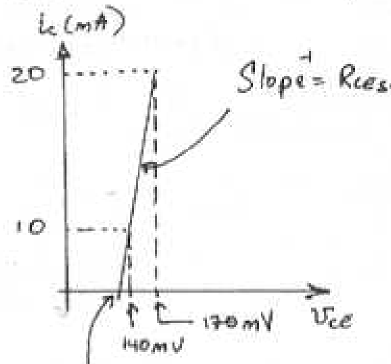
$$-(-1) = 25 \ln\left[\frac{1 + \frac{1}{24.5}}{1-0}\right] = 1\,mV.$$

5.52

Information given (where $\beta_{forced} = I_C/I_B$)

I_B (mA)	I_C (mA)	β_{forced}	V_{CEsat} (mV)
0.5	10	20	140
0.5	20	40	170

Given information displayed pictorally



$$\text{Slope}^{-1} = R_{CEsat} = \frac{170-140}{20-10} = \underline{\underline{3\,\Omega}}$$

$$V_{CE,off} = 140 - 10\,R_{CEsat} = 140 - 10 \times 3 = \underline{\underline{110\ mV}}$$

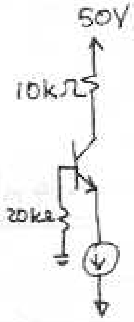
Inserting the given data into Eq (5.49) gives the following two equations

$$140 = 25 \ln\left[\frac{1 + \frac{21}{\beta_R}}{1 - 20/\beta_F}\right] \quad (1)$$

$$170 = 25 \ln\left[\frac{1 + 41/\beta_R}{1 - 40/\beta_F}\right] \quad (2)$$

which can be solved together to yield: $\beta_F = \underline{\underline{68.2}} \quad \beta_R = \underline{\underline{0.111}}$

5.53



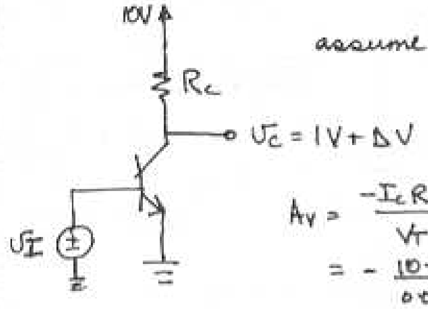
$BV_{CBO} = 30V$

$$V_B = \frac{50-30}{30} \times 20 = \underline{\underline{13.3V}}$$

$$V_C = 13.3 + 30 = \underline{\underline{43.3V}}$$

$$V_E = 13.3 - 0.7 = \underline{\underline{12.6V}}$$

5.54



assume $v_{CEsat} = 0.2V$

$$A_v = \frac{-I_C R_C}{V_T} = -\frac{10-1}{0.025} = \underline{\underline{-360\,V/V}}$$

At the verge of saturation $v_{CEsat} = 0.3V$

$\therefore V_C = 1 + \Delta V = 0.3.$

$$\Delta v = \underline{\underline{-0.7V}}$$

$\therefore v_o = 0.3V \quad i_C = \frac{10-0.3}{R_C}$

$$\frac{i_{C2}}{i_{C1}} = \frac{9.7/R_C}{(10-1)/R_C} = e^{\Delta v / V_T}$$

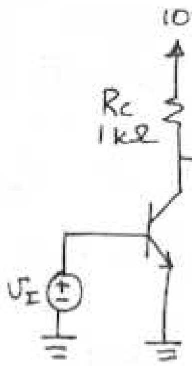
$\therefore$ Maximum Input signal

$$\Delta v = 0.025 \ln \frac{9.7}{9} = \underline{\underline{1.87mV}}$$

If we assume linear operation right to saturation we can use the gain A_v to calculate the maximum input swing. Thus for an output swing $\Delta v_o = 0.8$ we have

$$\Delta v_i = \frac{-\Delta v_o}{A_v} = \frac{-0.7}{-360} = \underline{\underline{1.94\,mV}}$$

5.55



$V_{CE} = 10 - 1.0 I_C \leftarrow$ DC collector voltage

$$A_v = -\frac{I_C R_C}{V_T} = -\frac{I_C \times 1.0}{0.025} = -40 I_C \quad (1)$$

- Assuming the output voltage $v_o = 0.3V$ is the lowest v_{CE} to stay out of saturation.

$\therefore v_o = 0.3 = 10 - i_C R_C$

$$= 10 - I_C R_C + \Delta v_o$$

$$\Delta v_o = -10 + 0.3 + I_C \times 1 \quad (2)$$

- Max output voltage before the transistor is cut off

$$V_{CE} + \Delta v_o = V_{CC}$$

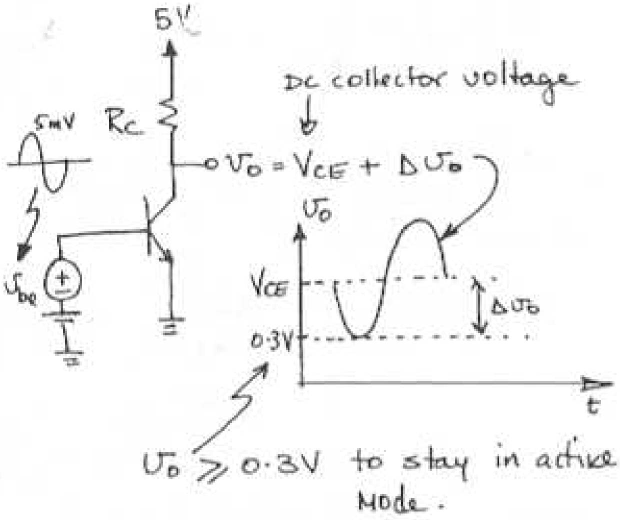
$$\Delta v_o = V_{CC} - V_{CE} = 10 - 10 + 1.0 I_C = 1.0 I_C \quad (3)$$

Use (1) to calculate the gain and (2), (3) to calculate the output limits in order to stay in active mode for a particular bias current I_C.

I_C (mA)	A_v (V/V)	Δv_o (V)
1	-40	-8. to 1
2	-80	-7. to 2
5	-200	-4.7 to 5
8	-320	-1.7 to 8
9	-360	-0.7 to 9

5.56

Since we are assuming linear operation we don't have to go to $i_c = I_s e^{V_{BE}/V_T}$ equation.



$$A_v = -\frac{I_C R_C}{V_T} = -\frac{V_{CC} - V_{CE}}{V_T}$$

On the verge of saturation

$$V_{CE} - \Delta v_o = 0.3V$$

for linear opearation $\Delta v_o = A_v v_{be}$

$$V_{CE} - |A_v v_{be}| = 0.3$$

$$(5 - I_C R_C) - A_v \times 5\times10^{-3} = 0.3$$

$$5 - |A_v V_T| - |A_v \times 5\times10^{-3}| = 0.3$$

$$|A_v(0.025 + 0.005)| = 5 - 0.3$$

$$|A_v| = 156.67$$ Note Av is negative.

$$\therefore A_v = -156.67 \text{ V/V}$$

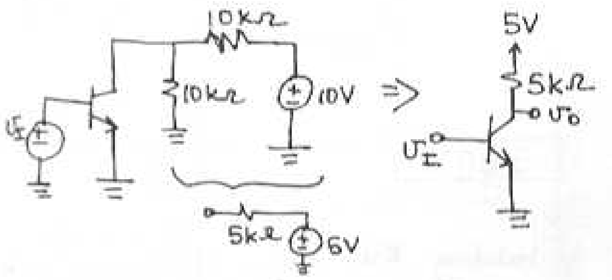
Now we can find the dc collector voltage. Refer to sketch of the output voltage, we see that

$$|\Delta v_o| = |(A_v \times 0.005)|$$

$$\therefore V_{CE} = 0.3 + |A_v| 0.005$$

$$= \underline{\underline{1.08V}}$$

5.57

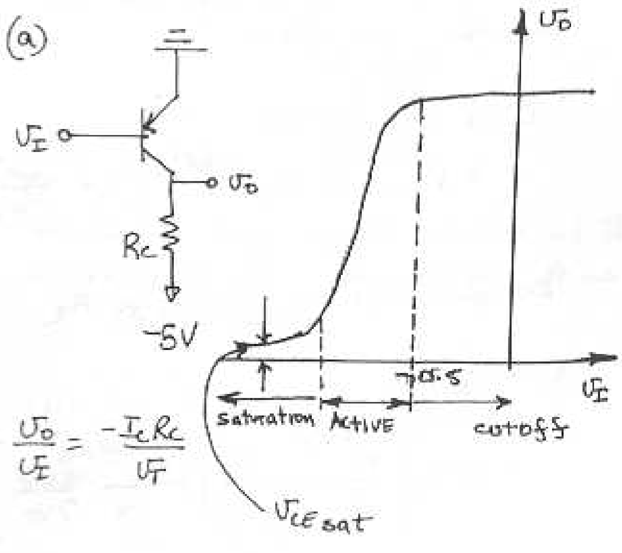


$$\frac{v_o}{v_I} = -\frac{I_C R_C}{V_T} = \frac{-0.5\times5}{0.026}$$

$$= \underline{\underline{-100 \text{ V/V}}}$$

5.58

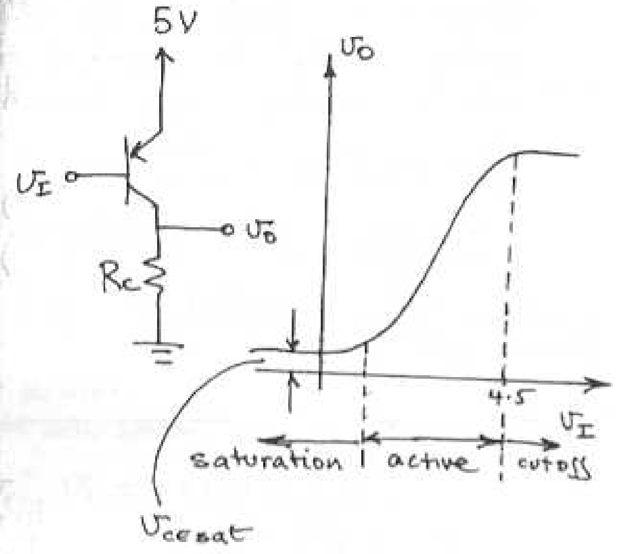
(a)



$$\frac{v_o}{v_I} = -\frac{I_C R_C}{V_T}$$

CONT.

(b)



5.59

Including the Early effect we note that:

$$i_c = I_s e^{v_{BE}/V_T}\left(1 + v_{CE}/V_A\right)$$

Also, note $I_c = I_s e^{v_{BE}/V_T}$ Eq (5.38b) is the value of the collector current with the Early voltage neglected.

Starting with the voltage at the collector we have:

$$v_O = V_{CC} - i_C R_C$$

$$= V_{CC} - R_C I_s e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

Take derivative to get gain Av

$$A_v = \frac{\partial v_O}{\partial v_I}$$

$$= -R_C I_s\left[\frac{e^{v_{BE}/V_T}}{V_T}\left(1 + \frac{v_{CE}}{V_A}\right) + \frac{e^{v_{BE}/V_T}}{V_A}\frac{\partial v_{CE}}{\partial v_I}\right]$$

$$A_v = -\frac{R_C I_s}{V_T} e^{v_{BE}/V_T}\left[1 + v_{CE}/V_A + \frac{V_T}{V_A}\frac{\partial v_{CE}}{\partial v_I}\right]$$

$$= \frac{-R_C I_C}{V_T}\left[1 + \frac{v_{CE}}{V_A} + \frac{V_T}{V_A} A_v\right]$$

$$-A_v\left[\frac{1}{\frac{R_C I_C}{V_T}} + \frac{V_T}{V_A}\right] = 1 + \frac{v_{CE}}{V_A} = \frac{V_A + v_{CE}}{V_A}$$

$$-A_v\left[\frac{V_A + R_C I_C}{\frac{R_C I_C V_A}{V_T}}\right] = \frac{V_A + V_{CE}}{V_A}$$

$$-A_v \Big/ \frac{R_C I_C}{V_T} = \frac{V_A}{V_A + R_C I_C} \times \frac{V_A + v_{CE}}{V_A}$$

$$= \frac{V_A + V_{CE}}{V_A + R_C I_C}$$ ÷ top & bottom by $V_A + V_{CE}$

$$= \frac{1}{\frac{V_A}{V_A + v_{CE}} + \frac{R_C I_C}{V_A + v_{CE}}}$$

This term is ≅ 1 ∵ $V_A >> v_{CE}$

$$\therefore A_v \cong \left[\frac{-R_C I_C / V_T}{\left(1 + \frac{R_C I_C}{V_A + v_{CE}}\right)}\right]$$

Q.E.D.

For $V_{CC} = 5V$ $V_{CE} = 2.5V$ $V_A = 100V$

Ignoring the Early Voltage:

$$A_v = \frac{-I_C R_C}{V_T} = \frac{V_{CC} - V_{CE}}{V_T} = \frac{5 - 2.5}{0.025} = 100\frac{V}{V}$$

With the Early voltage

$$A_v \cong \frac{-I_C R_C / V_T}{1 + \frac{R_C I_C}{V_A + v_{CE}}}$$

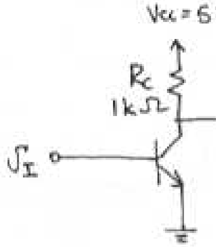
CONT.

But $v_{CE} = 0.5V$ & $\frac{I_C R_C}{V_T} = 100$ as shown above.

$$\therefore A_v = \frac{-100}{1 + \frac{2.5}{100+2.5}}$$

$$= -97.7 \text{ V/V}$$

5.60



$\downarrow I_C = \frac{5-2}{1} = 3mA$

$v_O = 2 \pm \Delta v_O$

Small signal voltage gain

$$A_v = -\frac{I_C R_C}{V_T} = \frac{-3 \times 1}{0.025} = -120 \text{ V/V}$$

If we assume linear operation around the bias point we can use A_v.

$\Delta v_O = -120 \times 5 \times 10^{-3} = -0.6V$ for $\Delta v_{BE} = 5mV$

$\Delta v_O = 120 \times 5 \times 10^{-3} = 0.6V$ for $\Delta V_{BE} = -5mV$

Using the transistor exponential characteristics as in Example 5.2: for $\Delta v_{BE} = 5mV$:

$$\frac{i_c}{3mA} = e^{\Delta v_{BE}/V_T} = e^{5/25}$$

$i_c = 3.66 mA$

$v_O = 5 - 3.66 = 1.34 V$

$\Delta v_O = 1.34 - 2 = -0.66V$

For $\Delta v_{BE} = -5mV$

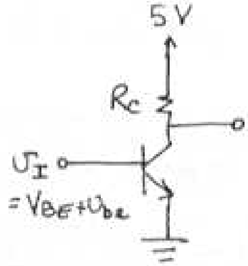
$i_c = 3 \times e^{-5/25} = 2.46mA$

$v_O = 5 - 2.46 \times 1 = 2.54V$

$\Delta v_O = 2.54 - 2 = 0.54V$

Δv_{BE} (mV)	Δv_O(V) Exponential	Δv_O(V) Linear
+0.5	−0.66	−0.6
−0.5	+0.54	+0.6

5.61



$v_O = V_{CE} + v_{ce}$

$v_I = V_{BE} + v_{be}$

(a) For maximum gain you would bias at the largest current since $A_v = -I_C R_C / V_T$. This also mean you would bias at the edge of saturation $A_v = \frac{-V_{CC} - V_{CEsat}}{V_T}$

$$= \frac{-5 - 0.3}{0.025}$$

$$= -188 \text{ V/V}$$

However any signal swing at the output would automatically drive it into saturation.

(b) for $A_v = -100$ V/V

$$A_v = \frac{V_{CC} - V_{CE}}{V_T} = \frac{5 - V_{CE}}{V_T} = 100$$

$V_{CE} = 2.5V$

CONT.

(c) For a dc collector current of 0.5mA

$$R_c = \frac{5-2.5}{0.5} = \underline{\underline{5\,k\Omega}}$$

(d) $I_S = 10^{-15}A \Rightarrow$

$$I_C = I_S e^{V_{BE}/V_T}$$

$$0.5\times10^{-3} = 10^{-15} e^{V_{BE}/0.025}$$

$$V_{BE} = \underline{\underline{0.673V}}$$

(e) If we assume linear operation we can use A_v to find the output change for $v_{be} = 5mV$

$$v_{ce} = A_v v_{be} = -100 \times 0.005$$
$$= -0.5\,V \quad \sim \text{peak sine wave.}$$

$\therefore$ the output is a $\underline{\underline{0.5V\ p \text{ sine wave}}}$

(f) for $v_{ce} = 0.5$

$$i_c = \frac{0.5}{5} = \underline{\underline{0.1mA \text{ peak}}}$$

This current is superimposed on I_C.

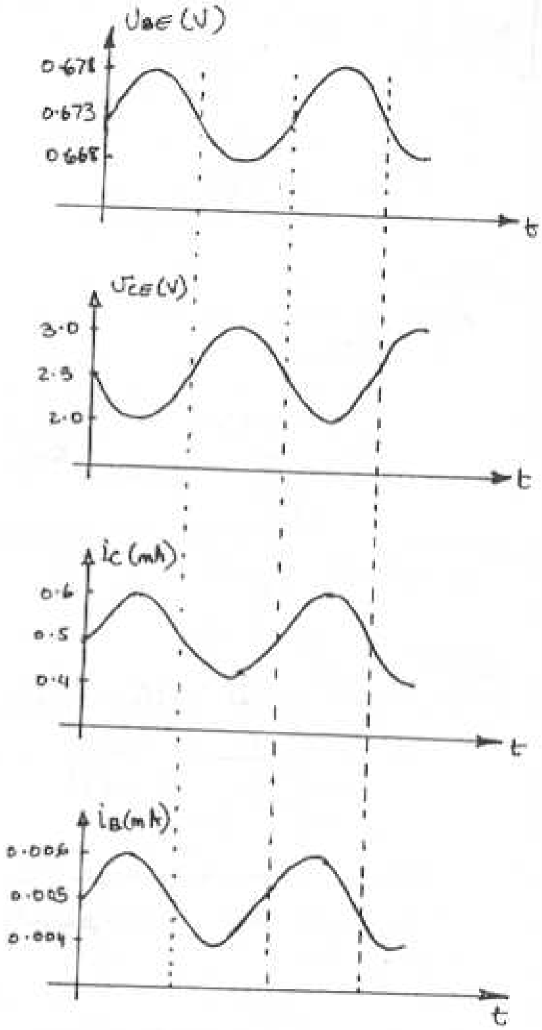
(g) $I_B = I_C/\beta = \frac{0.5}{100} = \underline{\underline{0.005mA}}$

$$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} = \underline{\underline{0.001\,mA_p}}$$

(h) $r_{in} = \frac{v_{be}}{i_b} = \frac{0.005}{0.001\times10^{-3}}$

$$= \underline{\underline{5k\Omega}}$$

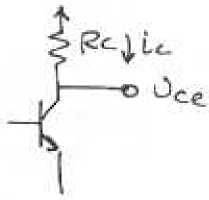
(i) See sketches that follow:



5.62

Eq (5.56)

$$A_v = \frac{v_{ce}}{v_{be}} = -\frac{I_C R_C}{V_T}$$



But $v_{ce} = -i_c R_c$

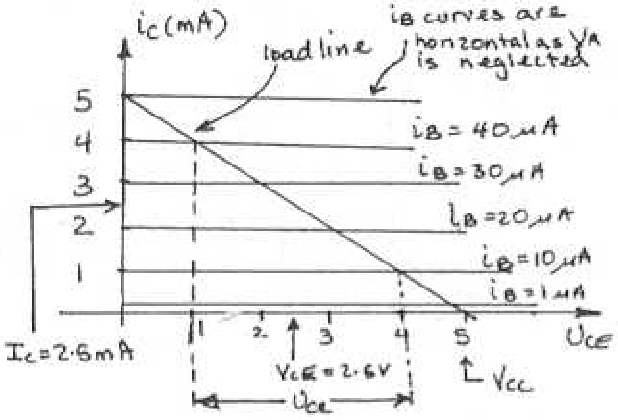
$$\therefore \frac{-i_c R_c}{v_{be}} = \frac{-I_C R_C}{V_T}$$

Now $g_m = \frac{\text{output current}}{\text{input voltage}} = \frac{i_c}{v_{be}}$

$$\therefore g_m R_c = \frac{I_C R_C}{V_T}$$

$$\underline{\underline{g_m = I_C/V_T}}$$

5.63



For i_B varying from $10\,\mu A$ to $40\,\mu A$, i_C varies from 1mA to 4mA ($\beta = 100$), and $v_C = V_{CC} - R_C i_C$ varies from 4 to 1V. Thus the peak-to-peak collector voltage swing is

$$v_{ce} = \underline{\underline{3V \text{ p-p}}}$$

For $V_{CE} = \frac{1}{2} V_{CC} = 2.5V$

$$I_C = \underline{\underline{2.5\text{mA}}}$$

$$\& \; I_B = \frac{2.5}{100} = \underline{\underline{25\,\mu A}}$$

$$V_{BB} = V_{BE} + I_B R_B$$
$$= 0.7 + 0.025 \times 100$$
$$= \underline{3.2V}$$

5.64

See the graphical construction that follows. For this circuit:

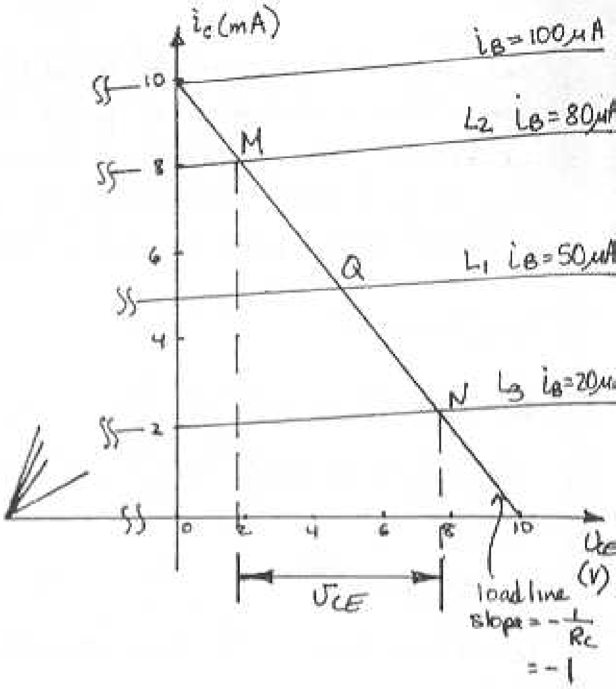
$V_{CC} = 10V$ $\qquad \beta = 100$

$R_C = 1k\Omega$ $\qquad V_A = 100V$

$I_B = 50\mu A$ - dc bias $\qquad i_C = \beta i_B$ at $V_{CE} = 0$

$\therefore I_C = 50 \times 100$

$= 5$mA - dc bias



- Given the base bias current of $50\,\mu A$ the dc or bias point of the collector current I_C & voltage V_{CE} can be found from the intersection of the load line & the transistor line L_1 of $i_B = 50\,\mu A$. Specifically:

Eq of $L_1 \Rightarrow i_C = I_C\left(1 + {v_{CE}}/{V_A}\right)$
$$= 5\left(1 + v_{CE}/100\right)$$
$$= 5 + 0.05\, v_{CE}$$

Eq of load line $\Rightarrow i_C = \dfrac{V_{CC} - v_{CE}}{R_C}$
$$= 10 - v_{CE}$$

$\therefore \; 10 - v_{CE} = 5 - 0.05\, v_{CE}$

$V_{CE} = v_{CE} = \underline{\underline{4.76\ V}}$

$I_C = i_C = 10 - v_{CE} = \underline{\underline{5.24\text{ mA}}}$

Now for a signal of $30\,\mu A$ peak superimposed on $I_B = 50\,\mu A$, the operating point moves along the

CONT.

load line between points N and M. To obtain the coordinates of point M, we solve the load line and line L_2 to find the intersection M and the load line and line L_3 to find N:

FOR POINT M:

$i_c = 8 + 8/100\, v_{CE}$ & $i_c = 10 - v_{CE}$

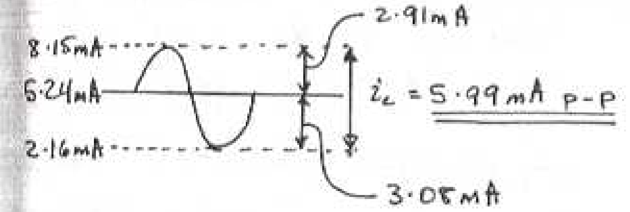
$\therefore\ i_C|_M = 8.15\text{mA} \quad V_{CE}|_M = 1.85\text{V}$

FOR POINT N:

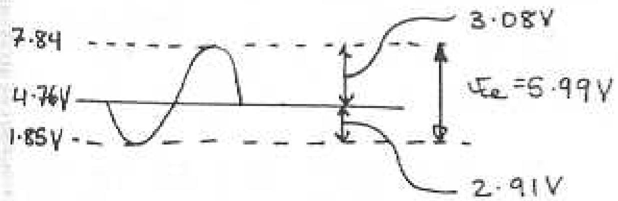
$i_c = 2 + 0.02 v_{CE}$ & $i_c = 10 - v_{CE}$

$v_{CE}|_N = 7.84\text{V} \quad i_c|_N = 2.16\text{mA}$

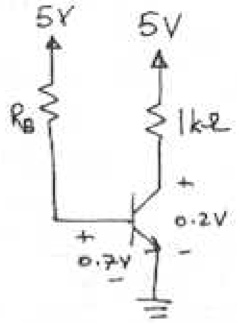
Thus the collector current varies as follows



And the collector voltage varies as:



5.65



$\dfrac{\beta_F}{\beta_{forced}}$ = overdrive factor = 10

$\therefore\ \beta_{forced} = \dfrac{\beta_F}{10} = \dfrac{20}{10} = \underline{\underline{2}}$

for $V_{CEsat} = 0.2$V

$5 - I_{Csat} \times 1 = 0.2$

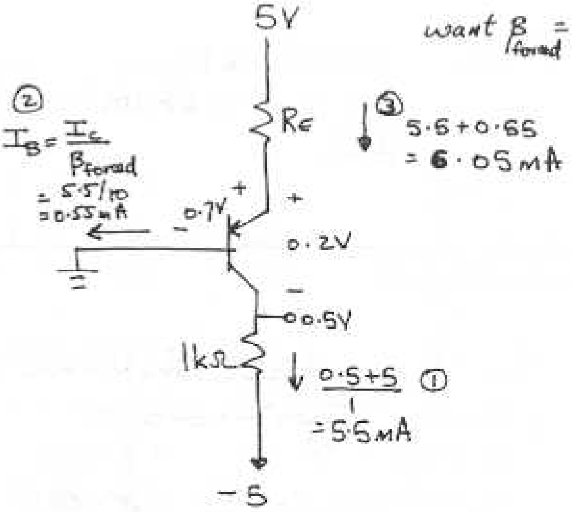
$I_{Csat} = 5 - 0.2 = 4.8\text{mA}$

$I_B = \dfrac{I_{Csat}}{\beta_{forced}} = \dfrac{4.8}{2} = 2.4\text{mA}$

$\therefore\ R_B = \dfrac{5-0.7}{2.4} = \dfrac{4.3}{2.4} = \underline{\underline{1.8\text{k}\Omega}}$

5.66

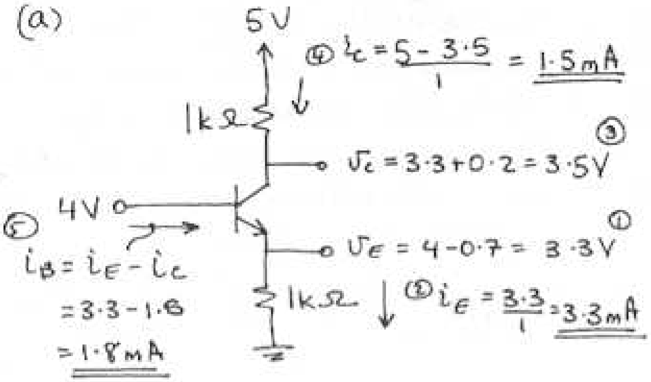
want $\beta_{forced} = 10$

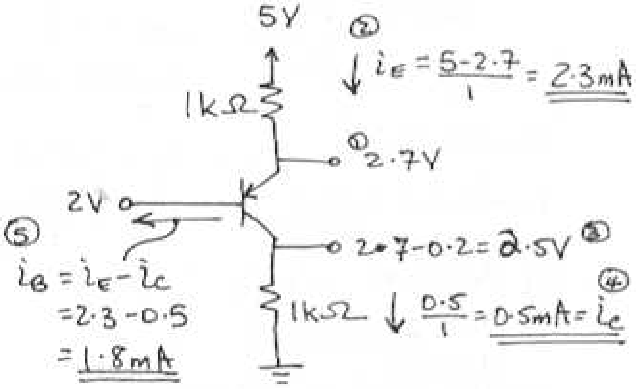


$R_E = \dfrac{5-0.7}{6.05} = \underline{\underline{710\,\Omega}}$

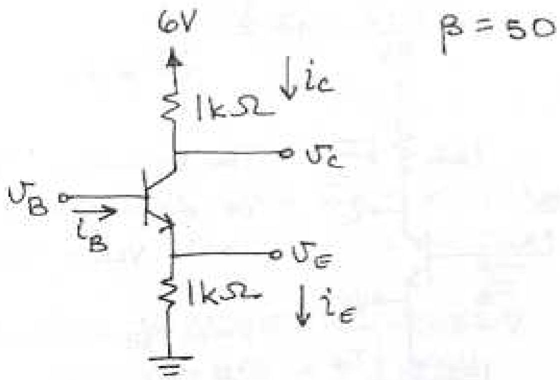
5.67

(a)





5.68



$\beta = 50$

Since conduction does not start until $v_{BE} = 0.5V$, i_C & i_B are essentially 0 for $v_B \leq 0.5V$

For:

$v_B = 1V$

$v_E = 1 - 0.3 = 0.7V$

$i_E = \frac{0.3}{1} = 0.3mA$

$i_C = \frac{\beta}{\beta+1} i_E$

$= \frac{50}{51} \times 0.3$

$= 0.2941mA$

$v_C = 6 - 0.29 \times 1$

$= 5.71V$

$i_B = i_E - i_C \left(\equiv \frac{i_C}{\beta}\right)$

$= 0.0059mA$

$V_B = 3V$

$V_E = 3 - 0.7 = 2.3V$

$i_E = 2.3mA$

$i_C = \frac{\beta}{\beta+1} i_E$

$= \frac{50}{51} \times 2.3$

$= 2.2549$

$v_C = 6 - 2.25$

$= 3.75V$

$i_B = 2.3 - 2.25$

$= 0.0451mA$

- let saturation begins at $v_B = x$
- at saturation starting
  $v_{BC} = 0.4V$

$v_C = x - 0.4 \Rightarrow i_C = \frac{6 - x + 0.4}{1}$

$v_E = x - 0.7 \Rightarrow i_E = \frac{x - 0.7}{1}$

Since we are still in active mode $i_C = \frac{50}{51} i_E$

$(6.4 - x) = \frac{50}{51}(x - 0.7)$

$101x = 361.4$

$\therefore x = v_B = 3.58V$

$i_C = 6 - x + 0.4 = 2.82mA$

$i_B = \frac{i_C}{\beta} = \frac{2.82}{50} = 0.056mA$

For $v_B = 4V$ and $6V$, the transistor is in saturation. Assume $v_{BE} = 0.7V$ & $V_{CEsat} = 0.7 - 0.6 = 0.1V$

CONT.

FOR:

$V_B = 4V$

$V_E = 4 - 0.7 = 3.3V$

$i_E = \frac{3.3}{1} = \underline{\underline{3.3\,mA}}$

$V_C = 3.3 + 0.1 = \underline{\underline{3.4V}}$

$i_C = \frac{6 - 3.4}{1} = \underline{\underline{2.6mA}}$

$i_B = i_E - i_C$
$= 3.3 - 2.6$
$= \underline{\underline{0.7mA}}$

$V_B = 6V$

$V_E = 6 - 0.7 = \underline{\underline{5.3V}}$

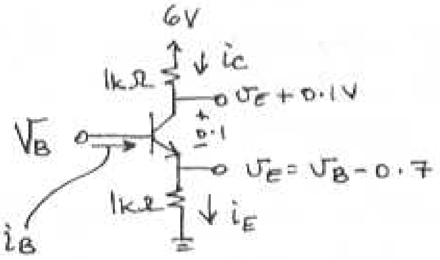
$i_E = \frac{5.3}{1} = \underline{\underline{5.3\,mA}}$

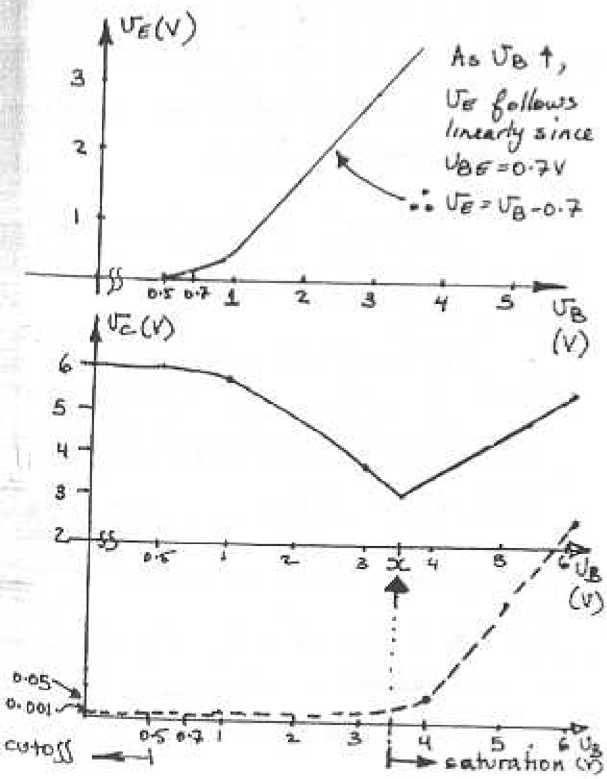
$V_C = 5.3 + 0.1 = \underline{\underline{5.4V}}$

$i_C = \frac{6 - 5.4}{1}$
$= \underline{\underline{0.6mA}}$

$i_B = 5.3 - 0.6$
$= \underline{\underline{4.7mA}}$

CIRCUIT IN SATURATION:





Notice for $v_B \leq 0.5V$ the transistor is essentially cutoff. Then at saturation, as v_B increases, both v_E & v_C increases linearly. This occurs as $v_{BE} = 0.7V$ & $V_{CE} = 0.1V$ in saturation

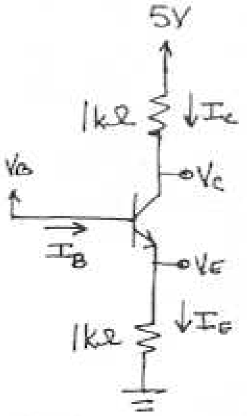
5.69

(a) $V_B = 2V$
$V_E = 2 - 0.7 = \underline{1.3V}$
$I_E = \frac{V_E}{1} = 1.3mA$
$I_C \cong 1.3\,mA$
$V_C = 5 - 1.3 = \underline{3.7V}$

(b) $V_B = 1V$
$V_E = 1 - 0.7 = \underline{0.3V}$
$I_E \cong I_C = 0.3mA$
$V_C = 5 - 0.3$
$= \underline{4.7V}$

(c) $V_B = 0V$ - cutoff
$V_E = \underline{0V}$
$I_E = 0A$
$V_C = \underline{5V}$

5.70



$\beta \sim$ high

The transistor stays in active mode until $V_{CB} = -0.4V$

$\therefore V_C - V_B = -0.4V$

$\because I_C = I_E$

$$\frac{5 - (V_B - 0.4)}{1} = \frac{V_B - 0.7}{1}$$

$5.7 + 0.4 = 2V_B$

$V_B = \underline{\underline{3.05V}}$

CONT.

For operation with $\beta_{forced} = 1 \Rightarrow$

$$\beta_{forced} = \frac{I_{csat}}{I_B} = 1 \qquad \left\{\begin{matrix} \text{assume} \\ V_{CE} = 0.2V \\ V_{CB} = -0.5V \end{matrix}\right\}$$

$$\therefore I_{csat} = I_B$$

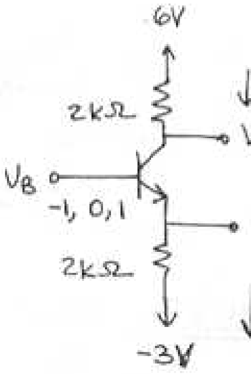
$$I_E = I_{csat} + I_B = 2I_{csat}$$

$$\frac{V_B - 0.7}{1} = 2 \times \frac{5 - (V_B - 0.5)}{1}$$

$$V_B - 0.7 = 2(5.5 - V_B)$$

$$V_B = \underline{\underline{3.9V}}$$

5.71



$\beta = \infty$

$I_C \cong I_E$

$V_C = 6 - 2I_E = \underline{\underline{4.7V}}, \underline{\underline{3.7V}}, \underline{\underline{2.7V}}$

$V_E = V_B - 0.7$
$= \underline{\underline{-1.7}}, \underline{\underline{-0.7}}, \underline{\underline{0.3}}$

$I_E = \frac{V_E + 3}{2}$
$= 0.65, 1.15, 1.65mA$

- Want V_B when $I_E = \frac{1}{10} \times 1.15mA$
$= 0.115mA$

$$V_E = -3 + 0.115 \times 2 = -2.77V$$
$$V_B = V_E + 0.7 = \underline{\underline{-2.07V}}$$

- Want V_B at the edge of conduction
At the edge of conduction assume $V_{BE} = 0.5V$

$\therefore V_B - 0.5 - 2I_E + 3 = 0 \leftarrow I_E = 0$ at edge of conduction

$V_B = \underline{\underline{-2.5V}}$

$V_E = V_B - 0.5 = \underline{\underline{-3V}}$

$I_C \cong 0A \quad \therefore V_C = \underline{\underline{6V}}$

At saturation assume $V_{CE} = 0.2V$
$V_{CB} = -0.5V$

$$\therefore I_E = \frac{V_B - 0.7 + 3}{2} \cong I_C = \frac{6 - (V_B - 0.5)}{2}$$

$$\therefore V_B + 2.3 = 6.5 - V_B$$
$$V_B = \underline{\underline{2.1V}}$$

$V_E = 2.1 - 0.7 = \underline{\underline{1.4V}} \quad V_C = V_B - 0.5 = \underline{\underline{1.6V}}$

- Want V_B at $\beta_{forced} = 2$, $V_{CE} = 0.2V$
$V_{CB} = -0.5V$

$$\beta_{forced} = \frac{I_{csat}}{I_B} = 2$$

$$I_E = I_B + I_{csat} = \frac{I_{csat}}{2} + I_{csat}$$
$$= \tfrac{3}{2} I_{csat}$$

$$V_E = V_B - 0.7 = -3 + 2I_E$$
$$= -3 + 3I_{csat}$$

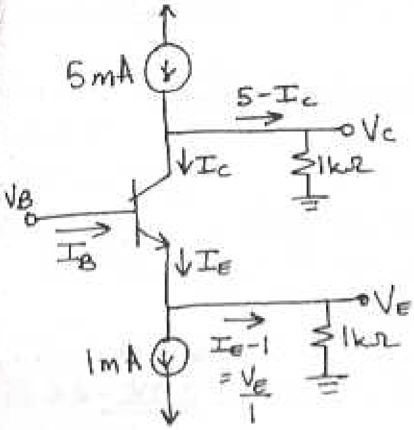
$$I_{csat} = \frac{2.3 + V_B}{3}$$

$$I_C = \frac{V_{CC} - (V_B - 0.5)}{2} = I_{csat}$$

$$6.5 - V_B = \frac{2}{3}(2.3) + \frac{2}{3}V_B$$

$$V_B = \frac{6.5 - \frac{2(2.3)}{3}}{1\frac{2}{3}} = \underline{\underline{2.98V}}$$

5.72



$\alpha \cong 1$
$V_{BE} = 0.7V$

Under normal conduction & $V_B = 0V$

$V_E = 0 - 0.7 = \underline{-0.7V}$

$I_E = 1 - \frac{0.7}{1} = 0.3mA \cong I_C$

$V_C = (5 - 0.3)\,1 = \underline{\underline{4.7V}}$

At cutoff ~ all of the current in the 1mA source is supplied through R_E
~ $V_{BE} = 0.5V$

$\therefore V_E = -1 \times 1 = -1V$ & $I_E = I_C = 0$

$\therefore V_B = V_E + 0.5 = -1 + 0.5$
$= \underline{\underline{-0.5V}}$

- For Saturation $V_{CE} = 0.2V$, $V_{CB} = -0.5V$

$V_C = V_E + 0.2$

$I_C = 5 - V_C/1 = 5 - \left(\frac{V_E - 0.2}{1}\right)$

$= 5.2 - V_E$

$I_E = \frac{V_E}{1} + 1$

At the edge of saturation $I_E \cong I_C$ still!

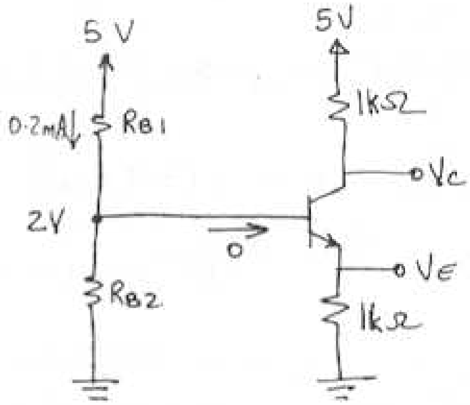
$\therefore 5.2 - V_E = V_E + 1$

$V_E = 4.2/2 = \underline{\underline{2.1\ V}}$

$V_B = V_E + 0.7 = \underline{\underline{2.8V}}$

$V_C = V_E + 0.2 = \underline{\underline{2.3V}}$

5.73



For $\beta = \infty$

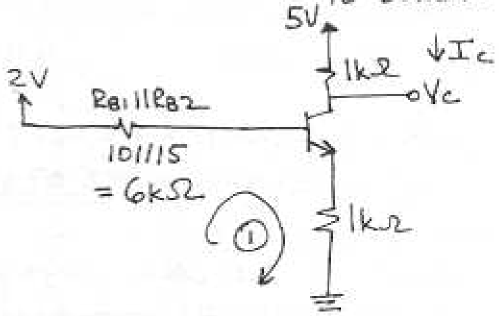
$\frac{5}{R_{B1} + R_{B2}} = 0.2$ & $\frac{R_{B2}}{R_{B1} + R_{B2}} 5 = 2$

$R_{B1} + R_{B2} = 25k\Omega$ sub

$\Rightarrow \frac{R_{B2}}{25} \times 5 = 2$

$R_{B2} = \underline{\underline{10k\Omega}}$ $R_{B1} = \underline{\underline{15k\Omega}}$

Now for $\beta = 100$, Use Thevenin's to obtain:



Loop ①: $2 - 6\left(\frac{I_E}{\beta + 1}\right) - 0.7 - I_E(1) = 0$ ($\beta + 1 = 101$)

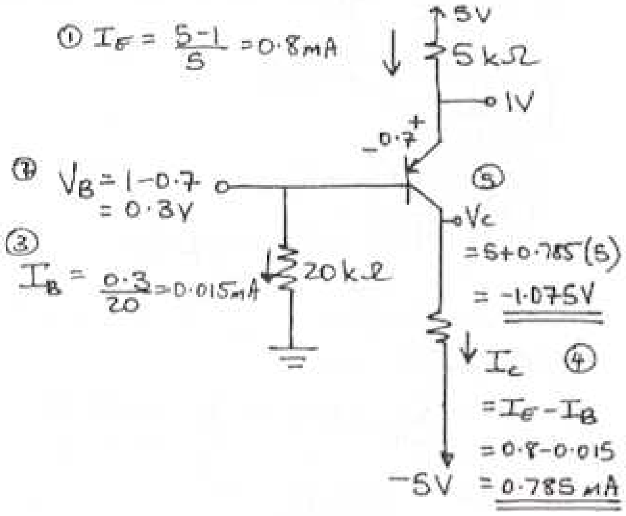
$I_E = 1.29mA$

CONT.

$I_C = \frac{100}{101} I_E = \frac{100}{101} \times 1.29 = \underline{\underline{1.28\,mA}}$

$V_C = 5 - 1.28(1) = \underline{\underline{3.72\,V}}$

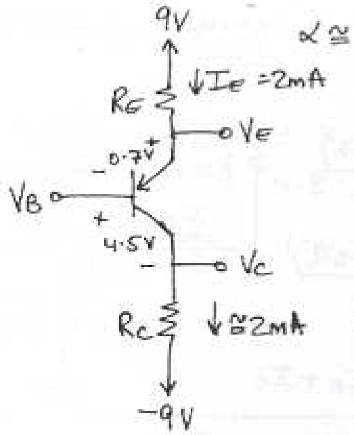
5.74



⑥ $\beta = \frac{I_C}{I_B} = \frac{0.785}{0.015} = \underline{\underline{52.3}}$

⑦ $\alpha = \frac{I_C}{I_E} = \frac{0.785}{0.8} = \underline{\underline{0.98}}$

5.75



$\alpha \cong 1$

$9 - 2R_E - 0.7 - 4.5 - 2R_C + 9 = 0$

$6.4 - R_E - R_C = 0$

LET $V_B = 0V$

$R_E = \frac{9-0.7}{2} = \underline{\underline{4.15\,k\Omega}}$

$R_C = \frac{-4.5+9}{2} = \underline{\underline{2.25\,k\Omega}}$

Using 5% resistor values

$R_E = 3.9\,k\Omega \quad R_C = 2.2\,k\Omega$

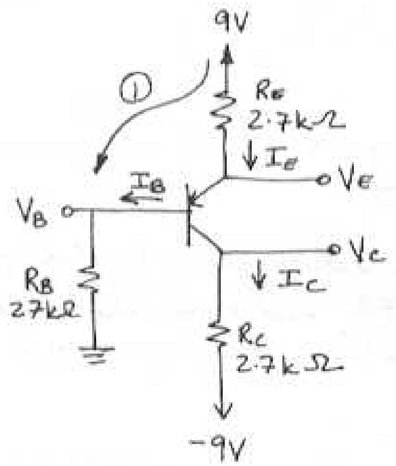
$I_E = \frac{9-0.7}{3.9} = \underline{\underline{2.12\,mA}}$

$V_C = -9 + 2.12 \times 2.2 = -4.3V$

$\therefore V_{BC} = \underline{\underline{4.3V}}$

5.76

$\beta = 30$



Loop ① $9 - 2.7I_E - 0.7 - \frac{I_E}{31} R_B = 0$

$I_E = 2.3243\,mA$

$V_B = R_B \times I_E/31 = \underline{\underline{2.02V}}$

$V_E = 9 - 2.7I_E = \underline{\underline{2.72V}}$

$V_C = -9 + \frac{30}{31} I_E(2.7) = \underline{\underline{-2.93V}}$

FOR $R_B = 270\,k\Omega$

Loop ① $9 - 2.7I_E - 0.7 - \frac{R_B}{31} I_E = 0$

$I_E = 0.7274\,mA$

CONT.

$V_B = R_B \times \frac{I_E}{31} = \underline{\underline{6.34V}}$

$V_E = 9 - 2.7I_E = \underline{\underline{7.04V}}$

$V_C = \frac{30}{31} I_E(2.7) - 9 = \underline{\underline{-7.10V}}$

To return the voltages to the ones first calculated we have

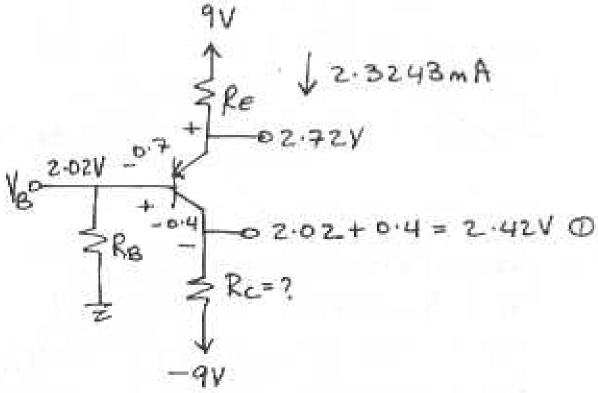
Loop ① $\sim I_E = 2.3243 \text{ mA}$

$9 - 2.7I_E - 0.7 - \frac{270}{\beta+1} I_E = 0$

$\beta = \underline{\underline{309}}$

5.77

Using the values from the first part of P5.76 and for the edge of saturation $V_{BC} > -0.4V$



CIRCUIT AT THE EDGE OF SATURATION

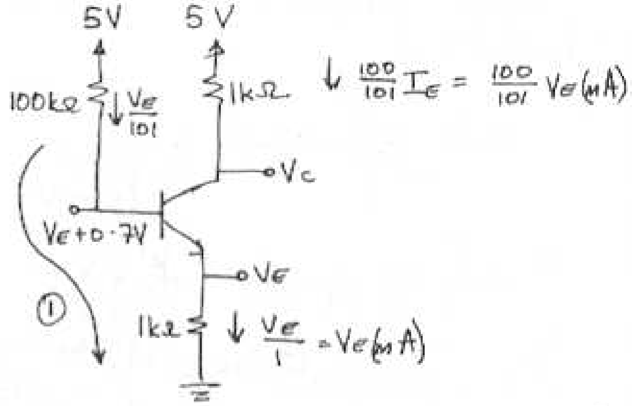
$I_C = \frac{30}{31} I_E = \frac{30}{31} \times 2.3243$

$R_C = \frac{2.42+9}{30/31 \times 2.3243} = \underline{\underline{5.08k\Omega}}$

5.78

$\beta = 100$

(a) $R_B = 100k\Omega$ — ∵ R_B is large assume active mode.



Loop ①

$5 - \frac{V_E}{101} \times 100 - 0.7 - V_E \times 1 = 0$

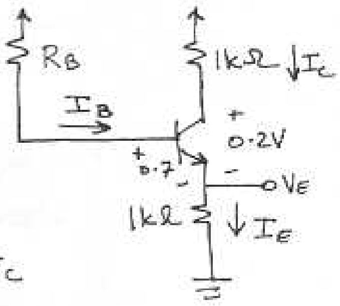
$V_E = \underline{\underline{2.16V}}$

$V_B = V_E + 0.7 = \underline{\underline{2.86V}}$

$V_C = 5 - 1 \times \frac{100}{101} V_E = \underline{\underline{2.86V}}$

Thus the BJT is in active mode as assumed.

(b) $R_B = 10k\Omega$ — assume saturation



$I_B = \frac{5-(V_E+0.7)}{R_B}$

$I_C = \frac{5-(V_E+0.2)}{1}$

$I_E = \frac{V_E}{1} = I_B + I_C$

$\therefore V_E = \frac{4.3-V_E}{10} + 4.8 - V_E$

$10V_E + V_E + 10V_E = 4.3 + 48$

CONT

$V_E = \underline{2.49V}$

$V_C = 2.49 + 0.2 = \underline{2.69V}$

$V_B = V_E + 0.7 = \underline{3.19V}$

Check : $I_C = \frac{5-2.69}{1} = 2.31mA$

$I_B = \frac{5-3.19}{10} = 0.181mA$

$\frac{I_C}{I_B} = \frac{2.31}{0.181} = 12.76 < 100$

Hence we are in saturation as assumed!

(C) $\underline{R_B = 1k\Omega}$ - expect saturation - use circuit in (b)

$I_B = \frac{5-(V_E+0.7)}{R_B} = \frac{4.3-V_E}{1}$

$I_C = \frac{5-(V_E+0.2)}{1} = \frac{4.8-V_E}{1}$

$I_E = I_B + I_C = V_E$

$4.3 - V_E + 4.8 - V_E = V_E$

$V_E = \underline{3V}$

$V_B = \underline{3.7V}$

$V_C = \underline{3.2V}$

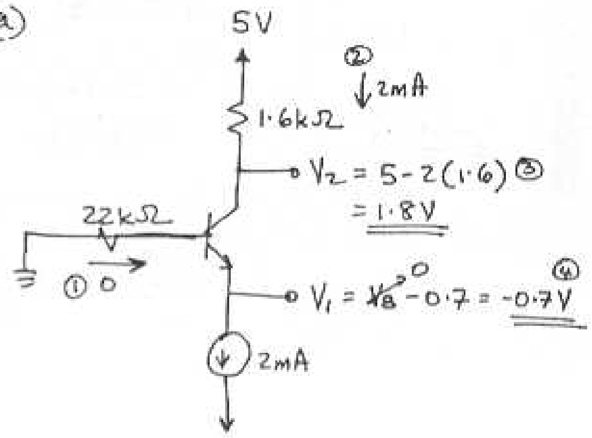
Check $I_B = 4.3 - 3 = 1.3mA$

$I_C = 4.8 - 3 = 1.8mA$

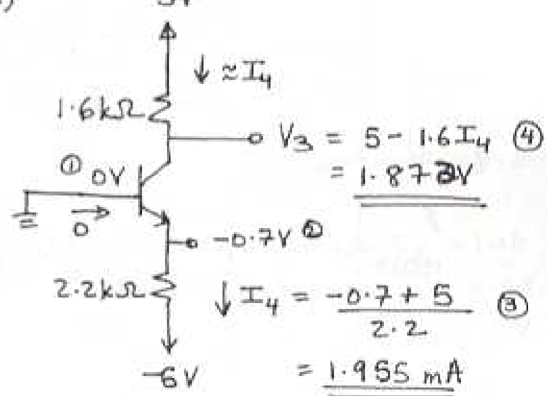
$\frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4 < 100$ ∴ SATURATION AS ASSUMED

5.79

(a)

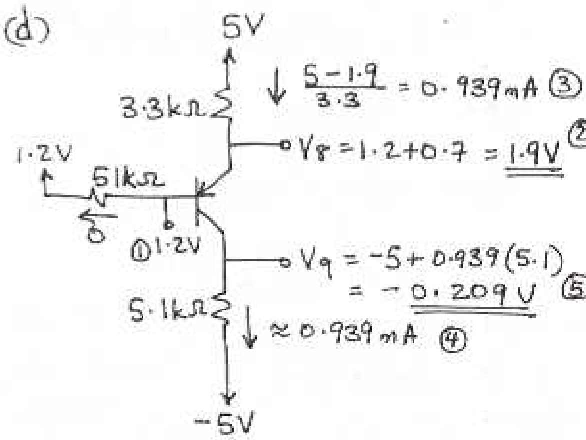


(b)



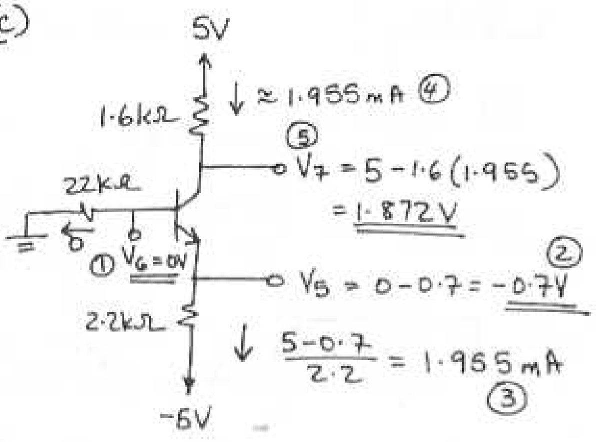
see below for part (c)

(d)

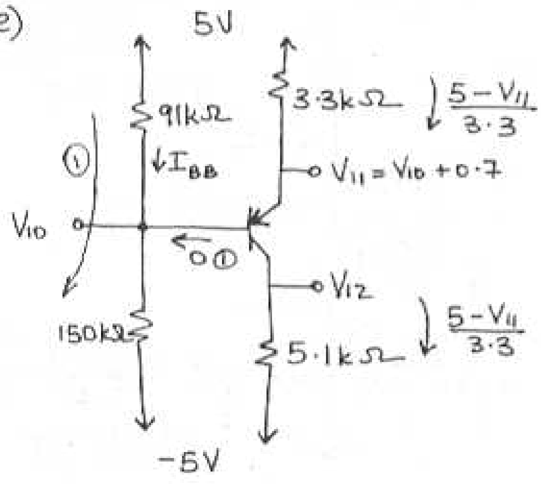


CONT.

(c)



(e)



Loop ①

$$5 - 91I_{BB} - 150I_{BB} + 5 = 0$$

$$I_{BB} = \frac{10}{91+150}$$

$$V_{10} = -5 + 150I_{BB}$$

$$= -5 + \frac{150}{91+150} \times 10$$

$$= \underline{\underline{1.224V}}$$

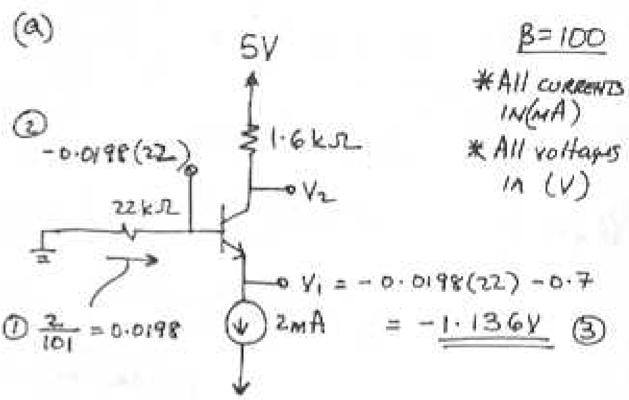
$$V_{11} = V_{10} + 0.7 = \underline{\underline{1.924V}}$$

$$\therefore I_C \approx I_E = \frac{5-V_{11}}{3.3}$$

$$V_{12} = -5 + \left(\frac{5-V_{11}}{3.3}\right)5.1 = \underline{\underline{-0.246V}}$$

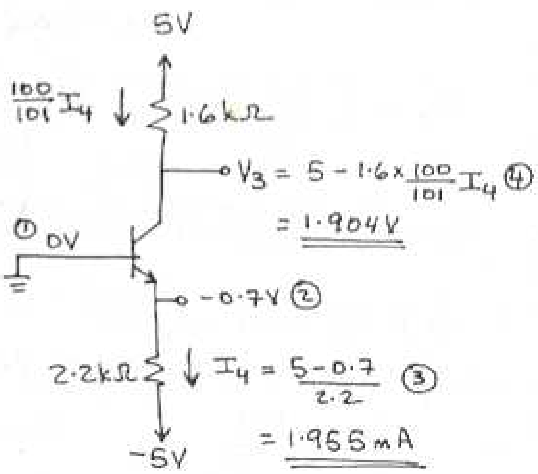
5.80

(a)

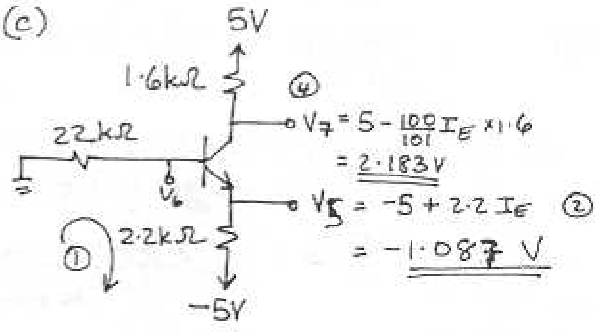


④ $V_2 = 5 - 2\left(\frac{100}{101}\right)1.6 = \underline{\underline{1.832V}}$

(b)



(c)



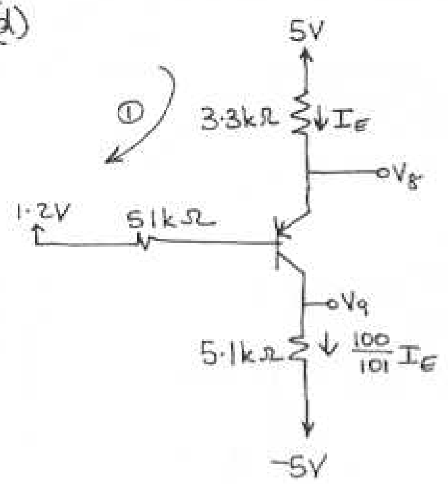
Loop ① $0 - \frac{I_E}{101}22 - 0.7 - 2.2I_E + 5 = 0$

$I_E = 1.778\,mA$

③ $V_6 = V_5 + 0.7 = \underline{\underline{-0.387\ V}}$

CONT

(d)



Loop ①

$$5 - 3.3I_E - 0.7 - \frac{I_E}{101} 51 - 1.2 = 0$$

$$I_E = 0.8147 \text{ mA}$$

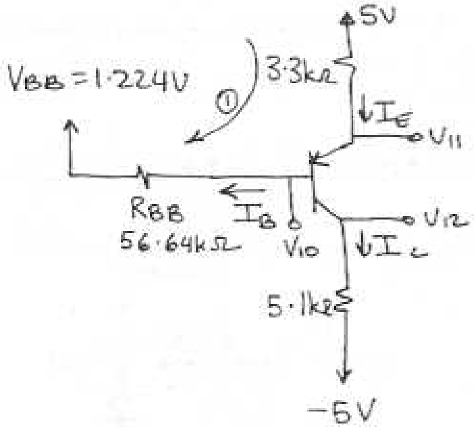
$$V_8 = 5 - 3.3 I_E = \underline{\underline{2.3114 \text{ V}}}$$

$$V_9 = -5 + 5.1 \times \frac{100}{101} I_E = \underline{\underline{-0.8862 \text{V}}}$$

(e) Use Thévenin's theorem to simplify the bias network:

$$V_{BB} = -5 + \frac{150}{150+91} \times 10 = 1.224 \text{ V}$$

$$R_{BB} = 150 \| 91 = 56.64 \text{ k}\Omega$$



Loop ①

$$5 - 3.3I_E - 0.7 - \frac{I_E}{101} R_{BB} - 1.224 = 0$$

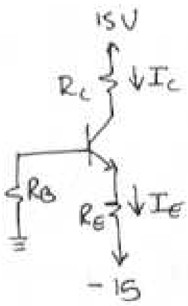
$$I_E = 0.7967 \text{ mA}$$

$$V_{11} = 5 - 3.3I_E = \underline{\underline{2.371 \text{V}}}$$

$$V_{12} = \frac{100}{101} I_E \times 5.1 - 5 = \underline{\underline{-0.977 \text{V}}}$$

$$V_{10} = V_{11} - 0.7 = \underline{\underline{1.67 \text{V}}}$$

5.81



Nominal $\beta = 100$.

Thus,

nominal $\alpha = \frac{100}{101} = 0.99$

nominal I_E = 1mA

nominal I_C = 0.99mA

nominal V_C = 5V

Thus, $R_C = \frac{15-5}{0.99} = 10.1\text{k}\Omega \overset{\text{use}}{\Longrightarrow} \underline{10\text{k}\Omega}$

$$I_E = 1 = \frac{15 - 0.7}{R_E + \frac{R_B}{\beta+1}}$$

$$= \frac{14.3}{R_E + \frac{R_B}{101}}$$

$$\Rightarrow R_E + \frac{R_B}{101} = 14.3 \quad (1)$$

As β varies from 50 to 150, need to limit the variation of I_E to ±10% of 1mA. One can reason that the maximum variation in I_E occurs for $\beta = 50$ (as opposed to $\beta = 150$). To see this more that when β decreases from 100 to 50 the base current doubles while a change in β from

CONT.

100 to 150 causes the base current to decrease to 2/3 its nominal value. Thus our decision will be based on imposing the 10% limit for $\beta = 50$.

$$0.9 = \frac{14.3}{R_E + \frac{R_B}{\beta+1}} = \frac{14.3}{R_E + \frac{R_B}{51}}$$

$$R_E + \frac{R_B}{51} = 15.89 \quad (2)$$

$$(2)-(1) \Rightarrow \quad R_B\left(\frac{1}{51} - \frac{1}{101}\right) = 1.59$$

$$\Rightarrow R_B = 163.8\,k\Omega \overset{\text{use}}{\Longrightarrow} \underline{\underline{164\,k\Omega}}$$

Sub into (1) gives

$$R_E = 12.7\,k\Omega \overset{\text{use}}{\Longrightarrow} \underline{\underline{13\,k\Omega}}$$

To find the expected range of I_C & V_C corresponding to β variation from 50 to 150 we use

$$I_C = \alpha \frac{14.3}{R_E + \frac{R_B}{\beta+1}}$$

$$\text{for } \beta = 50 \quad I_C = \frac{50}{51}\,\frac{14.3}{13 + \frac{164}{51}} = \underline{\underline{0.864\,mA}}$$

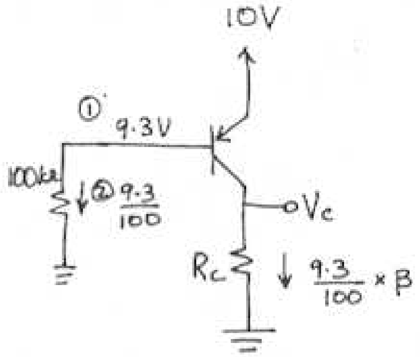
$$V_C = 15 - 0.864 \times 10 = \underline{\underline{6.36\,V}}$$

$$\text{for } \beta = 150 \quad I_C = \frac{150}{151} \times \frac{14.3}{13 + \frac{164}{151}}$$

$$= \underline{\underline{1.008\,mA}}$$

$$V_C = 15 - 1.008 \times 10 = \underline{\underline{4.92\,V}}$$

5.82



$$\text{For } V_C = 5\,V = \frac{9.3}{100} \times \beta \times R_C \qquad \beta = 50$$

$$R_C = \frac{500}{9.3 \times 50} = \underline{\underline{1.08\,k\Omega}}$$

For $\beta = 100$

$$V_C = \frac{9.3}{100} \times \beta \times R_C = \frac{9.3}{100} \times 100 \times 1.08$$

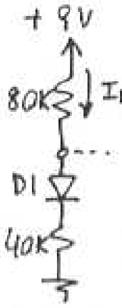
$$= \underline{\underline{10.04\,V}} \longleftarrow V_{BC} = 9.3 - 10.04 = -0.74$$

Since $V_{BC} < -0.4\,V$ the transistor $\underline{\underline{\text{saturates}}}$!

5.83

For $\beta = \infty$ and R open:

$i_{B1} = i_{B2} = 0$, $\alpha_1 = \alpha_2 = 1$



$I_{D1} = \dfrac{9-0.7}{80K+40K} = 0.07mA$

$V_{B1} = 0.7 + 0.07m \times 40K = \underline{3.5V}$

$V_{E1} = V_{B1} - 0.7V = 3.5 - 0.7 = \underline{2.8V}$

$I_{E1} = \dfrac{2.8V}{2K} = 1.4mA$

$I_{E1} = I_{C1}$ since $\alpha = 1$

$V_{C1} = 9V - 2K \times 1.4m - 0.7 = \underline{5.5V}$

$V_{CB} = 2V \rightarrow$ Transistor is in active mode.

$V_{B2} = V_{C1} = \underline{5.5V}$

$V_{E2} = 5.5 + 0.7 = \underline{6.2V}$

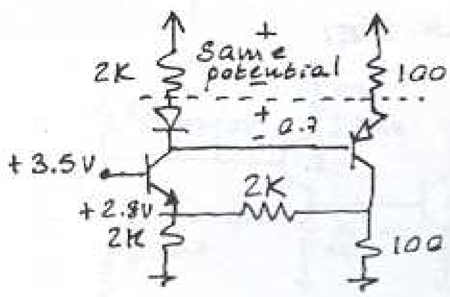
$I_{E2} = \dfrac{9-6.2}{100} = 28mA$

$I_{E2} = I_{C2}$ since $\alpha = 1$

$\rightarrow V_{C2} = 28mA \times 100\Omega = \underline{2.8V}$

For $\beta = \infty$ and R connected:

Still: $V_{B1} = 3.5V$, $V_{E1} = \underline{2.8V}$

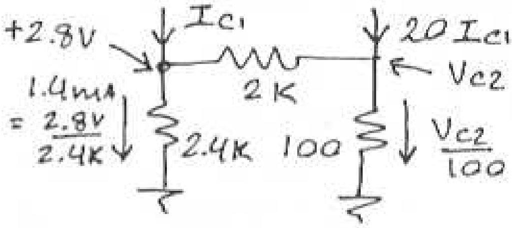


Since the voltage across the top two resistors is equal:

$I_{C1} \times 2K = I_{E2} \times 100$

$I_{E2} = 20 I_{C1}$

also: $I_{C1} = I_{E1}$, $I_{C2} = I_{E2}$



$1.4m - I_{C1} = 20 I_{C1} - \dfrac{V_{C2}}{100}$

$\rightarrow V_{C2} = 100 \times (21 . I_{C1} - 1.4m)$ (1)

also:

$\dfrac{V_{C2} - 2.8V}{2K\Omega} = 1.4mA - I_{C1}$

$\rightarrow V_{C2} = 5.6 - I_{C1} \times 2K$ (2)

Solving for I_{C1} from (1) & (2)

$I_{C1} = 1.4mA$

Substituting in either (1) v (2)

$V_{C2} = \underline{2.8V}$

and: $V_{E2} = 9 - 100 \times 1.4m = \underline{8.86V}$

$V_{B2} = V_{C1} = 8.86 - 0.7 = \underline{8.16V}$

For $\beta = 100$ and R open:

In the previous two cases $I_{D1} = 0.07mA$, $I_{E1} = 1.4mA$

if $\beta = 100 \rightarrow I_{B1} \simeq 0.014mA$ which is a significant amount compared to 0.07m

$\rightarrow$ must be taken into account

The bottom two resistors have equal voltage drops thus,

CONT.

$2K \times I_{E1} = 40K \times I_{D1}$

$\rightarrow I_{D1} = 0.05 \times I_{E1}$ (3)

also: $\underbrace{I_{D1} + I_{B1}}_{\text{FROM (3)}} = \frac{9 - V_{B1}}{80K}$

$I_{E1}\left(\frac{1}{\beta+1} + 0.05\right)$

for $\beta = 100$:

$0.06 \times I_{E1} = \frac{9 - V_{B1}}{80K}$

$\rightarrow V_{B1} = 9 - 4800 \times I_{E1}$ (4)

also: $V_{B1} = 0.7 + I_{E1} \times 2K$ (5)

From (4) ∧ (5) $\begin{cases} V_{B1} = 3.14V \\ I_{E1} = 1.22mA \end{cases}$

$V_{E1} = 1.22m \times 2K \rightarrow V_{E1} = \underline{2.44V}$

$I_{C1} = \alpha I_E = 0.99 \times 1.22m$
$= 1.2mA$

Again: voltage drop on top two resistors is equal

$2K \cdot I_{D2} = 100 \cdot I_{E2}$

$I_{D2} = 0.05\, I_{E2}$

but $I_{D2} = 1.2m - \frac{I_{E2}}{\beta+1}$

$\Rightarrow 1.2mA = \underbrace{\left(0.05 + \frac{1}{\beta+1}\right)}_{0.06} I_{E2}$

$I_{E2} = 20mA$

$V_{E2} = 9 - 100 \times 20m = \underline{7V}$

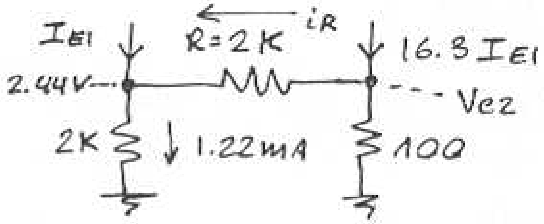
$V_{B2} = V_{C1} = 7 - 0.7 = \underline{6.3V}$

$I_{C1} = \alpha I_{E1} = 19.8mA$

$V_{C2} = 100 \times 19.8mA = \underline{1.98V}$

<u>For $\beta = 100$ and R connected:</u>

To simplify the solution: assume i on R is $\ll I_{E1} \rightarrow V_{E1} = 2.44V$



From top of circuit:

$I_{E2} = I_{C1}/0.06$

$I_{C2} = \frac{\alpha^2}{0.06} \cdot I_{E1}$

$I_{C2} = 16.3 \times I_{E1}$

To obtain I_{E1}:

$1.22m - I_{E1} = 16.3 I_{E1} - \frac{V_{C2}}{100}$

$V_{C2} = 100\,(17.2 I_{E1} - 1.22m)$ (6)

also:

$\frac{V_{C2} - 2.44}{2K} = 1.22m - I_{E1}$

$\rightarrow V_{C2} = 2.44 - 2000 I_{E1} + 2.44$
$= 4.88 - 2 \times 10^3 \cdot I_{E1}$ (7)

From (7) ∧ (8): $I_{E1} = 1.34mA$
$V_{C2} = \underline{2.18V}$

thus,

$I_{C1} = 1.33\, mA$

$I_{E2} = 22.1\, mA$

$V_{E2} = 9 - 22.1m \times 100 = \underline{6.79V}$

$V_{B2} = 6.79 - 0.7 = \underline{6.09V}$

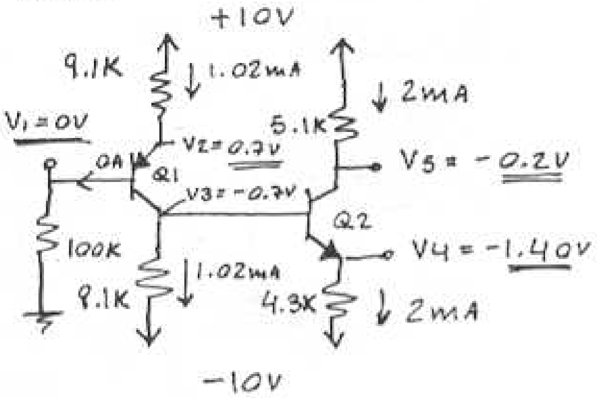
To confirm initial assumption on i of R:

$\frac{2.44 - 2.18}{2K} = 0.13mA$

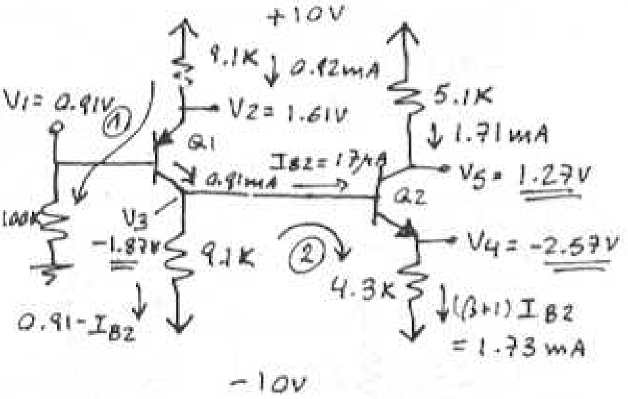
which is 10 times smaller than I_{E1}

5.84

(a) $\beta = \infty$



(b) $\beta = 100$



① $10 - 9.1(\beta+1)I_{B1} - 0.7 - 100 I_{B1} = 0$

$\rightarrow I_{B1} = 0.009 mA$

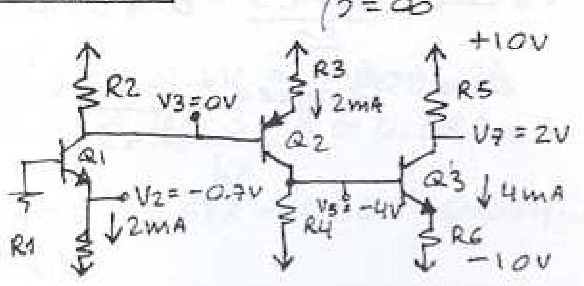
② $(0.91 - I_{B2}) \times 9.1 = 0.7 + (\beta+1) I_{B2} \times 4.3$

$\rightarrow I_{B2} = 0.017 mA$

$\rightarrow I_{E2} = 1.73 mA$

5.85

$\beta = \infty$



$$R_1 = \frac{9.3}{2} = \underline{\underline{4.7K\Omega}}$$

$$R_2 = \frac{10}{2} = 5 \rightarrow \underline{\underline{5.1K\Omega}}$$

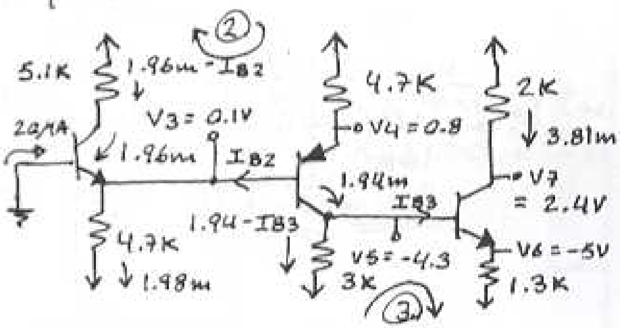
$$R_3 = \frac{9.3}{2} = \underline{\underline{4.7K\Omega}}$$

$$R_4 = \frac{6}{2} = \underline{\underline{3K\Omega}}$$

$$R_5 = \frac{8}{4} = \underline{\underline{2K\Omega}}$$

$$R_6 = \frac{10 - 4.7}{4} = \underline{\underline{1.3K\Omega}}$$

$\beta = 100$



② $(1.96 - I_{B2}) \times 5.1 = (\beta+1) I_{B2} \times 4.7 + 0.7$

$I_{B2} = 0.0194 mA$

$I_{E2} = 1.96 mA$

$V_3 = \underline{\underline{0.1V}}$ $\quad V_4 = \underline{\underline{0.8V}}$

③ $(1.94 - I_{B3}) \times 3 = 0.7 + 1.3 \times (\beta+1) \cdot I_{B3}$

$I_{B3} = 0.038 mA$

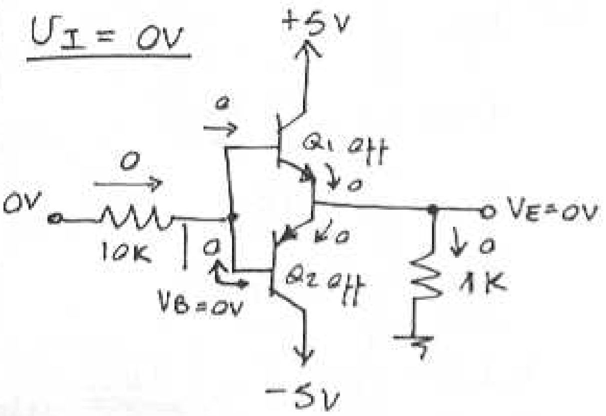
$I_{E3} = 3.85 mA$

$V_5 = \underline{\underline{-4.3V}}$ $\quad V_6 = \underline{\underline{-5V}}$

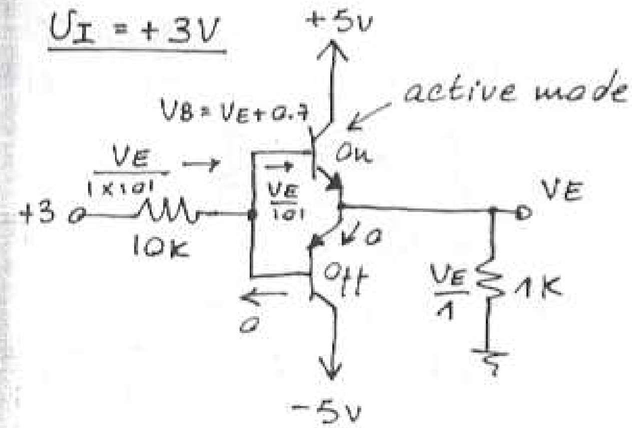
$V_7 = \underline{\underline{2.4V}}$

5.86

$v_I = 0V$



$v_I = +3V$

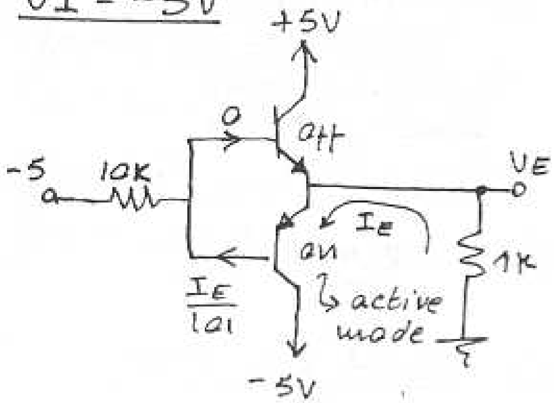


$$3 = \frac{V_E}{101} \times 10 + 0.7 + V_E$$

$$\Rightarrow V_E = \underline{\underline{2.09V}}$$

$$V_B = \underline{\underline{2.79V}}$$

$v_I = -5V$

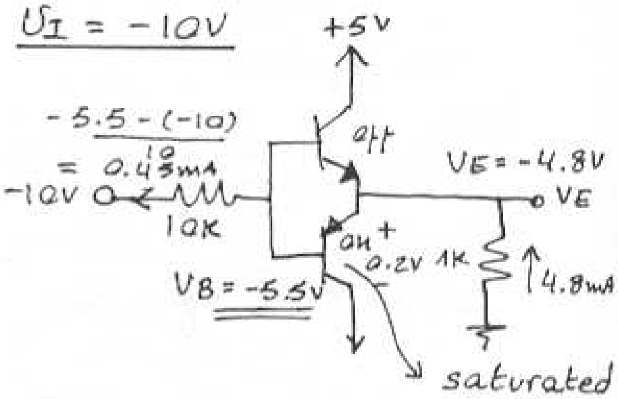


$$I_E = \frac{5-0.7}{1+10/101} = 3.91mA$$

$$V_E = -3.91V$$

$$V_B = \underline{\underline{-4.61V}}$$

$v_I = -10V$



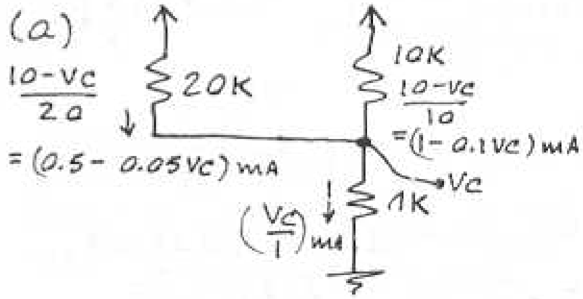
$$\frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7 < 100$$

thus, Q2 is saturated as assumed

$$V_E = \underline{\underline{-4.8V}} \quad V_B = \underline{\underline{-5.5V}}$$

5.87

(a)



$$(0.5 - 0.005V_C) + (1 - 0.1V_C) = V_C$$

$$V_C = \underline{\underline{1.3V}}$$

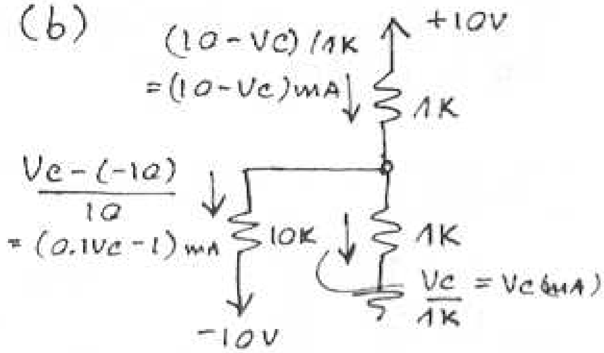
$$I_C = \frac{10-1.3}{10} = 0.87mA$$

$$I_B = \frac{10-1.3}{20} = 0.435mA$$

thus $\beta_{forced} = \frac{0.87}{0.435} = \underline{\underline{2}}$

CONT.

(b)



$$10 - V_C = (0.1V_C + 1) + (V_C)$$

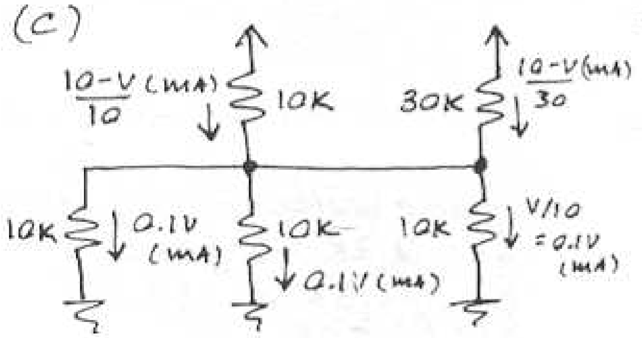
$$\Rightarrow V_C = +4.29\,V$$

$I_C = 4.29\,mA$

$I_B = \dfrac{4.29+10}{10} = 1.43\,mA$

$$\beta_{forced} = \frac{4.29}{1.43} = \underline{\underline{3}}$$

(c)



Node equation:

$$\frac{10-V}{10} + \frac{10-V}{30} = 0.1V + 0.1V + 0.1V$$

$$30 - 3V + 10 - V = 9V$$

$$40 = 13V$$

$$\Rightarrow V = \underline{\underline{3.08\,V}}$$

thus, $V_{C3} \simeq V_{C4} \simeq 3.08V$

$I_{B3} = 0.1V = 0.308\,mA$

$I_{E3} = \dfrac{10-3.08}{10} \simeq 0.692\,mA$

$I_{C3} = 0.692 - 0.308 = 0.384\,mA$

$$\beta_{3\,forced} = \frac{0.384}{0.308} = \underline{\underline{1.25}}$$

$I_{C4} = \dfrac{10-3.08}{30} = 0.231\,mA$

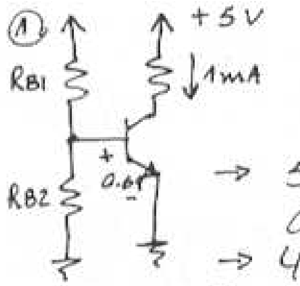
$I_{E4} = 0.1V = 0.308\,mA$

$I_{B4} = 0.308 - 0.231 = 0.077\,mA$

$$\beta_{4\,forced} = \frac{0.231}{0.077} = \underline{\underline{3}}$$

5.88

①



$$\frac{5 \times R_{B2}}{R_{B1}+R_{B2}} = 0.690$$

$\rightarrow 5R_{B2} = 0.69R_{B1} + 0.69R_{B2}$

$\rightarrow 4.31R_{B2} = 0.69R_{B1}$

$$\Rightarrow \frac{R_{B1}}{R_{B2}} = \underline{\underline{6.24}}$$

② Since $V_{BE} = \dfrac{5R_{B2}}{R_{B1}+R_{B2}}$

if both R_{B2} & R_{B1} are at 0.99 or 1.01 of their nominal value → V_{BE} will not be affected.

We must consider the cases when one resistor is at 0.99 and the other at 1.01 of their nominal value.

If: $R_{B2}' = 1.01\,R_{B2}$
$R_{B1}' = 0.99\,R_{B1}$

$\Rightarrow V_{BE} = 0.702V$

If: $R_{B2}' = 0.99\,R_{B2}$
$R_{B1}' = 1.01\,R_{B1}$

$\Rightarrow V_{BE} = 0.678V$

thus V_{BE} ranges from 0.678V to 0.702V

CONT.

For Ic: $I_c = I_s e^{V_{be}/V_T}$

for $V_{be} = 0.690 \rightarrow I_c = 1mA$

$\Rightarrow I_s = 1.032 \times 10^{-15}$

for $V_{BE} = 0.678 \rightarrow I_c = 0.618m$

$V_{BE} = 0.702 \rightarrow I_c = 1.62m$

Ic ranges from 0.618 mA to 1.62 mA.

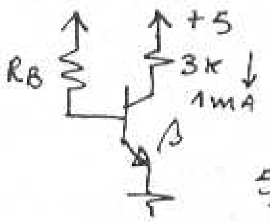
(3) If $R_C = 3K\Omega$

$V_{CE} = 5 - 3K \times 0.62m = 3.14V$

$V_{CE} = 5 - 3K \times 1.62m = 0.14V$

This circuit is too sensitive to parameter variations as shown here for a 1% resistor tolerance.

5.89



$R_B = ?$ if $\beta = 100$

$I_B \times \beta = I_C$

$$\frac{5-0.7}{R_B} = \frac{1m}{100}$$

$\rightarrow R_B = \underline{\underline{430K\Omega}}$

$V_{CE} = 5V - 3K \cdot 1mA = 2V$

If $\beta = 50$: $I_C = \frac{5-0.7}{430K} \times 50$

$I_C = 0.50mA$

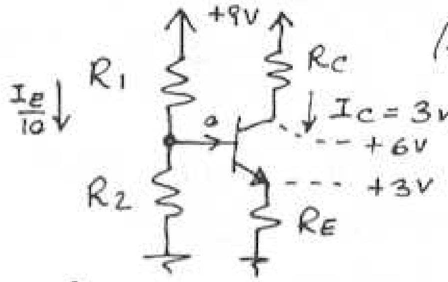
$\Rightarrow V_{CE} = 5 - 3K \times 0.5m = +3.5V$

If $\beta = 150$: $I_C = 1.5mA$

$V_{CE} = 0.5V$

This design is too sensitive to variations of β.

5.90



$\beta = \infty$

$$R_C = \frac{3V}{3mA} = 1K\Omega$$

$$R_E = \frac{3V}{3mA} = 1K\Omega$$

$V_b = 0.7 + 3 = 3.7V$

$$R_1 = \frac{9 - 3.7}{I_E/10} = 17.7K\Omega$$

$9V = (R_1 + R_2)\frac{I_E}{10} \rightarrow R_2 = 12.3K$

Choose suitable 5% resistors

$R_1 = 17.7K \rightarrow 18K\Omega$

$R_2 = 12.3K \rightarrow 13K\Omega$

$R_1 = R_2 = 1K$

$$V_{BB} = \frac{9 \times 13}{18+13} = 3.77V$$

For these values of R and $\beta = 90$: $R_B = 18 // 13 = 7.55K\Omega$

$$I_E = \frac{3.77 - 0.7}{1K + \frac{7.55K}{91}} = 2.83mA$$

$\alpha = 0.989 \Rightarrow I_C = 2.80mA$

If R_E is reduced by $\sim \frac{7.55K}{91}$

$\rightarrow R_E = 910\Omega$

$\Rightarrow I_E = 3.09mA$

$I_C = 3.05mA$.

5.91

For $\beta = \infty$ $\quad I_B = 0, I_E = 3mA$

$$R_1 = \frac{9 - 3.7}{I_E/2} = 3.5K\Omega$$

$$9V = (R_1 + R_2)\frac{I_E}{2} \Rightarrow R_2 = 2.5K\Omega$$

Suitable 5% resistors:

$R_1 = 3.5K \rightarrow 3.3K\Omega$
$R_2 = 2.5K \rightarrow 2.4K\Omega$
$R_E = R_C = 1K\Omega$

$$V_{BB} = \frac{9 \times 2.4}{3.3 + 2.4} = 3.79V$$

For $\beta = 90$:

$$R_B = (3.3 \| 2.4)K = 1.39K\Omega$$

$$I_E = \frac{3.79 - 0.7}{1K + \frac{1.39K}{91}} = 3.04mA$$

No need to adjust R_E. As the current from the voltage divider increases the effect of I_B is reduced.

5.92

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

(a) For $\beta = 100$, varying between 50 and 150 the maximum deviation in I_E (from the nominal value obtained for $\beta = 100$) occurs at the low end of β values ($\beta = 50$). Thus, to keep I_E within $\pm 5\%$ of nominal we must impose the constraint

$$I_E(\beta = 50) \geq 0.95 I_E(\beta = 100)$$

or, $$\frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{51}} \geq 0.95 \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}}$$

or, $$R_E + \frac{R_B}{101} \geq 0.95\left(R_E + \frac{R_B}{51}\right)$$

$$0.05 R_E \geq R_B\left(\frac{0.95}{51} - \frac{1}{101}\right)$$

$$\Rightarrow \frac{R_B}{R_E} \leq 5.73$$

Thus, the largest ratio of R_B/R_E is 5.73

(b) $I_E \cdot R_E = V_{CC}/3$

$$\rightarrow \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}} \cdot R_E = \frac{V_{CC}}{3}$$

$$\frac{V_{BB} - 0.7}{1 + \frac{R_B}{R_E} \cdot \frac{1}{\beta + 1}} = \frac{V_{CC}}{3}$$

$$V_{BB} = \frac{1}{3}V_{CC}\left(1 + \frac{5.73}{101}\right) + 0.7$$

$$\Rightarrow \underline{V_{BB} = 0.35\, V_{CC} + 0.7}$$

(c) $V_{CC} = 10V$

$$V_{BB} = 0.35 \times 10 + 0.7 = 4.2V$$

$$\rightarrow \frac{R_2}{R_1 + R_2} \times 10 = 4.2$$

$$\frac{R_2}{R_1 + R_2} = 0.42 \quad (1)$$

$$I_E \cdot R_E = \frac{1}{3}V_{CC}$$

CONT.

$2 \times R_E = \frac{1}{3} \times 10$

$\Rightarrow R_E = \underline{\underline{1.67\,K\Omega}}$

$R_B = 5.73 \times 1.67 = 9.55\,K\Omega$

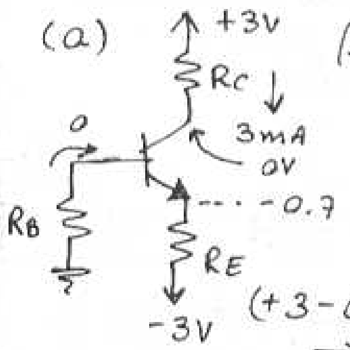
$\frac{R_1 . R_2}{R_1 + R_2} = 9.55$

Substituting from ① gives

$R_1 = \frac{9.55}{0.42} = \underline{\underline{22.7\,K\Omega}}$

5.93

(a)



$\beta = \infty, I_B = 0$

$I_E = I_C$

$3V = R_C \times 3mA$

$\rightarrow R_C = \underline{\underline{1K\Omega}}$

$(+3-0.7) = R_E \times 3mA$

$\rightarrow R_E = \underline{\underline{767\Omega}}$

(b) $\beta = 90 \quad \frac{V_{RE}}{10} = V_{RB}$

$I_B . R_B = \frac{I_E . R_E}{10}$

$\rightarrow \frac{I_E}{(\beta+1)} . R_B = \frac{I_E . R_E}{10}$

$\rightarrow R_B = \frac{(\beta+1)}{10} R_E$ ①

also, $0 = V_{RB} + 0.7 + V_{RE} - 3$

$2.3 = \frac{V_{RE}}{10} + V_{RE}$

$\rightarrow V_{RE} = \frac{2.3}{1.1} = 2.09V$

$2.09 = I_E \times R_E$ ②

but: $I_E = \frac{I_C}{\alpha} = \frac{3mA}{0.989} = 3.033m$

Substituting in ②:

$R_E = 689\Omega$

from ①:

$R_B = \underline{\underline{6269\Omega}}$

(c) Standard 5% values:

$R_C = 1K\Omega$

$R_E = 689\Omega \rightarrow 680\Omega$

$R_B = 6269\Omega \rightarrow 6.2K\Omega$

(d) $\beta = \infty: \quad I_B = 0$

$I_C = I_E$

$\underline{V_B = 0}$

$\underline{V_E = -0.7}$

$I_E = \frac{3-0.7}{R_E} = \frac{3-0.7}{680} = \underline{\underline{3.38mA}}$

$V_C = 3 - 3.38m \times 1K = \underline{\underline{-0.38V}}$

For $\beta = 90$:

$I_E = \frac{2.3}{680 + \frac{6.2K}{91}} = 3.07mA$

$I_C = \alpha I_E = \underline{\underline{3.04mA}}$

$V_B = \frac{R_B . I_E}{\beta+1} = -0.209$

$V_E = -0.209 - 0.7 = \underline{\underline{0.909}}\,V$

$V_C = 3 - I_C . R_C$

$= 3 - 3.04 \times 1 = \underline{\underline{-0.04V}}$

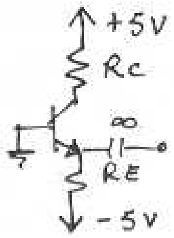
5.94

$V_E = -0.7V$

To obtain $I_E = 1mA$

$R_E = \frac{-0.7-(-5)}{1}$

$= 4.3\,K\Omega$



CONT.

To maximize gain while allowing ±1V signal at collector, design for a dc collector voltage of +1V.
Thus,

$$R_C = \frac{5-1}{I_C} \simeq \frac{4}{1} = \underline{\underline{4\,K\Omega}} \quad (\text{for } \alpha = 1)$$

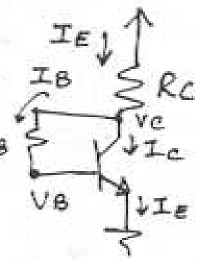
For 100°C rise in temperature, V_{BE} decreases by $2 \times 100 = 200\,mV$ and thus I_E increases by $\frac{0.2V}{R_E}$

$$= \frac{0.2V}{4.3K\Omega} = 0.047\,mA$$

i.e. an increase of $\underline{\underline{4.7\%}}$

The change in β from 50 to 150 causes α to change from 0.980 to 0.993 which implies an increase in collector current of $\underline{\underline{1.3\%}}$
Thus the overall increase in I_C is $\underline{\underline{6\%}}$

5.95



To allow a collector voltage swing of ±1V, we design for:

$$V_C = V_B + 1 = 0.7 + 1 = 1.7V$$

$I_E = 0.5\,mA$

$$\rightarrow R_C = \frac{5-1.7}{0.5} = \underline{\underline{6.6\,K\Omega}}$$

For $\beta = 100$:

$$I_B = \frac{I_E}{\beta+1} = \frac{0.5}{101} \simeq 5\,\mu A$$

$$I_B \cdot R_B = 1V$$

$$R_B = \frac{1V}{5\mu A} = \frac{1}{5}\,M\Omega = \underline{\underline{200\,K\Omega}}$$

Now, if the BJT used has $\beta = 50$, the emitter current resulting can be found from Eq (5.74)

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta+1}} = \frac{5 - 0.7}{6.6 + \frac{200}{51}} = \underline{\underline{0.41\,mA}}$$

and $I_B = \frac{0.41}{51} \simeq 8\,\mu A$

Thus the collector will be higher than the base by $8 \times 0.2 = 1.6V$, allowing for a $\underline{\underline{\pm 1.6V}}$ signal swing at the collector.

For $\beta = 150$:

$$I_E = \frac{5-0.7}{6.6 + \frac{200}{151}} = 0.54\,mA$$

$$I_B = \frac{0.54}{151} = 36\,\mu A$$

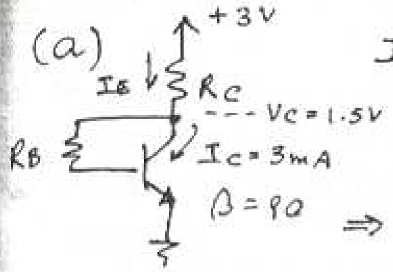
Thus the collector voltage will be higher than that of the base by

$$3.6 \times 0.2 \simeq 0.72V$$

allowing for only $\pm\, \underline{\underline{0.72V}}$ signal swing.

5.96

(a)



$I_E = \frac{1.5}{R_C} = \frac{I_C}{\alpha} \Rightarrow R_C = \frac{1.5\alpha}{3mA}$

$R_C = 495\Omega$

$1.5 = R_B I_B + 0.7$

$R_B = \frac{1.5 - 0.7}{I_C/\beta} = \underline{\underline{24K\Omega}}$

(b) Standard 5% values

$R_B = 24K\Omega$

$R_C = 495\Omega \rightarrow 510\Omega$

then, $I_E = \frac{3 - 0.7}{510 + \frac{24K}{91}} = 2.97mA$

$I_C = I_E \cdot \alpha = \underline{\underline{2.93mA}}$

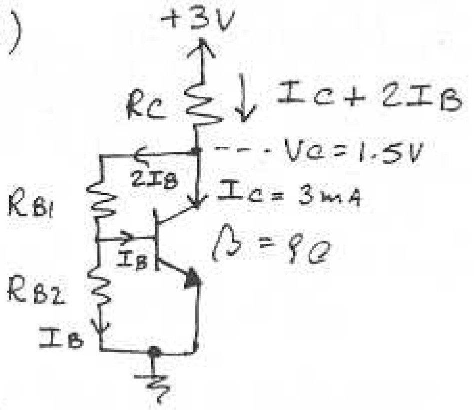
$V_C = 3 - 2.97m \times 510 = \underline{\underline{1.48V}}$

(c) $\beta = \infty \quad I_B = 0, V_B = 0.7V$

$I_C = I_E = \frac{3 - 0.7}{510} = \underline{\underline{4.5mA}}$

$V_C = 3 - 4.5m \times 510 = \underline{\underline{0.7V}}$

(d)



$I_B = \frac{I_C}{\beta} = \frac{3mA}{90} = 0.033mA$

$1.5V = 2I_B \times R_{B1} + 0.7$

$\rightarrow R_{B1} = 12.1K\Omega$

$0.7 = I_B \times R_{B2}$

$\rightarrow R_{B2} = 21.2K\Omega$

on R_C:

$I_C + 2I_B = 3.066mA$

$R_C = \frac{1.5V}{3.066mA} = 489\Omega$

Standard 5% values:

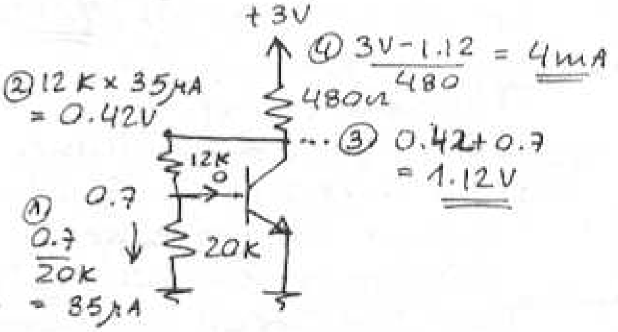
$R_{B1} = 12.1K \rightarrow 12K\Omega$

$R_{B2} = 21.2K \rightarrow 20K$

$R_C = 489.2 \rightarrow 480\Omega$

Re-evaluate if $\beta = \infty$:

$I_B = 0$



5.97

$I_B = I_C/\beta = 3mA/90 = 0.033m$

$V_C = R_B \cdot I_B + 0.7$

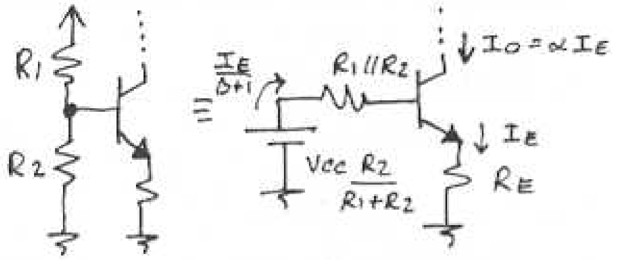
$V_C = 1.5V \rightarrow R_B = \underline{\underline{24.2K\Omega}}$

$I_E = \frac{I_C}{\alpha} = 3.03mA$

$I = I_C - I_B \equiv I_E$

$I = \underline{\underline{3.03mA}}$

5.98



$$V_{CC}\cdot\frac{R_2}{R_1+R_2} = \frac{I_E}{\beta+1}(R_1 /\!/ R_2)+V_{BE}+I_ER_E$$

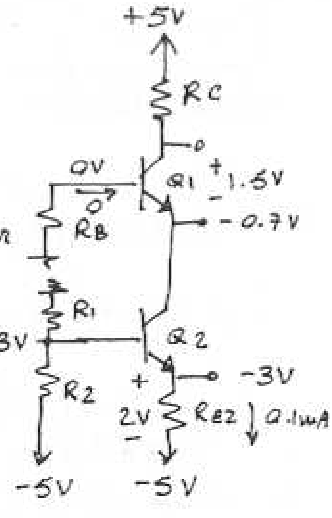
$$\Rightarrow I_E = \frac{V_{CC}\frac{R_2}{R_1+R_2} - V_{BE}}{R_E + \frac{R_1/\!/R_2}{\beta+1}}$$

Thus,

$$I_o = \alpha I_E = \alpha \cdot \frac{\left(V_{CC}\frac{R_2}{R_1+R_2} - V_{BE}\right)}{R_E + \frac{R_1/\!/R_2}{\beta+1}}$$

Q.E.D.

5.99



For $\beta = \infty$:

$R_{E2} = \frac{2}{0.1} = \underline{\underline{20\,K\Omega}}$

$R_C = \frac{5-0.8}{0.1} = 42\,K\Omega$

Select $R_C = \underline{\underline{43K}}$

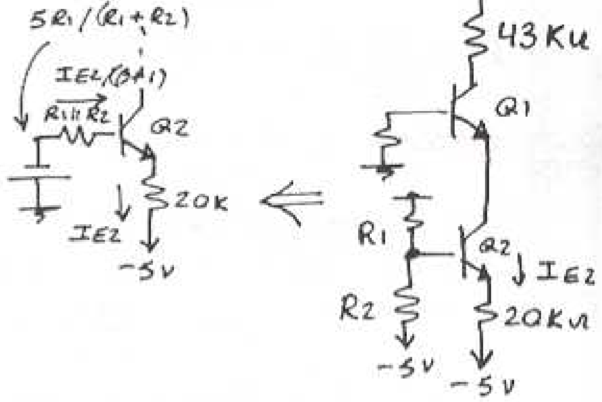
Also $V_{R1} = 2.3V$

$V_{R2} = 2.7V$

$\rightarrow \frac{R_1}{R_2} = \frac{2.3}{2.7}$

$\Rightarrow R_1 = \frac{23}{27}\cdot R_2$

For $\beta = 50$



$$I_{E2} = \frac{-5\cdot\frac{R_1}{R_1+R_2} - 0.7 + 5}{20 + \frac{R_1/\!/R_2}{\beta+1}}$$

$$= \frac{5\cdot\frac{R_1}{R_1+R_2} - 0.7}{20 + \frac{1}{51}\cdot\frac{R_1R_2}{R_1+R_2}}$$

Substitute $R_1 = \frac{23}{27}R_2$

$$I_{E2} = \frac{\frac{5}{1+23/27} - 0.7}{20 + \frac{1}{51}\cdot\frac{23/27\,R_2}{1+23/27}}$$

$$= \frac{2}{20\left[1 + \frac{1}{20\times 51}\times\frac{23R_2}{50}\right]}$$

$$V_{RE2} = I_{E2}\times 20 = \frac{2}{1 + \frac{0.46}{51\times 20}\cdot R_2}$$

For a reduction less than 5%

$$\frac{2}{1 + \frac{0.46\,R_2}{51\times 20}} \geq 0.95\times 2$$

$$R_2 \leq \left(\frac{1}{0.95} - 1\right)\frac{51\times 20}{0.46} = 116.7\,K\Omega$$

CONT.

Select $R_2 = \underline{\underline{120k\Omega}}$

$$R_1 = \frac{23}{27} R_2 = 102.2k\Omega$$

Select $R_1 = \underline{\underline{100k\Omega}}$

For these values,

$$\beta = \infty: I_{E2} = \frac{-5 \times \frac{100}{220} - 0.7 + 5}{20}$$

$$= \frac{2.027}{20} = 0.101mA$$

$V_{RE2} = 2.027V$

$$\beta = 50: I_{E2} = \frac{2.027}{20 + \frac{120 \times 100}{220 \times 51}}$$

$I_{E2} = 0.096mA$

$I_{C2} = 0.98 \times 0.096 = 0.094mA$

$I_{C1} = 0.98 \times 0.094 = 0.092mA$

To determine R_B:

$$V_{B1} = -I_{B1} . R_B = -\frac{I_{C1}}{\beta} . R_B$$

$$= -\frac{0.094}{50} . R_B$$

$$V_{E1} = V_{B1} - 0.7 = -\frac{0.094}{50} . R_B - 0.7$$

$$V_{C1} = 5 - R_C I_{C1} = 5 - 43 \times 0.092$$

$$= 1.044V$$

$$V_{CE1} = 1.044 + \frac{0.094}{50} . R_B + 0.7$$

$$= 2.5V$$

$\Rightarrow R_B = 402k\Omega$

Select $R_B = \underline{\underline{390k\Omega}}$

Now,

For $\beta = 50$: $I_{C1} = \underline{\underline{0.092mA}}$

$$V_{B1} = -\frac{0.092}{50} \times 390 = -0.717V$$

$V_{E1} = -1.417V$

$V_{CE1} = 1.044 + 1.417 = 2.46 \simeq \underline{\underline{2.5V}}$

For $\beta = 100$:

$$I_{E2} = \frac{2.027}{\frac{R_1 R_2}{R_1 + R_2} . \frac{1}{101} + 20} = 0.099mA$$

$I_{C1} = 0.099 \times 0.99 \times 0.99 = \underline{\underline{0.097mA}}$

$V_{C1} = 0.829V$

$V_{B1} = -0.378V$

$V_{E1} = -1.078V$

$V_{CE1} = \underline{\underline{1.91V}}$

For $\beta = 200$:

$$I_{E2} = \frac{2.027}{\frac{R_1 R_2}{R_1 + R_2} \cdot \frac{1}{201} + 20} = 0.1mA$$

$I_{C1} = 0.1 \times 0.995 \times 0.995 = \underline{\underline{0.099mA}}$

$V_{C1} = 0.743V$

$V_{B1} = -0.193V$

$V_{E1} = -0.893V$

$V_{CE1} = 1.636 \simeq \underline{\underline{1.64V}}$

5.100

Refer to Fig P5.100

Assuming $\beta = \infty \rightarrow I_{B3} = 0$

$$V_{B3} = \frac{V_{CC} - V_{BE1} - V_{BE2}}{R_1 + R_2} \times R_2 \ldots$$

$$+ V_{BE1} + V_{BE2}$$

$$V_{E3} = V_{B3} - V_{BE3}$$

$$I_O = \frac{\alpha V_{E3}}{R_E}$$

$$I_O = \frac{\alpha}{R_E}\left[\frac{(V_{CC} - V_{BE1} - V_{BE2}) R_2}{R_1 + R_2} + V_{BE1} + V_{BE2} - V_{BE3}\right]$$

CONT.

Now, if the circuit is designed so that all three transistors conduct equal currents, then $V_{BE1} = V_{BE2} = V_{BE3} = V_{BE}$

and I_O becomes:

$$I_O = \frac{\alpha}{R_E}\left[(V_{CC} - 2V_{BE})\frac{R_2}{R_1+R_2} + V_{BE}\right]$$

To eliminate the V_{BE} terms in this equation we select $R_1 = R_2$, resulting in

$$I_O = \frac{\alpha V_{CC}}{2R_E} \quad \text{Q.E.D.}$$

Now, to make the current through Q_1 and Q_2 equal to that through Q_3 (which is I_O/α)

$$\frac{V_{CC} - 2V_{BE}}{2R_1} = \frac{V_{CC}}{2R_E}$$

$$\Rightarrow R_1 = R_2 = R_E\left(1 - \frac{2V_{BE}}{V_{CC}}\right)$$

For $V_{CC} = 10V$, $\alpha = 1$
$I_O = 0.5mA$

$$0.5 = \frac{10}{2R_E} \Rightarrow R_E = \underline{10K\Omega}$$

$$R_1 = R_2 = 10\left(1 - \frac{2 \times 0.7}{10}\right)$$
$$= \underline{8.6K\Omega}$$

$$V_{C3}|_{min} = I_O . R_E + V_{BE}$$
$$= 0.5 \times 10 + 0.7$$
$$= \underline{5.7V}$$

5.101

Refer to Fig. P5.101

$$I_O = 2mA = \alpha \times \frac{5 - 0.7}{R} \simeq \frac{4.3}{R}$$

$$\Rightarrow R = \underline{2.15K\Omega}$$

$V_{C\,min} = \underline{0V}$ (In actual practice, $V_{C\,min} \simeq 0.4V$)

5.102

(a) Using the exponential characteristic:

$$i_C = I_C e^{v_{be}/V_T}$$

thus,

$$i_c = I_C e^{v_{be}/V_T} - I_C$$

giving

$$\frac{i_c}{I_C} = e^{v_{be}/V_T} - 1$$

(b) Using small-signal approximation:

$$i_c = g_m v_{be} = \frac{I_C}{V_T} \cdot v_{be}$$

Thus,

$$\frac{i_c}{I_C} = \frac{v_{be}}{V_T}$$

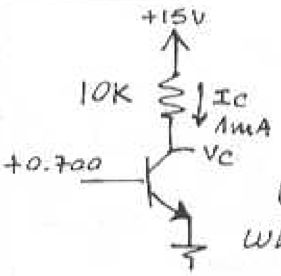
See table below

For signals of $\pm 5mV$, the error introduced by the small-signal approximation is 10%.
The error increases to above 20% for signals of $\pm 10mV$.

CONT.

v_{be} (mV)	i_c/I_C Expon.	i_c/I_C small-signal	% Error
+1	+0.041	+0.040	−2
−1	−0.039	−0.040	+2
+2	+0.083	+0.080	−4
−2	−0.077	−0.080	+4
+5	+0.221	+0.200	−9.5
−5	−0.181	−0.200	+10.3
+8	+0.377	+0.320	−15.2
−8	−0.274	−0.320	+16.8
+10	+0.492	+0.400	−18.7
−10	−0.330	−0.400	+21.3
+12	+0.616	+0.480	−22.1
−12	−0.381	−0.480	+25.9

5.103



$V_C = 15 - 1\times 10$
$= 5V$

$V_{BE} = 0.7 + v_{be}$
where $v_{be} = 0.005V$

$i_C \cong I_C\left(1 + \frac{v_{be}}{V_T}\right)$ Eq. (5.83)

$i_C \simeq I_C + i_c$ where:

$i_c = \frac{1m \times 0.005}{25m} = 0.2m$

$i_C = 1mA + 0.2mA$

$V_C = V_{CC} - i_C R_C$ Eq. (5.101)

$\Rightarrow V_C - \underbrace{i_c R_C}_{0.2m\times 10K}$

$V_C = 5V - 2V$

$\text{gain} = \frac{-2V}{0.005V} = -400\ V/V$

while $-g_m \cdot R_C = -\frac{1m}{25m}\cdot 10K = -400\ V/V$

5.104

$g_m = \frac{I_C}{V_T} = \frac{1.2mA}{25mV} = \underline{48\ \frac{mA}{V}}$

$r_\pi = \frac{\beta}{g_m} = \frac{120}{48\times10^{-3}} = \underline{\underline{2.5k\Omega}}$

$r_e = \frac{r_\pi}{\beta+1} = \frac{2500}{121} = 20.6\Omega$

For a bias current of $120\mu A$ i.e. 10 times lower:

$g_m = \frac{48}{10} = 4.8\ mA/V$

$r_\pi = 10\times 2.5 = 25K\Omega$

$r_e = 10\times 20.6 = 206\Omega$

5.105

$I_C = 2mA \Rightarrow g_m = \frac{2mA}{25mV}$

$g_m = 80mA/V$

$r_e = \frac{V_T}{I_E}$, $I_E = I_C\frac{(\beta+1)}{\beta}$

$I_E = 2mA \times \frac{51}{50} = 2.04mA$

$r_e = \frac{25m}{2.04m} = 12.25\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{50}{80\times10^{-3}} = 625\Omega$

gain: $-g_m \times R_C$

For $R_C = 5K\Omega$ and $\hat{v}_{be} = 5mV$

$\hat{v}_o = -80m \times 5K \times 5mV$
$= -2V$

5.106

$g_m = 50\,\frac{mA}{V} = \frac{I_C}{V_T}$

$\Rightarrow I_C = g_m \times V_T = 50m \times 25m = 1.25\,mA$

$r_\pi = 2K = \frac{\beta}{g_m} \Rightarrow \beta = 2K \times 50m$

$\underline{\underline{\beta = 100}} \rightarrow \alpha = \frac{100}{101} = 0.99$

$I_E = \frac{I_C}{\alpha} = \frac{1.25mA}{0.99} = \underline{\underline{1.26mA}}$

5.107

g_m varies from: $1.2 \times 60 = 72\,\frac{mA}{V}$
to $0.8 \times 60 = 48\,\frac{mA}{V}$

β varies from 50 to 200

$R_{in}|_{base} = r_\pi = \beta / g_m$

Largest value: $r_\pi = \frac{\beta_{max}}{g_{m\,min}} = \frac{200}{48m} = \underline{4.2K\Omega}$

Smallest value: $r_\pi = \frac{\beta_{min}}{g_{m\,max}} = \frac{50}{72m} = \underline{\underline{694\Omega}}$

5.108

Refer to Fig. 5.48.

$V_C = 2V \Rightarrow I_C = \frac{V_{CC} - V_C}{R_C}$

$I_C = \frac{5-2}{3K} = 1mA$

$g_m = \frac{I_C}{V_T} = \frac{1m}{25m} = 40\,\frac{mA}{V}$

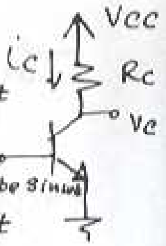
$i_C(t) = I_C + g_m v_{be}(t)$
$= 1m + 40 \cdot 10^{-3} \times 0.005\, \text{Sin}$
$= \underline{\underline{1 + 0.2\, \text{Sin}\, \omega t,\ mA}}$

$v_C(t) = 5 - R_C i_C(t)$
$= \underline{\underline{2 - 0.6\, \text{Sin}\, \omega t,\ V}}$

$i_B(t) = i_C(t) / \beta$
$= \frac{1 + 0.2 \cdot \text{Sin}\, \omega t}{100},\ mA$
$= \underline{\underline{10 + 2\, \text{Sin}\, \omega t,\ \mu A}}$

Voltage gain $= \frac{-0.6}{0.005} = \underline{\underline{-120}}\ V/$

5.109



$i_C = I_C + g_m \hat{v}_{be} \sin \omega t$

$v_C = V_{CC} - I_C R_C - g_m \hat{v}_{be} R_C \sin \omega t$

To maintain BJT in active region, $v_C \geq V_{BE}$, thus
$V_{CC} - I_C R_C - g_m R_C \hat{v}_{be} \geq V_{BE} + \hat{v}_{be}$

To obtain the largest possible output signal we design such that this constraint is satisfied with the equality sign, that is:
$V_{CC} - R_C I_C - g_m R_C \hat{v}_{be} = V_{BE} + \hat{v}_{be}$

substituting $g_m = \frac{I_C}{V_T}$, gives:

$V_{CC} - R_C I_C - R_C I_C \frac{\hat{v}_{be}}{V_T} = V_{BE} + \hat{v}_{be}$

$\Rightarrow R_C I_C \left(1 + \frac{\hat{v}_{be}}{V_T}\right) = V_{CC} - V_{BE} - \hat{v}_{be}$

CONT.

$$R_C I_C = \frac{(V_{CC} - V_{BE} - \hat{V}_{be})}{\left(1 + \frac{\hat{V}_{be}}{V_T}\right)} \quad \text{Q.E.D.}$$

$$\text{Voltage gain} = -g_m \cdot R_C$$
$$= -\frac{I_C}{V_T} \cdot R_C$$
$$= -\frac{V_{CC} - V_{BE} - \hat{V}_{be}}{V_T + \hat{V}_{be}}$$

For $V_{CC} = 5V$, $V_{BE} = 0.7V$ and $\hat{V}_{be} = 5mV$

$$R_C I_C = \frac{5 - 0.7 - 0.005}{1 + \frac{0.005}{0.025}} = 3.6V$$

Thus,

$$V_C = 5 - 3.6 = +\underline{1.4V}$$

Amplitude of output signal is

$$= 1.4 - (V_{BE} + \hat{V}_{be})$$
$$= 1.4 - 0.7 - 0.005$$
$$= \underline{0.695}\ V$$

$$\text{Voltage gain} = -\frac{0.695}{0.005} = \underline{-139\ \frac{V}{V}}$$

Check

$$\text{Voltage gain} = -\frac{(5 - 0.7 - 0.005)}{0.025 + 0.005}$$
$$= -143\ V/V$$

The difference is caused by decimal rounding-up of $R_C I_C$.

Otherwise:

$$\text{Voltage gain} = -\frac{0.716}{0.005}$$
$$= -143\ V/V$$

5.110

	a	b	c	d	e	f	g
α	1.000	0.990	0.98	1	0.990	0.90	0.941
β	∞	100	50	∞	100	9	16
I_C (mA)	1.00	0.99	1.00	1.00	0.248	4.5	17.5
I_E (mA)	1.00	1.00	1.02	1.00	0.25	5	18.6
I_B (mA)	0	0.010	0.020	0	0.002	0.5	1.10
g_m ($\frac{mA}{V}$)	40	39.6	40	40	0.01	180	700
r_e (Ω)	25	25	24.5	25	100	5	1.34
r_π (Ω)	∞	2.5K	1.25K	∞	10.1K	50	227

5.111

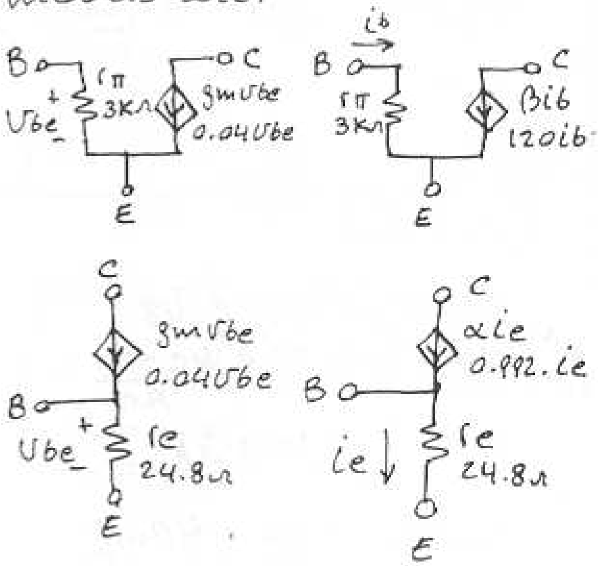
$I_C = 1mA$, $\beta = 120$ $\alpha = 0.992$

$$g_m = \frac{I_C}{V_T} = \frac{1}{25} = 40\frac{mA}{V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40 \times 10^{-3}} = 3K\Omega$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{0.992}{40 \times 10^{-3}} = 24.8\Omega$$

The four equivalent circuit models are:



5.112

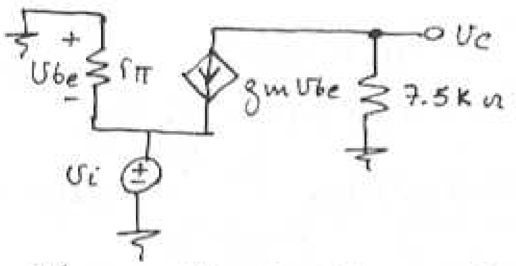
Refer to Fig P5.112

β very high $\rightarrow \alpha = 1$

$I_C = I_E = 0.5\,mA$

$V_C = 5 - 7.5 \times 0.5 = +\underline{\underline{1.25V}}$

$$g_m = \frac{I_C}{V_T} = \frac{0.5m}{25m} = \underline{\underline{20\frac{mA}{V}}}$$



Observe that $v_{be} = -v_i$

the output voltage v_c is found from:

$$v_c = -g_m v_{be} \times 7.5K$$

Thus the voltage gain is

$$\frac{v_c}{v_i} = g_m \times 7.5K$$

$$= 20 \times 7.5 = \underline{\underline{150}}\,V/V$$

5.113

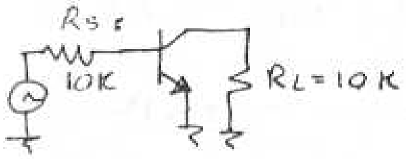
$$\frac{v_c}{v_{be}} = -g_m R_C \Rightarrow v_{be} = \frac{1}{50 \times 2}$$

$$= \underline{\underline{10mV}}\ p\text{-to-}p_K$$

$$i_b = \frac{v_{be}}{r_\pi} = \frac{10 \times 10^{-3}}{\beta / g_m} = \frac{0.01}{100/0.05}$$

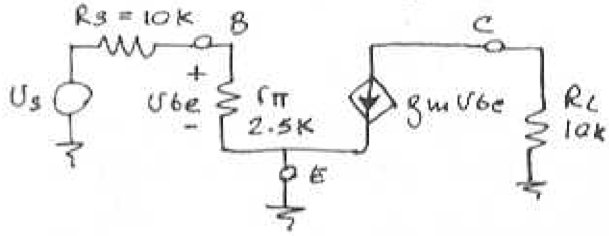
$$i_b = \underline{\underline{0.005}}\,mA\ p_K\text{-to-}p_K$$

5.114



$g_m = 40\frac{mA}{V}$, $r_\pi = 2.5K$

$$r_\pi = \beta / g_m \Rightarrow \beta = 2.5K \times 40m$$
$$= \underline{\underline{100}}$$



$$v_{be} = \frac{v_s \cdot r_\pi}{R_S + r_\pi} = 0.2 \times v_s$$

$$v_c = -g_m v_{be} \cdot R_L$$
$$= -g_m (0.2 v_s) \cdot R_L$$

$$v_c = -80\,v_s$$

$$\Rightarrow \text{gain } \frac{v_c}{v_s} = \underline{\underline{-80}}$$

<u>To double the gain:</u>

If I_C is fixed $\rightarrow g_m$ does not change.

$$\rightarrow \frac{v_{be}}{v_s} = 0.4 \quad (2 \times 0.2)$$

$$\rightarrow \frac{r_\pi}{R_S + r_\pi} = 0.4 \quad \wedge \quad R_S = 10K$$

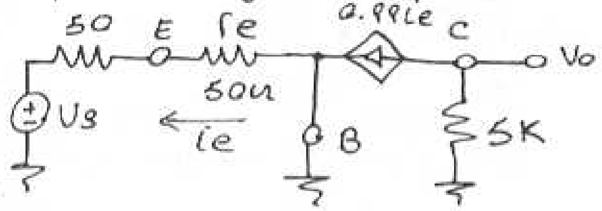
$$\Rightarrow r_\pi = 6.6K\Omega$$

Since $r_\pi = \frac{\beta}{g_m} \Rightarrow \beta = 6.6K \times 40m$

$$\beta = \underline{\underline{264}}$$

5.115

Refer to Fig. P5.115



$r_e = \frac{V_T}{I_E} = \frac{25mV}{0.5mA} = 50\Omega$

$R_{in} = R_s + r_e = \underline{\underline{100\Omega}}$

$V_o = -0.99\, i_e \times 5K$

but: $i_e = \frac{-v_s}{R_{in}} = \frac{-v_s}{100}$

$\Rightarrow V_o = +\frac{0.99 \times 5K}{100} . v_s$

$\frac{v_o}{v_s} = \underline{\underline{49.5}}\ V/V$

5.116

Refer to Fig P5.116

$\beta = 200 \rightarrow \alpha = 0.995$

$I_C = \alpha I_E = 0.995 \times 10m$
$= 9.95mA$

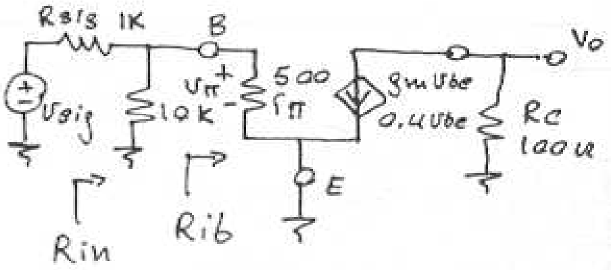
$V_C = 9.95m \times 100 = \underline{\underline{0.995V}}$

$I_B \simeq \frac{10m}{200} = 0.05mA$

$V_B = 1.5 - 10K \times 0.05m$
$= 1V$

$\Rightarrow V_{BC} = +0.005$

$\rightarrow$ Active region.



$g_m = \frac{I_C}{V_T} = \frac{9.95m}{25m} = 0.4\ A/V$

$r_\pi = \frac{\beta}{g_m} = \frac{200}{0.4} = 500\ \Omega$

$R_{ib} = r_\pi = \underline{\underline{500\Omega}}$

$R_{in} = 10K \parallel r_\pi = \underline{\underline{476\Omega}}$

$v_{be} = v_{sig} \times \frac{R_{in}}{R_{sig} + R_{in}} = v_{sig} \times 0.32$

also:

$v_o = -g_m v_{be} . R_C$
$= -g_m R_C \times 0.32\, v_{sig}$
$= -0.4 \times 100 \times 0.32\, v_{sig}$
$= -12.8\, v_{sig}$

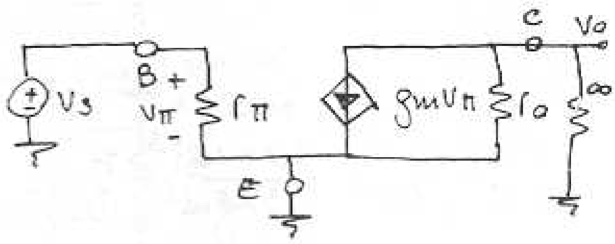
$\Rightarrow$ gain $\frac{v_o}{v_s} = -12.8 \simeq -13\ V/V$

If $v_o = \pm 0.4V$:

$\hat{v}_s = \frac{\hat{v}_o}{.13} = 30mV$

$\hat{v}_{be} = 0.32 \times 30m = 9.8mV$

5.117



$V_s = V_\pi \Rightarrow \frac{V_o}{V_s} = -g_m \times r_o$

CONT.

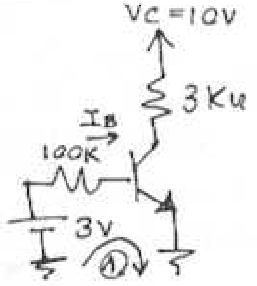
but: $r_o = \frac{V_A}{I_C} = \frac{V_A}{V_T \cdot g_m}$

$$\Rightarrow \frac{V_o}{V_s} = \frac{-V_A}{V_T}$$

If $V_A = 25V \Rightarrow \frac{V_o}{V_s} = -1000 \frac{V}{V}$

If $V_A = 250V \Rightarrow \frac{V_o}{V_s} = -10{,}000 \frac{V}{V}$

5.118



DC Analysis:

① $I_B = \frac{3-0.7}{100}$

$I_B = 0.023\,mA$

Saturation begins to occur when $V_C \leq 0.7V$

$\therefore I_C \geq \frac{10-0.7}{3} = 3.1\,mA$

$I_C = \beta I_B \rightarrow \beta \geq \frac{3.1}{0.023} = 135$

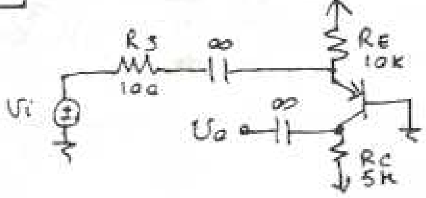
$\beta = 25$:

$r_e = \frac{V_T}{I_E} = \frac{V_T}{(\beta+1) I_B} = \frac{25\times10^{-3}}{26\times 0.023\times10^{-3}}$

$r_e = 41.8\,\Omega$

$g_m = \frac{\alpha}{r_e} = \frac{25/26}{41.8} = 23\,\frac{mA}{V}$

5.119



$\frac{V_o}{V_i} = \frac{\alpha\, r_e \| R_E}{R_E \| r_e + R_s} \cdot \frac{1}{r_e} \cdot R_C$

$= 0.99 \frac{(27\|10K)}{(27\|10K)+100} \times \frac{1}{27} \times 5\times10^3$

$= 38.9 \simeq 39\ V/V$

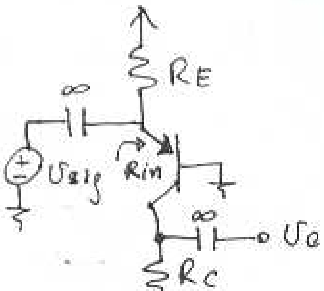
The amplifier clips when the peak output signal exceeds the difference between Vc (−5.4V) and the negative supply (−10V) i see Fig. 5.56.

That is when the peak output signal is 4.6V

This corresponds to a peak input signal of

$\hat{V}_i = \frac{4.6}{39} = 118\,mV$

5.120



$R_{in} = r_e \| R_E$

$\simeq r_e$

$= 75\,\Omega$

$I_E = \frac{25mV}{75\Omega} = 0.33\,mA$

$R_E = \frac{10-0.7}{0.33} = 28\,K\Omega$

$n = 2.8$

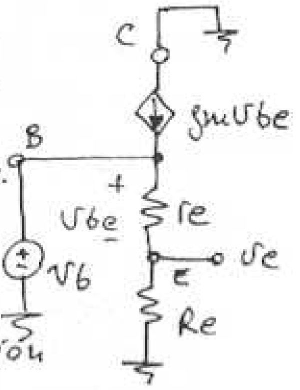
$R_C = 14\,K\Omega$

$\frac{V_o}{V_i} = \alpha \frac{R_C}{r_e} = \frac{14}{0.075} = 187\ V/V$

5.121

(a) Using the voltage divider rule:

$$\frac{v_e}{v_b} = \frac{R_e}{R_e + r_e} \quad \text{Q.E.D.}$$



(b) Node equation at B:

$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$
$$= \frac{v_{be}}{r_e}(1 - g_m r_e)$$
$$= \frac{v_{be}}{r_e}\left(1 - g_m \frac{\alpha}{g_m}\right)$$
$$= \frac{v_{be}}{r_e}(1-\alpha)$$
$$= \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{\beta+1}\right)$$
$$= \frac{v_{be}}{r_e} \cdot \frac{1}{(\beta+1)}$$

But, from voltage-divider rule

$$v_{be} = v_b \cdot \frac{r_e}{r_e + R_e}$$

$$\Rightarrow i_b = \frac{1}{(\beta+1) r_e} \cdot \frac{v_b \cdot r_e}{r_e + R_e}$$

from which we find

$$R_{in} = \frac{v_b}{i_b} = (\beta+1)(R_e + r_e)$$

Q.E.D.

For $R_e = 1\,K\Omega$, $\beta = 100$ and $I_E = 1\,mA$

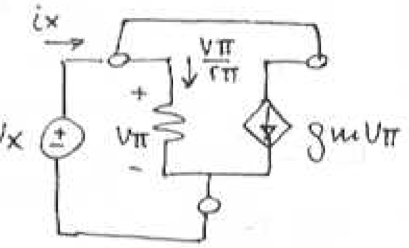
$$r_e = \frac{V_T}{I_E} = \frac{25\,mV}{1\,mA} = 25\,\Omega$$

Thus,

$$\frac{v_e}{v_b} = \frac{1000}{1000+25} = \underline{\underline{0.976\ V/V}}$$

$$R_{in} = (100+1)(1000+25)\,\Omega$$
$$= 101 \times 1.025\,K\Omega$$
$$= \underline{\underline{103.5\,K\Omega}}$$

5.122



$$i_x = \frac{V_\pi}{r_\pi} + g_m V_\pi$$
$$= \frac{V_\pi}{r_\pi}(1 + g_m r_\pi)$$
$$= \frac{V_\pi}{r_\pi}(1+\beta)$$

But $V_\pi = V_x$

$$\rightarrow R_{in} = \frac{V_x}{i_x} = \frac{V_\pi}{i_x} = \frac{r_\pi}{\beta+1}$$

$$R_{in} = \underline{\underline{r_e}}$$

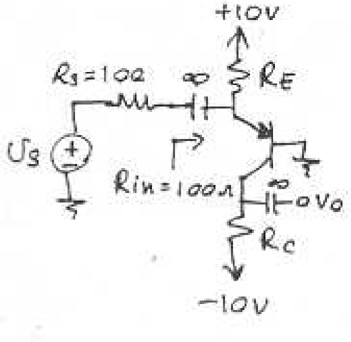
5.123

$R_{in} = R_E \parallel r_e \qquad r_e \simeq 100\Omega$

thus, $\frac{V_T}{I_E} = 100 \rightarrow I_E = 0.25\,mA$

$V_E = 0.7\,V$

$$R_E = \frac{10 - 0.7}{1m}$$
$$= \underline{\underline{9.3\,K\Omega}}$$



CONT.

Selection of a value for R_C:

The voltage gain is directly proportional to R_C,

$$\frac{v_o}{v_s} = \frac{v_e}{v_s} \cdot \frac{v_o}{v_e}$$
$$= \frac{R_{in}}{R_s + R_{in}} \cdot \frac{\alpha R_C}{r_e}$$
$$\simeq \frac{100}{100+100} \cdot \frac{R_C}{0.1}$$
$$= 5R_C \text{ , } R_C \text{ in } K\Omega.$$

For an emitter-base signal as large as 10mV, the signal at the collector will be $g_m R_C \times 0.010$ volts. Thus the maximum collector voltage in the positive direction will be:

$$v_C|_{max} = V_C + 0.01\, g_m \cdot R_C$$
$$= -10 + I_C R_C + 0.01 \times \frac{1}{0.1} \times R_C$$
$$= -10 + 0.25 R_C + 0.1 R_C$$
$$= -10 + 0.35 R_C$$

To prevent saturation,
$v_C|_{max} \leq V_B$ which is 0V.

Thus to obtain maximum gain while allowing an emitter-base signal as large as 10mV and at the same time keeping the transistor in the active mode we select R_C from:

$$-10 + 0.35 R_C = 0$$
$$\Rightarrow R_C = \underline{\underline{28.6\ K\Omega}}$$

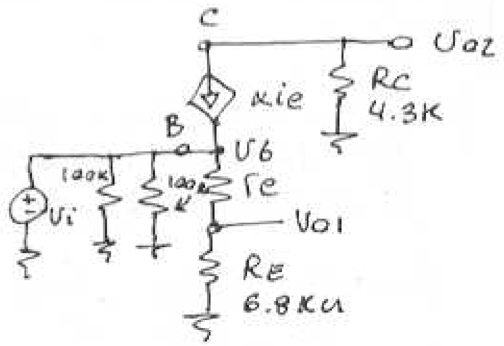
Voltage gain $= \frac{v_o}{v_s} = 5R_C = 143\ V/V$

5.124

Refer to Fig P5.124
For large β, the DC base current will be ~ 0
Thus the DC voltage at the base can be found directly using the voltage-divider rule

$$V_B = 15 \cdot \frac{100}{100+100} = 7.5V$$

$\because V_{BE} = 0.7$
$$V_E = 7.5 - 0.7 = 6.8V$$
$$\rightarrow I_E = \frac{6.8V}{6.8K\Omega} = \underline{\underline{1mA}}$$



$$v_b = v_i$$
$$\rightarrow \frac{v_{o1}}{v_i} = \frac{R_E}{R_E + r_e} \qquad Q.E.D.$$

Also,
$$i_e = \frac{v_b}{r_e + R_E} = \frac{v_i}{r_e + R_E}$$

and,
$$v_{o2} = -\alpha i_e \cdot R_C$$
$$= -\frac{\alpha R_C v_i}{r_e + R_E}$$

Thus,
$$\frac{v_{o2}}{v_i} = \frac{-\alpha R_C}{R_E + r_e} \qquad Q.E.D.$$

CONT.

Substituting $r_e = \frac{V_T}{I_E} = 25\Omega$

and $R_E = 6.8K\Omega$, $R_C = 4.3K\Omega$ and $\alpha \simeq 1$ gives

$$\frac{v_{o1}}{v_i} = \frac{6.8}{0.025+6.8} = \underline{\underline{0.996\ V/V}}$$

$$\frac{v_{o2}}{v_i} = -\frac{4.3}{6.8+0.025} = \underline{\underline{0.63\ V/V}}$$

If the node labeled v_{o2} is connected to ground:
$R_E = 0$

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{r_e}$$

5.125

If: $R_i = 10K\Omega$, $A_{vo} = 100\ V/V$
$R_o = 100\Omega$, $R_{sig} = 2K\Omega$ and
$R_{in} = 8K\Omega$ when $R_L = 1K\Omega$
then:

$$G_m = \frac{A_{vo}}{R_o} = \frac{100}{100} = \underline{\underline{1A/V}}$$

$$A_v = A_{vo}.\frac{R_L}{R_L+R_o} = \frac{100\times 1K}{1K+100} = \underline{\underline{91V/V}}$$

$$G_{vo} = \frac{R_i}{R_i+R_{sig}}.A_{vo} = \frac{10K}{10K+2K}\times 100 = \underline{\underline{83.3\ V/V}}$$

$$G_v = \frac{R_{in}}{R_{in}+R_{sig}} \times A_{vo} \times \frac{R_L}{R_L+R_o}$$

$$= \frac{8K}{8K+2K} \times 100 \times \frac{1K}{1K+100} = \underline{\underline{72.7\ \frac{V}{V}}}$$

$$R_{out} = \frac{G_{vo}}{G_v}.R_L - R_L = \frac{8.3}{72.7}\times 1K - 1K$$

$$= \underline{\underline{146\Omega}}$$

$$A_i = \frac{v_o/R_L}{v_i/R_{in}} = A_v.\frac{R_{in}}{R_L} = 91\times\frac{8K}{1K}$$

$$= 728\ A/A\ //$$

5.126

Refer to Fig. P5.126
$R_i = 900K\Omega$
$A_{vo} = 10\ V/V$
$R_o = 1.43K\Omega$
$R_{in} = 400K\Omega$ when $R_L = 10K\Omega$

(a) $i_i \equiv v_i/R_{in}$

$$\Rightarrow \frac{v_i}{R_{in}} = \frac{v_i}{R_i} - f.i_o$$

where $i_o = \frac{A_{vo}v_i}{R_o+R_L}$

$$\rightarrow \frac{v_i}{R_{in}} = \frac{v_i}{R_i} - f.\frac{A_{vo}v_i}{R_o+R_L}$$

Solving for f:

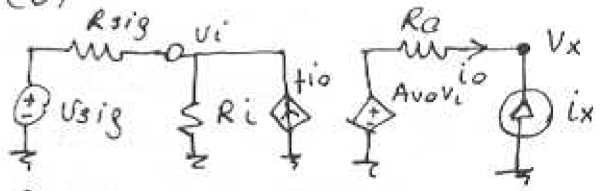
$$f = \left(\frac{1}{R_i} - \frac{1}{R_{in}}\right)\cdot\frac{R_o+R_L}{A_{vo}}$$

Thus,

$$f = \left(\frac{1}{900K} - \frac{1}{400K}\right)\cdot\frac{(1.43K+10K)}{10}$$

$$f = -1.\underline{6\times 10^{-3}}$$

(b)



$$R_{out} = \frac{v_x}{i_x}\Big|_{v_{sig}=0}$$

If $v_{sig}=0$: on the left hand side

$$v_i = f.i_o\ (R_{sig}\,//\,R_i)$$

$$v_i = (-1.6\times 10^{-3}\times(100K//900K)\,i_o$$

$$= -144.i_o$$

On the right hand side:

$$V_x = R_o i_x + A_{vo}.v_i$$

$$= R_o.i_x + A_{vo}(-144\,i_o)$$

$$= R_o\,i_x + A_{vo}(+144)\,i_x$$

CONT.

$\rightarrow V_x = i_x\,(R_o + A_{vo} \times 144)$

$R_{out} = \dfrac{V_x}{i_x} = 1.43K + 10 \times 144$

$R_{out} = \underline{\underline{2.87 K\Omega}}$

Which is the same value as in Example 5.17

(c) For $R_{sig} = \infty$

$$\frac{R_{in}}{R_i} = \frac{R_L + R_o}{R_L + R_{out}}$$

$$R_{out} = (R_L + R_o)\frac{R_i}{R_{in}} - R_L \;//$$

In Example 5.17:

$$R_{out} = (10K + 1.43K)\frac{900}{400} - 10$$

$$= \underline{\underline{15.7 K\Omega}}$$

5.127

$$G_v = \frac{R_{in}}{R_{in}+R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L+R_o} = G_{vo} \cdot \frac{R_L}{R_L+R_{out}}$$

but $G_{vo} = \dfrac{R_i}{R_i+R_{sig}} \cdot A_{vo}$

Substituting G_{vo}:

$$\frac{R_{in}}{R_{in}+R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L+R_o} = \frac{R_i}{R_i+R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L+R_{out}}$$

$$\rightarrow \frac{R_{in}}{R_i}\,\frac{(R_{sig}+R_i)}{(R_{sig}+R_{in})} = \frac{R_L+R_o}{R_L+R_{out}}$$

Q.E.D

(a) for $R_L = \infty$

$$\frac{R_{in}}{R_i}\,\frac{(R_{sig}+R_i)}{(R_{sig}+R_{in})} = 1$$

$$R_{in}R_{sig} + R_{in}R_i = R_iR_{sig} + R_iR_{in}$$

$$R_{in} \cdot R_{sig} = R_i \cdot R_{sig}$$

$$\rightarrow R_{in} = R_i$$

Q.E.D

(b) for $R_{sig} = 0$

$$\frac{R_{in}}{R_i} \cdot \frac{R_i}{R_{in}} = \frac{R_L+R_o}{R_L+R_{out}}$$

$$\rightarrow R_{out} = R_o$$

Q.E.D.

5.128

Refer to Fig. 5.60(a)

$I_C = 0.2mA \Rightarrow I_E = \dfrac{I_C}{\alpha}$

$I_E = \dfrac{0.2}{\beta} \cdot (\beta+1) = 0.202\,mA$

$r_e = \dfrac{V_T}{I_E} = \dfrac{0.025}{0.202m} = 123.76\Omega$

$R_i = (\beta+1)r_e = \underline{\underline{12{,}5K\Omega}}$

$$\frac{v_o}{v_s} = -g_m\,(R_C \| r_o) \cdot \frac{R_i}{R_i+R_s}$$

$$\simeq -\frac{\alpha}{r_e} \cdot R_C \cdot \frac{R_i}{R_i+R_s}$$

$$= -\frac{100}{101} \times \frac{24K}{123.76} \times \frac{12.5K}{12.5K+10K}$$

$$\frac{v_o}{v_s} = -\underline{\underline{106.7}}\ V/V$$

$R_o = R_C \| r_o \simeq R_C = \underline{\underline{24 K\Omega}}$

$$\frac{v_o}{v_s} = \left.\frac{v_o}{v_s}\right|_{no\,load} \times \frac{10}{10+24}$$

$$= -106.7 \times \frac{10}{34} = \underline{\underline{-31.4}}\ V/V$$

5.129

$I_C = 0.2\,mA \Rightarrow r_e = 123.76\,\Omega$

$R_i = (\beta+1)(r_e + R_E)$

$= (101)(123.76 + 125) = \underline{25.1\,K\Omega}$

$$\frac{v_o}{v_s} = \frac{-\alpha R_C}{r_e + R_E} \times \frac{R_i}{R_i + R_s}$$

$$= \frac{-100}{101} \times \frac{24K}{(123.76+125)} \times \frac{25.1K}{25.1K + 10K}$$

$= \underline{-68.30}\ V/V$

$R_o = R_C = \underline{24K\Omega}$

With $10K\Omega$ load

$$\frac{v_o}{v_s} = -68.30 \times \frac{10}{10+24} = \underline{-20\,\frac{V}{V}}$$

Without R_e $\quad V_\pi \leq 5\,mV$

$$V_\pi = \frac{(\beta+1)\,r_e}{(\beta+1)r_e + R_s} \cdot V_s$$

$$= \frac{(101)\cdot(123.76)}{(101)\cdot(123.76)+(10K)} \cdot V_s \leq 5\,mV$$

$\Rightarrow V_s \leq \underline{9\,mV}$

With R_e

$$V_\pi = \frac{(\beta+1)r_e \cdot V_s}{(\beta+1)(r_e + R_E) + R_s} \leq 5\,mV$$

$$V_s \leq \frac{5m[(101)(123.76+125)+10K]}{101 \times 123.76}$$

$V_s \leq \underline{14\,mV}$

5.130

Refer to Fig. P5.130

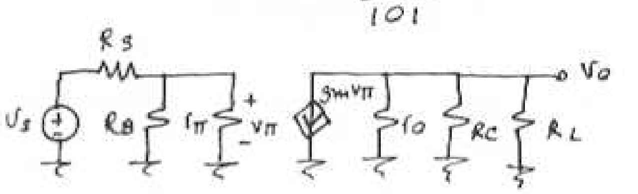
$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B/(\beta+1)}$$

where, $V_{BB} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$

$$= 9 \cdot \frac{15}{27+15} = 3.21\,V$$

$R_B = R_1 \| R_2 = 15 \| 27 = 9.64\,K\Omega$

Thus, $I_E = \dfrac{3.21 - 0.7}{1.2 + \frac{9.64}{101}} = \underline{1.94\,mA}$



$$g_m = \frac{I_C}{V_T} = \frac{0.99 \times 1.94}{0.025} = 76.8\,\frac{mA}{V}$$

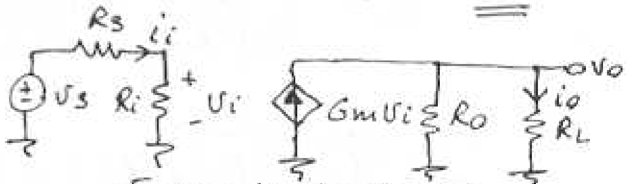
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{76.8} = 1.3\,K\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.99 \times 1.94} = 52.1\,K\Omega$$

$R_i = R_B \| r_\pi = 9.64 \| 1.3 = \underline{1.15\,K\Omega}$

$G_m = -g_m = \underline{-76.8\,\frac{mA}{V}}$

$R_o = R_C \| r_o = 2.2 \| 52.1 = \underline{2.11}\,K\Omega$



Equivalent circuit

$$A_v = \frac{v_o}{v_s} = \frac{v_i}{v_s} \cdot \frac{v_o}{v_i}$$

$$= \frac{R_i}{R_s + R_i} \cdot \frac{G_m (R_o \| R_L)\, v_i}{v_i}$$

$$= \frac{-1.15}{10+1.15} \times 76.8 \times (2.11 \| 2)$$

$= \underline{-8.13}\ V/V$

$$A_i = \frac{i_o}{i_i} = \frac{v_o \cdot R_L}{v_s / (R_s + R_i)}$$

CONT.

$$\rightarrow A_i = \frac{v_o}{v_s} \cdot \frac{R_s + R_i}{R_L}$$
$$= -8.13 \times \frac{(10 + 1.15)}{2}$$
$$= \underline{\underline{-45.3}}\ A/A$$

5.131

Refer to Fig. P5.130.

$V_{CC} = 9V \quad V_{BB} = \frac{1}{3} V_{CC} = 3V$

Neglecting the base current,

$$R_1 + R_2 = \frac{9}{0.2} = 45K\Omega$$

$$\frac{R_2}{R_1 + R_2} = \frac{1}{3}$$

$$\Rightarrow \underline{\underline{R_2 = 15K\Omega}} \quad , \quad R_1 = \underline{\underline{30K\Omega}}$$

$$R_B = R_1 // R_2 = \frac{30 \times 15}{45} = 10K\Omega$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

$$2 = \frac{3 - 0.7}{R_E + 10/101} \Rightarrow R_E = \underline{\underline{1.05K\Omega}}$$

Use $R_E = \underline{\underline{1K\Omega}}$

The resulting I_E will be

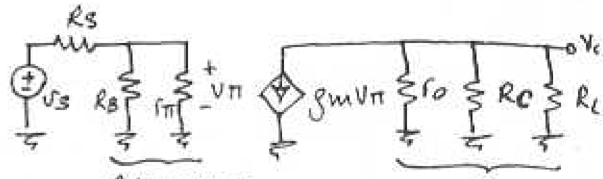
$$I_E = \frac{3 - 0.7}{1 + 10/101} = 2.09\,mA$$

$$I_C = \alpha I_E = 0.99 \times 2.09 = 2.07\,mA$$

$$g_m = \frac{I_C}{V_T} = \frac{2.07}{0.025} = 82.9\,\frac{mA}{V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{82.9} = 1.21\,K\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{2.07} = 48.3\,K\Omega.$$



$$\frac{v_o}{v_s} = \frac{v_\pi}{v_s} \cdot \frac{v_o}{v_\pi} = \frac{R_i}{R_s + R_i} \cdot \frac{-g_m v_\pi R}{v_\pi}$$

$$= \frac{-1.08}{10 + 1.08} \times 82.9 \times R$$

To obtain $\frac{v_o}{v_s} = -8\frac{V}{V}$ we use:

$$R = \frac{8 \times 11.08}{1.08 \times 82.9} = 0.99K\Omega$$

Now $R = r_o // R_C // R_L$

$$0.99 = 48.3 // R_C // 2$$

$$\Rightarrow R_C = 2.04\,K\Omega$$

use $R_C = \underline{\underline{2K\Omega}}$

Check: $V_C = 9 - 2.07 \times 2 = 4.86V$ while $V_B \simeq 3V$. Thus in active mode as assumed.

5.132

Refer to Fig. P5.130

$$V_{BB} = 9 \cdot \frac{47}{82 + 47} = 3.28\,V$$

$$R_B = 47 // 82 = 29.88\,K\Omega$$

$$I_E = \frac{3.28 - 0.7}{3.6 + \frac{29.88}{101}} = 0.66\,mA$$

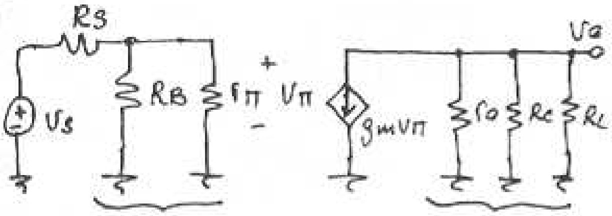
$$I_C = 0.99 \times 0.66 = 0.65\,mA$$

$$g_m = \frac{0.65}{0.025} = 26\,\frac{mA}{V}$$

$$r_\pi = \frac{100}{26} = 3.85\,K\Omega$$

$$r_o = \frac{100}{0.66} = 151.5\,K\Omega$$

CONT.



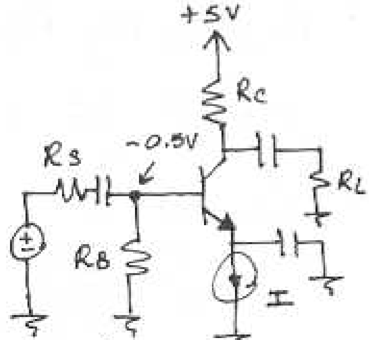
$R_i = 29.88 \,||\, 3.85 = 3.41K\Omega$ $\qquad 151.5 \,||\, 6.8 \,||\, 2 = 1.53K\Omega$

$A_v = \frac{v_o}{v_s} = \frac{3.41}{10+3.41} \times -26 \times 1.53$

$= \underline{-10.1\ V/V}$ Which is about 25% higher than in the original design. The improvement is not as large as might have been expected because although R_i increases, g_m decreases by about the same factor. Indeed most of the improvement is due to the increase in R_C and hence in the effective load resistance.

5.133

$\beta = 100$
$r_o = \infty$



$R_{in} = 5K\Omega$, $R_{in} = R_B \,||\, r_\pi$

$\Rightarrow 5K = \frac{R_B \cdot r_\pi}{R_B + r_\pi}$

$5K r_\pi = R_B(r_\pi - 5K)$

but: $r_\pi = \frac{V_T}{I_B}$ and $R_B \cdot I_B = 0.5$

$\rightarrow 5K \cdot \frac{V_T}{\cancel{I_B}} = \frac{0.5}{\cancel{I_B}}(r_\pi - 5K)$

thus, $r_\pi = 5250\Omega$
then $R_B = 105K$
choose $R_B = \underline{100K\Omega}$

and $I_B = 4.76\mu A$
$I_E = (\beta+1)I_B = 101 \times 4.76\mu A$
$I_E = 0.48mA$
$I = I_E \rightarrow I \simeq \underline{0.5mA}$

To avoid saturation:
$V_C - V_B \geqslant -0.5$
$V_C = 5V - R_C[I_C + g_m v_{be}]$

$I_C = I \cdot \alpha = 0.5m \times 100/101$
$= 0.49mA$

$g_m = \frac{V_T}{I_C} = \frac{25m}{0.49m} \simeq 50\frac{mA}{V}$

$v_{be} = 0.005V$

$\rightarrow V_C = 5 - R_C[0.49m + 50m \times 5m]$
$= 5 - 0.74 \times 10^{-3} \times R_C$

Then:
$V_C - V_B = (5 - 0.74m R_C) - (-0.5 + v_{be})$
$= 5.495 - 0.74m R_C \geqslant -0.5$
$R_C \leqslant \underline{8.1K\Omega}$.

Base-to-Collector open circuit gain:

$\frac{v_c}{v_b} = -g_m R_C = -50m \times 8.1K$
$= \underline{-405\ V/V}$

For $R_s = 10K$, $R_L = 10K$

$\frac{v_o}{v_b} = -g_m(R_C \,||\, R_L)$
$= -50m \times 4.47K$
$= -223\ V/V$

$\frac{v_c}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b} = \frac{5}{5+10} \times -223$
$= \underline{-74.3\ V/V}$

5.134

Refer to Fig P5.134

$I_E = 0.5\,mA$

(a) $I_E = \dfrac{15-0.7}{R_E + \dfrac{R_S}{\beta+1}}$

$0.5 = \dfrac{14.3}{R_E + \dfrac{2.5}{100}}$

$\Rightarrow R_E = \underline{\underline{28.57\,K\Omega}}.$

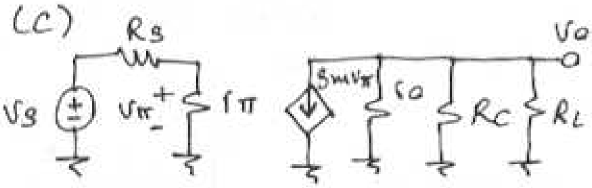
(b) $V_C = 15 - R_C \cdot I_C$

$5 = 15 - R_C \times 0.99 \times 0.5\,m$

$\Rightarrow R_C = 20.2\,K\Omega$

$\simeq \underline{\underline{20\,K\Omega}}$

(c)



$R_L = 10\,K\Omega\ ,\ R_S = 2.5K$

$r_o = 200\,K\Omega$

$g_m = \dfrac{I_C}{V_T} \simeq \dfrac{0.5m}{25m} = 20\,\dfrac{mA}{V}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{20} = 5\,K\Omega$

$A_v = \dfrac{v_o}{v_s} = \dfrac{v_\pi}{v_s} \times \dfrac{v_o}{v_\pi}$

$= \dfrac{r_\pi}{r_\pi + R_S} \times -g_m\,(r_o \,//\, R_C \,//\, R_L)$

$= -\dfrac{5}{5+2.5} \times 20\,(200 \,//\, 20 \,//\, 10)$

$= \underline{\underline{-86}}\ V/V$

5.135

Refer to Fig. P5.135

(a) For each transistor

$V_{BB} = 15 \times \dfrac{47}{100+47} = 4.8\,V$

$R_B = R_1 \,//\, R_2 = 100 \,//\, 47 = 32\,K\Omega$

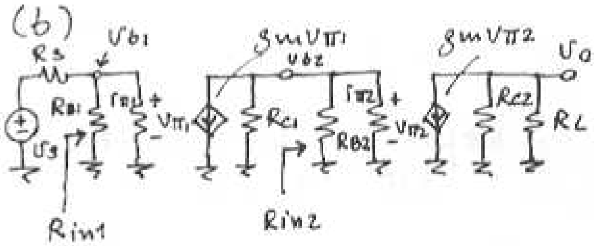
$I_E = \dfrac{4.8 - 0.7}{3.9 + \dfrac{32}{101}} = 0.97\,mA$

$I_C = 0.99 \times 0.97 = \underline{\underline{0.96\,mA}}$

$V_C = V_{CC} - I_C \times R_C$

$= 15 - 0.96 \times 6.8 = \underline{\underline{8.5\,V}}$

(b)



$R_{B1} = R_{B2} = R_B = 32\,K\Omega$

$g_{m1} = g_{m2} = \dfrac{0.96}{0.025} = 38.4\,\dfrac{mA}{V}$

$r_{\pi 1} = r_{\pi 2} = \dfrac{100}{38.4} = 2.6\,K\Omega$

$R_{C1} = R_{C2} = 6.8\,K\Omega$

$r_{o1} = r_{o2} = \infty$

(c) $R_{in1} = R_{B1} \,//\, r_{\pi 1}$

$= 32 \,//\, 2.6 = \underline{\underline{2.4\,K\Omega}}$

$\dfrac{v_{b1}}{v_s} = \dfrac{R_{in1}}{R_s + R_{in1}}$

$= \dfrac{2.4}{5+2.4} = \underline{\underline{0.32}}\ V/V$

(d) $R_{in2} = R_{B2} \,//\, r_{\pi 2}$

$= 32 \,//\, 2.6 = \underline{\underline{2.4\,K\Omega}}$

CONT.

$$v_{b2} = -g_{m1} v_{\pi 1} (R_{C1} \,\|\, R_{in2})$$
$$= -38.4\, v_{b1} (6.8 \,\|\, 2.4)$$

$$\frac{v_{b2}}{v_{b1}} = \underline{-68.1} \text{ V/V}$$

(e) $$v_o = -g_{m2} v_{\pi 2} (R_{C2} \,\|\, R_L)$$
$$= -38.4\, v_{b2} (6.8 \,\|\, 2)$$
$$\frac{v_o}{v_{b2}} = \underline{-59.3} \text{ V/V}$$

(f) $$\frac{v_o}{v_s} = \frac{v_{b1}}{v_s} \times \frac{v_{b2}}{v_{b1}} \times \frac{v_o}{v_{b2}}$$
$$= 0.32 \times -68.1 \times -59.3$$
$$= \underline{1292} \text{ V/V}$$

5.136

Refer to the circuit in Fig. P5.136

$$R_{in} = (\beta+1)(r_e + 250)$$
$\beta = 100$ $\quad r_e = \frac{V_T}{I_e} = \frac{0.025}{0.1} = 250\Omega$
$$R_{in} = 101 \times (250 + 250)$$
$$= \underline{50.5 K\Omega}$$

$$\frac{v_b}{v_s} = \frac{R_{in}}{R_s + R_{in}} = \frac{50.5}{20 + 50.5}$$
$$= 0.72 \text{ V/V}$$
$$\frac{v_o}{v_b} = \frac{-\alpha (20 \,\|\, 20)}{(r_e + R_E)}$$
$$= -\frac{0.99 \times 10}{0.250 + 0.250} = \underline{-19.8} \text{ V/V}$$

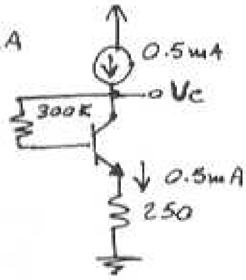
Thus, $\frac{v_o}{v_s} = 0.72 \times -19.8 = \underline{-14.2 \text{ V/V}}$

For $v_{be} = 5mV$, $v_e = 5mV$ also (since $R_e = r_e = 250\Omega$)

Thus,
$$v_b = 5 + 5 = 10mV$$
$$v_s = \frac{10m}{0.72} = \underline{13.88} mV$$
$$v_o = 13.88 \times 14.2 = \underline{197.2} mV$$

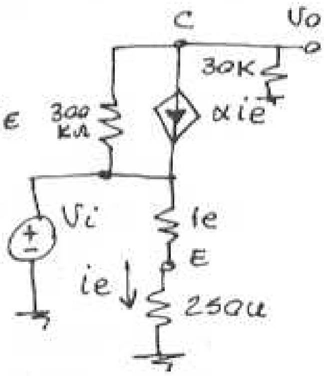
5.137

(a) $I_C = 0.99 \times 0.5$ mA
$= 0.495$mA



$V_C = I_E R_E + V_{BE} \ldots$
$+ I_B R_B$
$= 0.5 \times 0.175 + 0.7$
$+ 0.005 \times 300$
$= \underline{2.28V}$

(b) $i_e = \frac{v_i}{r_e + R_E}$



$r_e = \frac{V_T}{I_E} = 50\Omega$

$\rightarrow i_e = \frac{v_i}{50 + 250}$

$i_e = \frac{v_i}{300}$

Node equation at c:
$$\frac{v_o - v_i}{300K} + \alpha i_e + \frac{v_o}{30K} = 0$$
$$\frac{v_o - v_i}{300K} + \frac{\alpha v_i}{(250+50)} + \frac{v_o}{30K} = 0$$
$$\Rightarrow \frac{v_o}{v_i} = \underline{-90} \text{ V/V}$$

5.138

(a) Without R_e,
$$A_v \simeq \frac{-\beta (R_C \,\|\, R_L \,\|\, r_o)}{r_\pi + R_s}$$
CONT.

Since no value is specified for r_o (or V_A) we shall neglect its effect, thus

$$A_v = -\frac{\beta (R_C // R_L)}{r_\pi + R_s}$$

Substituting $r_\pi = \beta / g_m$ yields

$$A_v = -\frac{(R_C // R_L)}{\frac{\beta}{\beta+1}\cdot\frac{1}{g_m} + \frac{R_s}{\beta}}$$

$$= \frac{-(R_C // R_L)}{r_e + \frac{R_s}{\beta}}$$

Maximum gain is obtained at the high β,

$$A_{v\,max} = -\frac{(R_C // R_L)}{0.050 + \frac{10}{150}}$$

$r_e \rightarrow 0.050$

$$= -8.57\,(R_C // R_L)$$

The minimum gain is obtained for β at its lowest value

$$A_{v\,min} = -\frac{(R_C // R_L)}{0.050 + \frac{10}{50}}$$

$$= -4\,(R_C // R_L)$$

Thus, $\frac{A_{v\,max}}{A_{v\,min}} = \frac{8.57}{4} = \underline{\underline{2.14}}$

(b) With R_e

$$A_v \simeq -\frac{\beta (R_C // R_L)}{r_\pi (1 + g_m R_e) + R_s}$$

$$= -\frac{(R_C // R_L)}{\frac{1}{g_m}(1 + g_m R_e) + \frac{R_s}{\beta}}$$

$$A_v = \frac{-(R_C // R_L)}{\frac{1}{g_m} + R_e + \frac{R_s}{\beta}}$$

$$\frac{A_{v\,max}}{A_{v\,min}} = \frac{\frac{1}{g_m} + R_e + \frac{R_s}{\beta_{min}}}{\frac{1}{g_m} + R_e + \frac{R_s}{\beta_{max}}}$$

Thus,

$$1.2 = \frac{50 + R_e + \frac{10,000}{50}}{50 + R_e + \frac{10,000}{150}}$$

$$\Rightarrow R_e = \underline{\underline{550\,\Omega}}$$

(c) For $\beta = 100$:

(i) Without R_e,

$$A_v = \frac{-(R_C // R_L)}{r_e + R_s/\beta}$$

$$= \frac{-(R_C // R_L)}{0.050 + 10/100}$$

$$= -6.66\,(R_C // R_L)$$

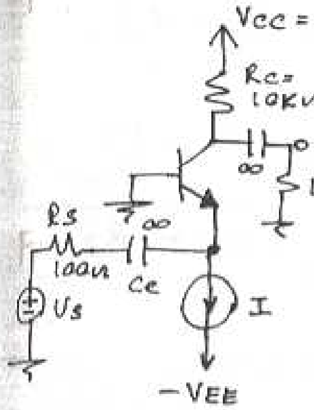
(ii) With $R_e = 550\,\Omega$

$$A_v \simeq \frac{-(R_C // R_L)}{0.050 + 0.550 + \frac{10}{100}}$$

$$= -1.43\,(R_C // R_L)$$

Thus including R_e reduces the gain by a factor of $\frac{6.66}{1.43} = \underline{\underline{4.6}}$

5.139



$R_i = r_e$

$r_e = \frac{25m}{I} = 100$

$I = \frac{25m}{100} = \underline{\underline{0.25m}}$

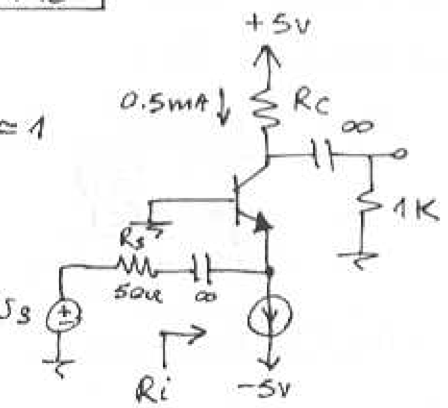
$\frac{v_o}{v_s} = -\alpha \frac{(R_C \| R_L)}{R_s + r_e}$

assuming $\alpha = 1$

$\frac{v_o}{v_s} = \frac{10 \| 10}{(0.100)\,2} = \underline{\underline{-25\ \frac{V}{V}}}$

5.140

$\alpha \simeq 1$



$R_i = \frac{V_T}{I} = 50\Omega \Rightarrow I = \underline{\underline{0.5mA}}$

$V_C = 5 - 0.5.R_C$

$V_{Cmin} = V_C - 0.01\, g_m\, (R_C \| 1K)$

To prevent saturation $V_{Cmin} = 0$

$\rightarrow 0 = V_C - 0.01 \times 20\, (R_C // 1)$

$= 5 - 0.5\ \frac{R_C}{R_C + 1}$

$5R_C + 5 - 0.5R_C^2 - 0.5R_C - 0.2R_C = 0$

$0.5R_C^2 - 4.3R_C + 5 = 0$

$R_C = \frac{4.3 + \sqrt{4.3^2 + 10}}{1}$

$= 9.64\,K\Omega$

Select $R_C = \underline{\underline{9.1K\Omega}}$

$V_C = 0.45V$

$\frac{v_o}{v_s} = \frac{R_i}{R_s + R_i}\, g_m\, (R_C \| 1)$

$= \frac{50}{50+50} \times 20 \times (9.1 // 1)$

$= \underline{\underline{9}}\ V/V$

For $v_{be\,max} = 10mV$

$v_{s\,max} = 20mV$

$V_{C\,max} = 180mV$

Thus the collector voltage swings from

$(0.45 - 0.18)V$ to $(0.45 + 0.18)V$

i.e $\underline{\underline{\text{from } 0.27V \text{ to } 0.63V}}$

5.141

Refer to Fig P5.141

$R_i = r_e = \frac{V_T}{I_E} = \frac{V_T}{0.5} = \underline{\underline{50\Omega}}$

To find the voltage gain v_o/v_s first note that

$\frac{v_e}{v_s} = \frac{R_i}{R_s + R_i} = \frac{50}{50+50} = 0.5$

Then,

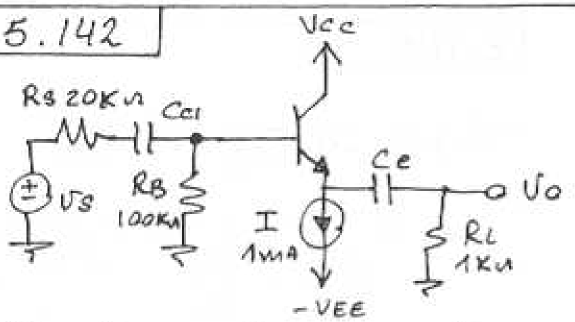
$\frac{v_c}{v_e} = \frac{\alpha \times (\text{Total resistance at c})}{r_e}$

$\simeq \frac{1 \times (100K\Omega // 1K\Omega)}{50\Omega}$

$= 19.8\ V/V$

Thus, $\frac{v_o}{v_s} = 19.8 \times 0.5 = \underline{\underline{9.9}}\ V/V$

5.142



$V_A = 100 \Rightarrow r_o = \dfrac{V_A}{I_o} = \dfrac{100}{\frac{\beta}{\beta+1} \times 1}$

$r_o = 101\,K\Omega$

(a) $R_{in} = R_B \| \left[(\beta+1)(r_e + (R_L \| r_o))\right]$

$R_{in} = 100K \| \left[(101)(25 + (1K \| 101K))\right]$

$R_{in} = \underline{\underline{50.6\,K\Omega}}$

$\dfrac{v_b}{v_s} = \dfrac{R_{in}}{R_{in}+R_s} = \underline{\underline{0.717}}\ V/V$

$\dfrac{v_o}{v_s} = \dfrac{v_b}{v_s} \cdot \dfrac{(R_L \| r_o)}{(R_L \| r_o) + r_e} = \underline{\underline{0.975}}\ V/V$

(b) The peak value of v_o occurs when the current flowing from ground through R_L equals the bias current I on a negative signal swing

$\therefore \dfrac{\hat{v}_o}{R_L} = I = 1mA$

$\hat{v}_o = 1V \Rightarrow \hat{v}_s = \dfrac{1}{0.975} = \underline{\underline{1.025\,V}}$

$v_\pi = v_s' \times \dfrac{(\beta+1) r_e}{(\beta+1) r_e + (\beta+1)(R_L \| r_o)}$

= : Where v_s' is

$v_s' = v_s \times \dfrac{R_{in}}{R_{in}+R_s} = v_s \times 0.717$

$v_\pi = \dfrac{(1.025 \times 0.717) \times 101 \times 25}{101[25+990]}$

$= \underline{\underline{18.10\,mV}}$

(c) $\hat{v}_\pi = 10mV \Rightarrow$

$v_s = \dfrac{10m \times 101\,[25+990]}{0.717 \times 101 \times 25}$

$= \underline{\underline{0.566\,V}}$

$v_o = \dfrac{v_o}{v_s} \times 0.566 = \underline{\underline{0.552}}\ V$

(d) $R_o = \left(r_e + \dfrac{(R_s \| R_B)}{\beta+1}\right) \| r_o$

$= \left(25 + \dfrac{(20K \| 100K)}{101}\right) \| 101K$

$= \underline{\underline{190\,\Omega}}$

Open circuit voltage gain

$A_v\big|_{R_L=\infty} = \dfrac{R_B}{R_s+R_B} \cdot \dfrac{r_o}{r_o + r_e + \frac{R_s \| R_B}{\beta+1}}$

$= \dfrac{100}{20+100} \cdot \dfrac{101K}{101K + 25 + \frac{(100 \| 20)K}{101}}$

$= \underline{\underline{0.832}}\ V/V$

$A_v = A_v\big|_{R_L=\infty} \times \dfrac{R_L}{R_L+R_o}$

$= 0.832 \times \dfrac{500}{500+190}$

$= 0.\underline{603}\ V/V$

5.143

Refer to Fig. P5.143

(a) $I_E = \dfrac{9-0.7}{1+100/(\beta+1)}$ CONT.

For $\beta = 40$, $I_E = \dfrac{8.3}{1+\frac{100}{41}} = \underline{\underline{2.41}}\ mA$

$V_E = 1 \times 2.41 = \underline{2.41\ V}$

$V_B = 2.41 + 0.7 = \underline{3.11\ V}$

For $\beta = 200$, $I_E = \dfrac{8.3}{1+\frac{100}{201}} = \underline{5.54}\ mA$

$V_E = +\underline{5.54V}$

$V_B = +\underline{6.24}V$

(b) $R_i = 100K\Omega // (\beta+1)[r_e + (1//1)]$
$= 100 // (\beta+1)[r_e + 0.5]$

For $\beta = 40$, $I_E = 2.41 mA$
$\rightarrow r_e = 10.37\ \Omega$

thus $R_i = 100 // 41 \times (0.01037 + 0.5)$
$= 100 // 21$
$= \underline{17.30}\ \Omega$

For $\beta = 200$, $I_E = 5.54 mA$
$\rightarrow r_e = 4.51\ \Omega$

thus $R_i = 100 // 201 (0.0045 + 0.5)$
$= 100 // 101.4$
$= \underline{50.3}\ K\Omega$

(c) $\dfrac{v_o}{v_s} = \dfrac{v_b}{v_s} \cdot \dfrac{v_o}{v_b}$
$= \dfrac{R_i}{R_s + R_i} \cdot \dfrac{(1//1)}{(1//1) + r_e}$

For $\beta = 40$,
$\dfrac{v_o}{v_s} = \dfrac{17.3}{10 + 17.3} \times \dfrac{0.5}{0.5 + 0.01037}$
$= \underline{\underline{0.621}}\ V/V$

For $\beta = 200$,
$\dfrac{v_o}{v_s} = \dfrac{50.3}{10 + 50.3} \cdot \dfrac{0.5}{0.5 + 0.0045}$
$= \underline{\underline{0.827}}\ V/V$

5.144

Refer to Fig. P5.144

$I_E = \dfrac{5 - 0.7}{3.3 + \frac{100}{101}} = \underline{\underline{1.00}}\ mA$

$r_e = \dfrac{25}{1.00} = 25\Omega$

$R_i = (\beta+1)[r_e + (3.3//1)]$
$= \underline{\underline{80.0}}\ K\Omega$

$\dfrac{v_o}{v_s} = \dfrac{v_b}{v_s} \cdot \dfrac{v_o}{v_b} = \dfrac{R_i}{R_s + R_i} \ \dfrac{(3.3//1)}{r_e + (3.3//1)}$

Thus,

$\dfrac{v_o}{v_s} = \dfrac{80}{100 + 80} \times \dfrac{(3.3//1)}{0.025 + (3.3//1)}$
$= \underline{\underline{0.430}}\ V/V$

$\dfrac{i_o}{i_i} = \dfrac{v_o / R_L}{v_s / (R_s + R_i)}$
$= \dfrac{v_o}{v_s} \cdot \dfrac{(R_s + R_i)}{R_L}$
$= 0.43 \times \dfrac{(100 + 80)}{1}$
$= \underline{\underline{77.4}}\ A/A$

$R_{out} = 3.3 // \left[r_e + \dfrac{100}{\beta+1}\right]$
$= 3.3 // \left[0.025 + \dfrac{100}{101}\right]$
$= \underline{\underline{0.776}}\ K\Omega$

5.145

Refer to Fig 5.63(a)

$R_o = r_o // [r_e + 10K / \beta+1]$

CONT.

$$R_o = \frac{V_A}{I_C} \;//\; \left[\frac{V_T}{I_E} + \frac{10\,K\Omega}{\beta+1}\right]$$

$$R_o = \frac{125}{0.99\times2.5} \;//\; \left[\frac{0.025}{2.5} + \frac{10}{100+1}\right]$$
$$= 50.5 \;//\; [0.010 + 0.099]$$
$$= 0.109\,K\Omega = \underline{109\,\Omega}$$

With no load:

$$R_i = (\beta+1)[r_e + r_o]$$

For $\beta = 100$, $r_e = 10\Omega$, $r_o = 50.5\,K\Omega$

$$R_i = 101 \times (0.010 + 50.5)$$
$$= \underline{5.1}\,M\Omega$$

$$\frac{v_o}{v_s} = \frac{v_b}{v_s}\cdot\frac{v_o}{v_b} = \frac{R_i}{R_s+R_i} \times \frac{r_o}{r_o+r_e}$$
$$= \frac{5100}{10+5100}\cdot\frac{50.5}{50.5+0.01}$$
$$= \underline{0.988}\,V/V$$

With $R_L = 1\,K\Omega$:

$$\frac{v_o}{v_s} = 0.998 \times \frac{R_L}{R_L+R_o}$$
$$= 0.998 \times \frac{1}{1+0.109} = \underline{0.890}\,V/V$$

Largest negative output signal occurs when the BJT cuts off, thus:

$$v_{omin} = -2.5\,mA \times 1\,K\Omega$$
$$= \underline{-2.5\,V}$$

Largest positive output signal occurs when $v_B = +3.4\,V$. The corresponding value of v_E is $\simeq 3.4 - 0.7 = +2.7\,V$ (where we have neglected the signal component of v_{BE})

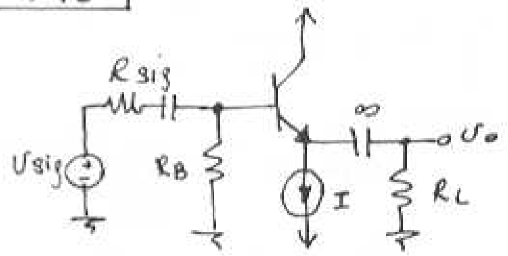
Now, the DC level at the emitter with zero input signal is

$$V_E = -10\,K\Omega \times I_B - 0.7$$
$$= -10 \times \frac{2.7}{101} - 0.7$$
$$\simeq -0.97\,V$$

Thus the signal component is:

$$v_{e\,max} = v_E - V_E$$
$$= 2.5 - (-0.97)$$
$$= \underline{3.47\,V}$$

5.146



$$G_v\big|_{R_L=\infty} = \frac{R_B}{R_{sig}+R_B}\cdot\underbrace{\frac{r_o}{(R_{sig}//R_B + r_e + r_o)}}_{\simeq 1 \text{ for very large } r_o} = 0.99$$

$$\Rightarrow \frac{R_B}{R_{sig}+R_B} = 0.99$$

$$\rightarrow \frac{R_B}{10K+R_B} = 0.99 \Rightarrow R_B = 990\,K\Omega$$

$$R_{out} = r_o \,//\, \left(r_e + \frac{R_s//R_B}{\beta+1}\right)$$

for r_o large: $R_{out} \simeq r_e + \dfrac{R_s//R_B}{\beta+1}$

$$r_e + \frac{(10//990)K}{\beta+1} = 200\,\Omega$$

$$r_e + \frac{(20//990)K}{\beta+1} = 300\,\Omega$$

CONT.

Then,

$$r_e + \frac{9.9K}{\beta+1} = 200 \quad (1)$$

$$r_e + \frac{19.6K}{\beta+1} = 300 \quad (2)$$

Solving Eqs. (1) and (2)

$$\beta+1 = 97 \rightarrow \beta = 96$$

and $r_e = 98\,\Omega$

If: $R_{sig} = 30K\Omega$ and $R_L = 1K\Omega$

$$G_v = \frac{R_B}{R_{sig}+R_B} \cdot \frac{(r_o \| R_L)}{\frac{R_{sig}\|R_B}{\beta+1} + r_e + (r_o\|R_L)}$$

If r_o is large:

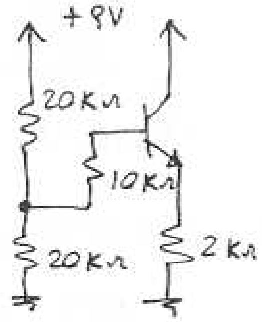
$$G_v = \frac{R_B}{R_{sig}+R_B} \cdot \frac{R_L}{\frac{R_{sig}\|R_B}{\beta+1} + r_e + R_L}$$

$$= \frac{990}{30+990} \cdot \frac{1}{\frac{(30\|990)}{97} + 0.098 + 1}$$

$$= \underline{0.7\ V/V}$$

5.147

(a) DC Analysis



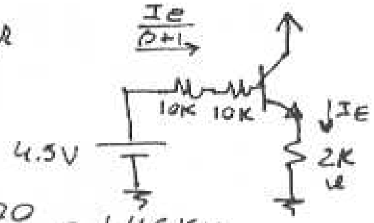
$$I_E = \frac{4.5-0.7}{2 + \frac{10+10}{101}}$$

$$= \underline{1.73mA}$$

$$I_C = 0.99 \times 1.73 = \underline{1.71\,mA}$$

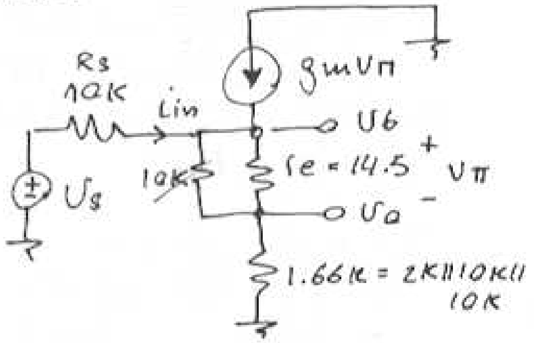
$$g_m = \frac{I_C}{V_T} = 68.5\ mA/V$$

$$r_e = \frac{V_T}{I_e} = \underline{14.5\,\Omega}$$



$$r_\pi = \frac{\beta}{g_m} = \frac{100}{68.5} = \underline{1.46\,K\Omega}$$

(b) Simplified equivalent circuit:



$$v_o \cong 1.66K \times \left(\frac{V_\pi}{14.5}\right) = 114.48\,V_\pi$$

$$\frac{V_\pi}{14.5} = i_{in} + 68\times10^{-3}\times V_\pi$$

$$\rightarrow V_\pi\left(\frac{1}{14.5} - 68\times10^{-3}\right) = i_{in}$$

$$i_{in} = \frac{V_\pi}{1.035K}$$

$$v_b = V_\pi + v_o = 115.48\,V_\pi$$

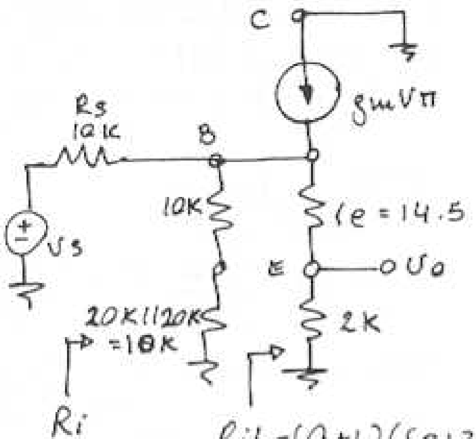
$$R_i \equiv \frac{v_b}{i_{in}} = \frac{115.48\,V_\pi}{V_\pi/1.035} = \underline{120\,K\Omega}$$

$$\frac{v_o}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b} = \frac{R_i}{R_i+R_s} \cdot \frac{114.48\,V_\pi}{115.48\,V_\pi}$$

$$= \frac{120}{120+10} \times \frac{114.48}{115.48} = \underline{0.92\,V/V}$$

(c) With C_B open-circuited so that bootstraping is eliminated, we obtain the following equivalent circuit model:

CONT.



$R_{ib} = (\beta+1)(r_e + 2K)$
$= 203.46\,K\Omega$

$R_i = (10K + 20K \| 20K) \,\|\, 203.46K$
$= \underline{\underline{18.21\,K\Omega}}$ (much lower than the value obtained with bootstrap.)

$$v_o = \frac{V_\pi}{14.5} \times 2K = 138 \times V_\pi$$

$$v_b = v_o + v_\pi = (1 + 138) V_\pi = 139\, V_\pi$$

$$\frac{v_o}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b} = \frac{R_i}{R_s + R_i} \cdot \frac{v_o}{v_b}$$

$$= \frac{18.21}{10 + 18.21} \cdot \frac{138}{139} = \underline{\underline{0.64}}\ V/V$$

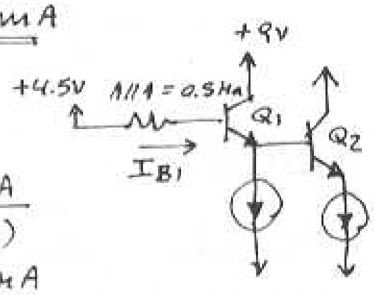
Much lower than the value obtained with bootstrapping. This is due to the lower R_i. Bootstrapping raises the component of input resistance due to the base biasing network.

5.148

Refer to the circuit in Fig P5.148

(a) $I_{E2} = \underline{5\,mA}$
$\beta_1 = 50, \quad \beta_2 = 100$

$$I_{E1} = 50\mu + I_{B2} = 50 + \frac{I_{E2}}{\beta_2 + 1} = 50 + \frac{5000}{101}$$
$$\simeq \underline{\underline{0.1\,mA}}$$



$$I_{B1} = \frac{0.1\,mA}{(50+1)} = 1.96\,\mu A$$

$$V_{B1} = 4.5 - 0.5 \times 1.96 = \underline{\underline{3.52V}}$$

$$V_{B2} = 3.52 - 0.7 = \underline{\underline{2.82}}\ V$$

(b) Refer to Fig. P.5.148

$$\frac{v_o}{v_{b2}} = \frac{R_L}{R_L + r_{e2}}$$

$R_L = 1\,K\Omega \quad r_{e2} = \frac{25}{5} = 5\Omega$

$$\frac{v_o}{v_{b2}} = \frac{1}{1 + 0.005} = \underline{\underline{0.995}}\ V/V$$

$$R_{ib2} = (\beta_2 + 1)(r_{e2} + R_L) = (101) \times (1.005) = \underline{\underline{101.5\,K\Omega}}$$

(c) $$\frac{v_{e1}}{v_{b1}} = \frac{R_{ib2}}{R_{ib2} + r_{e1}}$$

$$r_{e1} = \frac{V_T}{100\,\mu A} = 250\Omega$$

$$\rightarrow \frac{v_{e1}}{v_{b1}} = \frac{101.5}{101.5 + 0.25} = \underline{\underline{0.997}}\ \frac{V}{V}$$

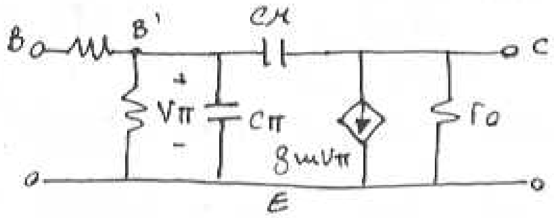
$$R_i = 1M\Omega \| 1M\Omega \| (\beta_1 + 1)(r_{e1} + R_{ib2})$$
$$= 1 \| 1 \| 51 \times (0.25 + 101.5)\,K\Omega$$
$$= 1 \| 1 \| 5.2\,M\Omega$$
$$= 0.499\,M\Omega = \underline{\underline{499\,K\Omega}}$$

(d) $$\frac{v_{b1}}{v_s} = \frac{R_i}{R_s + R_i} = \frac{499}{100 + 499} = 0.833\ V/V$$

CONT.

(e) $\frac{v_o}{v_s} = \frac{v_{b1}}{v_s} \cdot \frac{v_{e1}}{v_{b1}} \cdot \frac{v_o}{v_{e1}}$
$= 0.833 \times 0.997 \times 0.995$
$= \underline{\underline{0.826}}$ V/V

5.149



$r_x = \underline{\underline{100\Omega}}$

$g_m = \frac{I_C}{V_T} = \frac{0.5mA}{25mV} = \underline{\underline{20\ mA/V}}$

$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{20} = \underline{\underline{5K\Omega}}$

$r_o = \frac{V_A}{I_C} = \frac{50V}{0.5mA} = \underline{\underline{100K\Omega}}$

$C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^{0.5}} = \frac{30}{\left(1+\frac{2}{0.75}\right)^{0.5}} = \underline{\underline{15.7}}\ fF$

$C_{je} \simeq 2C_{jeo} = 2\times 20 = 40 fF$

$C_{de} = \tau_F g_m = 30\times10^{-12}\times 20\times10^{-3} = 600 fF$

$C_\pi = C_{je} + C_{de} = \underline{\underline{0.640}}\ pF$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{20\times10^{-3}}{2\pi(0.64+0.016)\times10^{-12}}$
$= \underline{\underline{4.85}}$ GHz

5.150

$|h_{fe}| \simeq f_T/f$

• At $I_C = 0.2mA$, $|h_{fe}| = 2.5$ at $f = 500 MHz$, thus:

$f_T = 2.5\times500 = \underline{\underline{1.25}}$ GHz

• At $I_C = 1.0mA$, $|h_{fe}| = 11.6$ at $f = 500 MHz$, thus:

$f_T = 11.6 \times 500 = \underline{\underline{5.8}}$ GHz

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} \rightarrow C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$

$C_\pi(I_C = 0.2mA) = \frac{8\times10^{-3}}{2\pi\times1.25\times10^9} - 0.05.10^{-12}$
$= 0.9686\ pF$

$C_\pi(I_C = 1.0mA) = \frac{40\times10^{-3}}{2\pi\times5.8\times10^9} - 0.05\times10^{-12}$
$= 1.0476\ pF$

Since $C_\pi = C_{je} + \tau_F g_m$,

$C_{je} + 8\times10^{-3}\tau_F = 0.9686\times10^{-12}$ (1)

$C_{je} + 40\times10^{-3}\tau_F = 1.0476\times10^{-12}$ (2)

Solving Eqn. (1) and (2) together yields,

$C_{je} = \underline{\underline{0.95}}pF$, $\tau_F = \underline{\underline{2.47}}ps$

5.151

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$
$= \frac{80\times10^{-3}}{2\pi(10+1)\times10^{-12}}$
$= \underline{\underline{4.24GHz}}$

$f_\beta = f_T/\beta_0 = (4.24/150)\times10^9$
$= \underline{\underline{28.26MHz}}$

5.152

At $I_C = 2mA$ the diffusion part of C_π is: $10-2 = 8\ pF$

CONT.

At $I_C = 0.2\,mA$ the diffusion capacitance becomes 0.8 pF and thus,

$$C_\pi = 0.8 + 2 = 2.8\,pF$$

and:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{8\times10^{-3}}{2\pi(2.8+1)\times10^{-12}} = \underline{\underline{335\,MHz}}$$

5.153

$$\omega_T = g_m/(C_\pi + C_\mu)$$

$$2\pi\times5\times10^9 = \frac{20\times10^{-3}}{(C_\pi+0.1)\times10^{-12}}$$

$$C_\pi + 0.1 = \frac{20}{10\pi} = 0.64\,pF$$

$$C_\pi = \underline{\underline{0.54}}\,pF$$

$$g_m = \underline{\underline{20\,mA/V}}$$

$$r_\pi = \beta/g_m = 150/20 = \underline{\underline{7.5\,k\Omega}}$$

$$f_\beta = f_T/\beta = \frac{5\times10^9}{150} = \underline{\underline{33.3\,MHz}}$$

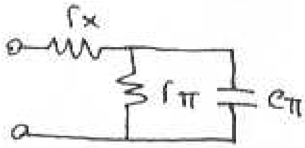
5.154

$|h_{fe}|$ becomes 20 at:

$$f_T/20 = \frac{1\times10^9}{20} = \underline{\underline{50}}\,MHz$$

$$f_\beta = f_T/\beta_0 = \frac{1000\,MHz}{200} = \underline{\underline{5\,MHz}}$$

5.155



$$Z = r_x + \frac{1}{\frac{1}{r_\pi} + j\omega C_\pi}$$

$$= r_x + \frac{r_\pi}{1 + j\omega C_\pi r_\pi}$$

$$Z = r_x + \frac{r_\pi}{1 + j(\omega/\omega_\beta)}$$

$$= r_x + \frac{r_\pi(1 - j\omega/\omega_\beta)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$= r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} - j\cdot\frac{r_\pi(\omega/\omega_\beta)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$\mathrm{Re}[Z] = r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

For $\mathrm{Re}[Z]$ to be an estimate of r_x good to within 10% we must keep

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \le \frac{r_x}{10}$$

But $r_x \le r_\pi/10$

Thus,

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \le \frac{r_\pi}{100}$$

$$1 + \left(\frac{\omega}{\omega_\beta}\right)^2 \ge 100$$

or $\omega \ge 10\,\omega_\beta$ (approx.)

5.156

See completed table below

CONT.

	I_E (mA)	r_e (Ω)	g_m (mA/V)	r_π (KΩ)	β_0
(a)	1	25	40	2.5	100
(b)	1	25	40	3.13	125.3
(c)	0.99	25.3	39.6	2.525	100
(d)	10	2.5	400	0.25	100
(e)	0.1	250	4	25	100
(f)	1.0	25	40	0.25	10
(g)	1.25	20	50	0.20	10

CONT.	f_T (MHz)	C_π (pF)	C_μ (pF)	f_β (MHz)
(a)	400	2	13.9	4
(b)	501.3	2	10.7	4
(c)	400	2	13.8	4
(d)	400	2	157	4
(e)	100	2	4.4	1
(f)	400	2	13.9	40
(g)	800	1	9	80

5.157

From Example 5.18:

$I = 2mA$, $\beta = 100$, $f_T = 800MHz$

$R_B = 50K\Omega$, $R_C = 4K\Omega$, $r_x = 50\Omega$

$V_A = 100$, $C_\mu = 1pF$, $R_{sig} = 5K\Omega$

$R_L = 5K\Omega$

$$g_m = \frac{2mA}{25mV} = 80\frac{mA}{V}$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{80m} = 1250\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100V}{2mA} = 50K\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{80\times10^{-3}}{2\pi\times800\times10^6} = 16pF$$

$$C_\mu = 1pF \Rightarrow C_\pi = 15pF$$

$$A_M = \frac{-R_B}{R_B+R_{sig}} \cdot \frac{r_\pi \times g_m R_L'}{(r_\pi + r_x + (R_B /\!/ R_{sig}))}$$

where $R_L' = r_o /\!/ R_C /\!/ R_L$

$= (50/\!/4/\!/5)\ K\Omega$

$= 2.1 K\Omega$

$$A_M = \frac{-50}{50+5} \cdot \frac{1250 \times 168}{1250+50+(50/\!/5)K\Omega}$$

where $168 = g_m \times R_L'$

$= 80\times10^{-3} \times 2.1 \cdot 10^3$

Then: $A_M = -32.6$

$20 \log |A_M| = \underline{\underline{30.3dB}}$

$$C_{in} = C_\pi + C_\mu(1 + g_m R_L')$$

$= 15 + 1(1+168) = 184pF$

$R'_{sig} = r_\pi /\!/ [r_x + (R_B /\!/ R_{sig})]$

$= 1250 /\!/ [50 + (50K /\!/ 5K)]$

$= 983\Omega$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} = \frac{1}{2\pi\times184\times10^{-12}\times983}$$

$= \underline{\underline{880KHz}}$

Gain-bandwidth product

$GB = |A_M| \times f_H = 32.6 \times 880\times10^3$

$= 29\times10^6$

Previously, in example 5.18

$GB = 39\times754\times10^3 = 29\times10^6$

Thus, the designer traded gain for bandwidth by increasing I. However, by doubling I the dissipation increased by a factor of 2, since:

$$Power = \underset{2I}{\underline{I}} \times V_{supply}$$

5.158

Refer to Fig. 5.71(a)

$R_B \gg R_{sig}$, $r_x \ll R_{sig}$

$R_{sig} \gg r_\pi$, $g_m R_L' \gg 1$

$g_m R_L' C_\mu \gg C_\pi$ CONT.

$$A_M = \underbrace{\frac{-R_B}{R_B + R_{sig}}}_{④} \cdot \frac{r_\pi \cdot g_m \cdot R_L'}{\underbrace{r_\pi + \underbrace{r_x + \underbrace{(R_{sig} \| R_B)}_{①}}_{②}}_{③}}$$

① $= R_{sig}$, since $R_B >> R_{sig}$
② $= R_{sig}$, since $R_{sig} >> r_x$
③ $= R_{sig}$, since $R_{sig} >> r_\pi$
④ $= 1$, since $R_B >> R_{sig}$

Thus,

$$A_M = -\frac{r_\pi \cdot g_m \cdot R_L'}{R_{sig}}$$

but $r_\pi = \frac{\beta}{g_m}$

$$\Rightarrow A_M = -\frac{\beta R_L'}{R_{sig}} \quad \text{Q.E.D}$$

(b) $$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R'_{sig}}$$

$$C_{in} = \underbrace{C_\pi + C_\mu\,(1 + \underbrace{g_m R_L'}_{①})}_{②}$$

① $= g_m R_L'$, since $R_L' >> 1$
② $= C_\mu g_m R_L'$, since $C_\mu g_m R_L' >> C_\pi$

$\Rightarrow C_{in} = C_\mu \cdot g_m \cdot R_L'$

$$R'_{sig} = \underbrace{r_\pi \;\|\; [\underbrace{r_x + \underbrace{(R_B \| R_{sig})}_{①}}_{②}]}_{③}$$

① $= R_{sig}$, since $R_B >> R_{sig}$
② $= R_{sig}$, since $R_{sig} >> r_x$
③ $= r_\pi$, since $R_{sig} >> r_\pi$

$\Rightarrow R'_{sig} = r_\pi$

Thus,

$$f_H = \frac{1}{2\pi \cdot C_\mu \cdot g_m \cdot R_L' \cdot r_\pi}$$

Q.E.D.

(c)

$$GB = |A_M| \times f_H = \beta \frac{R_L'}{R_{sig}} \frac{1}{2\pi C_\mu \beta R_L'}$$

$$= \frac{1}{2\pi} \frac{1}{C_\mu \cdot R_{sig}} \quad \text{Q.E.D.}$$

For: $C_\mu = 1pF$ and $R_{sig} = 25K\Omega$

$$GB = \frac{1}{2\pi \times 10^{-12} \times 25 \times 10^3} \approx 6.36 \times 10^6$$

For: $I_C = 1mA$, $\beta = 100$, $R_{sig} = 25K\Omega$

1) If $R_L' = 25K\Omega$

$$A_M = -100 \cdot \frac{25K}{25K} = \underline{\underline{-100}}$$

$$20 \log|100| = 40dB$$

$$f_H = \frac{1}{2\pi \times 10^{-12} \times 100 \times 25 \times 10^3} = \underline{\underline{63.6KHz}}$$

2) If $R_L' = 2.5K\Omega$

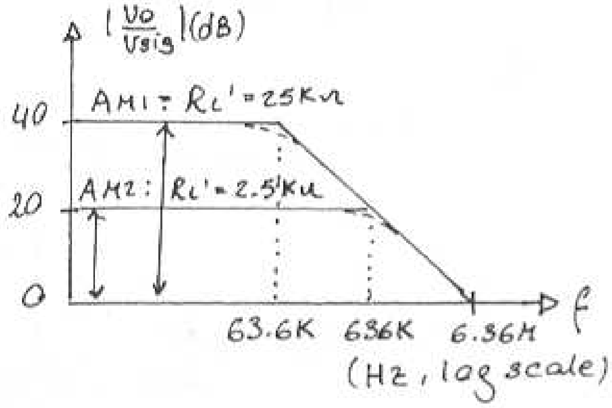
$$A_M = -100 \cdot \frac{2.5K}{25K} = \underline{\underline{-10}}$$

$$20 \log|10| = 20$$

CONT.

$$f_H = \frac{1}{2\pi \times 10^{-12} \times 100 \times 2.5.10^3}$$

$$f_H = \underline{\underline{636}} \text{ KHz}$$



$GP = 6.36 \times 10^6 = A_M \times f_H$

when $A_M = 1 \Rightarrow f_H = \underline{\underline{6.36.10^6}}$ Hz

$$R_L' = \frac{1}{2\pi(6.36 \times 10^6)\underbrace{C_\mu}_{1\times10^{-12}} \times \underbrace{\beta}_{100}}$$

$R_L' = \underline{\underline{250\Omega}}$

5.159

Refer to Fig. P5.159

$R_{in} = R_1 \,//\, R_2 \,//\, (r_x + r_\pi)$

where $R_1 = 33K\Omega$, $R_2 = 22K\Omega$

Next →

CONT.

$r_x = 50$ and,

$r_\pi = \frac{\beta_0}{g_m} = \frac{120}{0.3\times 40} = \frac{120}{12} = 10K\Omega$

$R_{in} = 33 \| 22 \| 10.05 = \underline{\underline{5.7K\Omega}}$

$A_M = \frac{-R_{in}}{R_{in}+R_S} \cdot \frac{r_\pi}{r_\pi + r_x} \cdot g_m\,(R_C \| R_L \| r_o)$

$= -\frac{5.7}{5.7+5} \cdot \frac{10}{10+0.05} \cdot 12\,(4.7 \| 5.6 \| 300)$

$= \underline{\underline{-16.11}}$ V/V

$R'_{sig} = r_\pi \| [r_x + (R_1 \| R_2 \| R_{sig})]$

$= 10K \| [50 + (33 \| 22 \| 5)K]$

$= \underline{\underline{2.69K\Omega}}$

$R'_L = r_o \| R_C \| R_L = 300 \| 4.7 \| 5.6 (K\Omega)$

$= 2.53K\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi \cdot f_T} = \frac{12 \cdot 10^{-3}}{2\pi \times 700 \cdot 10^6} =$

$= 2.73pF$

$C_\pi = (2.73 - 1)pF$

$= 1.73pF$

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 1.73p + 1p(1 + 12\times 2.53)$

$= 33pF$

$f_H = 1/2\pi C_{in} \cdot R'_{sig}$

$= 1/2\pi \times 33 \cdot 10^{-12} \times 2.69 \cdot 10^3$

$= \underline{\underline{1.79MHz}}$

5.160

$R_{in} = R_1 \| R_2 \| r_\pi$

where $r_\pi = \frac{\beta}{g_m}$ and $g_m = \frac{I_C}{V_T}$

$g_m = \frac{0.8}{0.025} = 32mA/V$

$r_\pi = \frac{200}{32} = 6.25K\Omega$

$R_{in} = 68 \| 27 \| 6.25 = 4.72K\Omega$

$R'_L = R_C \| R_L = 4.7 \| 10 = 3.2K\Omega$

$A_M = \frac{R_{in}}{R_S + R_{in}} \times -g_m R'_L$

$= \frac{-4.72}{10+4.72} \times 32 \times 3.2$

$= \underline{\underline{-32.8}}$ V/V

$C_T = C_\pi + C_\mu(1 + g_m R'_L)$

where $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{32\times 10^{-3}}{2\pi \times 10^9}$

$= 5.1pF$

$C_\pi = 5.1 - 0.8 = 4.3pF$

$C_T = 4.3 + 0.8(1 + 32\times 3.2)$

$= 87pF$

The resistance seen by C_T is R_T,

$R_T = r_\pi \| R_1 \| R_2 \| R_S$

$= 6.25 \| 68 \| 27 \| 10 = 3.2K\Omega$

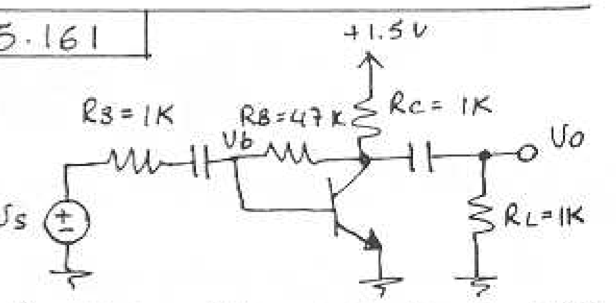
Thus

$f_H \simeq \frac{1}{2\pi C_T R_T}$

$= \frac{1}{2\pi \times 87\times 10^{-12} \times 3.2\times 10^3}$

$= \underline{\underline{572}}$ KHz

5.161



$\beta = 100$, $C_\mu = 0.8pF$, $f_T = 600MHz$

CONT.

(a) $1.5V = 1K(I_C + I_B) + 47K \cdot I_B + 0.7V$

$= I_C + \frac{I_C}{\beta} + 47\frac{I_C}{\beta} + 0.7$

$I_C = \frac{0.8}{1 + \frac{48}{\beta}} = \frac{0.8}{1 + \frac{48}{100}} = \underline{\underline{0.54 mA}}$

(b) $g_m = \frac{I_C}{V_T} = 40 \times 0.54 = \underline{\underline{21.6 \frac{mA}{V}}}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{21.6} = \underline{\underline{4.63 K\Omega}}$

(c) $V_o \simeq -g_m (R_C \parallel R_L) \cdot v_b$

$= -21.6 (1 \parallel 1) v_b$

$= -10.8 v_b$

$\Rightarrow \frac{v_o}{v_b} = \underline{\underline{-10.8}} \ V/V$

(d) Using Miller's theorem to find R_i:

$R_i = \frac{47}{1 - \frac{V_o}{v_b}} = \frac{47}{1 + 10.8} = \underline{\underline{4 K\Omega}}$

(e) $A_M = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b} = \frac{R_i}{R_S + R_i} \times \frac{v_o}{v_b}$

$= \frac{4K}{1K + 4K} \times -10.8 = \underline{\underline{-8.64}}$

(f) $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^{-3}}{2\pi \cdot 600 \times 10^6}$

$= 5.73 pF$

$C_\pi = 5.73 - 0.8 = 4.93 pF$

$C_{in} = C_\pi + C_\mu (1 + g_m R_L')$

$= 4.93 + 0.8 (1 + 21.6 \times (1 \parallel 1))$

$= 14.37 pF$

(g) $f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{in}}$

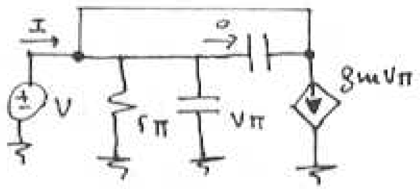
$R_{in} = R_i \parallel R_S = 4K \parallel 1K$

$= 0.8 K\Omega$

$\Rightarrow f_H = \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.8 \times 10^3}$

$f_H = \underline{\underline{13.84 MHz}}.$

5.162



$I = \frac{V}{r_\pi} + sC_\pi V_\pi + g_m V_\pi$

$Y_{in} = (g_m + \frac{1}{r_\pi}) + sC_\pi$

$Z_i = \frac{1}{(g_m + \frac{1}{r_\pi}) + sC_\pi}$

$= \frac{1}{\frac{1}{r_e} + sC_\pi} = \frac{r_e}{1 + sC_\pi r_e}$

$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)}$

Since C_π contains a component that is proportional to the bias current, it follows that at high currents $C_\pi \gg C_\mu$ and

$f_T \simeq \frac{g_m}{2\pi C_\pi} \simeq \frac{1}{2\pi \cdot C_\pi r_e}$

Thus,

$Z_i = \frac{r_e}{1 + s/\omega_T}$ (at high currents)

The phase angle will be -45° at $\omega = \omega_T$, or

$f = f_T = \underline{\underline{400 MHz}}$

CONT.

For a lower bias current so that $C_\pi = C_\mu$,

$$f_T = \frac{1}{4\pi C_\pi r_e}$$

and $$Z_i = \frac{r_e}{1 + \frac{s}{2\omega_T}}$$

$-45°$ angle is obtained at $\omega = 2\omega_T$ or $f = 2f_T$
$= \underline{\underline{800\,MHz}}$

(Assuming f_T remains constant which is not necessarily true)

$\boxed{5.163}$

Refer to Fig. P5.159 and Problem 5.159

$\omega_L = \omega_{C1} + \omega_{C2} + \omega_{CE}$

$\omega_{C1} = \frac{1}{C_{C1} \cdot R_{C1}} \Rightarrow f_{C1} = \frac{1}{2\pi C_{C1} \cdot R_{C1}}$

$R_{C1} = R_S + [R_1 \| R_2 \| (r_x + r_\pi)]$
$= 5K + [33K \| 22K \| (50 + 10K)]$
$= 10.7\,K\Omega$

$C_{C1} = 1\mu F$

$\Rightarrow f_{C1} = \frac{1}{2\pi \cdot 1 \times 10^{-6} \times 10.7 \cdot 10^3}$
$= \underline{\underline{14.87\,Hz}}$

$f_{C2} = 1/2\pi \cdot C_{C2} \cdot R_{C2}$

$R_{C2} = R_L + (R_C \| r_o)$
$= 5.6K + (4.7K \| 300K)$
$= \underline{10.23\,K\Omega}$

$C_{C2} = 1\mu F$

$\Rightarrow f_{C2} = \frac{1}{2\pi \times 1 \cdot 10^{-6} \times 10.23 \cdot 10^3}$
$= \underline{\underline{15.55\,Hz}}$

$f_{CE} = 1/2\pi C_E R_E'$

$R_E' = R_E \| \left[\frac{r_\pi + r_x + (R_1 \| R_2 \| R_S)}{\beta_0 + 1}\right]$
$= 3.9 \| \left(\frac{10K + 50 + (33 \| 22 \| 5)K}{121}\right)$
$= 109.8\,\Omega$

$C_E = 10\mu F$

$\Rightarrow f_{CE} = \frac{1}{2\pi \times 10 \times 10^{-6} \times 109.8}$
$= \underline{\underline{144.95\,Hz}}$

$f_L = \frac{\omega_L}{2\pi} = 14.87 + 15.55 + 109.8$

$f_L = \underline{\underline{140.22\,Hz}}$

$\boxed{5.164}$

Refer to Fig P5.159 and Problem 5.163.

To select C_E so that it contributes 90% of the value of ω_L:

$\frac{1}{2\pi C_E \cdot R_E'} = 0.9 \times 100$

$R_E' = 109.8\,\Omega$ (From problem 5.163)

$\Rightarrow C_E = \frac{1}{2\pi \cdot 109.8 \times 90}$
$= 16.1\mu F$

To select C_1 so that it contributes 5% of f_L:

$R_{C1} = 10.7\,K\Omega$ (From P 5.163)

$\Rightarrow C_1 = \frac{1}{2\pi \times 10.7 \cdot 10^3 \times 0.05 \times 100}$

$C_1 = 2.97\mu F$

CONT.

To select C_2 so that it contributes to 5% of f_L:

$R_{C2} = 10.23\,K\Omega$ (From P5.163)

$$\Rightarrow C_2 = \frac{1}{2\pi \times 10.23.10^3 \times 0.05 \times 100}$$

$$= \underline{3.11\,\mu F}$$

5.165

Refer to Fig. P5.159.

$$R_{C1} = R_S + [R_B \,//\, (r_x + r_\pi)]$$

$$= 10 + [10 \,//\, (0.1 + 1)]$$

$$= 10.99\,K\Omega$$

$$R_E' = R_E \,//\, \frac{r_\pi + r_x + (R_B // R_S)}{\beta_0 + 1}$$

$$\simeq 1 \,//\, \frac{1 + 0.1 + (10//10)}{100 + 1}$$

$$\simeq 57\,\Omega$$

For C_E and C_{C1} to contribute equally to the determination of f_L,

$$C_E R_E' = C_{C1} \cdot R_{C1}$$

$$\Rightarrow \frac{C_E}{C_{C1}} = \frac{R_{C1}}{R_E'} = \frac{10.99}{0.057} = \underline{193}$$

5.166

Refer to Fig P5.166.

(a) At midband frequencies

$$I_B = \frac{V_S}{R_S + r_\pi}$$

$$I_C = \beta \cdot I_b = \frac{\beta V_S}{R_S + r_\pi}$$

$$V_o = -I_C \cdot (R_C // R_L)$$

$$= -\frac{\beta (R_C // R_L)}{R_S + r_\pi} \cdot V_S$$

$$A_M = \frac{V_o}{V_S} = -\beta \frac{(R_C // R_L)}{R_S + r_\pi}$$

(b) Pole due to C_E:

$$\omega_{PE} = \frac{1}{C_E \left(r_e + \frac{R_S}{\beta + 1}\right)} \;//$$

Pole due to C_C:

$$\omega_{PC} = \frac{1}{C_C (R_C + R_L)} \;//$$

Zeros are both at $\underline{s = 0}$

(c)

$$A(s) = A_M \cdot \frac{s^2}{(s + \omega_{PE})(s + \omega_{PC})}$$

$$A(s) = -\frac{\beta (R_C // R_L)}{R_S + r_\pi} \cdot \frac{s^2}{\left[s + \frac{1}{C_E \left(r_e + \frac{R_S}{\beta + 1}\right)}\right] \times \left[s + \frac{1}{C_C (R_C + R_L)}\right]}$$

(d)

$$A_M = -\frac{100\,(10//10)}{10 + \frac{100}{40}}$$

$$= -\underline{40}\ V/V$$

(e) Since the resistance that forms the pole ω_{PE} is very small, we choose to make ω_{PE} the dominant pole, thus:

$$f_{PE} = f_L = 100 = \frac{1}{2\pi C_E \left(25 + \frac{10K}{101}\right)}$$

$$\Rightarrow C_E = \frac{1}{2\pi \times 100 \times (0.025 + 0.100) \cdot 10^3}$$

$$= \underline{12.7\,\mu F}$$

$f_{PC} = 10\,Hz \Rightarrow$

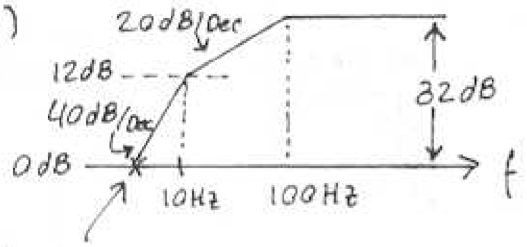
CONT.

$$10\,Hz = \frac{1}{2\pi C_C (R_L + R_C)}$$

$$\Rightarrow C_C = \frac{1}{2\pi \times 10 (10+10) 10^3}$$

$$= \underline{\underline{0.8\,\mu F}}$$

(f)



Unity-gain frequency must be an octave lower than 10 Hz. i.e. at $\underline{\underline{5\,Hz}}$

(g) $$A(j\omega) = -A_M \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$$

$$= +40 \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$$

Thus $$\phi = \tan^{-1}\left(\frac{\omega}{\omega_{PE}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{PC}}\right)$$

$$= -\left[\tan^{-1}\frac{f}{f_{PE}} + \tan^{-1}\frac{f}{f_{PC}}\right]$$

$$= -\left[\tan^{-1}\frac{f}{100} + \tan^{-1}\frac{f}{10}\right]$$

Thus at $f = 100\,Hz$,

$$\phi = -[\tan^{-1} 1 + \tan^{-1} 10]$$

$$\simeq -\underline{\underline{129.3^\circ}}$$

$\boxed{5.167}$

Refer to Fig. P5.167

(a) $$I_e = \frac{V_s}{r_e + R_e + \frac{1}{sC_E}}$$

$$I_c \simeq I_e$$

$$V_o = -R_C I_c = \frac{-R_C}{r_e + R_e + \frac{1}{sC_E}} \cdot V_s$$

$$A(s) \equiv \frac{V_o}{V_s} = \frac{-R_C}{r_e + R_e + \frac{1}{sC_E}}$$

$$= \frac{-R_C}{r_e + R_e} \cdot \frac{s}{s + \frac{1}{C_E (r_e + R_e)}}$$

Thus, $$A_M = \frac{-R_C}{r_e + R_e}$$

$$\omega_L = \frac{1}{C_E \cdot (r_e + R_e)}$$

(b) A_M is reduced by the factor $\frac{r_e + R_e}{r_e}$

$$= \underline{1 + \frac{R_e}{r_e}}$$

(c) ω_L is reduced by the factor $\left(1 + \frac{R_e}{r_e}\right)$

which is the same as the gain reduction factor. Thus, the value of R_e can be used as the parameter for exercising the gain-bandwidth trade off.

(d) $R_e = 0$:

$$|A_M| = \frac{R_C}{r_e} = \frac{10{,}000}{25} = \underline{400\,V/V}$$

$$f_L = \frac{1}{2\pi C_E r_e} = \frac{1}{2\pi \times 100 \times 10^{-6} \times 25}$$

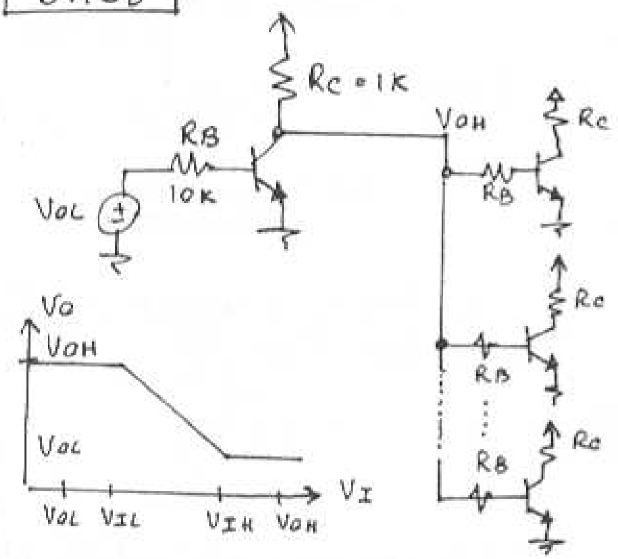
$$= \underline{\underline{63.7\,Hz}}$$

To lower f_L by a factor of 5 use:

CONT.

$R_e = 4r_e = \underline{100\Omega}$. The gain is also lowered by a factor of 5 to $\underline{80}$ V/V

5.168



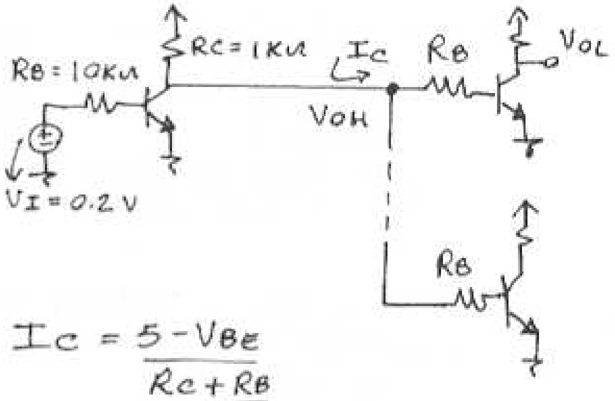
$$V_{IH} = V_{BE} + I_{B(EOS)} R_B$$
$$= 0.7 + \left[\left(\frac{V_{CC} - V_{CEsat}}{R_C}\right)/100\right] R_B$$
$$= 0.7 + \frac{5 - 0.2}{100} \times 10$$
$$= 1.18\,V$$

$\therefore$ For $NM_H = V_{OH} - V_{IH} = 1$
$$V_{OH} = 1 + 1.18 = 2.18\,V$$
$\therefore$ $V_{OH} = V_{CC} - R_C \cdot \dfrac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{N}}$
$$= 5 - 1\left(\frac{5 - 0.7}{1 + \frac{10}{N}}\right) > 2.18$$

$N < 19.05 \Rightarrow N = \underline{19}$

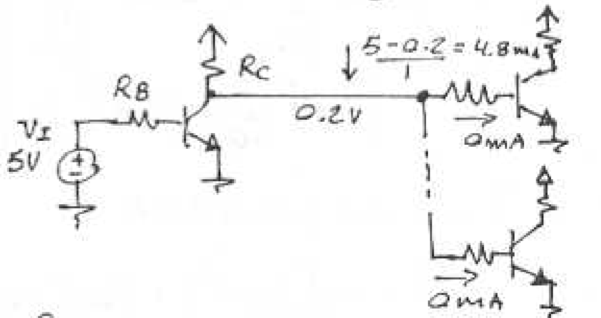
5.169

(a) Input is low:



$$I_C = \frac{5 - V_{BE}}{R_C + \frac{R_B}{10}}$$
$$= \frac{5 - 0.7}{1 + 10/10} = \underline{2.15\,mA}$$
$$PD_{V_{I\,LOW}} = I_C^2 \cdot R_C = 2.15^2 \times 1$$
$$= \underline{4.62\,mW}$$

(b) Input is high:

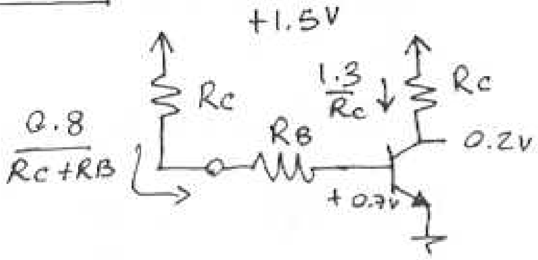


Power dissipated neglecting the base circuit

$$PD_{V_{I\,HIGH}} = V_{CC} I_C$$
$$= 5 \times 4.8 = \underline{24\,mW}$$

(c) $PD_{AVG} = \frac{1}{2}(4.62 + 24)$
$$= \underline{14.31\,mW}$$

5.170



$$\beta_{Forced} = 10 = \frac{1.3/R_C}{0.8/R_C+R_B} \quad (1)$$

$$\frac{80}{13} = \frac{R_C+R_B}{R_C} = 1 + \frac{R_B}{R_C}$$

$$\frac{R_B}{R_C} = \frac{67}{13} \Rightarrow R_B = 5.15 R_C \quad (2)$$

Total current from supply

$$= \frac{1.3}{R_C} + \frac{0.8}{R_C+R_B}$$

From Eqn. (1) $\frac{0.8}{R_C+R_B} = \frac{1.3/R_C}{10}$

Thus $I_{TOTAL} = \frac{1.3}{R_C} + \frac{1}{10}\frac{1.3}{R_C}$

$$= \frac{1.1 \times 1.3}{R_C}$$

$$P_{D\,TOTAL} = I_{TOTAL} \times 1.5V$$

$$= \frac{1.1 \times 1.3 \times 1.5}{R_C} = 1\,mW$$

$$R_C = \frac{1.1 \times 1.3 \times 1.5}{1} = 2.145\,K\Omega$$

Choose $R_C = \underline{\underline{2.2K\Omega}}$

$R_B = 5.15 \times 2.145 = \underline{\underline{11K\Omega}}$

5.171

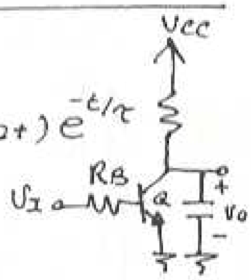
Refer to Fig. P5.171

V_x	V_y	V_z
0.2	0.2	5
0.2	5	0.2
5	0.2	0.2
5	5	0.2

These are 4 input combinations. When any input is high, (V_x and/or V_y)$_{high}$, $V_Z = \underline{0.2V}$

When both inputs are low, $(V_x \text{ and } V_y)_{LOW} \rightarrow$ $\underline{\underline{V_Z \text{ is high}}}$

5.172



$$V_O(t) = V_\infty - (V_\infty - V_{O+})e^{-t/\tau}$$

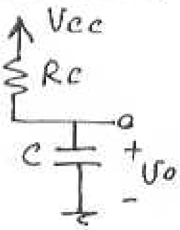
$V_\infty = V_{CC}$

$V_{O+} = V_{CE\,sat}$

$\tau = C \cdot R_C$

Thus,

$$\underline{\underline{V_O(t) = V_{CC} - (V_{CC} - V_{CE\,sat})e^{-t/R_C C}}}$$



$V_O(t) =$

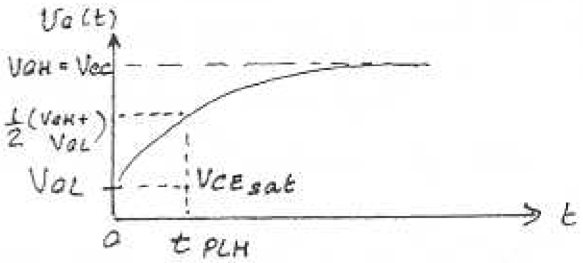
$$5 - (5-0.2)e^{-t/10\times10^{-12}\times1\times10^{3}}$$

$$\frac{1}{2}(V_{OH} + V_{OL}) = 2.6V$$

$$2.6 = 5 - 4.8e^{-\frac{t_{PLH}}{10^{-8}}}$$

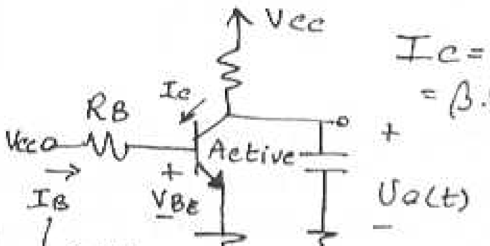
$t_{PLH} = 0.69 \times 10^{-8}\,s.$

$t_{PLH} = \underline{\underline{6.9\,ns}}$



5.173

At $t = 0+$

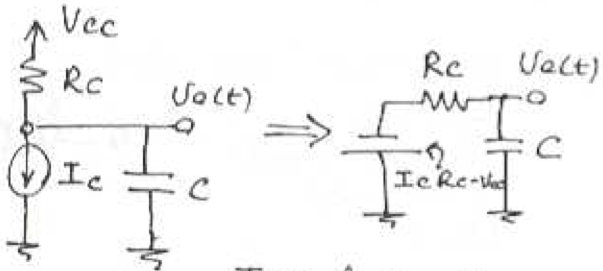


$$I_C = \beta I_B = \beta \cdot \frac{(V_{CC} - V_{BE})}{R_B}$$

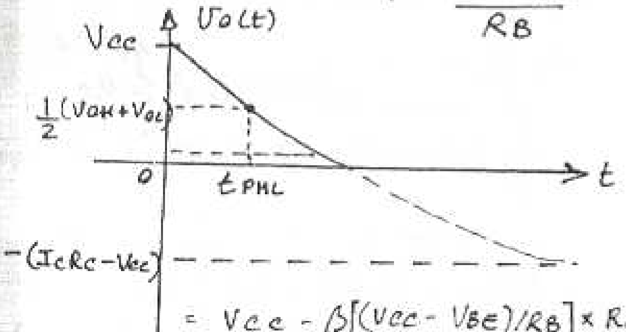
$\hookrightarrow \frac{V_{CC} - V_{BE}}{R_B}$

$$v_O(0+) = V_{CC}$$

Thus the equivalent circuit for the capacitor discharge will be:



$$I_C = \beta \frac{V_{CC} - V_{BE}}{R_B}$$



$= V_{CC} - \beta[(V_{CC} - V_{BE})/R_B] \times R_C$

$$v_O(t) = v_\infty - (v_\infty - v_{0+}) e^{-t/RC}$$

$$= V_{CC} - \beta \frac{(V_{CC} - V_{BE})}{R_B} R_C \ldots$$

$$+ \beta \frac{(V_{CC} - V_{BE})}{R_B} R_C \cdot e^{-t/CR_C}$$

$$= V_{CC} - \beta (V_{CC} - V_{BE}) \frac{R_C}{R_B} (1 - e^{-t/RC})$$

At $t = t_{PHL}$, $v_O = \frac{1}{2}(V_{OH} + V_{OL})$

$$= \frac{1}{2}(V_{CC} + V_{CE_{sat}})$$

$$= \frac{1}{2}(5 + 0.2) = 2.6\text{V}$$

Thus,

$$2.6 = 5 - 50(5 - 0.7)\frac{1}{10}(1 - e^{-\frac{t_{PHL}}{CR_C}})$$

$$1 - e^{-\frac{t_{PHL}}{CR_C}} = \frac{0.48}{4.3}$$

$$t_{PHL} = 0.118 \times 10 \times 10^{-12} \times 1 \times 10^{3}$$

$$= \underline{\underline{1.18\text{ ns}}}$$

Comparing this value to that of t_{PLH} found in Problem 5.172 we observe that t_{PHL} is much smaller than t_{PLH}.

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(6.9 + 1.18) = \underline{\underline{4\text{ns}}}$$

Chapter 6 _ Problems

6.1

Assume 0.18 μm CMOS process and refer to Table 6.1:

$\mu_n C_{ox} = 387\ \mu A/_{V^2}$ Also assume $\frac{W}{L} = 10$

Assuming operation in saturation mode:

$V_{ov} = 0.15\,V \Rightarrow I_D = \frac{1}{2} K_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 387 \times 0.15^2 \times 10$

$I_D = 43.5\ \mu A$

$V_{ov} = 0.4\,V \Rightarrow I_D = 309.6\ \mu A$

Therefore:

$0.15^V \leq V_{ov} \leq 0.4\,V \Rightarrow \underline{43.5\,\mu A \leq I_D \leq 309.6\,\mu A}$

Now if we consider the same range of current for I_C of a BJT and we assume an npn transistor in a standard high-voltage process: (refer to Table 6.2)

$I_S = 5 \times 10^{-15}\,A = 5 \times 10^{-9}\,\mu A$

$I_C = I_S e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln \frac{I_C}{I_S}$

$I_C = 43.5\,\mu A \Rightarrow V_{BE} = 0.025 \ln \frac{43.5}{5 \times 10^{-9}} = 0.572\,V$

$I_C = 309.6\,\mu A \Rightarrow V_{BE} = 0.621\,V$

Therefore:

$43.5\,\mu A \leq I_C \leq 309.6\,\mu A \Rightarrow \underline{0.572^V \leq V_{BE} \leq 0.621^V}$

6.2

If the area of the emitter-base junction is changed by a factor of 10, then I_S is changed by the same factor. If V_{BE} is kept constant, then I_C is also changed by the same factor:

$I_C = I_S e^{V_{BE}/V_T}$ $\quad I_S \propto A,\ I_C \propto I_S \Rightarrow I_C \propto A$

$A_2 = 10 A_1 \Rightarrow I_{C2} = 10 I_{C1}$

If I_C is kept constant, then V_{BE} changes:

$I_{S2} = 10 I_{S1} \Rightarrow I_S e^{V_{BE1}/V_T} = 10 I_S e^{V_{BE2}/V_T}$

$e^{\frac{V_{BE1} - V_{BE2}}{V_T}} = 10 \Rightarrow V_{BE1} - V_{BE2} = V_T \ln 10 = 0.058^V$ or $\underline{58\,mV}$

6.3

$\frac{W}{L} = 10$, $I_D = 100\,\mu A$, $I_D = \frac{1}{2} k' \frac{W}{L} V_{ov}^2$

	0.8 μm		0.5 μm		0.25 μm		0.18 μm	
	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PM
$V_{OV}(V)$	0.4	-0.59	0.32	-0.54	0.27	-0.46	0.23	-0.41
$V_{GS}(V)$	1.1	-1.29	1.02	-1.34	0.7	-1.08	0.71	-0.9

$V_{ov} = \sqrt{\frac{2 I_D}{K_n' \frac{W}{L}}} = \sqrt{\frac{2 \times 100}{K_n \times 10}} = \sqrt{\frac{20}{K_n'}}$

$V_{GS} = V_t + V_{ov}$

6.4

$|V_{ov}| = 0.25\,V$, $I_D = 100\,\mu A$

$I_D = \frac{1}{2} K_n' \frac{W}{L} V_{ov}^2 \Rightarrow \frac{W}{L} = \frac{2 I_D}{K_n' V_{ov}^2}$

For NMOS:

$K_n' = 267 \frac{\mu A}{V^2} \Rightarrow \left(\frac{W}{L}\right)_n = \frac{2 \times 100}{267 \times 0.25^2} = \underline{11.98 \simeq}$

For PMOS:

$K_n' = 93 \frac{\mu A}{V^2} \Rightarrow \left(\frac{W}{L}\right)_p = \frac{2 \times 100}{93 \times 0.25^2} = \underline{34.4 \simeq 3}$

6.5

$i_{Dn} = i_{Dp} \Rightarrow \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n V_{ovn}^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p V_{ovp}^2$

we also have $g_{mn} = g_{mp}$

$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow V_{ovn} = V_{ovp}$ ②

①, ② $\Rightarrow \frac{\left(\frac{W}{L}\right)_p}{\left(\frac{W}{L}\right)_n} = \frac{\mu_n}{\mu_p} = \frac{460}{160} = 2.88$

6.6

$g_m = 10\,mA/_V$, $V_{ov} = 0.2^V$

$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow I_D = \frac{g_m V_{ov}}{2} = \frac{10 \times 0.2}{2} = \underline{1\,mA}$

$I_D = \frac{1}{2} K_n' \frac{W}{L} V_{ov}^2 \Rightarrow \frac{W}{L} = \frac{2 I_D}{K_n' V_{ov}^2} = \frac{2 \times 1 \times 10^3}{387 \times 0.2^2} = 12$

$\underline{\frac{W}{L} \simeq 129}$

for an npn transistor: $g_m = \frac{I_C}{V_T} \Rightarrow I_C = 10 \times 0.025 = \underline{0.25}$

6.7

$\frac{W}{L} = 10 \quad I_D = 100\,\mu A$

	0.8 μm		0.5 μm		0.25 μm		0.18 μm	
	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS
g_m (mA/V)	0.5	0.34	0.62	0.37	0.73	0.43	0.88	0.41

$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{\mu_n C_{ox}}\sqrt{2\times10\times100} = 44.7\sqrt{\mu_n C_{ox}}\ \frac{\mu A}{V}$

6.8

$V_{ov} = 0.25\,V$

For an npn transistor: $g_m = \frac{I_C}{V_T} = \frac{0.1}{0.025} = 4\,mA/V$

For an NMOS with the same g_m, i.e. $g_m = 4\,mA/V$

we will have: $g_m = \frac{2I_D}{V_{ov}} \Rightarrow I_D = g_m \times \frac{V_{ov}}{2} = 0.5\,mA$

$I_D = 0.5\,mA$

6.9

Assuming large r_o for both transistors, for case (a) we have $r = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}}$

$r = \frac{10^3}{\sqrt{2\times200\times10\times0.1\times10^3}} = 1.58\,k\Omega$

For case (b) we have $r = r_\pi \,\|\, \frac{1}{g_m} = \frac{\beta}{(\beta+1)g_m}$

$r = \frac{\beta V_T}{(\beta+1)I_C} \simeq \frac{V_T}{I_C} = \frac{0.025}{0.1} = 0.25\,k\Omega$

$r = 250\,\Omega$

6.10

$g_m = \frac{2I_D}{V_{ov}} = \frac{2\times100\times10^{-3}}{0.5} = 0.4\,mA/V$

$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{25\times1}{0.1} = 250\,k\Omega$

$A_o = g_m r_o = 0.4\times250 = 100\,V/V$

$g_m = \mu_n C_{ox}\frac{W}{L}V_{ov} \Rightarrow W = \frac{g_m\times L}{\mu_n C_{ox}\times V_{ov}} = \frac{0.4\times1}{127\times10^{-3}\times0.5}$

$W = 6.3\,\mu m$

6.11

$L = 0.3\,\mu m, \quad I_D = 100\,\mu A, \quad V_{ov} = 0.2\,V$

$g_m = \frac{2I_D}{V_{ov}} = \frac{2\times100\times10^{-3}}{0.2} = 1\,mA/V$

$r_o = \frac{V_A}{I_D} = \frac{V_A' \times L}{I_D} = \frac{5\times0.3}{0.1} = 15\,k\Omega$

$A_o = g_m r_o = 1\times15 = 15\,V/V$

$g_m = \mu_n C_{ox}\frac{W}{L}V_{ov} \Rightarrow W = \frac{g_m\times L}{\mu_n C_{ox} V_{ov}} = \frac{1\times0.3}{387\times10^{-3}\times0.2}$

$W = 3.88\,\mu m$

6.12

BJT: $\beta = 100 \quad V_A = 100\,V$

MOSFET: $\mu_n C_{ox} = 200\,\mu A/V^2, \ \frac{W}{L} = 40, \ V_A = 10\,V$

Device	BJT		MOSFET	
Bias current (mA)	I_C 0.1	1	I_D 0.1	1
g_m (mA/V)	4	40	1.26	4
r_o (kΩ)	1000	100	100	10
A_o (V/V)	4000	4000	126	40
R_i (kΩ)	25	2.5	∞	∞

$R_i = r_\pi = \beta/g_m$ for BJT

6.13

$L = 0.3\,\mu m, \ W = 6\,\mu m, \ V_{ov} = 0.2\,V$

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2 = \frac{1}{2}\times387\times\frac{6}{0.3}\times0.2^2 = 155\,\mu A$

$I_D = 0.155\,mA$

$g_m = \frac{2I_D}{V_{ov}} = 1.55\,mA/V$

$C_{gs} = \frac{2}{3}\frac{W}{L}C_{ox} + C_{ov} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$

$C_{gs} = \frac{2}{3}\times6\times0.3\times8.6 + 6\times0.37 = 12.54\,fF$

$C_{gd} = C_{ov}W = 0.37\times6 = 2.22\,fF$

$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{1.55\times10^{-3}}{2\pi(12.54+2.22)\times10^{-15}} = 16.7\,GHz$

If we use the approximation formula:

$f_T \simeq \frac{1.5\,\mu_n V_{ov}}{2\pi L^2}$ when $C_{gs} \gg C_{gd}$, $C_{gs} \simeq \frac{2}{3}WLC_{ox}$

$f_T \simeq \frac{1.5\times450\times10^{-4}\times0.2}{2\times\pi\times0.3^2\times10^{-12}} = 23.9\,GHz$

The approximation formula overestimates

Cont.

f_T because it ignores $WL_{ov}C_{ox}$ or C_{ov} in C_{gs} and C_{gd} calculation.

6.14

$L = 0.3\,\mu m \quad W = 6\,\mu m \quad V_{ov} = 0.2 \quad C_L = 100\,fF$

$A_o = \frac{2V'_A L}{V_{ov}} = \frac{2 \times 5 \times 0.3}{0.2} = 15\,V/V$

$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 387 \times \frac{6}{0.3}\, 0.2^2 = 0.155\,mA$

$r_o = \frac{V'_A L}{I_D} = \frac{5 \times 0.3}{0.155} = 9.7\,k\Omega$

$f_p = \frac{1}{2\pi C_L r_o} = \frac{1}{2\pi \times 9.7 \times 100 \times 10^{-12}} = 164.2\,MHz$

$g_m = \frac{2I_D}{V_{ov}} = 1.55\,mA/V$

$f_T = \frac{g_m}{2\pi C_L} = 2.5\,GHz$

In order to double f_T or equivalently double $g_m, \sqrt{I_D}$ has to be doubled or I_d has to be multiplied by 4: $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$, $f_T \propto g_m \propto \sqrt{I_D}$

$I_D = 4 \times 0.155 = 0.62\,mA$

In that case: $A_o \propto \frac{1}{V_{ov}} \propto \frac{1}{\sqrt{I_D}} \Rightarrow A_o = 15 \times \frac{1}{2} = 7.5\,V/V$

$f_p \propto \frac{1}{r_o} \propto I_D \Rightarrow f_p = 164.2 \times 4 \Rightarrow$

$f_p = 656.8\,MHz$

6.15

$I_C = 10\,\mu A$, High-voltage process:

$g_m = \frac{I_C}{V_T} = \frac{10 \times 10^{-3}}{0.025} = 0.4\,mA/V$

$C_{de} = \tau_F g_m = 0.35 \times 10^{-9} \times 0.4 \times 10^{-3} = 140 \times 10^{-15}\,F = 140\,fF$

$C_{je} = 2C_{jeo} = 2 \times 1 = 2\,pF = 2000\,fF$

$C_\pi = C_{de} + C_{je} = 2140\,fF$

$C_\mu \simeq C_{\mu o} = 0.3\,pF = 300\,fF$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{0.4 \times 10^{-3}}{2\pi(2140 + 300) \times 10^{-15}} = 26.1\,MHz$

$I_C = 100\,\mu A$, High-Voltage process:

$g_m = 10 \times 0.4 = 4\,mA/V$, $C_{de} = 10 \times 140 = 1400\,fF$

$C_\pi = 3400\,fF \Rightarrow f_T = \frac{4 \times 10^{-3}}{2\pi(3400 + 300) \times 10^{-15}} = 172.1\,MHz$

$I_C = 10\,\mu A$, Low-Voltage process

$g_m = \frac{10 \times 10^{-3}}{0.025} = 0.4\,mA/V$

$C_{de} = 10 \times 10^{-12} \times 0.4 \times 10^{-3} = 4\,fF$

$C_{je} = 2 \times 5\,fF = 10\,fF$

$C_\pi = C_{de} + C_{je} = 14\,fF$

$C_\mu \simeq C_{\mu o} = 5\,fF$

$f_T = \frac{g_m}{2\pi(C_\mu + C_\pi)} = \frac{0.4 \times 10^{-3}}{2\pi(5 + 14) \times 10^{-15}} = 3.35\,GHz$

$I_C = 100\,\mu A$, Low-Voltag process

$g_m = \frac{100 \times 10^{-3}}{0.025} = 4\,mA/V$

$C_{de} = 10 \times 4 = 40\,fF$

$C_\pi = 40 + 10 = 50\,fF$, $C_\mu = 5\,fF$

$f_T = \frac{4 \times 10^{-3}}{2\pi(50 + 5) \times 10^{-15}} = 11.6\,GHz$

In Summary:

	Standard High-Voltage npn		Standard low-Volta[ge] npn	
	$I_C = 10\,\mu A$	$I_C = 100\,\mu A$	$I_C = 10\,\mu A$	I_C = [100 μA]
f_T	26.1 MHz	172.1 MHz	3.35 GHz	11.6 GH[z]

6.16

$L = 1\,\mu m$, $I_D = 100\,\mu A$, $0.8\,\mu m$ - NMOS

a) $V_{ov} = 0.25\,V$

$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{ov}^2 \Rightarrow W = \frac{2L I_D}{\mu_n C_{ox} V_{ov}^2} = \frac{2 \times 1 \times 100}{127 \times 0.25^2}$

$\Rightarrow W = 25.2\,\mu m$

$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 100 \times 10^{-3}}{0.25} = 0.8\,mA/V$

$r_o = \frac{V'_A L}{I_D} = \frac{25 \times 1}{0.1} = 25\,k\Omega$

$A_o = g_m r_o = 20\,V/V$

$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox} = \frac{2}{3} \times 25.2 \times 1 \times 2.3 + 25.2 \times 0.2$

$C_{gs} = 43.68\,fF \simeq 44\,fF$

$C_{gd} = WL_{ov}C_{ox} = 25.2 \times 0.2 = 5.04\,fF$

$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.8 \times 10^{-3}}{2\pi(44 + 5.04) \times 10^{-15}} = 2.6\,GHz$

b) $f_T \propto g_m \propto V_{ov}$ therefore in order to double f_T, V_{ov} has to be doubled: $V_{ov} = 0.5\,V$. Consequently,

$W = 25.2/4 = 6.3\,\mu m$ $\quad r_o, C_{gs}, C_{gd}$ unchanged

$g_m = \frac{0.8}{2} = 0.4\,mA/V$, $A_o = \frac{20}{2} = 10\,V/V$

6.17

$I_C = 1mA \Rightarrow g_m = \frac{I_C}{V_T} = 40 mA/V$

For pnp:

$C_{de} = \tau_F g_m = 30^n \times 40^m = 1200pF$

$C_{je} = 2C_{jeo} = 2\times 0.3 = 0.6pF$

$C_\pi = 1200.6pF$

$C_\mu \simeq 1pF$

$\Big\} \Rightarrow f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{40^m}{2\pi(1200.6+1)^{pF}}$

$\underline{f_T = 5.3MHz}$

For npn:

$C_{de} = \tau_F g_m = 0.35^n \times 40^m = 14\ pF$

$C_{je} = 2\times 1 = 2pF$

$C_\mu \simeq 0.3pF$

$C_\pi = 14+2 = 16pF$

$\Big\} \Rightarrow f_T = \frac{40^m}{2\pi(16+0.3)^{pF}} = \underline{391^{MHz}}$

6.18

$A_0 = g_m r_o = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}} = \frac{2V_A'L}{V_{OV}}$

Therefore A_0 is only determined by setting values for L and V_{OV}.

$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{2I_D/V_{OV}}{2\pi(\frac{2}{3}WLC_{ox}+C_{ov}+C_{ov})}$

If we assume that C_{ov} is very small or equivalently $C_{gs} >> C_{gd}$ and $C_{gs} = \frac{2}{3}WLC_{ox}$:

(replace I_D with $\frac{1}{2}k_n'\frac{W}{L}V_{ov}^2$)

$f_T \simeq \frac{k_n' W/L\, V_{OV}}{2\pi \times \frac{2}{3}WLC_{ox}} = \frac{3\mu_n V_{OV}/L^2}{4\pi} = \frac{3}{4\pi}\mu_n\frac{V_{OV}}{L^2}$

As we can see f_T can be determined after knowing V_{ov} and L, it is not dependent on either I_D or W.

6.19

$V_{ov} = 0.2V$, $L = 0.2\mu m, 0.3\mu m, 0.4\mu m$

$A_0 = g_m r_o = \frac{2V_A}{V_{OV}} = \frac{2V_A'L}{V_{OV}} = \frac{2\times5\times L}{0.2} = 50L\ V/V$

$f_T \simeq \frac{1.5\mu_n V_{OV}}{2\pi L^2} = \frac{1.5\times450\times10^{-4}\times0.2}{2\times3.14\times L^2\times10^{-12}} = \frac{2.15}{L^2}\ GHz$

$L(\mu m)$	0.2	0.3	0.4
$A_0(V/V)$	10	15	20
$f_T(GHz)$	53.75	23.9	13.4

6.20

$L = 0.5\mu m$, $V_{ov} = 0.3V$, $C_L = 1PF$, $f_T = 100MHz$

$f_T = \frac{g_m}{2\pi C_L} \Rightarrow g_m = 2\pi C_L f_T = 2\pi\times1^P\times100^M = 628\ \mu A/V$

$g_m = \frac{2I_D}{V_{OV}} \Rightarrow I_D = g_m \times V_{OV}/2 = 628\times\frac{0.3}{2}$

$\underline{I_D = 94.2\mu A}$

$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{ov}^2 \Rightarrow W = \frac{2LI_D}{k_n' V_{ov}^2} = \frac{2\times0.5\times94.2}{190\times0.3^2} = 5.51\ \mu m$

$\underline{W = 5.51\mu m}$

$r_o = \frac{V_A}{I_D} = \frac{V_A'L}{I_D} = \frac{20\times0.5}{94.2\times10^{-3}} = 106.2K\Omega$

$A_0 = g_m r_o = \frac{628}{1000}\times106.2 = 66.7V/V$

$f_{3db} = \frac{1}{2\pi C_L r_o} = \frac{1}{2\pi\times1^P\times106.2^K} = \underline{1.5^{MHz}}$

6.21

$I_o = I_{REF} = 50\mu A$, $L = 0.5\mu m$, $W = 5\mu m$, $V_t = 0.5^V$

$I_o = I_D = \frac{1}{2}k_n'\frac{W}{L}V_{ov}^2$ $\quad k_n' = 250\mu A/V^2$

$50 = \frac{1}{2}\times250\times\frac{5}{0.5}(V_{GS}-0.5)^2 \Rightarrow V_{GS} = 0.7^V, 0.3^V$

$V_{GS} = 0.3V < V_t$ is not acceptable, therefore

$\underline{V_{GS} = 0.7V}$

$I_D = I_{RE} = \frac{V_{DD}-V_{GS}}{R} \Rightarrow \frac{1.8-0.7}{R} = 0.050 \Rightarrow \underline{R = 22K\Omega}$

Q_1 and Q_2 have the same V_{GS}. The lowest value of V_o or V_{DS2} is when $V_{DS} = V_{GS} - V_t = 0.7-0.5 = 0.2^V$

hence $V_{omin} = 0.2^V$

$r_o = \frac{V_A}{I_D} = \frac{V_A'L}{I_D} = \frac{20\times0.5}{0.05} = 200K\Omega$

$\Delta I_o \simeq \frac{\Delta V_o}{r_o} = \frac{1}{200^K} = 5\mu A \Rightarrow \underline{\Delta I_o = 5\mu A}$

6.22

$\mu_n C_{ox} = 250\,\mu A/V^2$, $V_A' = 20V/\mu m$, $V_t = 0.6V$

$\frac{\Delta I_o}{I_o} = 5\% \Rightarrow \Delta I_o = 5\mu A$ For $\Delta V_o = 1.8 - 0.25 = 1.55^V$

$r_o = \frac{\Delta V_o}{\Delta I_o} = \frac{1.55}{5\mu} = 310\,k\Omega$

$r_o = \frac{V_A' L}{I_O} \Rightarrow L = I_D \times \frac{r_o}{V_A'} = 0.1 \times \frac{310}{20} = \underline{\underline{1.55\,\mu m}}$

$V_{omin} = V_{GS} - V_t = 0.25 \Rightarrow V_{GS} = 0.25 + 0.6 = 0.85^V$

$R = \frac{V_{DD} - V_{GS}}{I_D} = \frac{1.8 - 0.85}{0.1} = \underline{\underline{9.5\,k\Omega}}$

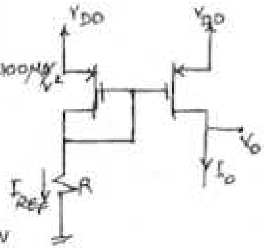
$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)^2 \Rightarrow W = \frac{2 L I_D}{\mu_n C_{ox}(V_{GS} - V_t)^2}$

$\Rightarrow W = \frac{2 \times 1.55 \times 100}{250\,(0.85 - 0.6)^2} = \underline{\underline{19.84\,\mu m}}$

6.23

$V_{DD} = 1.8V$, $|V_t| = 0.6V$, $\mu_p C_{ox} = 100\,\mu A/V^2$

$I_{REF} = 80\,\mu A$ $V_{omax} = 1.6^V$



$V_{DS} \leq V_{GS} - V_t$

$V_{omax} = V_{DSmax} = V_{GS} - V_t \Rightarrow$

$1.6 - 1.8 = V_{GS} + 0.6 \Rightarrow V_{GS} = -0.8^V$

$\Rightarrow V_G = 1.8 - 0.8 = 1^V$

$R = \frac{V_G}{I_D} = \frac{1}{0.080} = \underline{12.5\,k\Omega}$

$I_D = \frac{1}{2}\mu_p C_{ox}(V_{GS} - V_t)^2 \frac{W}{L} \Rightarrow W = \frac{2 L I_D}{\mu_p C_{ox}(V_{GS} - V_t)^2}$

$\frac{W}{L} = \frac{2 \times 80}{100(-0.8 + 0.6)^2} = \underline{40}$

6.24

$W_2 = 4W_1$ $L_1 = L_2$ $V_{ov} = 0.3^V$, $I_{REF} = 20\,\mu A$

$I_o = I_{REF} \frac{(W/L)_2}{(W/L)_1} = 20 \times 4 = \underline{80\,\mu A}$

$V_{omin} = V_{ov} = \underline{0.3V}$

$V_t = 0.5V$. According to Eq. 6.11 $I = \frac{(W/L)_2}{(W/L)_1} I_{REF} \left(1 + \frac{V_o - V_{GS}}{V_{A2}}\right)$

$V_{ov} = V_{GS} - V_t \Rightarrow V_{GS} = 0.3 + 0.5 = 0.8V$

$1 + \frac{V_o - V_{GS}}{25} = 1 \Rightarrow \underline{V_o = 0.8V}$

Or we could simply say $V_{DS1} = V_{DS2} = V_o$ and

Since $V_{DS1} = V_{GS1} = 0.8^V \Rightarrow \underline{V_o = 0.8^V}$

$r_{o2} = \frac{V_A}{I_{D2}} = \frac{25}{0.08} = 312.5\,k\Omega$

$r_{o2} = \frac{\Delta V_o}{\Delta I_o} = \frac{1}{\Delta I_o} \Rightarrow \Delta I_o = \frac{1}{312.5^K} = \underline{3.2\,\mu A}$

6.25

Refer to Fig. P6.25

$V_{GS1} = V_{GS2} \Rightarrow \frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1} \Rightarrow I_{D2} = I_{REF}\frac{(W/L)_2}{(W/L)_1}$

$I_{D2} = I_{D3}$

$V_{GS3} = V_{GS4} \Rightarrow I_{D3}/I_{D4} = \frac{(W/L)_3}{(W/L)_4} \Rightarrow \frac{I_{D2}}{I_{D4}} = \frac{(W/L)_3}{(W/L)_4}$

$\underline{I_{D4} = I_{REF}\frac{(W/L)_2}{(W/L)_1}\frac{(W/L)_4}{(W/L)_3} = I_o}$

6.26

Refer to Fig. P6.26:

$V_{DS2} \leq V_{GS2} - V_{tp} \Rightarrow V_{DSmax} = V_{GS2} - V_t$

$(1.3 - 1.5) = V_{GS2} - (-0.6) \Rightarrow V_{GS2} = -0.8V = V_{GS1}$

For Q_1:

$I_{D1} = 20\,\mu A = \frac{1}{2} \times 80 \times \frac{W_1}{0.8} \times (-0.8 + 0.6)^2$

$\Rightarrow \underline{W_1 = 10\,\mu m}$

$I_2 = 100\,\mu A = 5\,I_{REF} \Rightarrow W_2 = 5W_1 = \underline{50\,\mu m}$

$I_3 = I_{REF} \Rightarrow W_3 = W_1 = 10\,\mu m$

$I_3 = I_4 \Rightarrow \frac{W_3}{W_4} = \frac{\mu_n}{\mu_p} \Rightarrow W_4 = 10 \times \frac{80}{200} = \underline{4\,\mu m}$

For Q_5: $V_{DS5} = V_{GS5} - V_{tn}$ For lowest V_o

$(-1.3 - (-1.5)) = V_{GS5} - 0.6 \Rightarrow V_{GS5} = 0.8^V$

$I_5 = 50\,\mu A = \frac{1}{2} \times 200 \times \frac{W_5}{0.8} \times (0.8 - 0.6)^2 \Rightarrow \underline{W_5 = 10\,\mu m}$

Now we calculate R:

$V_{GS2} = -0.8^V \Rightarrow V_{G2} = 1.5 - 0.8 = 0.7^V$

$R = V_{G2}/I_{REF}$

$R = \frac{0.7}{20\,\mu A} = 35\,k\Omega$

$\underline{R = 35\,k\Omega}$

$r_{o2} = \frac{V_A' L}{I_2} = \frac{12 \times 0.8}{0.1} = \underline{96\,k\Omega}$, $r_{o5} = \frac{10 \times 0.8}{0.05} = \underline{160\,k\Omega}$

6.27

If the transistor with $W = 10$ is diode-connected, then:

$$I_2 = 100 \times \frac{20}{10} = 200\,\mu A$$
$$I_3 = 100 \times \frac{40}{10} = 400\,\mu A$$

If the transistor with $W = 20$ is diode-connected then:

$$I_2 = 100 \times \frac{10}{20} = 50\,\mu A$$
$$I_3 = 100 \times \frac{40}{20} = 200\,\mu A$$

If the transistor with $W = 40$ is diode-connected, then:

$$I_2 = 100 \times \frac{10}{40} = 25\,\mu A$$
$$I_3 = 100 \times \frac{20}{40} = 50\,\mu A$$

So for cases that only one transistor is diode connected, 4 different output currents are possible (depending on the configuration we choose).

If 2 transistors are diode-connected, then they act as an equivalent transistor whose width is the sum of the widths of each transistor:

If $W_{eff} = 10 + 20$ then $I_o = 100 \times \frac{40}{30} = 133\,\mu A$

If $W_{eff} = 20 + 40$ then $I_o = 100 \times \frac{10}{60} = 16.7\,\mu A$

If $W_{eff} = 40 + 10$ then $I_o = 100 \times \frac{20}{50} = 40\,\mu A$

So 3 different output currents are possible depending on which two transistors are diode-connected. Now we calculate V_{SG}:

$100 = \frac{1}{2} \times 80 \times \frac{30}{1}(V_{SG} - 0.7)^2 \Rightarrow V_{SG} = 1^V$ for $W_{eff} = 30\,\mu m$

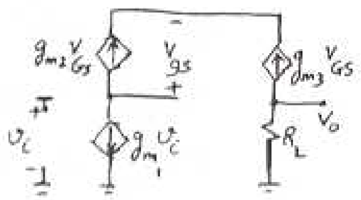
all have the same V_{SG} for any given configuration.

For $W_{eff} = 60 \Rightarrow 100 = \frac{1}{2} \times 80 \times \frac{60}{1}(V_{SG} - 0.7)^2$
$\Rightarrow V_{SG} = 0.9^V$

for $W_{eff} = 50 \Rightarrow 100 = \frac{1}{2} \times 80 \times \frac{50}{1}(V_{SG} - 0.7)^2$
$\Rightarrow V_{SG} = 0.93V$

6.28

$$-g_{m2} V_{gs} = g_{m1} V_i$$
$$\Rightarrow V_{GS} = -\frac{g_{m1}}{g_{m2}} V_i$$
$$V_o = -R_L g_{m3} V_{gs}$$
$$V_o = -R_L g_{m3} \frac{-g_{m1}}{g_{m2}} V_i$$



$$\frac{V_o}{V_i} = +R_L g_{m1} \frac{g_{m3}}{g_{m2}} = R_L g_{m1} \frac{W_3}{W_2}$$

Note that since Q_2, Q_3 have the same V_{GS} and therefore the same V_{OV}: $\frac{g_{m3}}{g_{m2}} = \frac{I_3}{I_2} = \frac{W_3}{W_2}$

6.29

$I_S = 10^{-15} A$

a) $I_{REF} = I_S e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln \frac{I_{REF}}{I_S}$

$I_{REF} = 10\,\mu A \Rightarrow V_{BE} = 0.025 \ln \frac{10 \times 10^{-6}}{10^{-15}} = 0.576^V$

$I_{REF} = 10\,mA \Rightarrow V_{BE} = 0.025 \ln \frac{10 \times 10^{-3}}{10^{-15}} = 0.748^V$

Therefore:

$$10\,\mu A \leq I_{REF} \leq 10\,mA \Rightarrow 0.576^V \leq V_{BE} \leq 0.748^V$$

Since β is very high, I_B is negligible and hence $I_o \simeq I_{REF}$: $\underline{10\,\mu A \leq I_o \leq 10\,mA}$

b) $I_o = I_{REF} \frac{1}{1 + 2/\beta}$ (Eq. 6.21)

For $0.1^{mA} \leq I_C \leq 5^{mA}$, β remains constant at 100.

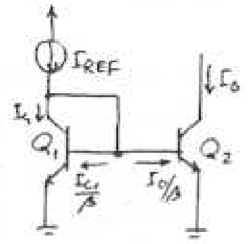
$I_{REF} = 10\,mA \Rightarrow I_o = \frac{10}{1 + \frac{2}{70}} = 9.72\,mA$

$I_{REF} = 0.1\,mA \Rightarrow I_o = \frac{0.1}{1 + 2/100} = 0.098\,mA$

$I_{REF} = 1\,mA \Rightarrow I_o = \frac{1}{1 + 2/100} = 0.98\,mA$

$I_{REF} = 10\,\mu A \Rightarrow$

6.30



$I_{S2} = I_{S1} \times m \quad , I_{C1} = I_C$

$I_{REF} = I_C + \frac{I_C}{\beta} + \frac{I_O}{\beta}$ ①

$V_{BE1} = V_{BE2} \Rightarrow$

$V_T \ln \frac{I_C}{I_{S1}} = V_T \ln \frac{I_O}{I_{S2}}$

$\Rightarrow \frac{I_O}{I_C} = \frac{I_{S2}}{I_{S1}} = m \Rightarrow I_C = I_O/m$

by substituting for I_C in ①:

$I_{REF} = \frac{I_O}{m} + \frac{I_O}{m\beta} + \frac{I_O}{\beta} \Rightarrow \frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1}{\beta} + \frac{m}{\beta}}$

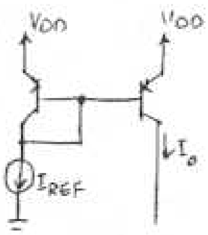
$\frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1+m}{\beta}}$

This result is the same as Eq. 6.22.

For large β, $I_O/I_{REF} = m$, with finite β this ratio drops to $I_O/I_{REF} = \frac{m}{1 + \frac{1+m}{\beta}}$. To keep the introduced error within 5%: $0.95m = \frac{m}{1 + \frac{1+m}{\beta}}$

$\beta_{min} = 80 \Rightarrow 0.95 = \frac{1}{1 + \frac{1+m}{80}} \Rightarrow \underline{m = 3.21}$

6.31



The transfer ratio is the same as Eq. 6.21:

$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}}$

$\beta = 20 \Rightarrow \frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{20}} = 0.91$

6.32

$I_O = I_{REF} = 2mA$

$r_{O2} = \frac{V_{A2}}{I_O} = \frac{90}{2} = 45K\Omega$

$r_{O2} = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{10-1}{\Delta I_O} = 45 \Rightarrow \underline{\Delta I_O = 0.2mA}$

$\frac{\Delta I_O}{I_O} = \frac{0.2}{2} = \underline{10\% \text{ change}}$

6.33

$I_S = 10^{-15}A \quad \beta = 50$

$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}} \Rightarrow I_{REF} = I_O(1 + \frac{2}{\beta}) = 1 \times (1 + \frac{2}{50})$

$I_{REF} = 1.02\ mA$

$V_{BE} = V_T \ln \frac{I_O}{I_S(1 + V_{CE}/V_A)} = 0.025 \frac{10^{-3}}{10^{-15}(1 + \frac{2}{50})} = 0.689^V$

$V_C = V_B = 5 - 0.689 = 4.31V$

$V_C = R \times I_{REF} \Rightarrow R = \frac{4.31}{1.02} = 4.2K\Omega$

$r_O = \frac{V_A}{I_O} = \frac{50}{1} = 50K\Omega$

V_{Omax} occurs when Q_2 is on the edge of saturation or $V_{CE} = 0.3^V$. Therefore V_{Omax} is $5 - 0.3 = 4.7^V$

$r_O = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{4.7 - (-5)}{\Delta I_O} = 50^K \Rightarrow \Delta I_O = 0.194mA$

$\frac{\Delta I_O}{I_O} \times 100 = \frac{0.194}{1} \times 100 = \underline{19.4\%}$ change in I_O

6.34

$I_{C1} = I_{C2} = I_{R1}$

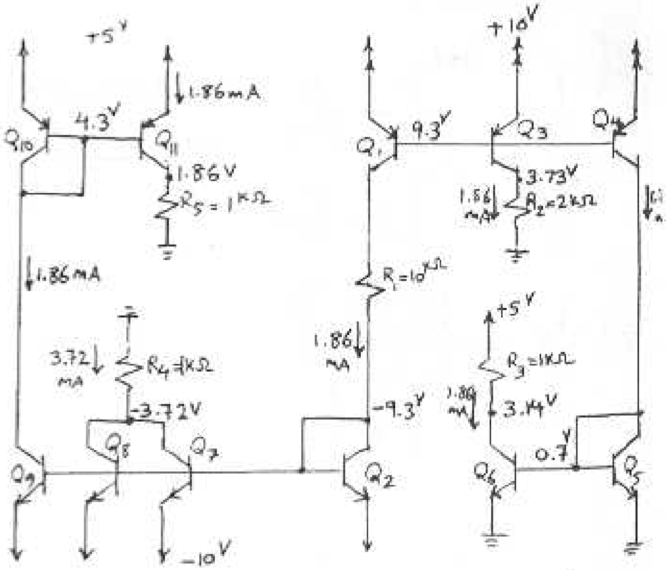
$V_{B1} = 10 - 0.7 = 9.3V$, $V_{B2} = -10 + 0.7 = -9.3^V$, $I_{R_1} = \frac{9.3 + 9.3}{10}$

$\Rightarrow I_{R_1} = 1.86\ mA = I_{C1} = I_{C2} = I_{C3} = I_{C4} = I_{C5} = I_{C6}$

$V_{C3} = 1.86^m \times 2^K = 3.72V$, $V_{C5} = 0.7^V$

$V_{C6} = 5 - 1.86 \times 1 = 3.14V$, $I_{C9} = I_{C8} = I_{C7} = I_{C2} = 1.86^{mA}$

$I_{R4} = 2 \times 1.86 = 3.72mA \Rightarrow V_{C7} = -3.72 \times 1 = -3.72V$



Cont.

$I_{C10} = I_{C9} = 1.86mA$

$V_{C9} = V_{C10} = V_{B10} = 5 - 0.7 = 4.3^V$

$I_{C11} = I_{C10} = 1.86mA$

$V_{C11} = 1.86 \times 1 = 1.86V$

6.35

a) Refer to Fig. P6.35

$R = 10K\Omega$

$V_1 = -0.7^V \Rightarrow I_{C_1} = \frac{-0.7 - (-10.7)}{10^K} = 1mA$

$\underline{I_{C_1} = 1mA}$

$V_2 = 5.7 - 0.7 = 5V$

$I = I_{C3} + I_{C4}$, $I_{C3} = I_{C4} = I_{C_1} \Rightarrow I = 2 \times 1 = \underline{2mA}$

$V_3 = 0 + 0.7 = 0.7^V$

$V_4 = -10.7 + 1 \times 10^K = -0.7V$

$V_5 = -10.7 + 1 \times \frac{10^K}{2} = -5.7^V$

b) $R = 100K\Omega$

$V_1 = -0.7V \Rightarrow I_{C_1} = \frac{-0.7 + 10.7}{100^K} = \underline{0.1mA}$

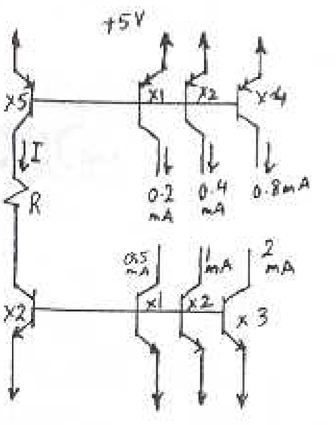
$I = 2 I_{C_1} = \underline{0.2mA}$

$V_3 = 0.7V$, $V_2 = 5.7 - 0.7 = 5^V$

$V_4 = -10.7 + \frac{1}{10} \times 100 = -0.7^V$

$V_5 = -10.7 + 0.1 \times \frac{100}{2} = -5.7^V$

6.36



$I = \frac{10 - 1.4}{R} = 1mA$

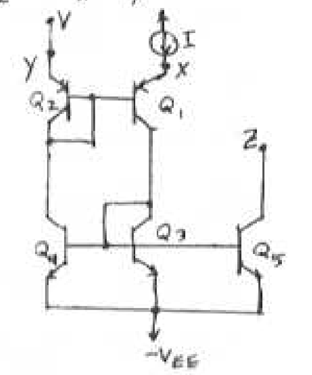
$\Rightarrow \underline{R = 8.6K\Omega}$

6.37

a) Refer to fig. P6.37.

Since Q_1, Q_3 have equal currents, I, and Q_3 and Q_4 have equal V_{BE} which results in equal currents, then Q_2 has the same current as Q_1.

Therefore: $V_{BE1} = V_{BE2} \Rightarrow V_x = V_y = V$



Also, $V_{BE5} = V_{BE3}$

$\Rightarrow I_5 = I_3 = I$

or $I_Z = I$.

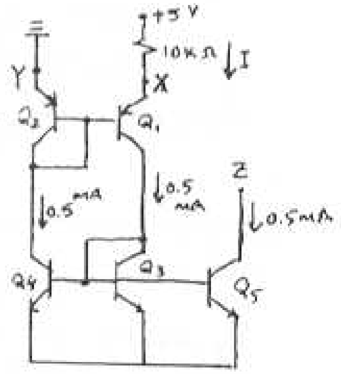
b) $I_2 = I_4$, $I_1 = I_3$

Since $V_{BE4} = V_{BE3}$ then $I_3 = I_4$ and hence $I_1 = I_2$.

From $I_1 = I_2$ we conclude that $V_{BE1} = V_{BE2}$ and

Since $V_{B1} = V_{B2}$ then V_{E1} has to be the same as V_{E2}: $V_{E_1} = V_{E_2} = 0$

$VX = 0$



$I = \frac{5.0}{10^K} = 0.5mA$

6.38

$40\,db = 20 \log A_o \Rightarrow A_o = 100\,V/V$

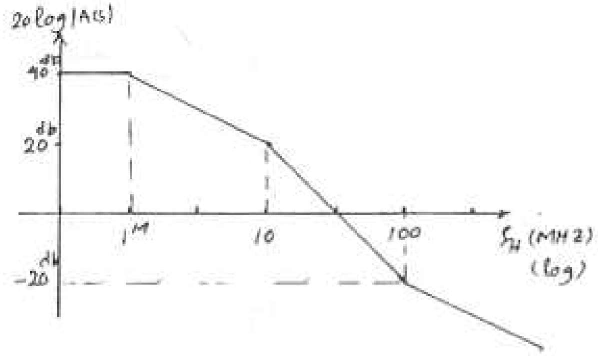
$$A(s) = +100 \frac{(1 + s/100\times10^6\times2\pi)}{(1 + \frac{s}{2\pi\times10^3})(1 + \frac{s}{2\pi\times10^6})}$$

$$A(s) = +100 \frac{(1 + \frac{s}{2\pi\times10^8})}{(1 + \frac{s}{2\pi\times10^3})(1 + \frac{s}{2\pi\times10^6})}$$

$$Eq.6.36: \omega_H = 1\Big/\sqrt{\left(\left(\frac{1}{2\pi\times10^3}\right)^2 + \left(\frac{1}{2\pi\times10^6}\right)^2 - 2\left(\frac{1}{2\pi\times10^8}\right)^2\right)}$$

$f_H = 0.995\,MHz$

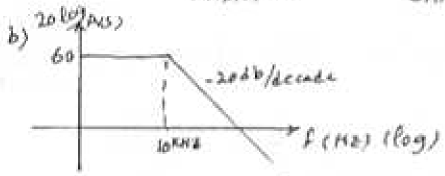
Cont.

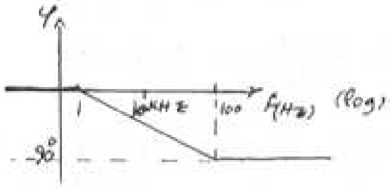


6.39

a) $A(s) = \frac{1000}{(1+\frac{s}{2\pi\times 10\times 10^3})} = \frac{1000}{(1+\frac{s}{2\pi\times 10^4})}$

b)





6.40

$F_H(s) = \frac{1}{(1+\frac{s}{\omega_{P1}})(1+\frac{s}{\omega_{P2}})}$ $\quad \omega_{P1} < \omega_{P2}$

using dominant-pole approximation: $\omega_H \simeq \omega_{P1}$

using the root sum of squares formula:

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}}} = \frac{\omega_{P1}}{\sqrt{1+(\frac{\omega_{P1}}{\omega_{P2}})^2}}$$

The difference between the two estimates for ω_H is:

$$\Delta\omega_H = \omega_{P1} - \frac{\omega_{P1}}{\sqrt{1+(\frac{\omega_{P1}}{\omega_{P2}})^2}}$$

IF $n = \frac{\omega_{P2}}{\omega_{P1}}$:

$$\frac{\Delta\omega_H}{\omega_{P1}} = 1 - \frac{1}{\sqrt{1+\frac{1}{n^2}}}$$

for $\frac{\Delta\omega_H}{\omega_{P1}} = 10\% = 0.1 \Rightarrow \underline{n = 2.07}$

for $\frac{\Delta\omega_H}{\omega_{P1}} = 1\% = 0.01 \Rightarrow \underline{n = 7.02}$

6.41

$$A(s) = -100\,\frac{1+s/10^6}{(1+s/10^5)(1+s/10^7)}$$

a) $\omega_H \simeq 10^5$ rad/s

b) $\omega_H \simeq \frac{1}{\sqrt{(\frac{1}{10^5})^2+(\frac{1}{10^7})^2-2(\frac{1}{10^6})^2}} = 101$ Krad/s

IF the pole at 10^6 rad/s is lowered to 10^5 rad/s the transfer function becomes:

$A(s) = \frac{-100}{1+s/10^7} \Rightarrow f_H = \frac{10^7}{2\pi}$ Hz

6.42

$30^\circ = 3\tan^{-1}\frac{\omega}{\omega_p} = 3\tan^{-1}\frac{10^6}{\omega_p} \Rightarrow \omega_p = 5.67\times10^6$ rad/s

6.43

$$\omega_H \simeq \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{C_{gs}R_{gs}+C_{gd}R_{gd}}$$

$\omega_H = \frac{1}{C_{gs}R' + C_{gd}(R'+R_L'+g_m R_L' R')}$ (From Example 6.1

For $C_{gs}=C_{gd}=1$ PF, $R_L' = 3.33$ kΩ, $g_m = 4$ mA/V

$$\omega_H = \frac{1}{10^{-12}R' + 10^{-12}(R'+3.33\times10^3+4\times3.33\times R')}$$

To obtain $\omega_H = 2\pi\times150\times10^3$

$2\pi\times150\times10^3 = \frac{10^{12}}{3.33\times10^3+15.32R'} \Rightarrow R' = 69.04K$

$R' = R \,\|\, R_{in} = R \,\|\, 420^K = 69.04 \Rightarrow \underline{R = 82.6\,k\Omega}$

6.44

Pole at $\frac{1}{2\pi\times10^4\times5\times10^{-12}} = 3.18$ MHz and

Cont

pole at $\dfrac{1}{2\pi \times 20\times10^3 \times 2\times10^{-12}} = 3.98\,MHz$

Since both poles are relatively close together, we use the root-sum-of-squares formula:

$$f_H = \frac{1}{\sqrt{\frac{1}{3.18^2}+\frac{1}{3.98^2}}} = 2.5\,MHz$$

Manufactured design:

Poles at $\dfrac{1}{2\pi\times10^4\times15\times10^{-12}} = 1.06\,MHz$ and at

$$\frac{1}{2\pi\times20\times10^3\times12\times10^{-12}} = 0.66\,MHz$$

$$f_H = \frac{1}{\sqrt{\frac{1}{1.06^2}+\frac{1}{0.66^2}}} = 0.56\,MHz$$

6.45

Refer to solution of Example 6.6:

$$A_M = \frac{V_o}{V_i} = -\frac{R_{in}}{R_{in}+R}\, g_m R_L' = -\frac{1.2}{1.2+0.1}(2\times12)$$

$A_M = -22.2\,V/V$

$R_{gs} = R_{in} \| R = 1.2^M \| 0.1^M = 92.3\,k\Omega$

$\tau_{gs} = R_{gs} C_{gs} = 1\times10^{-12}\times 92.3\times10^3 = 92.3\,ns$

$R_{gd} = R' + R_L' + g_m R_L' R'$ where $R' = R_{in}\|R = 92.3\,k\Omega$

$R_{gd} = 92.3 + 12 + 2\times12\times92.3 = 2.32\,M\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 1\times10^{-12}\times 2.32\times10^6 = 2320\,ns$

$$\omega_H = \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{(92.3+2320)\times10^{-9}} = 414.5\,krad/s$$

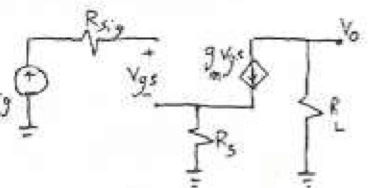
$f_H = 66\,KHz$

6.46

a) $V_o = -g_m R_L V_{gs}$ ①

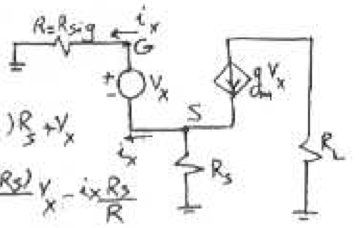
$V_{gs} = V_{sig} - R_s \times g_m V_{gs}$

$V_{gs}(1+g_m R_s) = V_{sig}$



① $\Rightarrow V_o = \dfrac{-g_m R_L}{1+g_m R_s} V_{sig} \Rightarrow \dfrac{V_o}{V_{sig}} = \dfrac{-g_m R_L}{1+g_m R_s}$

b) $V_S = (g_m V_x - i_x) R_s$

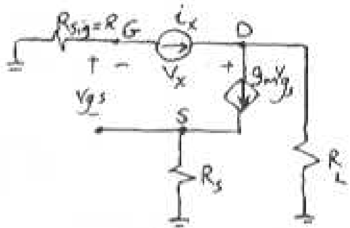


$V_G = V_S + V_x = (g_m V_x - i_x) R_s + V_x$

$\Rightarrow i_x = \dfrac{V_G}{R} = \dfrac{(1+g_m R_s)}{R} V_x - \dfrac{i_x R_s}{R}$

$$R_{gs} = \frac{V_x}{i_x} = \frac{1+R_s/R}{\frac{1+g_m R_s}{R}} = \frac{R+R_s}{1+g_m R_s} \quad (R \text{ is } R_{sig})$$

to calculate R_{gd}:



$V_G = -R i_x$

$V_S = -R i_x - V_{gs}$

$V_S = R_s \times g_m V_{gs}$ $\Big\} \Rightarrow$

$R_s g_m V_{gs} = -R i_x - V_{gs} \Rightarrow V_{gs} = \dfrac{-R i_x}{1+g_m R_s}$

At D: $i_x = g_m V_{gs} + \dfrac{V_x - R \times i_x}{R_L}$

substitute V_{gs}: $i_x = -\dfrac{g_m R i_x}{1+g_m R_s} + \dfrac{V_x}{R_L} - \dfrac{R i_x}{R_L}$

$$i_x\left[1+\frac{g_m R}{1+g_m R_s}+\frac{R}{R_L}\right] = \frac{V_x}{R_L}$$

$$R_{gd} = \frac{V_x}{i_x} = R_L + R + \frac{g_m R R_L}{1+g_m R_s} \quad (R \text{ is } R_{sig})$$

c) $R_S = 0$:

$$\frac{V_o}{V_{sig}} = \frac{-4\times5^K}{1+4\times0} = -20\,V/V$$

$R_{gs} = R_{sig} = 100\,K\Omega$

$R_{gd} = 5^K + 100K + 4\times5\times100 = 2105\,K\Omega$

$$\omega_H \simeq \frac{1}{C_{gs}R_{gs}+C_{gd}R_{gd}} = \frac{1}{10^{-12}\times100\times10^3+10^{-12}\times2105\times10^3}$$

$\omega_H \simeq 453.5\,Krad/s$

$|Gain| \times Bandwidth = 20\times453.5 = 9.07\,Mrad/s$

$R_S = 100\,\Omega$:

$$\frac{V_o}{V_{sig}} = \frac{-4\times5}{1+4\times0.1} = -14.3\,V/V$$

$R_{gs} = \dfrac{100+0.1}{1+4\times0.1} = 71.5\,K\Omega$

$R_{gd} = 5 + 100 + \dfrac{4\times5\times100}{1+4\times0.1} = 1533.6\,K\Omega$

$$\omega_H = \frac{1}{10^{-12}\times71.5\times10^3+10^{-12}\times1533.6\times10^3} = 623\,K\,rad/s$$

Cont.

$|Gain| \times Bandwidth = 14.3 \times 623^k = \underline{8.91^{Mrad/s}}$

$\underline{R_S = 250\Omega}$:

$$\frac{v_o}{v_{sig}} = \frac{-4\times5}{1+4\times0.25} = -10^{V}/_{V}$$

$$R_{gs} = \frac{100+0.25}{1+4\times0.25} = 50.1K\Omega$$

$$R_{gd} = 5+100+\frac{4\times5\times100}{1+4\times0.25} = 1105 \ K\Omega$$

$$\omega_H = \frac{1}{10^{-12}\times50.1\times10^3+10^{-12}\times1105\times10^3} = 865.7 \, Krad/s$$

$|gain| \times Bandwidth = 10 \times 865.7^K = \underline{8.66 \ Mrad/s}$

Summary table:

$R_s^{(\Omega)}$	Gain $(V/_V)$	ω (krad/s)	Gain.BW product (Mrad/s)
0	-20	453.5	9.07
100	-14.3	623.0	8.91
250	-10	865.7	8.66

The GainxBandwidth is approximately constant.

6.47

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_{in}}{R_{in}+R_{sig}}(g_m R_L') = -\frac{5}{5+1}(0.3\times100^k)$$

$A_M = -25 \, V/_V$ Now refer to Example 6.6.

$R_{gs} = R_{in} \, // \, R_{sig} = 5^{M\Omega} // 1^{M\Omega} = 0.83^{M\Omega}$

$\tau_{gs} = R_{gs} C_{gs} = 0.2\times10^{-12}\times0.83\times10^6 = 166.7 \, ns$

$R_{gd} = R' + R_L' + g_m R_L' R'$

$R' = R_{in} // R_{sig} = 0.83 \, M\Omega$ $\Big\} \Rightarrow R_{gd} = 0.83+0.1+0.83\times0.3\times100$

$R_{gd} = 25.92 M\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 25.92\times10^6\times0.1\times10^{-12} = 2592 \, ns$

$$\omega_H \simeq \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{(166.7+2592)\times10^{-9}} = 362.5^{krad/s}$$

$f_H = 57.7 \, KHZ$

6.48

If we assume that capacitors are perfect open circuits for midband, then:

$$A_M = \frac{V_o}{V_{sig}} = \frac{-R_{in}}{R_{in}+R_{sig}}(g_m R_L') = \frac{-650}{650+150}(5\times10) = 40.6 V/_V$$

$\tau_{gs} = C_{gs} R_{gs} = C_{gs}(R_{in} // R_{sig}) = 2^P \times (150^K // 650K)$

$\tau_{gs} = 243.75 \, ns$

$\tau_{gd} = C_{gd} R_{gd}$, Refer to Example 6.6

$R_{gd} = R' + R_L' + g_m R_L' R'$

$R' = 150^K // 650K = 121.9K\Omega$ $\Rightarrow R_{gd} = 121.9 + 10 + 5\times10\times121.9$

$R_{gd} = 6.2 M\Omega$

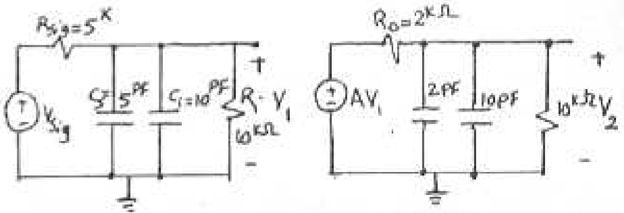
$\tau_{gd} = C_{gd} R_{gd} = 0.5^P \times 6.2^M = 3100 \, ns$

$\tau_L = R_L' C_L = 10^k \times 3^P = 30 \, ns$

$$\omega_H = \frac{1}{\tau_{gs}+\tau_{gd}+\tau_L} = \frac{1}{30^n+3100^n+243.75^n} = 296.4 \, Krad/s$$

$f_H = 47.2 \, KHZ$

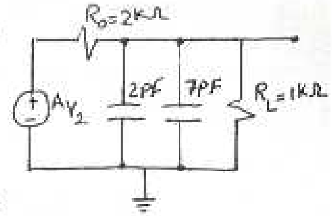
6.49



$\tau_1 = (5^{PF}+10^{PF})\times(5^K // 10^k)$

$\tau_1 = 50 ns$

$\omega_1 = \frac{1}{\tau_1} = 20 Mrad/s$



$\tau_2 = (2^{PF}+10^{PF})(10^k // 2^K)$

$\tau_2 = 20 ns$

$\omega_2 = \frac{1}{\tau_2} = 50 \, Mrad/s$

$\tau_3 = (2^{PF}+7^{PF})(1^K // 2^K)$

$\tau_3 = 6 ns$

$\omega_3 = 166^{Mrad/s}$

$$\omega_H \simeq \frac{1}{\tau_1+\tau_2+\tau_3} = \frac{1}{50^n+20^n+6^n} = 13.2 \, MHZ$$

$\underline{f_H = 2.1 \, MHZ}$

6.50

Eq. 6.41 $\omega_H \simeq \frac{1}{\sum RC}$

a) $\omega_H = \frac{1}{20+5+1^{ns}} = 38.46\,\text{Mrad/s} \Rightarrow f_H = 6.12\,\text{MHz}$

b) $\omega_H = \frac{1}{\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \frac{1}{\omega_{p3}}}$ (Eq. 6.38)

$$\omega_H = \frac{2\pi}{\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}} = \frac{2\pi}{\frac{1}{50}+\frac{1}{200}+\frac{1}{1000}} = 241.5\ \text{Mrad/s}$$

$f_H = 38.46\,\text{MHz}$

c) $\omega_H = \frac{1}{\frac{1}{50}+\frac{1}{200}+\frac{1}{1000}} = 38.46\,\text{Mrad/s}$

$f_H = 6.12\,\text{MHz}$

d) $\omega_H = \frac{1}{1000+200+200} = 714\,\text{Krad/s} \Rightarrow f_H = 113.7\,\text{KHz}$

e) $\omega_H = \frac{1}{0.4+1} = 714\,\text{Krad/s} \Rightarrow f_H = 113.7\,\text{KHz}$

f) $\omega_H = \frac{1}{1+0.2+0.15} = 741\,\text{Krad/s} \Rightarrow f_H = 118\,\text{KHz}$

g) $\omega_H = \frac{2\pi}{\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}} = \frac{2\pi}{\frac{1}{1}+\frac{1}{2}+\frac{1}{5}+\frac{1}{5}}$

$\omega_H = 3.3\,\text{Grad/s} \Rightarrow f_H = 526\,\text{MHz}$

6.51

$$R_{in} = \frac{R}{1-\text{Gain}} = \frac{100}{1-0.95} = 2000\,\text{K}\Omega = 2\,\text{M}\Omega$$

6.52

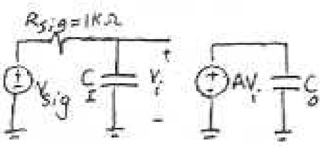
Eq. 6.44a $Z_I = Z/_{1-K} \Rightarrow C_I = 0.1\times(1-(-1000))$

$\Rightarrow C_I = 100.1\,\text{pF}$

$C_o = 0.1\times\left(\frac{-1}{1000}+1\right)$

$C_o = 99.9\,\text{fF}$

(using Miller's Theorem)



$$V_o = AV_i = A\times V_{sig}\,\frac{1/C_I s}{R_{sig}+\frac{1}{C_I s}} \Rightarrow \frac{V_o}{V_{sig}} = \frac{A}{1+C_I R_{sig} s}$$

$$\omega_H = \frac{1}{C_I R_{sig}} = \frac{1}{100.1^{p}\times 1^{K}} = 9.99\,\text{Mrad/s} \Rightarrow f_H = 1.59\,\text{MHz}$$

To calculate unity gain frequency:

$|\text{Gain}| = 1$

$$\frac{V_o}{V_i} = \frac{A}{1+C_I R_{sig} s} = \frac{-1000}{1+100.1\times10^{-9} s} \quad (s=j\omega)$$

$$\frac{1000}{\sqrt{1+(100.1\times10^{-9}\times\omega_T)^2}} = 1 \Rightarrow \omega_T \simeq 10\,\text{Grad/s}$$

$f_T \simeq 1.59\,\text{GHz}$

As we can see $f_T \simeq f_H \times A$

6.53

Using Miller's Theorem, in each case the capacitance at the input is $C(1-A)$ and the capacitance at the output is $C(1-\frac{1}{A})$. Thus:

a) $A = -1000\,\text{V/V}$ and $C = 1\,\text{pF}$

$C_i = 1.001\,\text{nF}$ and $C_o = 1.001\,\text{pF}$

b) $A = -10\,\text{V/V}$ and $C = 10\,\text{pF}$

$C_i = 110\,\text{pF}$ and $C_o = 11\,\text{pF}$

c) $A = -1\,\text{V/V}$ and $C = 10\,\text{pF}$

$C_i = 20\,\text{pF}$ and $C_o = 20\,\text{pF}$

d) $A = 1\,\text{V/V}$ and $C = 10\,\text{pF}$

$C_i = 0\,\text{pF}$ and $C_o = 0\,\text{pF}$

e) $A = 10\,\text{V/V}$ and $C = 10\,\text{pF}$

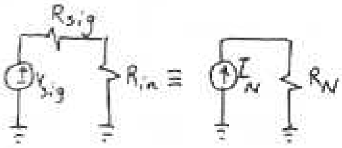
$C_i = -90\,\text{pF}$ and $C_o = 9\,\text{pF}$

In (e) the negative capacitance at the input can be used to cancel the effect of the input capacitance of the amplifier.

6.54

a) $R_{in} = \frac{R}{1-A} = \frac{R}{1-2} = -R$ (Miller's theorem)

b) $I_N = \frac{V_{sig}}{R_{sig}}$



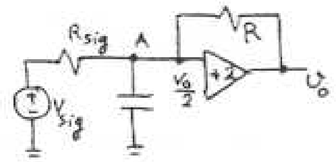
$R_N = R_{sig} \,||\, R_{in}$

If $R_{sig} = R$ then:

$$R_N = R\,||\,(-R) = \infty \Rightarrow I_L = I_N = \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$$

Cont.

KCL at A:



$$\frac{\frac{V_o}{2} - V_{sig}}{R_{sig}} + \frac{v_o}{2} \times CS + \frac{-v_o}{2R} = 0$$

If $R_{sig} = R \Rightarrow \frac{+v_{sig}}{R} = \frac{v_o}{2} CS \Rightarrow \frac{v_o}{v_{sig}} = \frac{2}{RCS}$

6.55

From Table 6.3 we have:

Intrinsic gain $= g_m r_o = \frac{2V_A' L}{V_{OV}}$

$\rightarrow$ Intrinsic gain $= \frac{2\times10\times1}{0.2} = 100\,V/V$

$g_m r_o = 100\,V/V$

$g_m = \frac{2I_D}{V_{OV}} \Rightarrow I_D = \frac{g_m V_{OV}}{2} = \frac{2\times0.2}{2} = 0.2\,mA$

$g_m = \mu_n C_{ox}\frac{W}{L}V_{OV} \rightarrow W = \frac{g_m L}{k_n' V_{OV}} = \frac{2\times1\times10^3}{125\times0.2} = 80\,\mu m$

6.56

$A_o = g_m r_o = 100\,V/V = \frac{2V_A' L}{V_{OV}}$

$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 = 100\,\mu A$

$\frac{A_{o2}}{A_{o1}} = \frac{V_{OV1}}{V_{OV2}} = \sqrt{\frac{I_{D1}}{I_{D2}}} \Rightarrow A_{o2} = A_{o1}\sqrt{\frac{I_{D1}}{I_{D2}}}$

Now if I_D is $25\,\mu A$ then: $A_{o2} = 100\sqrt{\frac{100}{25}} = 200\,V/V$

$I_D = 400\,\mu A \Rightarrow A_{o2} = 100\sqrt{\frac{100}{400}} = 50\,V/V$

$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} \Rightarrow \frac{g_{m2}}{g_{m1}} = \sqrt{\frac{I_{D2}}{I_{D1}}} \Rightarrow g_{m2} = g_{m1}\sqrt{\frac{I_{D2}}{I_{D1}}}$

For $I_D = 25\,\mu A$ $\quad g_{m2} = g_{m1}\sqrt{\frac{25}{100}} \Rightarrow g_{m2} = g_{m1}/2$

$I_D = 400\,\mu A \Rightarrow g_{m2} = g_{m1}\sqrt{\frac{400}{100}} \Rightarrow g_{m2} = 2g_{m1}$

6.57

Refer to Fig. P6.57.

a) if we neglect the current in feedback resistor: $I_D = 200\,\mu A = \frac{1}{2}k'\frac{W}{L}V_{OV}^2 = \frac{1}{2}\times2\times V_{OV}^2 \Rightarrow$

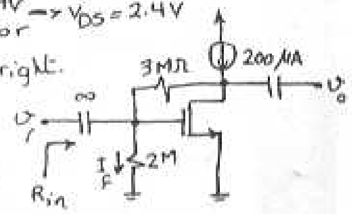
$V_{OV}^2 = 200\times10^{-3} \Rightarrow V_{OV} = 0.45V \Rightarrow V_{GS} = V_t + V_{OV}$

$\Rightarrow V_{GS} = 0.5 + 0.45 = 0.95V$

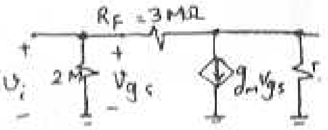
$\Rightarrow V_G = 0.95V \Rightarrow I_F = \frac{0.95}{2M} = 0.48\,\mu A \ll 200\,\mu$

$\Rightarrow V_{DS} = 0.48\times5 = 2.4V \Rightarrow V_{DS} = 2.4V$

So our assumption for neglecting I_F was right.



b) To find the small-signal gain we write a KCL at the output node:



$\frac{v_o}{r_o} + g_m v_{gs} + \frac{v_o - v_i}{R_F} = 0$

$v_{gs} = v_i$

$\frac{v_o}{r_o} + g_m v_i + \frac{v_o}{R_F} - \frac{v_i}{R_F} = 0 \Rightarrow \frac{v_o}{v_i} = \frac{1/R_F - g_m}{\frac{1}{R_F} + \frac{1}{r_o}}$

$g_m = \frac{2I_D}{V_{OV}} = \frac{2\times200\mu}{0.45} = 0.89\,mA/V$

$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100\,k\Omega$

$\frac{v_o}{v_i} = \frac{1/3000 - 0.89}{\frac{1}{3000} + \frac{1}{100}} = -86.1\,V/V$

$V_{DSmin} = V_{GS} - V_t = V_{OV} = 0.45V$

In our case $V_{DS} = 2.4^V$, therefore the largest signal at the output is 2.4-0.45= 1.95V. The corresponding input signal is

$\frac{v_o}{A} = \frac{1.95}{-86.1} = 0.023V$ or 23mV.

c) $R_{in} = \frac{v_x}{i_x}$

$i_x = \frac{v_x}{R_i} + \frac{v_x - v_o}{R_F}$ ①

$\frac{v_x - v_o}{R_F} = \frac{v_o}{r_o} + g_m v_x \Rightarrow v_o = v_x \frac{1/R_F - g_m}{\frac{1}{r_o} + \frac{1}{R_F}}$

① $\Rightarrow i_x = \frac{v_x}{R_i} + \frac{v_x}{R_F} - \frac{v_x}{R_F}\,\frac{1/R_F - g_m}{1/r_o + \frac{1}{R_F}}$

$$\frac{v_x}{i_x} = R_{in} = \frac{1}{\frac{1}{R_i} + \frac{1}{R_F} - \frac{1}{R_F(\frac{R_F}{r_o}+1)} + \frac{g_m}{(\frac{R_F}{r_o}+1)}}$$

$R_{in} = R_i \,//\, R_F \,//\, \frac{1 + R_F/r_o}{g_m - 1/R_F}$

$R_{in} = 2^M // 3^M // \frac{1 + 3000/100}{0.89 - 1/3000} = 33.9^{k\Omega}$

6.58

Refer to Fig. 6.18a and assume Q_2 and Q_3 are matched:

$k_n' = 2.5\,k_p' = 250\,\mu A/V^2$, $V_{An} = |V_{Ap}| = 10^V$

$R_{out} = 100^{k\Omega} = r_{o1} \| r_{o2} = \frac{10}{I_{D1}} \| \frac{10}{I_{D2}}$

Since $I_{D1} = I_{D2} \Rightarrow \frac{10}{I_{D1}} = 200 \Rightarrow I_{D1} = I_{D2} = 0.05\ mA$

$I_{REF} = I_{D2} = 0.05\,mA$

Eq. 6.49 : $A_v = -g_m (r_{o1} \| r_{o2}) \rightarrow -40 = -g_m \times 100^k$

$\Rightarrow g_{m1} = 0.4\ mA/V$

$g_{m1} = \sqrt{2K_n'(\frac{W}{L})_1 I_{D1}} \Rightarrow (\frac{W}{L})_1 = \frac{0.4^2}{2 \times 250 \times 10^{-3} \times 0.05} = 6.4$

$(\frac{W}{L})_1 = 6.4$

$g_{m1} = \frac{2I_{D1}}{V_{OV1}} \Rightarrow V_{OV1} = \frac{2 \times 0.05}{0.4} = 0.25\ V$

If Q_2 and Q_3 have the same V_{OV} as Q_1, then $|V_{OV2}| = 0.25\ V$

$I_{D2} = \frac{1}{2} k_p' (\frac{W}{L})_2 V_{OV}^2 \Rightarrow (\frac{W}{L})_2 = \frac{0.05 \times 2}{100 \times 10^{-3} \times 0.25^2} = 16$

$(\frac{W}{L})_2 = (\frac{W}{L})_3 = 16$ (Q_2 and Q_3 are matched)

6.59

As discussed in Example 6.8, the transfer characteristic of the amplifier over the desired region (segment III) is quite linear. Therefore the DC bias component of the input signal (for maximum output swing) should be chosen at the midpoint between V_{IA} and V_{IB} that is:

input dc bias $= \frac{V_{IA} + V_{IB}}{2} = \frac{0.88 + 0.93}{2} = 0.905\ V$

The corresponding amplitude of the resulting output sinusoid is:

output sinusoid amplitude $= \frac{V_{OA} + V_{OB}}{2} = \frac{2.47 + 0.33}{2}$

output Sinusoid amplitude $= 1.4V$

6.60

As discussed in Example 6.8, the transfer characteristic of the amplifier over the region labeled as segment III, is quite linear.

$V_{OA} = V_{DD} - V_{OV3} = 5 - 0.53 = 4.47^V$

Now to find the linear equation for segment III, we can write $i_{D1} = i_{D2}$:

$\frac{1}{2} k_n' (\frac{W}{L})_1 (v_I - V_{tn})^2 (1 + \frac{v_O}{V_{An}}) = \frac{1}{2} k_p' (\frac{W}{L})_2 (V_{SG} - |V_{tp}|)^2 (1 + \frac{V_{DD} - v_O}{V_{Ap}})$

$\Rightarrow 200 (v_I - 0.6)^2 (1 + \frac{v_O}{20}) = 65 \times 0.53^2 \times (1 + \frac{V_{DD} - V_O}{10})$

$\frac{200}{65 \times 0.53^2} (v_I - 0.6)^2 = \frac{1.5 - v_O/10}{1 + \frac{v_O}{20}}$

$7.3 (v_I - 0.6)^2 = \frac{1 - v_O/15}{1 + \frac{v_O}{20}} = \frac{1 - 0.067 v_O}{1 + 0.05 v_O} \simeq 1 - 0.117 v_O$

$\Rightarrow v_O = 8.57 - 62.57 (v_I - 0.6)^2$ (1)

If we substitute for $V_{OA} = 4.47^V$, then $V_{IA} = 0.86^V$

To determine coordinates of B, note that

$V_{IB} - V_{tn} = V_{OB}$ or $V_{IB} - 0.6 = V_{OB}$

substitute in (1):

$V_{OB} = 8.57 - 62.57 V_{OB}^2 \Rightarrow V_{OB} = 0.36^V$

$V_{IB} = 0.6 + 0.36 = 0.96V$

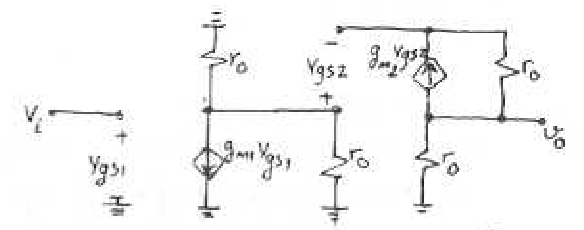
Therefore the linear region is:

$0.86^V \leq v_I \leq 0.96^V$ or $0.36^V \leq v_O \leq 4.47^V$

6.61

Since $V_{An} = |V_{Ap}|$ and the drain current of both Q_1 and Q_2 is equal to I, their output resistances are equal. That is $r_{o1} = r_{o2} = r_o$

The small-signal model of this amplifier is:



Cont.

$v_{gs2} = -g_{m_1} v_{gs_1}(r_o \| r_o)$ and $v_{gs_1} = v_i$

$v_{gs2} = -g_{m_1} \frac{r_o}{2} v_i$

$v_o = -g_{m2} v_{gs2}(r_o \| r_o) = -g_{m2}(-g_{m_1} \frac{r_o}{2} v_i)\frac{r_o}{2}$

$A_v = \frac{v_o}{v_i} = g_{m_1} g_{m2} \frac{r_o^2}{4}$

6.62

Refer to Fig. 6.18a.

Note that Q_2, Q_3 are not matched:

$I_{D1} = 100\,\mu A$

a) $I_{D2} = I_{D1} = 100\,\mu A$ $\quad \frac{I_{D3}}{I_{D2}} = \frac{(W/L)_3}{(W/L)_2} = \frac{W_3}{W_2}$

(Note that $V_{SG2} = V_{SG3}$)

$\Rightarrow I_{D3} = 100\,\mu A \; \frac{10}{40} = 25\,\mu A \Rightarrow I_{REF} = 25\,\mu A$

b) By referring to Fig. 6.18d, you notice that in Segment III, both Q_1 and Q_2 are in saturation and the transfer characteristic is quite linear. The output voltage in this segment is limited between V_{OA} and V_{OB}:

Coordinates of point A: $v_{OA} = V_{DD} - V_{OV3}$

$V_{OV3}^2 = \frac{I_{D3}}{\frac{1}{2}k_p'(\frac{W}{L})_3} = \frac{25}{\frac{1}{2} \times 50 \times \frac{10}{1}} = 0.1^V \Rightarrow |V_{OV3}| = 0.32^V$

$v_{OA} = 3.3 - 0.32 = 2.98^V$

At point B: $V_{OB} = V_{IB} - V_{tn}$

Now we find the transfer equation for the linear section: (Refer to Example 6.8)

$i_{D1} = i_{D2} \Rightarrow$ (Note that $V_{OV2} = V_{OV3}$)

$k_n'(\frac{W}{L})_1 (v_I - V_{tn})^2 (1 + \frac{v_O}{V_{An}}) = k_p'(\frac{W}{L})_2 V_{OV3}^2 (1 + \frac{V_{DD} - v_O}{|V_{Ap}|})$

$100 \times \frac{20}{1}(v_I - 0.8)^2 (1 + \frac{v_O}{100}) = 50 \times \frac{40}{1} \times 0.32^2 (1 + \frac{3.3 - v_O}{50})$

$(v_I - 0.8)^2 = 0.32^2 (1.066 - \frac{v_O}{50}) / (1 + \frac{v_O}{100})$

$(v_I - 0.8)^2 = 0.11 \left(\frac{1 - 0.019 v_O}{1 + 0.01 v_O}\right) \simeq 0.11(1 - 0.03 v_O)$

$(v_I - 0.8)^2 = 0.11(1 - 0.03 v_O)$ ①

Now if we solve for $V_{OB} = V_{IB} - 0.8$

$V_{OB}^2 + 0.0033 v_{OB} - 0.11 = 0 \Rightarrow V_{OB} = 0.33^V$

Therefore the extreme values of v_O for which Q_1 and Q_2 are in saturation: $0.33^V \leq v_O \leq 2.98^V$

c) From (b) we can find V_{IA} and V_{IB}:

$V_{IB} = V_{OB} + V_t = 0.33 + 0.8 = 1.13V$

If we solve ① for $V_{OA} = 2.98V$ then:

$(V_{IA} - 0.8)^2 = 0.11(1 - 0.03 \times 2.98) \Rightarrow V_{IA} = 1.116^V$

Large-signal voltage gain $= \frac{\Delta v_O}{\Delta v_I} = \frac{2.98 - 0.33}{1.13 - 1.116}$

$\frac{\Delta v_O}{\Delta v_I} = -189.3\ V/V$

d) $v_O = \frac{V_{DD}}{2} = \frac{3.3}{2} = 1.65V$

Differentiating both sides of ①: $(\frac{\partial}{\partial v_I})$

$2(v_I - 0.8) = 0.11 \times (-0.03) \frac{\partial v_O}{\partial v_I}$

$\Rightarrow \frac{\partial v_O}{\partial v_I} = -606.1(v_I - 0.8)$

For $v_O = 1.65^V$, from ① we have:

$(v_I - 0.8)^2 = 0.11(1 - 0.03 \times 1.65) \Rightarrow v_I = 1.123$

$\left.\frac{\partial v_O}{\partial v_I}\right|_{v_I = 1.123} = -195.8\ V/V$

e) $R_{out} = r_{o1} \| r_{o2}$

$r_{o1} = \frac{V_{An}}{I_{D1}} = \frac{100}{0.1^m} = 1\,M\Omega$

$r_{o2} = \frac{V_{Ap}}{I_{D2}} = \frac{50}{0.1^m} = 500\,k\Omega$

$\Rightarrow R_{out} = 500^K \| 1$ $\quad R_{out} = 333$

$g_{m_1} = \sqrt{2k_n'(\frac{W}{L})_1 I_{D1}} = \sqrt{2 \times 100^\mu \times \frac{20}{1} \times 100^\mu} = 0.632\,m$

$A_v = -g_{m_1}(r_{o1} \| r_{o2}) = -210.6\ V/V$

6.63

a) When D and G are open, since the gates do not draw any current, therefore no current g through R and $V_D = V_G$, $I_{D1} = I_{D2}$.

$\frac{1}{2}k_n'(\frac{W}{L})_1 (V_G - (-1.5) - 0.5)^2 = \frac{1}{2}k_p'(\frac{W}{L})_2 (1.5 - V_G$

$(V_G + 1)^2 = (1 - V_G)^2 \Rightarrow V_G + 1 = 1 - V_G \Rightarrow V_G = 0$

$I_{D1} = \frac{1}{2} \times 1 \times (0 + 1.5 - 0.5)^2 = 0.5\,mA$

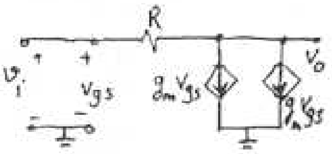
$I_{D1} = I_{D2} = 0.5\,mA$

Cont

b) $v_o = v_i - R(2g_m v_{gs})$

$v_{gs} = v_i$

$v_o = v_i - 2g_m R v_i$

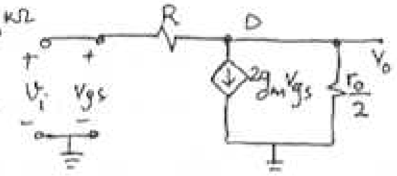


$A_v = \frac{v_o}{v_i} = 1 - 2g_m R$

$g_m = \frac{2I_D}{V_{GS}-V_t} = \frac{2\times 0.5}{1.5-0.5} = 1\ mA/V$

$A_v = 1 - 2\times 1\times 1000 = -1999\ V/V$

c) $r_o = \frac{V_A}{I_D} = \frac{20}{0.5} = 40^{k\Omega}$



If we write KCL at D:

$\frac{v_i - v_o}{R} = 2g_m v_{gs} + \frac{v_o}{r_o/2}$, $v_{gs} = v_i$

$\frac{v_i}{R} - 2g_m v_i = \frac{v_o}{R} + \frac{2v_o}{r_o} \Rightarrow \frac{v_o}{v_i} = A_v = \frac{1-2g_m R}{1+\frac{2R}{r_o}}$

or $A_v = \frac{1-2\times 1\times 1000}{1+\frac{2\times 1000}{40}} = -39.2\ V/V$

$R_{in} = \frac{v_x}{i_x}$: $v_x - Ri_x = \frac{r_o}{2}(i_x - 2g_m v_{gs})$

$v_{gs} = v_x$

$v_x + 2g_m \frac{r_o}{2} v_x = Ri_x + \frac{r_o}{2} i_x \Rightarrow R_{in} = \frac{v_x}{i_x} = \frac{R + r_o/2}{1+g_m r_o}$

$R_{in} = \frac{1000 + 40/2}{1 + 1\times 40} = 24.9 \simeq 25\ k\Omega$

d) $\frac{v_o}{v_{sig}} = A_v \frac{R_{in}}{R_{in}+R_{sig}} = -39.2 \times \frac{25^k}{100^k + 25^k}$

$\frac{v_o}{v_{sig}} = -7.84\ V/V$

e) In order for both Q_1 and Q_2 to remain in the saturation region:

$V_{DS} \geq V_{GS} - V_t \Rightarrow V_D + 1.5 \geq V_G + 1.5 - V_t$

$\Rightarrow V_D \geq V_G - 0.5$

For Q_2 : $V_{SD} \geq V_{SG} - |V_t| \Rightarrow 1.5 - V_D \geq 1.5 - V_G - 0.5$

$\Rightarrow V_D \leq V_G + 0.5$

In (a) we showed that the gate is at 0 volt, therefore $V_D \geq -0.5$ and $v_D \leq 0.5$

$\Rightarrow -0.5 \leq v_o \leq 0.5$

6.64

$R_i = r_\pi = \frac{\beta}{g_m} = \frac{\beta}{I_C/V_T} = \frac{100}{1} \times 0.025 = 2.5\ K\Omega$

$g_m = \frac{I_C}{V_T} = 40\ mA/V$, $r_o = \frac{V_A}{I_C} = 100\ K\Omega$

$A_{vo} = -g_m r_o = -40 \times 100 = -4000\ V/V$

$R_o = r_o = 100\ K\Omega$

If R_i is multiplied by 4, since $R_i \propto \frac{1}{I_C}$, then I_C has to be divided by 4: $I_C = \frac{1\ mA}{4} = 0.25\ mA$

For $I_C = 0.25\ mA$: $g_m = \frac{0.25}{0.025} = 10\ mA/V$

$r_o = \frac{V_A}{I_C} = \frac{100}{0.25} = 400\ K\Omega$

$A_{vo} = -10 \times 400 = -4000\ V/V$

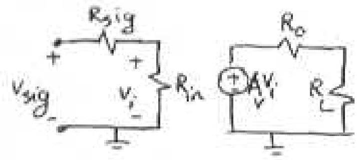
In general $A_{vo} = -g_m r_o = -\frac{I_C}{V_T} \times \frac{V_A}{I_C} = -\frac{V_A}{V_T}$

and A_{vo} is not dependent on I_C.

$R_o = r_o = 400\ K\Omega$, $R_i = 4\times 2.5 = 10^{K\Omega}$

For $I_C = 1\ mA$ and $R_{sig} = 5\ K\Omega$, $R_L = 500\ K\Omega$

$v_i = v_{sig} \frac{R_{in}}{R_{in}+R_{sig}}$



$G_v = \frac{R_{in}}{R_{in}+R_{sig}} A_{vo} \frac{R_L}{R_L+R_o}$

$G_v = -4000 \times \frac{2.5\ K\Omega}{5+2.5} \times \frac{500}{500+100} = -1111\ V/V$

$G_v = -1111\ V/V$

For $I_C = 0.25\ mA$: $R_{in} = 10^{K\Omega}$, $R_o = 400$

$G_v = \frac{10}{10+5} \times (-4000) \times \frac{500}{500+400} = -1481.5\ V/V$

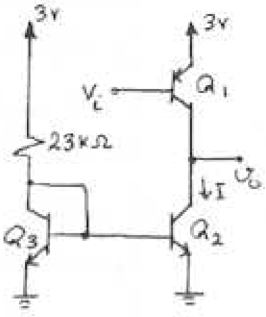
$G_v = -1481.5\ V/V$

6.65

a) $I_{REF} = I_{C3} = \frac{3 - V_{BE3}}{23K\Omega}$

$I_{REF} = \frac{3-0.7}{23}$

$I_{REF} = 0.1mA$



$\frac{I_{C2}}{I_{C3}} = \frac{\text{Area of } Q_2}{\text{Area of } Q_3} = 5$

$\Rightarrow I_{C2} = 5 I_{C3}$

$I_{C2} = I = 0.5mA \Rightarrow I = 0.5mA$

b) $|V_A| = 50V \Rightarrow r_{o1} = \frac{|V_A|}{I} = \frac{50}{0.5} = 100K\Omega$

$r_{o2} = \frac{50}{0.5} = 100K\Omega$

Total resistance at the collector of Q_1 is equal to $r_{o1} \| r_{o2}$, thus: $r_{tot} = 100^K \| 100^K = 50K\Omega$

$r_{tot} = 50K\Omega$

c) $g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.5}{0.025} = 20 mA/V$

$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{20} = 2.5K\Omega$

d) $R_{in} = r_{\pi 1} = 2.5K\Omega$

$R_o = r_{o1} \| r_{o2} = 100^K \| 100^K = 50K\Omega$

$A_v = -g_{m1} R_o = -20 \times 50 = -1000 V/V$

6.66

$A_M = -g_m R_L' = -5 \times 20 = -100 V/V$

$C_{in} = C_{gs} + C_{gd}(1 + g_m R_L')$ (Eq. 6.55)

$C_{in} = 2 + 0.1(1 + 5 \times 20) = 12.1 pF$

$f_H \simeq \frac{1}{2\pi C_{in} R_{sig}}$ (Eq. 6.54)

$f_H \simeq \frac{1}{2\pi \times 12.1 \times 10^{-12} \times 20k} = 658 KHz$

6.67

Eq. 6.57, $\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{CL}$

$\tau_H = C_{gs} R_{sig} + C_{gd}[R_{sig}(1 + g_m R_L') + R_L'] + C_L R_L'$

$\tau_H = 2^P \times 20^K + 0.1^P [20^K (1 + 5 \times 20) + 20^K] + 1^P \times 20^K$

$\tau_H = 264 ns$

$f_H \simeq \frac{1}{2\pi \tau_H} = 603 KHz$

$A_M = -g_m R_L' = -5 \times 20 = -100 V/V$

τ_{gs} : 15.1%
τ_{gd} : 77.3%
τ_L : 7.6%

Contribution of each time-constant to the overall τ_H.

If we compare f_H to the one obtained in Problem 6.66, we notice that Problem 6.66 has a larger f_H due to neglecting the time constants of C_L and C_{gs}.

6.68

From Eq. 6.60 we have: $\omega_Z = g_m / C_{gd}$

$\Rightarrow f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{5^m}{2\pi \times 0.1^P} = 7.96 GHz$

f_{P1} and f_{P2} are the poles of the transfer function of equation (6.60), whose denominator is a quadratic polynomial with coefficients

s:

$= [C_{gs} + C_{gd}[1 + g_m R_L']] R_{sig} + (C_L + C_{gd}) R_L'$

$= [2 + 0.1(1 + 5 \times 20)] 20 + (1 + 0.1) \times 20$

$= 264 ns = 264 \times 10^{-9} sec$

Coefficient of s^2:

$= [(C_L + C_{gd}) C_{gs} + C_L C_{gd}] R_{sig} R_L'$

$= [(1 + 0.1) 2 + 1 \times 0.1] 20^K \times 20^K =$

$= 920 \times 10^{-18} (sec)^2$

Therefore the quadratic equation is:

$1 + 264 \times 10^{-9} s + 920 \times 10^{-18} s^2 = 0$

Denoting the frequencies of the roots of this equation with ω_{P1} and ω_{P2}, we have:

$\omega_{P1} = 3.84 \times 10^6 \ rad/s \Rightarrow f_{P1} = \frac{\omega_{P1}}{2\pi} = 611.15 KHz$

$\omega_{P2} = 283.12 \times 10^6 \ rad/s \Rightarrow f_{P2} = \frac{\omega_{P2}}{2\pi} = 45.06 MHz$

Cont.

Since $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, a good estimate for f_H is f_{P_1}:

$$f_H \simeq f_{P_1} = 611.15 \text{ KHz}$$

Approximate value of f_{P_1} obtained using (Eq. 6.66) is:

$$f_{P_1} \cong \frac{1}{2\pi[(C_{gs}+C_{gd}(1+g_m R_L'))R_{sig}+(C_L+C_{gd})R_L']}$$

$f_{P_1} \simeq 603.16$ KHZ

Approximate value of f_{P2} obtained using (Eq. 6.67) is:

$$f_{P_2} = \frac{[C_{gs}+C_{gd}(1+g_m R_L')]R_{sig}+(C_L+C_{gd})R_L'}{2\pi[(C_L+C_{gd})C_{gs}+C_L C_{gd}]R_L' R_{sig}}$$

$f_{P2} = 45.67$ MHz

The estimate of f_{P_1} using Eq. 6.66 is 1.3% lower than the exact value, while the estimate of f_{P2} is about 1.3% higher than its exact value.

6.69

$R_L' = 5 k\Omega$:

$A_M = -g_m R_L' = -5\times 5 = -25$ V/V

using (Eq. 6.66):

$$f_{P_1} \cong \frac{1}{2\pi[(C_{gs}+C_{gd}(1+g_m R_L'))R_{sig}+(C_L+C_{gd})R_L']}$$

$$f_{P_1} = \frac{1}{2\pi[(2+0.1\times(1+5\times5))20+(1+0.1)\times5]}$$

$f_{P_1} = 1.63$ MHz

using (Eq. 6.67):

$$f_{P2} = \frac{[C_{gs}+C_{gd}(1+g_m R_L')]R_{sig}+(C_L+C_{gd})R_L'}{[(C_L+C_{gd})C_{gs}+C_L C_{gd}]R_L' R_{sig}\times 2\pi}$$

$$f_{P2} = \frac{(2+0.1(1+5\times5))20+(1+0.1)5}{((1+0.1)\times2+1\times0.1)5\times20\times2\pi}$$

$f_{P2} = 67.5$ MHz

Eq. 6.63: $s_Z = \frac{g_m}{C_{gd}} \Rightarrow f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{5^m}{2\pi 0.1^F} = 7.96^{GHz}$

$f_{P_1} \ll f_{P2}$ and $f_{P_1} \ll f_Z \rightarrow f_{P_1}$ is the dominant pole.

$f_H \simeq f_{P_1} = 1.63$ MHz

|Gain| × Bandwidth = $25\times1.63 = 40.75$ MHz

$f_t = |A_M| f_H = 40.75$ MHz

Since $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, a dominant pole exists.

$R_L' = 10 k\Omega$:

$A_M = -5\times10 = -50$ V/V

$$f_{P1} = \frac{1}{2\pi[(2+0.1(1+5\times10))20+(1+0.1)\times10]} = 1.04 \text{ MHz}$$

$$f_{P2} = \frac{(2+0.1(1+5\times10))20+(1+0.1)\times10}{[(1+0.1)2+1\times0.1]10\times20\times2\pi} = 52.96 \text{ MHz}$$

$$f_Z = \frac{5}{2\pi\times0.1} = 7.96 \text{ GHz}$$

$f_{P_1} \ll f_{P2}$, $f_{P_1} \ll f_Z \Rightarrow f_{P_1}$ is the dominant pole and therefore $f_H \simeq f_{P_1} = 1.04$ MHz

$|A_M|\cdot f_H = 50\times1.04 = 52$ MHz

Since f_{P2} is still slightly greater than $|A_M|\cdot f_H$, therefore: $f_T \simeq 52$ MHz

$R_L' = 20 k\Omega$:

$A_M = -5\times20 = -100$ V/V, from Problem 6.68 we have:

$f_{P_1} = 603.16$ KHz

$f_{P2} = 45.67$ MHz

$f_Z = 7.96$ GHz

Again $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, therefore f_{P_1} is the dominant pole and f_H can be approximated by f_{P_1}. $f_H \simeq f_{P_1} = 603.16$ KHz

$|A_M| f_H = 60.32$ MHz

Since $f_{P2} < |A_M|\cdot f_H$, therefore f_T is smaller than $|A_M|\cdot f_H$

The results are summarized in this table:

R_L'	$5^{k\Omega}$	$10^{k\Omega}$	$20^{k\Omega}$
A_M (V/V)	-25	-50	-100
f_{P_1} (MHz)	1.63	1.04	0.60
$\lvert A_M'\rvert\cdot f_H$ (MHz)	40.75	52.00	60.32

6.70

Using Eq. 6.70: $A_M = -\frac{r_\pi}{R_{sig}+r_x+r_\pi}(g_m R_L')$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5K\Omega \Rightarrow A_M = -\frac{5}{1+0.2+5}(20\times 5)$

$\underline{A_M = 80.65\ V/V}$

Using Miller's Theorem and Eq. 6.71:

$C_{in} = C_\pi + C_\mu(1+g_m R_L') = 10+0.5(1+20\times5) = 60.5\ pF$

Eq. 6.69: $R'_{sig} = r_\pi \| (R_{sig}+r_x) = 5^K \| (1^K+0.2)$

$R'_{sig} = 0.97K\Omega$

Eq. 6.72: $f_H \simeq \frac{1}{2\pi C_{in} R'_{sig}} = \frac{1}{2\pi \times 60.5^p \times 0.97^K} \Rightarrow$

$\underline{f_H = 2.71MHz}$

6.71

From Problem 6.70 we have: $r_\pi = 5K\Omega$ and

$A_M = -80.65\ V/V$, $R'_{sig} = 0.97K\Omega$

Eq. 6.75: $f_Z = \frac{g_m}{2\pi C_\mu} = \frac{20^m}{2\pi \times 0.5^p} = 6.37GHz$

Eq. 6.76: $f_{P_1} \simeq \frac{1}{2\pi[(C_\pi + C_\mu(1+g_m R_L'))R'_{sig} + (C_L+C_\mu)R_L']}$

$f_{P_1} \simeq \frac{1}{2\pi[(10+0.5(1+20\times5))0.97 + 2.5\times 5]}$

$f_{P_1} = 2.24MHz$

Eq. 6.77: $f_{P_2} = \frac{(C_\pi + C_\mu(1+g_m R_L'))R'_{sig} + (C_L+C_\mu)R_L'}{2\pi[C_\pi(C_L+C_\mu)+C_L C_\mu]R'_{sig}R_L'}$

$f_{P_2} = \frac{(10+0.5(1+20\times5))0.97+2.5\times5}{2\pi(10(2+0.5)+2\times0.5)0.97\times5}$

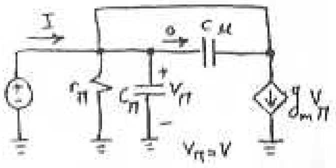
$f_{P_2} = 89.89MHz$

Since $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, we can approximate f_H by f_{P_1}: $f_H \simeq f_{P_1} = 2.24MHz$

If we compare f_H to the results obtained from applying Miller's Theorem in Problem 6.70, then our results are 17% lower.

6.72

$I = \frac{V}{r_\pi} + sC_\pi V_\pi + g_m V_\pi$



$Y_{in} = (g_m + \frac{1}{r_\pi}) + sC_\pi$

$Z_i = \frac{1}{(g_m+\frac{1}{r_\pi})+sC_\pi} = \frac{1}{\frac{1}{r_e}+sC_\pi} = \frac{r_e}{1+sC_\pi r_e}$

$\underline{Z_i = \frac{r_e}{1+sC_\pi r_e}}$

$f_T = \frac{g_m}{2\pi(C_\pi+C_\mu)}$ Since $C_\pi = C_{de}+C_{je} = \tau_F g_m + C_{je}$,

Therefore C_π has a component that depends on the bias current and at high currents $C_\pi \gg C_\mu$

and $f_T \simeq \frac{g_m}{2\pi C_\pi} \simeq \frac{1}{2\pi C_\pi r_e}$

Thus: $Z_i \simeq \frac{r_e}{1+\frac{s}{\omega_T}}$ at high currents

The phase angle will be $-45°$ at $\omega = \omega_T$ or $f = f_T = 400MHz$.

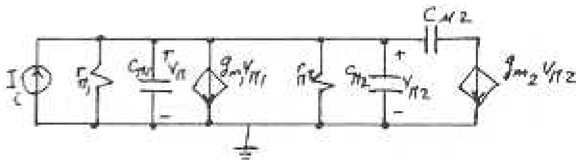
For lower bias currents, so that $C_\pi = C_\mu$

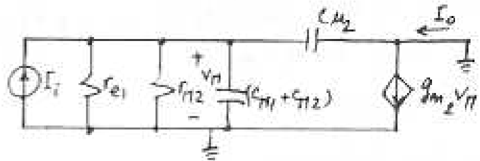
$f_T \simeq \frac{1}{4\pi C_\pi r_e}$ and $Z_i = \frac{r_e}{1+\frac{s}{2\omega_T}}$ and $-45°$

phase is obtained at $\omega = 2\omega_T$ or $f = 2f_T = 800MHz$

(Assuming that f_T remains constant which is not necessarily true!!)

6.73





$V_\pi = \frac{I_i}{(\frac{1}{r_{e1}}+\frac{1}{r_{\pi 2}}) + s(C_{\pi 1}+C_{\pi 2}+C_{\mu 2})}$

Cont.

$$I_o = g_{m2} V_\pi - C_\mu s V_\pi = \frac{(g_{m2} - C_\mu s) I_i}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

$$\frac{I_o}{I_i} = \frac{g_{m2} - C_\mu s}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

$I_{C_1} = I_{C_2} \Rightarrow r_{\pi 1} = r_{\pi 2}$, $g_{m1} = g_{m2}$, $C_{\pi 1} = C_{\pi 2}$

$$\frac{I_o}{I_i} = \frac{g_m - C_\mu s}{\left(\frac{1}{r_e} + \frac{1}{r_\pi}\right) + (C_\mu + 2C_\pi) s} = \frac{1 - \frac{C_\mu}{g_m} s}{\left(\frac{1}{g_m r_e} + \frac{1}{g_m r_\pi}\right) + s \frac{C_\mu + 2C_\pi}{g_m}}$$

$$g_m r_e = \frac{I_C}{V_T} \frac{V_T}{I_E} = \alpha = \frac{\beta}{\beta + 1}$$

$g_m r_\pi = \beta$

$$\Rightarrow \frac{I_o}{I_i} = \frac{1 - \frac{C_\mu}{g_m} s}{1 + \frac{1}{\beta} + \frac{1}{\beta} + s(2C_\pi + C_\mu)/g_m}$$

$$\frac{I_o}{I_i} = \frac{1}{1 + 2/\beta} \quad \frac{1 - s\, C_\mu/g_m}{1 + s \frac{(2C_\pi + C_\mu)}{g_m(1 + \frac{2}{\beta})}}$$

If the circuit is biased at 1mA and $\beta = \infty$, $f_T = 400$ MHz and $C_\mu = 2$pF:

$$g_m = \frac{1}{0.025} = 40 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \frac{40^m}{2\pi \times 400^M} = 15.9\text{pF}$$

$C_\pi = 15.9 - 2 = 13.9$ pF

Pole frequency: $$f_p = \frac{g_m}{2\pi(2C_\pi + C_\mu)} = \frac{40 \times 10^{-3}}{2\pi(2 \times 13.9 + 2)^p}$$

$$\underline{f_p = 213.74 \text{ MHz}}$$

Zero frequency: $$f_z = \frac{g_m}{2\pi C_\mu} = \frac{40^m}{2\pi \times 2^p} = \underline{3.18 \text{GHz}}$$

6.74

$$A_M = -g_m R_L' = -5 \times 20 = \underline{-100 \text{ V/V}}$$

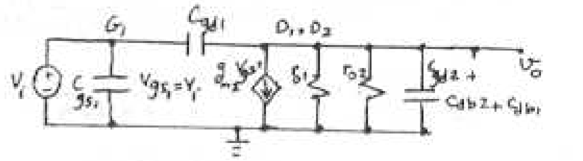
$$f_H = \frac{1}{2\pi(C_L + C_{gd}) R_L'} = \frac{1}{2\pi(1 + 0.1) \times 20} = 7.23 \text{ MHz}.$$

(Note that in this case there is no Rsig and we used Eq. 6.79)

$f_{3db} = f_H = \underline{7.23 \text{ MHz}}$

$f_t = |A_M| \cdot f_H = \underline{723 \text{ MHz}}$

6.75



$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2 \times 90 \times \frac{100}{1.6} \times 100} = 1060 \, \mu\text{A/V}$$

$g_m = 1.06$ mA/V

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{k}\Omega$$

$$\text{DC gain} = -g_{m1}(r_{o1} \| r_{o2}) = -1.06 \times (128 \| 192)$$
$$= -81.4 \text{ V/V}$$

Total capacitance between output node and ground $= C_{gd2} + C_{db1} + C_{db2} = 0.015 + 0.020 + 0.036$

$C_L = 0.071$ pF

Write a KCL at output:

$$s C_{gd1} (V_i - V_o) = g_m V_i + \frac{V_o}{r_{o1}} + \frac{V_o}{r_{o2}} + V_o s C_L$$

$$\frac{V_o}{V_i} = -\frac{g_m - s C_{gd1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + (C_L + C_{gd1}) s}$$

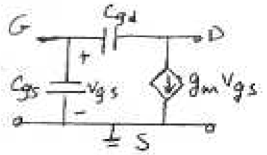
Thus: $$f_Z = \frac{g_m}{2\pi C_{gd1}} = \frac{1.06^m}{2\pi \times 0.015^p} = \underline{11.3 \text{GHz}}$$

$$f_P = \frac{1}{2\pi} \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{C_L + C_{gd1}} = \frac{\frac{1}{128^k} + \frac{1}{192^k}}{2\pi(0.071 + 0.015)^p}$$

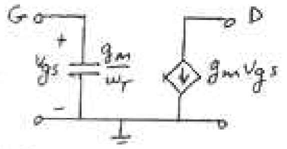
$$\underline{f_P = 24.1 \text{ MHz}}$$

6.76

a) For small C_{gd} and low gain from G to D, we can neglect the miller effect and C_{gd}.

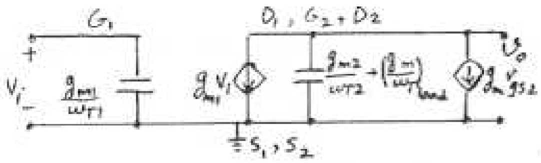


$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \simeq \frac{g_m}{C_{gs}} \quad \text{Thus } C_{gs} \simeq \frac{g_m}{\omega_T}$$



b) replace the controlled source $g_{m2} V_{gs2}$ with a resistance $\frac{1}{g_{m2}}$. (Source absorbtion theory)

Cont.



$$V_o = -g_{m1} v_i \frac{1}{g_{m2} + s\left(\frac{g_{m2}}{\omega_{T2}} + \frac{g_{m load}}{\omega_{T load}}\right)}$$

Since the load device is identical to Q_1,

$g_{mload} = g_{m1}$ and $\omega_{Tload} = \omega_{T1} = \omega_T$.

Thus:

$$\frac{v_o}{v_i} = \frac{-g_{m1}/g_{m2}}{1 + \frac{s}{\omega_T}\left(1 + \frac{g_{m1}}{g_{m2}}\right)}$$

$$\frac{g_{m1}}{g_{m2}} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{ov}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{ov}} = \frac{W_1}{W_2}$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{-A_o}{1 + \frac{s}{\omega_T}(1 + A_o)} \quad \text{where } A_o = \frac{W_1}{W_2} = \frac{g_{m1}}{g_{m2}}$$

c) $A_o = 3\,V/V$, $W_2 = 25\,\mu m$

$A_o = \frac{W_1}{W_2} \Rightarrow W_1 = 3 \times 25 = 75\,\mu m$

$I_{D_1} = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L}(V_{GS} - V_t)^2 = \frac{1}{2} \times 200^{\mu} \times \frac{75}{0.5} 0.3^2$

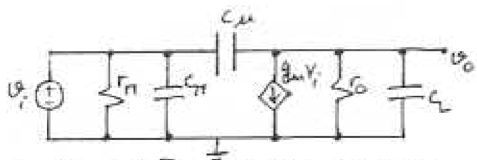
$I_{D_1} = 1.35\,mA$

$I_{D2} = \frac{1}{2} \times 200\,\mu A/V^2 \times \frac{25}{0.5} \times 0.3^2 = 0.45\,mA$

Thus: $I = I_{D1} + I_{D2} = 1.35 + 0.45 = 1.8\,mA$

$$f_{3db} = \frac{f_T}{1 + A_o} = \frac{12 \times 10^9}{1 + 3} = 3\,GHz$$

6.77



writing a node equation at the output yields:

$$sC_\mu(v_i - v_o) = g_m v_i + \frac{v_o}{r_o} + v_o C_L s$$

$$\frac{v_o}{v_i} = \frac{C_\mu s - g_m}{\frac{1}{r_o} + (C_L + C_\mu)s} = -g_m r_o \frac{1 - sC_\mu/g_m}{1 + s(C_L + C_\mu)r_o}$$

for small C_μ, $\omega C_\mu << g_m$: $\frac{v_o}{v_i} = \frac{-g_m r_o}{1 + s(C_\mu + C_L)r_o}$

For $I_C = 200\,\mu A$, $V_A = 100\,V$: $g_m = \frac{200^{\mu}}{0.025} = 8\,mA/V$

$r_o = \frac{100}{200} = 0.5\,M\Omega$

Thus DC-Gain $= -g_m r_o = -8 \times 0.5 \times 10^3 = -4000\,V/V$

For $C_L = 1\,pF$, $C_\mu = 0.2\,pF$:

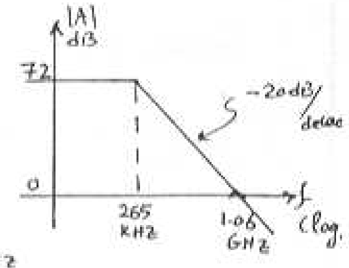
$$\omega_{3db} = \frac{1}{(C_L + C_\mu)r_o} = \frac{1}{(1 + 0.2)^p \times 0.5^M} = 1.67\,M\,rad/s$$

$f_{3db} = 265.4\,kHz = f_H$

$f_T = |A_o| f_H = 4000 \times 265.4 = 1.06\,GHz$

Bode Plot for $|A|$:

$4000\,V/V = 72\,dB$



Check $f_Z = \frac{g_m}{2\pi C_\mu}$

$$f_Z = \frac{8^m}{2\pi \times 0.2^p} = 6.4\,GHz$$

6.78

$f_T = \frac{g_m}{2\pi(C_L + C_{gd})}$, $g_m = 1\,mA/V$, $f_T = 2\,GHz$

$$\rightarrow C_L + C_{gd} = \frac{1 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 79.61\,fF$$

To have $f_{T2} = 1\,GHz$, we need:

$$C_L + C_{gd} = \frac{1 \times 10^{-3}}{2\pi \times 1 \times 10^9} = 159.23\,fF$$

Thus we need an additional capacitance of

$159.23 - 79.61 = \underline{79.61\,fF}$

6.79

$$g_m = \sqrt{2k_n' \frac{W}{L} I_D} = \sqrt{2 \times \frac{160}{1000} \times \frac{50}{0.5} \times 0.5} = 4\,mA/V$$

$g_{mb} = \chi g_m = 0.2 \times 4 = 0.8\,mA/V$

$r_o = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.5} = 20\,k\Omega$

$R_o = r_o = 20\,k\Omega$

$A_{v_o} = 1 + (g_m + g_{mb})r_o = 1 + (4^m + 0.8^m) \times 20^k = 97\,V/V$

$R_{out} = r_o + A_{v_o} R_s$ (Eq. 6.102)

$R_{out} = 20^k + 97 \times 20^k = 1.96\,M\Omega$

$R_{in} = \frac{r_o + R_L}{A_{v_o}}$ (Eq. 6.86)

$R_{in} = \frac{20 + 20}{97} = 0.41\,k\Omega$

$A_v = A_{v_o}\frac{R_L}{R_L + r_o} = 97\,\frac{20}{20 + 20} = 48.5\,V/V$

Cont.

$$G_v = \frac{v_o}{v_{sig}} = A_{v_o}\frac{R_L}{R_L + r_o + A_{v_o}R_s} \quad \text{(Eq. 6.95)}$$

$$G_v = \frac{97\times 20^K}{20^K + 20^K + 97\times 20^K} = 0.98 \text{ V/V}$$

$$G_{is} = A_{v_o}\frac{R_s}{R_{out}} = 97\times\frac{20}{1960} = 0.99 \text{ A/A}$$

$$G_i = G_{is}\frac{R_{out}}{R_{out}+R_L} = 0.99\times\frac{1960}{1960+20} = 0.98 \text{ A/A}$$

6.80

$R_L = r_o$

$A_v = 100$ V/V $= \frac{v_o}{v_i} = A_{v_o}\frac{R_L}{R_L+r_o} = A_{v_o}\frac{r_o}{r_o+r_o}$

$\Rightarrow A_v = A_{v_o}/2 = 100 \Rightarrow A_{v_o} = 200$ V/V

$R_{in} = 2K\Omega$

$R_{in} = \frac{r_o+R_L}{A_{v_o}} = \frac{r_o+r_o}{A_{v_o}} \Rightarrow 2^K = \frac{2r_o}{200} \Rightarrow r_o = 100\ K\Omega$

$r_o = \frac{V_A}{I_D} \Rightarrow I_D = \frac{V_A}{r_o} = \frac{20}{100} = 0.2$ mA

$A_{v_o} = 1 + (g_m+g_{mb})r_o = 1 + (1+\chi)g_m r_o$

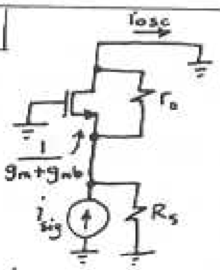
$200 = 1+(1+0.2)g_m\times 100 \Rightarrow g_m = 1.66$ mA/V

$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2\times0.2}{1.67} = 0.24^V$

$I_D = \frac{1}{2} k'_n \frac{W}{L}V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k'_n V_{OV}^2} = \frac{2\times0.2}{0.1\times0.24^2}$

$\frac{W}{L} = 69.4$

6.81



$$i_{osc} = \frac{R_s\, i_{sig}}{R_s + (r_o \| \frac{1}{g_m+g_{mb}})} = \frac{R_s}{R_s + \frac{r_o}{1+(g_m+g_{mb})r_o}}\, i_{sig}$$

$$G_{is} = \frac{i_{osc}}{i_{sig}} = \frac{R_s}{R_s+\frac{r_o}{A_{vo}}} = \frac{A_{vo}R_s}{r_o + A_{vo}R_s}$$

Using (6.102) we have: $G_{is} = \frac{A_{vo}R_s}{R_{out}}$

If $A_{vo}R_s \gg r_o$ that is $[1+(g_m+g_{mb})r_o]R_s \gg r_o$

or $(g_m+g_{mb})R_s \gg 1$ then $G_{is}\simeq 1$

6.82

$R_{in} = \frac{r_o+R_L}{A_{v_o}}$, $A_{v_o} = 1+(g_m+g_{mb})r_o$

$A_{v_o} \simeq (g_m+g_{mb})r_o \equiv A_o$

$\Rightarrow R_{in} \simeq \frac{1}{g_m+g_{mb}} + \frac{R_L}{A_o}$

IF $R_L = A_o r_o$ then $R_{in} = \frac{1}{g_m+g_{mb}} + r_o$

6.83

$g_m = \frac{2I_D}{V_{OV}} = \frac{2\times1^m}{0.8-0.55} = 8$ mA/V

$g_{mb} = \chi g_m = 0.2\times 8 = 1.6$ mA/V

$r_o = \frac{V_A}{I_D} = \frac{20}{1^m} = 20K\Omega$

From Eq. 6.101 we have: $R_{out} = r_o + [1+(g_m+g_{mb})r_o]R_s$

$R_{out} = 20 + (1+(8+1.6)\times20)R_s = 200\ K\Omega$

$R_s = 932.64\Omega$

$V_{BIAS} = I_D R_s + V_{GS} = 1^m\times 932.64\times10^{-3} + 0.8 = 1.73^V$

$V_{BIAS} = 1.73^V$

6.84

$V_s \times (g_m+g_{mb}) = i_{osc} - \frac{V_s}{r_o}$

$\Rightarrow i_{osc} = V_s\left(\frac{1}{r_o} + g_m + g_{mb}\right)$ (1)

$V_s = V_{sig} - R_s i_{osc}$

If we substitute V_s in (1):

$i_{osc} = (V_{sig} - R_s i_{osc})\left(\frac{1}{r_o} + g_m+g_{mb}\right)$

$i_{osc}\left(1+\frac{R_s}{r_o} + R_s(g_m+g_{mb})\right) = V_{sig}\frac{(1+(g_m+g_{mb})r_o)}{r_o}$

$i_{osc} = V_{sig}\frac{1+(g_m+g_{mb})r_o}{r_o+R_s+R_s r_o(g_m+g_{mb})}$

Since $A_{v_o} = G_{v_o} = 1+(g_m+g_{mb})r_o$ (Eq. 6.88)

and $R_{out} = r_o + A_{v_o}R_s$ (Eq. 6.102), then

$i_{osc} = V_{sig}\frac{A_{v_o}}{R_{out}}$

6.85

a) $V_{GS2} = V_{GS3} \Rightarrow I_{D2} = I_{D3}$. Also $I_{D1} = I_{D2}$
$\Rightarrow I_{D1} = 100\mu A$.

$I_{D1} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS_1} - V_t)^2 \Rightarrow 0.1 = \frac{1}{2} \times 4 \times (V_{GS_1} - 0.8)^2$

$V_{GS_1} - 0.8 = \pm 0.224 \Rightarrow V_{GS_1} = 1.024^V$, $V_{OV_1} = V_{OV_2} = 0.224^V$

$V_{BIAS} = R_S I_{D1} + V_{GS} = 0.05 \times 0.1 + 1.024 = 1.029^V$

b) $g_{m_1} = \frac{2 I_{D1}}{V_{OV}} = \frac{0.1 \times 2}{0.224} = 0.89 mA/V$

$g_{m_2} = \sqrt{2 k_n' \frac{W}{L} I_D} = \sqrt{2 \times 4 \times 0.1} = 0.89 mA/V$

$g_{m3} = g_{m2} = 0.89 mA/V$

$g_{mb} = \chi g_m \Rightarrow g_{mb1} = g_{mb2} = g_{mb3} = 0.2 \times 0.89 = 0.18 \ mA/V$

$r_{o_1} = r_{o_2} = r_{o_3} = \frac{V_A}{I_D} = \frac{20}{0.1} = 200 k\Omega$

c) $R_{in} = \frac{r_o + R_L}{A_{v_o}}$ where $A_{v_o} = 1 + (g_m + g_{mb}) r_o$

$A_{v_o} = 1 + (0.89 + 0.18) 200$

$A_{v_o} = 215 \ V/V$

In this case R_L is in fact R_{out} of the Active load in the drain of Q_1, which is $r_{o2} = 200 k\Omega$.

Thus: $R_{in} = \frac{200 + 200}{215} = 1.86 k\Omega$

d) $R_{out} = r_o + A_{v_o} R_S = 200 + 215 \times 0.05 = 210.75 \ k\Omega$

e) $A_v = \frac{v_o}{v_i} = A_{v_o} \frac{R_L}{R_L + r_o} = 215 \frac{200}{200 + 200} = 107.5 V/V$

$G_v = A_{v_o} \frac{R_L}{R_L + R_{out}} = 215 \frac{200}{200 + 210.75} = 104.7 \ V/V$

f) For Q_2 to stay in saturation region, v_o can go as high as $V_{DD} - V_{OV2}$, Thus $v_o \leq 3.3 - 0.224$ or $v_o \leq 3.076^V$.

For Q_1 to stay in saturation region, we need to have $V_{GD1} \leq V_t \Rightarrow V_{BIAS} - v_o \leq 0.8 \Rightarrow$
$v_o \geq 1.029 - 0.8$ or $v_o \geq 0.229^V$
hence $0.229^V \leq v_o \leq 3.076^V$. This implies that the peak-to-peak output swing is
$3.076 - 0.229 = 2.847^V$.

Since $G_v = \frac{v_o}{v_{sig}} = 104.7$, then the maximum peak-to-peak value of v_{sig} is $\frac{2.847}{104.7}$ that is 27mV.

6.86

From Fig. 6.31b the low-frequency gain $\frac{v_o}{v_{sig}}$ can be written as:

$$\frac{v_o}{v_{sig}} = \frac{\frac{1}{g_m + g_{mb}}}{R_s + \frac{1}{g_m + g_{mb}}} \times (g_m + g_{mb}) \times R_L'$$

$$\frac{v_o}{v_{sig}} = \frac{(g_m + g_{mb}) R_L'}{1 + (g_m + g_{mb}) R_{sig}} = \frac{(5 + 0.2 \times 5) \times 20}{1 + (5 + 0.2 \times 5) \times 1} = 17.14 ,$$

$\frac{v_o}{v_{sig}} = 17.14 V/V$

From Eq. 6.105 we have $f_{P1} = \frac{1}{2\pi C_{gs}(R_{sig} \| \frac{1}{g_m + g_{mb}})}$

$f_{P1} = \frac{1}{2\pi \times 2pF (1K \| \frac{1}{5 + 0.2 \times 5})} = 557 MHz$

From Eq. 6.106 we have:

$f_{P2} = \frac{1}{2\pi (C_{gd} + C_L) R_L'} = \frac{1}{2\pi (0.1^P + 2^P) 20^K} = 3.79 MHz$

Since $f_{P2} \ll f_{P1}$, then f_{P2} is the dominant pole and
$f_H \simeq f_{P2} = 3.79 MHz$

6.87

$A_{v_o} = 1 + (g_m + g_{mb}) r_o = 1 + (5 + 0.2 \times 5) 20^K = 121 V/V$

$R_{out} = r_o + A_{v_o} R_S = 20 + 121 \times 1 = 141 K\Omega$

$G_v = G_{vo} \frac{R_L}{R_L + R_{out}} = A_{v_o} \frac{R_L}{R_L + R_{out}} = 121 \frac{20^K}{20^K + 141^K}$

$G_v = \frac{v_o}{v_{sig}} = 15 V/V$

$R_{in} = \frac{r_o + R_L}{A_{v_o}} = \frac{20 + 20}{121} = 0.33 K\Omega$

$R_{gs} = R_S \| R_{in} = 1^K \| 0.33^K = 0.25 K\Omega = 250 \Omega$

$R_{gd} = R_L \| R_{out} = 20^K \| 141^K = 17.5 K\Omega$

$\tau_H = C_{gs} R_{gs} + (C_L + C_{gd}) R_{gd} = 2^P \times 0.25^K + (2 + 0.1) 17.5^K = 37.25 \ ns$

$f_H \simeq \frac{1}{2\pi \tau_H} = 4.27 MHz$

Comparing with results in Problem 6.86, we notice that gain is reduced by 12.5%, while f_H is increased by 12.7%, when r_o is taken into account.

6.88

$i_o = i_i - v_i/r_\pi$ Eq. 6.110

If we write a KCL at the emitter node, we will have: $\frac{v_i - i_o R_L}{r_o} + \frac{v_i}{r_e} = i_i$

Substitude for i_o from Eq. 6.110, then:

$$\frac{v_i - R_L i_i + R_L/r_\pi\, v_i}{r_o} + \frac{v_i}{r_e} = i_i$$

$$v_i\left(\frac{1}{r_o} + \frac{R_L}{r_\pi r_o} + \frac{1}{r_e}\right) = i_i\left(1 + \frac{R_L}{r_o}\right)$$

$$R_{in} = \frac{v_i}{i_i} = \frac{1 + R_L/r_o}{\frac{1}{r_o} + \frac{1}{r_e} + \frac{R_L}{r_\pi r_o}} = \frac{r_o + R_L}{1 + \frac{r_o}{r_e} + \frac{R_L}{r_\pi}}$$

$r_\pi = (\beta+1) r_e$ therefore:

$$R_{in} = \frac{r_o + R_L}{1 + \frac{r_o}{r_e} + \frac{R_L}{(\beta+1) r_e}}$$

This is the same as equation 6.111.

6.89

$\beta = 100$, $R_{in} \simeq r_e \frac{r_o + R_L}{r_o + R_L/(\beta+1)}$

$$R_{in} \simeq r_e \frac{1 + R_L/r_o}{1 + \frac{R_L}{r_o}\frac{1}{\beta+1}}$$

R_L/r_o	0	1	10	100	1000
R_{in}/r_e	1	1.98	10	50.75	91.83

6.90

$I = 1\text{mA}$, Intrinsic gain $= A_o = \frac{V_A}{V_T} = 2000\,V/V$

$V_A = 2000\,V_T = 50V$

$r_o = \frac{V_A}{I_C} = \frac{50}{1} = 50k\Omega$

From Eq. 6.112 we have: $R_{in} \simeq r_e \frac{r_o + R_L}{r_o + R_L/(\beta+1)}$

Assuming β is very large so that:

$\frac{R_L}{\beta+1} \ll r_o$, we have: $R_{in} \simeq r_e \frac{r_o + R_L}{r_o}$

For R_{in} to be $2r_e$, we need:

$2r_e = r_e \frac{r_o + R_L}{r_o} \Rightarrow r_o = R_L \Rightarrow \underline{R_L = 50k\Omega}$

6.91

Refer to Fig. 6.34:

write KCL at the emitter:

$$\frac{v}{r_e} + \frac{v}{R_e} = i_x + g_m v \Rightarrow v = \frac{i_x}{\frac{1}{r_e} - g_m + \frac{1}{R_e}}$$

Note that $\frac{1}{r_e} - g_m = \frac{\beta+1}{r_\pi} - g_m = \frac{\beta+1-\beta}{r_\pi} = \frac{1}{r_\pi}$

therefore: $v = \frac{i_x}{\frac{1}{r_\pi} + \frac{1}{R_e}} = (r_\pi \| R_e)\, i_x$

$v = R'_e i_x$ (1) where $R'_e = r_\pi \| R_e$

Now we write the equation for v:

$v = v_x - (i_x + g_m v) r_o$

if we substitude v from (1):

$R'_e i_x = v_x - i_x r_o - g_m R'_e i_x r_o$

$i_x (R'_e + r_o + g_m R'_e r_o) = v_x$

$R_{in} = \frac{v_x}{i_x} = r_o + (1 + g_m r_o) R'_e$ where $R'_e = R_e \| r_\pi$

(Same as Eq. 6.117)

6.92

Eq. 6.118: $R_{out} \simeq (1 + g_m R'_e) r_o$

$\Rightarrow \frac{R_{out}}{r_o} \simeq 1 + g_m R'_e$ where $R'_e = R_e \| r_\pi$

$$R'_e = \frac{r_\pi R_e}{r_\pi + R_e} = \frac{(\beta+1) r_e R_e}{(\beta+1) r_e + R_e} = \frac{(\beta+1) r_e}{\frac{(\beta+1) r_e}{R_e} + 1}$$

$$m = \frac{R_e}{r_e} \Rightarrow R'_e = \frac{(\beta+1) r_e}{\frac{\beta+1}{m} + 1}$$

$$\frac{R_{out}}{r_o} = 1 + g_m \frac{r_e(\beta+1)}{\frac{\beta+1}{m} + 1} = 1 + \frac{g_m \frac{r_\pi}{\beta+1}(\beta+1)}{\frac{\beta+1}{m} + 1}$$

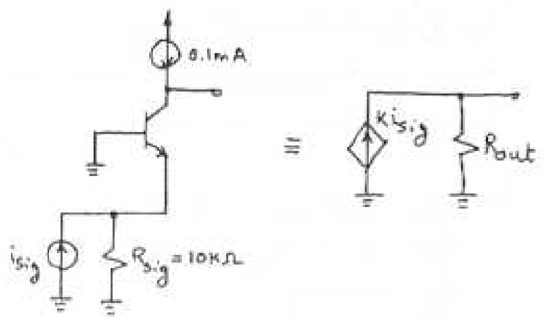
$$\frac{R_{out}}{r_o} = 1 + \frac{\beta}{\frac{\beta+1}{m} + 1} = \frac{(\beta+1)(m+1)}{\beta+1+m} = \frac{101(m+1)}{101+m}$$

$m = \frac{R_e}{r_e}$	1	2	10	$\beta/2$	1000
$\frac{R_{out}}{r_o}$	1.98	2.94	10.01	$\beta/3 + 1$	91.74

6.93

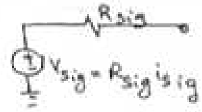
$R_{sig} = 10k\Omega$, $\beta = 100$, $V_A = 50V$, $I_C = 0.1mA$

Cont.



$$r_o = \frac{V_A}{I_C} = \frac{50}{0.1^m} = 500K\Omega$$
$$g_m = \frac{I_C}{V_T} = \frac{0.1}{0.025} = 4\ mA/V$$
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25K\Omega$$

The Thevenin equivalent circuit of the source is:



Thus using equations 6.121 and 6.114 we can write:

$$\frac{V_{out}}{V_{sig}} = \frac{r_\pi}{r_\pi + R_{sig}} A_{v_o} = \frac{r_\pi}{r_\pi + R_{sig}}(1 + g_m r_o)$$

$$\frac{V_{out}}{i_{sig}} = \frac{r_\pi R_{sig}}{r_\pi + R_{sig}}(1 + g_m r_o) = (r_\pi \| R_{sig})(1 + g_m r_o)$$

$$\frac{V_{out}}{i_{sig}} = 14.29 \times 10^6\ V/A$$

From Eq. 6.117a we have $R_{out} = r_o + (1 + g_m r_o) R'_e$ where $R'_e = r_\pi \| R_{sig}$.

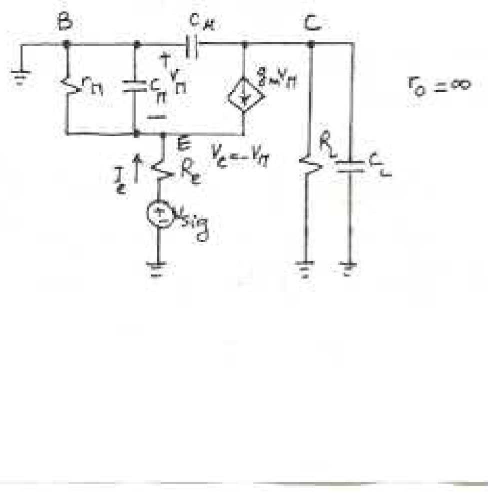
$$R_{out} = 500^k + (1 + 4^m \times 500^k)(25^k \| 10^k) = 14.79 M\Omega$$

$R_{out} = 14.79 M\Omega$

$$V_{out} = K i_{sig} R_{out} \Rightarrow K = \frac{V_{out}}{i_{sig}} \times \frac{1}{R_{out}} = \frac{14.29 \times 10^6}{14.79 \times 10^6}$$

$\Rightarrow K = 0.97$

6.94

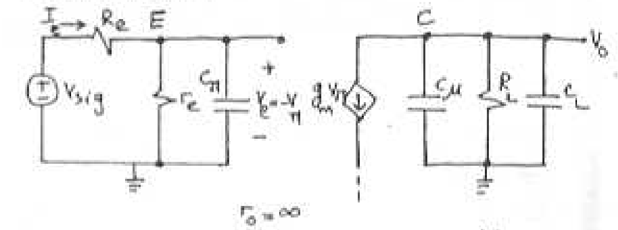


we observe that V_e, the voltage at the emitter is equal to $-V_\pi$. We can write a node equation at the emitter:

$$I_e = -V_\pi\left(\frac{1}{r_\pi} + sC_\pi\right) - g_m V_\pi = V_e\left(\frac{1}{r_\pi} + g_m + sC_\pi\right)$$

Thus, the input admittance looking into the emitter is: $\frac{I_e}{V_e} = \frac{1}{r_\pi} + g_m + sC_\pi = \frac{1}{r_e} + sC_\pi$

Therefore we can replace the transistor at the input of the circuit by this admittance as shown below:



a) As we can see above, the circuit can be seperated into two parts with each part having its own pole.

$$f_{P1} = \frac{1}{2\pi C_\pi (R_e \| r_e)} \quad \text{(input part)}$$

$$f_{P2} = \frac{1}{2\pi (C_\mu + C_L) R_L} \quad \text{(output part)}$$

If we compare the poles to Eq. 6.105 and 6.106 for MOSFETS, we observe that these equations are the bipolar counterparts of those ones.

$$f_{P1} = \frac{1}{2\pi C_{gs}\left(R_s \| \frac{1}{g_m + g_{mb}}\right)} \quad (6.105)$$

$$f_{P2} = \frac{1}{2\pi (C_{gd} + C_L) R_L} \quad (6.106)$$

b) For $C_\pi = 14pF$, $C_\mu = 2pF$, $C_L = 1pF$, $I_C = 1mA$, $R_{sig} = 1K\Omega$, $R_L = 10K\Omega$. $\Rightarrow g_m = \frac{1}{0.025} = 40\ mA/V$

$$f_{P1} = \frac{1}{2\pi \times 14^P (1^K \| \frac{100}{40})} \quad (\text{Assuming } \beta = 100)$$

$f_{P1} = 15.9 MHz$

$$f_{P2} = \frac{1}{2\pi(2^P + 1^P)10^K} = 5.3\ MHz$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{40^m}{2\pi(14+2)^P} = 398.1\ MHz$$

f_T is much greater than the poles. $f_T \gg f_{P1}$
$f_T \gg f_{P2}$

6.95

In Fig.6.32, if we replace the NMOS with an npn transistor, then we can write:

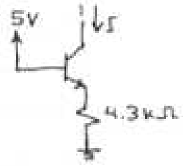
$R_{be} = R_e \| R_{in}$ where $R_{in} = r_e \dfrac{r_o + R_L}{1 + \frac{r_o}{r_e} + \frac{R_L}{(\beta+1) r_e}}$

$R_{bc} = R_L \| R_{out}$ where $R_{out} = (1 + g_m R_e') r_o$

Using the open-circuit time constants method to evaluate f_H:

$$f_H = \frac{1}{2\pi \left(C_\pi (R_e \| R_{in}) + (C_L + C_\mu)(R_L \| R_{out}) \right)}$$

6.96



$\beta = 100$, $V_A = 100^V$

$I_C \simeq I_E = \dfrac{5 - V_{BE}}{4.3^k} = \dfrac{5 - 0.7}{4.3^k} = 1mA$

$I = 1mA$

From Eq. 6.117a we have:

$R_{out} = r_o + (1 + g_m r_o)(r_\pi \| R_e)$

$r_o = \dfrac{V_A}{I_C} = \dfrac{100}{1} = 100 k\Omega$

$g_m = \dfrac{I_C}{V_T} = \dfrac{1}{0.025} = 40 mA/V$

$r_\pi = \dfrac{\beta}{g_m} = 2.5 k\Omega$, $R_e = 4.3 k\Omega$

Thus: $R_{out} = 100 + (1 + 40 \times 100)(2.5^k \| 4.3^k)$

$\underline{R_{out} = 6.43^{M\Omega}}$

If the collector voltage undergoes a change of 10V while the BJT remains in the active mode, the corresponding change in the collector current is: $\Delta I = \dfrac{\Delta V}{R_{out}} = \dfrac{10}{6.43^M} = 1.56 \mu A$

$\underline{\Delta I = 1.56 \mu A}$

6.97

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 160^\mu \times 100 \times 0.2^2 = \underline{320 \mu A}$

b) $g_m = \dfrac{2 I_D}{V_{ov}} = \dfrac{2 \times 0.32}{0.2} = \underline{3.2 mA/V} = g_{m1} = g_{m2}$

$g_{mb} = \chi g_m = 0.2 \times 3.2 = \underline{0.64 mA/V} = g_{mb1} = g_{mb2}$

$r_o = \dfrac{V_A}{I_D} = \dfrac{1}{\lambda I_D} = \dfrac{1}{0.05 \times 0.32} = \underline{62.5 k\Omega} = r_{o1} = r_{o2}$

$A_o = g_m r_o = 3.2^m \times 62.5^k = \underline{200 V/V}$

$A_{vo2} = 1 + (g_{m2} + g_{mb2}) r_{o2}$ (Eq. 6.123)

$A_{vo2} = 1 + (3.2 + 0.64) \times 62.5 = \underline{241 \ V/V}$

c) $A_{vo} = -A_{o1} A_{vo2}$ (Eq. 6.128)

$A_{vo} = -g_{m1} r_{o1} A_{vo2} = -3.2^m \times 62.5 \times 241 = 48200 V/V$

$\underline{A_{vo} = 48200 \ V/V}$

d) $G_m = \dfrac{A_{o1} A_{vo2}}{r_{o2} + A_{vo2} r_{o1}}$ (Eq. 6.130)

$G_m = \dfrac{200 \times 241}{62.5 + 241 \times 62.5} = 3.19 mA/V \simeq g_m$

$R_{out} = r_{o2} + [1 + (g_{m2} + g_{mb2}) r_{o2}] r_{o1}$ (Eq. 6.126)

$R_{out} = 62.5 + [1 + (3.2 + 0.64) 62.5] 62.5$

$\underline{R_{out} = 15.125 M\Omega}$

e) $A_v = -A_o^2 \dfrac{R_L}{R_L + A_o r_o}$ (Eq. 6.131)

$A_v = -(200)^2 \dfrac{10^M}{10^4 + 200 \times \frac{62.5^k}{1000}} = 17778 \ V/V$

f) $V_{DS} \geq V_{ov}$ for Q_1. Therefore $V_{DSmin} = V_{ov}$

$V_{DSmin1} = 0.2^V$

$V_{BIAS} = V_{GS2} + V_{DSmin1} = (0.2 + 0.6) + 0.2 = 1^V$

$V_{BIAS} = 1^V$

6.98

$v_y = i_x \times r_{o1} = \dfrac{v_x}{R_{out}} \times r_{o1} \Rightarrow \dfrac{v_y}{v_x} = \dfrac{r_{o1}}{R_{out}}$

$\dfrac{v_y}{v_x} = \dfrac{r_{o1}}{r_{o2} + [1 + (g_{m2} + g_{mb2}) r_{o2}] r_{o1}}$

If we use equation 6.127 to approximate R_{out}, then $\dfrac{v_y}{v_x} \simeq \dfrac{r_{o1}}{g_{m2} r_{o2} r_{o1}} = \dfrac{1}{g_{m2} r_{o2}}$

6.99

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 \Rightarrow$ For same I: $\dfrac{V_{ov b}^2}{V_{ov a}^2} = \dfrac{(\frac{W}{L})_a}{(\frac{W}{L})_b}$

For same I, if $\frac{W}{L}$ is divided by 4, then V_{ov}^2 is multiplied by 4, or equivalently

Cont.

V_{ov} is doubled.

$g_m = \mu_n C_{ox} \frac{W}{L} V_{ov}$. Thus g_m for circuit (b) is half of the one for circuit (a).

$A_o = g_m r_o = \frac{2I_D}{V_{ov}} \times \frac{V_A}{I_D} = \frac{2V_A' L}{V_{ov}}$. Thus if L is multiplied by 4 and V_{ov} is halved, then A_o is doubled for circuit (b).

In summary, for circuit (b), V_{ov} is doubled, g_m is halved, A_o is doubled.

b) Each transistor in circuit (c) has the same overdrive voltage as the one in circuit (a). Referring to Eq. 6.129 and 6.130:

$A_{v_o} = -A_o^2 = -(g_m r_o)^2$

$G_m \simeq g_{m1} = g_m$ (same as circuit (a))

Note that for the transistors in circuit (c) the g_m and r_o are the same as the ones in circuit (a). Thus the intrinsic gain for circuit (c), $A_{v_o} = -A_o^2$ where A_o is the intrinsic gain for circuit (a).

In general, circuit (c) has a higher output resistance and for the same V_{ov} of transistors it has lower output swing. The output swing is limited to $2V_{ov}$ on the low side for circuits (b) and (c) while it is only limited to V_{ov} for circuit (a).

6.100

a) $A_M = -g_m R_L' = -5 \times (20^K \| 20^K) = \underline{-50\ V/V}$

$R_{gs} = R_{sig} = 20\,K\Omega$

$R_{gd} = R_{sig}(1 + g_m R_L') + R_L' = 20^K(1 + 5 \times 20^K \| 20^K) + 20^K \| 20^K$

$R_{gd} = 1030\,K\Omega = 1.03\,M\Omega$

We use Eq. 6.57:

$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_L'$

$\tau_H = 2^P \times 20^K + 0.2^P \times 1030^K + 1^P \times (20^K \| 20^K) = 256\,ns$

$f_H = \frac{1}{2\pi \tau_H} = \underline{622\ KHz}$

$\underline{|A_M| \cdot f_H = 31.1\ MHz}$

b) For the cascode amplifier:

$A_{01} = g_{m1} r_{01} = 5 \times 20 = 100\,V/V$

$A_{V02} = 1 + (g_{m2} + g_{mb2}) r_{02} = 1 + (5 + 0.2 \times 5) \times 20^K$

$A_{V02} = 121\,V/V$

$R_{out} = r_{02} + A_{V02} r_{01} = 20^K + (121 \times 20^K) = 2.44\,M\Omega$

$A_V = A_{V_o} \frac{R_L}{R_L + R_{out}} = -121 \times 100 \times \frac{20}{20 + 2440} = \underline{-98.4\,V/V}$

Using Eq. 6.137,

$\tau_H = R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})] + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) + (R_L \| R_{out})(C_L + C_{gd2})$

$R_{d1} = r_{o1} \| \left(\frac{1}{g_{m2} + g_{mb2}} + \frac{R_L}{A_{V02}} \right)$ (Eq. 6.124)

$R_{d1} = 20^K \| \left(\frac{1}{5 + 0.2 \times 5} + \frac{20}{121} \right) = 0.327\,K\Omega$

$\tau_H = 20^K[2 + 0.2(1 + 5 \times 0.327)] + 0.327(0.2^P + 0.2^P + 2) + (20^K \| 2.44^M)(1 + 0.2)$

$\tau_H = 75.1\,ns$

$f_H = \frac{1}{2\pi \tau_H} = \underline{2.12\,MHz}$

$\underline{|A_V| \cdot f_H = 208.61\,MHz}$

6.101

$A_V = 66\,dB = 1995\,V/V$

$A_V = A_{V_o} \frac{R_L}{R_L + R_{out}}$ and $R_L = R_{out} \Rightarrow A_V = A_{V_o} \times \frac{1}{2}$

$A_{V_o} = (1 + g_{m2} r_{o2}) g_{m1} r_{o1} \simeq g_m^2 r_o^2 = \left(\frac{2I_D}{V_{ov}} \times \frac{V_A}{I_D}\right)^2 = \left(\frac{2V_A}{V_{ov}}\right)^2$

$\Rightarrow 1995 = \frac{1}{2} \times \left(\frac{2 \times 10}{V_{ov}}\right)^2 \Rightarrow V_{ov} = 0.317\,V$

$\Rightarrow I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 200 \times 10^{-3} \times 10 \times 0.317^2 = 0.1\,mA$

Since R_{sig} is small:

$\tau_H \simeq (C_L + C_{gd})(R_L \| R_{out})$

$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100\,k\Omega$, $g_m = \frac{2I_D}{V_{ov}} = 0.631\,mA/V$

$R_{out} = A_{V2} r_{01} + r_{02} = (1 + g_m r_o) r_o + r_o$

$R_{out} = 6510\,K\Omega \simeq 6.5\,M\Omega$, $R_L = R_{out}$.

$\tau_H \simeq (1\,pF + 0.1\,pF)\left(\frac{6510^K}{2}\right) = 3580.5\,ns$

$f_H = 44.5\,KHz$

$f_t \simeq |A_V| \cdot f_H = 1995 \times 44.5 = 88.8\,MHz$

If the cascode transistor is removed, then we have a common-source configuration.

Cont.

$A_M = -g_m(r_o \| R_L) = -0.637(100^K \| 6510^K)$

$A_M = -62.74\ V/V$

$f_H = \dfrac{1}{2\pi(C_L + C_{gd})R_L'} = \dfrac{1}{2\pi(1+0.1)(100^K \| 6510^K)} = 1.47^{MHz}$

$f_H = 1.47\ MHz$

$|A_M| \cdot f_H = 92.2\ MHz \simeq f_T$

Note that the unity-gain stays nearly unchanged. The result is the same as Fig 6.39.

6.102

$R_L = \beta r_o$, $\beta = 100$, $|V_A| = 100^V$, $I = 0.1mA$

From Fig. 6.40, we can write:

$R_{in} = r_{\pi 1} = \dfrac{\beta}{g_{m1}} = \dfrac{\beta V_T}{I_C} = \dfrac{100 \times 0.025}{0.1} = 25K\Omega$

$R_{in} = 25\ K\Omega$

$R_{out} \simeq \beta r_{o2}$, $r_{o2} = \dfrac{V_A}{I_C} = \dfrac{100}{0.1} = 1^{M\Omega}$

$R_{out} = 100 \times 1^M = 100\ M\Omega$

$A_{v_o} = -\beta A_{vo2} = -\beta g_{m2} r_{o2} = -100 \times \dfrac{0.1^m}{0.025} \times 1^M \Rightarrow$

$A_{v_o} = -400 \times 10^3\ V/V$

$G_m = \dfrac{A_{v_o}}{R_{out}} = \dfrac{400 \times 10^3}{100 \times 10^6} = 4\ mA/V \simeq g_m$

$G_m = 4\ mA/V$

Since $R_{out} = R_L = \beta r_o$ we have

$\dfrac{v_o}{v_i} = -G_m(R_L \| R_{out}) = -G_m \dfrac{\beta r_o}{2} = -4\dfrac{100 \times 1^M}{2}$

$\dfrac{v_o}{v_i} = -200 \times 10^3\ V/V$

From Fig. 6.41 the gain of the CE stage is

$A_{CE} = -g_{m1}\left(r_{o1} \| r_{e2}\dfrac{r_{o2} + R_L}{r_{o2} + \frac{R_L}{\beta+1}}\right)$

$r_{o1} = r_{o2} = r_o$, $R_L = \beta r_o$

$A_{CE} = -g_{m1}\left(r_o \| r_{e2}\dfrac{r_o + \beta r_o}{r_o + \frac{\beta r_o}{r_o + 1}}\right)$

$A_{CE} = -g_{m1}\left(r_o \| r_{e2}\dfrac{(\beta+1)r_o}{2r_o}\right)$

$A_{CE} = -g_{m1}\left(r_o \| \dfrac{r_\pi}{2}\right) = -\dfrac{I}{V_T}\left(1^M \| \dfrac{25^K}{2}\right)$

$A_{CE} = -50\ V/V$

6.103

$R_{sig} = 4K\Omega$, $R_L = 2.4K\Omega$, $I = 1mA$, $\beta = 100$, $r_o = 100^{K\Omega}$

Refer to Fig. 6.42:

$A_M = -\dfrac{r_\pi}{r_\pi + r_x + R_{sig}} \times g_m(\beta r_o \| R_L)$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{1/0.025} = 2.5K\Omega$ $\quad g_m = \dfrac{I_C}{V_T} = 40\ mA/V$

$A_M = -\dfrac{2.5}{2.5 + 0.05 + 4} \times 40 \times (100 \times 100 \| 2.4^K)$

$A_M = -36.6\ V/V$

$R'_{sig} = r_\pi \| (r_x + R_{sig}) = 2.5^K \| (0.05 + 4^K)$

$R'_{sig} = 1.55K\Omega$

$R_{\pi 1} = R'_{sig} = 1.55K\Omega$

$R_{\mu 1} = R'_{sig}(1 + g_m R_{e1}) + R_{e1}$

$R_{e1} = r_{o1} \| r_{e2}\left(\dfrac{r_o + R_L}{r_o + R_L/\beta+1}\right) = 100^K \| \dfrac{100^K}{101}\left(\dfrac{100 + 2.4}{100 + \frac{2.4}{101}}\right)$

$R_{e1} = 1K\Omega$

$R_{\mu 1} = 1.55(1 + 40 \times 1) + 1 = 64.55^{K\Omega}$

$R_{out} = \beta r_o = 10M\Omega$

$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + (C_{cs1} + C_{\pi 2})R_{e1} + (C_L + C_{cs2} + C_{\mu 2})(R_L \| R_{out})$

$\tau_H = 14 \times 1.55 + 2 \times 64.55 + (0 + 14) \times 1 + (0 + 2)(2.4^K \| 10^M)$

$\tau_H = 169.6ns$

$f_H = 939KHz$

6.104

a) If we employ Miller's theorem to $C_{\mu 1}$:

$\dfrac{1}{C_{\mu 1}S}\dfrac{1}{1-A} = \dfrac{1}{C_\mu S}\dfrac{1}{1-(-1)} = \dfrac{1}{2C_\mu S}$

or $2C_\mu$ appears in parallel with C_π. Thus the time constant due to $(C_\pi + 2C_\mu)$ is:

$R'_{sig}(C_\pi + 2C_\mu)$ which results in:

$f_{P1} = \dfrac{1}{2\pi R'_{sig}(C_\pi + 2C_\mu)}$

If we refer to Fig. 6.42, we'll see that the output pole is: $f_{P2} = \dfrac{1}{2\pi(C_L + C_{cs2} + C_{\mu 2})R_L}$

b) $R_{sig} = 1K\Omega \Rightarrow R'_{sig} = r_\pi \| R_{sig} = \dfrac{100}{1/0.025} \| 1^K$

$\Rightarrow R'_{sig} = 0.714K\Omega$

$f_{P1} = \dfrac{1}{2\pi \times 0.714^K(5 + 2 \times 1)^P} = 31.85\ MHz$

$f_{P2} = \dfrac{1}{2\pi(0 + 0 + 1) \times 10} = 15.9\ MHz$

(Assume $R_L = 10\ K\Omega$)

Cont.

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P_1}}\right)^2+\left(\frac{1}{f_{P_2}}\right)^2}} = 14.2\ \text{MHz}$$

IF $R_{sig} = 10\ K\Omega$:

$R'_{sig} = 2.5^K \parallel 10^K = 2K\Omega$

$f_{P_1} = \frac{1}{2\pi(5+2)2} = 11.4\ \text{MHz}$

f_{P_2} is the same: $f_{P_2} = 15.9\ \text{MHz}$

$f_H = 9.26\ \text{MHz}$

6.105

Refer to Fig. 6.43.

$I = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}V_{ov}^2 \Rightarrow 100\mu A = \frac{1}{2}\times 60\frac{W}{L}0.2^2$

$\Rightarrow \frac{W}{L} = 83.3$

$V_{SG1} = V_{DD} - V_{BIAS1} = 3.3 - V_{BIAS1}$

$V_{ov} = V_{SG1} - |V_{tp}| = 0.2 \Rightarrow 0.2 = 3.3 - V_{BIAS1} - 0.8$

$\Rightarrow \underline{V_{BIAS1} = 2.3V}$

For Maximum swing: $V_{SD_1} = V_{ov} \Rightarrow V_{D1} = 3.3 - 0.2 = 3.1^V$

$\Rightarrow V_{D1} = 3.1^V$

then: $V_{SG2} - |V_{tp}| = V_{ov} \Rightarrow 3.1 - V_{BIAS2} - 0.8 = 0.2$

$\underline{V_{BIAS2} = 2.1V}$

The highest allowable voltage at the output is $V_{DD} - V_{ov} - V_{ov} = 3.3 - 0.2 - 0.2 = \underline{2.9^V}$

$R_o = g_{m2}\, r_{o2}\, r_{o1}$ (Eq. 6.141)

$g_m = \frac{I_C}{V_T} = \frac{0.1}{0.025} = 4\ mA/V$ $\quad r_o = \frac{V_A}{I_C} = \frac{5}{0.1} = 50k\Omega$

$R_o = 4\times 50\times 50 = 10\ M\Omega$

$\underline{R_o = 10M\Omega}$

6.106

$I_D = 0.2\ mA \Rightarrow g_m = \frac{2I_D}{V_{ov}} = \frac{2\times 0.2}{0.25} = 1.6\ mA/V$

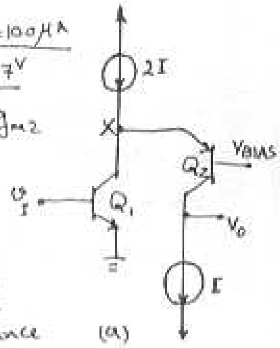
$r_o = \frac{V_A}{I_D} = \frac{5}{0.2} = 25k\Omega$

$R_{out} = A_o^2 r_o$, $A_o = g_m r_o = 1.6\times 25 = 40\ V/V$

$R_{out} = 40\times 40\times 25 = \underline{40M\Omega}$

6.107

$I = 100\mu A$, $V_{BIAS} = 1^V$



a) $I_{C_1} = 2I - I_{E2} = 2I - I = \underline{I = 100\mu A}$

b) $V_X = V_{BIAS} + V_{BE} = 1 + 0.7 = \underline{1.7^V}$

c) $g_{m_1} = \frac{I}{V_T} = \frac{0.1}{0.025} = \underline{4mA/V} = g_{m2}$

$r_{o1} = r_{o2} = \frac{V_A}{I_C} = \frac{100}{0.1} = \underline{1M\Omega}$

d) $V_{omax} = V_X - V_{CEsat}$

$V_{omax} = 1.7 - 0.2 = \underline{1.5V}$

e) $R_i = r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = \underline{25K\Omega}$

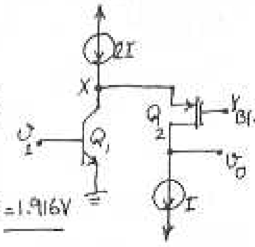
f) Since the output resistance of the current source is equal to R_o , then the overall output resistance $R_{out} = \frac{R_o}{2}$.

$R_o = \beta r_o = 100\times 1^M = 100M\Omega \Rightarrow \underline{R_{out} = 50M\Omega}$

g) $A_M \approx g_m\,(\beta r_o \parallel R_L)$ where $R_L = \beta r_o$

$A_M = g_m\frac{\beta r_o}{2} = 4\times 100\times\frac{1^M}{2} = 200\times 10^6\ V/V$

The corren source 2I should ideally have infinite output resistance.



a) $I_1 = 2I - I = I = 100\mu A$

b) $V_X = V_{BIAS} + V_{SG}$

$I = \frac{1}{2}k'_p\frac{W}{L}V_{ov}^2$

$0.1 = \frac{1}{2}\times 2\times V_{ov}^2 \Rightarrow V_{ov} = 0.316^V$

$V_{SG} = |V_t| + V_{ov} = 0.6 + 0.316$

$V_{SG} = 0.916V \Rightarrow V_X = 1^V + 0.916 = \underline{1.916V}$

c) $g_{m1} = \frac{0.1}{0.025} = 4mA/V$

$g_{m2} = \frac{2I_D}{V_{ov}} = \frac{2\times 0.1}{0.316} = 0.633\ mA/V$

$r_{o1} = \frac{100}{0.1} = 1M\Omega$, $r_{o2} = \frac{5}{0.1} = 50K\Omega$

d) $V_{omax} = V_X - V_{SDmin} = V_X - V_{ov} = 1.916 - 0.316 = 1.6^V$

e) $R_i = r_{\pi_1} = \frac{\beta}{g_{m1}} = \frac{100}{4} = 25K\Omega$

f) $R_{out} = R_{out1} + [1 + (g_m + g_{mb})R_{out1}]\,r_{o2}$

Cont.

$R_{out1} = r_{o1}$

$R_{out} = r_{o2} + [1+g_{m2} r_{o2}]\, r_{o1} = 50^K + 0.633^m \times 1^M \times 50^K$

$R_{out} = 31.7\ M\Omega$

f) First note that R_{L2} or the output resistance of the current source is equal to R_{out} or 32.6 MΩ

$R_{L2} = 32.6 M\Omega$ or $R_{L2} \simeq g_{m2} r_{o1} r_{o2}$

$\frac{v_x}{v_i} = -g_{m1} (r_{o1} \| R_{in2})$

$R_{in2} = \frac{1}{g_{m2}} + \frac{R_{L2}}{A_{o2}} = \frac{1}{g_{m2}} + \frac{g_{m2} r_{o1} r_{o2}}{g_{m2} r_{o2}} = \frac{1}{g_{m2}} + r_{o1}$

$\frac{v_x}{v_i} = -g_{m1} \left(r_{o1} \| (r_{o1} + \frac{1}{g_{m2}})\right) \simeq -g_{m1} \times \frac{r_{o1}}{2} = -2000 V/V$

$\frac{v_o}{v_x} = (1+g_{m2} r_{o2}) \frac{R_{L2}}{R_{L2}+R_{out}} = (1+0.633\times 50)\frac{1}{2} = 16.3 V/V$

$\frac{v_o}{v_i} = -2000 \times 16.3 = -32600 V/V$

In order for the output resistance of the current-source to reduce the gain by 1%:

$\frac{v_x}{v_i} = -g_{m1} \left(r_{o1} \| (r_{o1} + \frac{1}{g_{m2}}) \| R\right) \simeq -g_{m1} \left(\frac{r_{o1}}{2} \| R\right)$

For $\frac{v_x}{v_i} = -\frac{99}{100} \times 2000 = 1980$ we should have:

$\frac{r_{o1}}{2} \| R = \frac{1980}{4} = 495^K \Rightarrow 500^K \| R = 495^K \Rightarrow$

$R = 49.5 M\Omega$ (This is the output resistance of the 2I current-source)

(Note that $\frac{v_o}{v_x}$ did not depend on R)

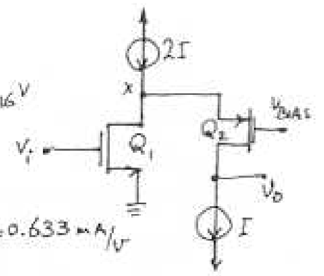
a) $I_1 = 2I - I = 100\ \mu A$

b) $V_X = V_{BIAS} + V_{SG}$

$I = \frac{1}{2} k'_p \frac{W}{L} V_{ov}^2 \Rightarrow V_{ov} = 0.316^V$

$V_{SG} = 0.6 + 0.316 = 0.916^V$

$V_X = 1 + 0.916 = 1.916V$



c) $g_{m1} = g_{m2} = \frac{2I_D}{V_{ov}} = \frac{0.1\times 2}{0.316} = 0.633\ mA/V$

$r_{o1} = r_{o2} = \frac{5}{0.1} = 50 k\Omega$

d) $v_{omax} = V_X - V_{DSmin} = V_X - V_{ov} = 1.916 - 0.316 = 1.6^V$

e) $R_{in} = \infty$

f) $R_{out} = r_{o2} + (1+g_{m2} r_{o2}) r_{o1} = 50 + (1+0.633\times 50)\times 50$

$R_{out} = 1682.5K = 1.68 M\Omega = R_{L2} \simeq g_{m2} r_{o2} r_{o1}$

g) $\frac{v_x}{v_i} = -g_{m1}(r_{o1} \| R_{in2})$

$R_{in2} = \frac{1}{g_{m2}} + \frac{R_{L2}}{A_{o2}} \simeq \frac{1}{g_{m2}} + \frac{g_{m2} r_{o2} r_{o1}}{g_{m2} r_{o2}} = \frac{1}{g_{m2}} + r_{o1}$

Note that R_{L2} is the output resistance of the current source I:

$\frac{v_x}{v_i} = -g_{m1}\left(r_{o1} \| (\frac{1}{g_{m2}} + r_{o1})\right) = -0.633 \times \left(50^K \| (\frac{1}{0.633} + 50^K)\right)$

$\frac{v_x}{v_i} = -16.07 V/V$

$\frac{v_o}{v_x} = (1+g_{m2} r_{o2}) \frac{R_{L2}}{R_{L2}+R_{out}} = (1+0.633\times 50)\times \frac{1}{2}$

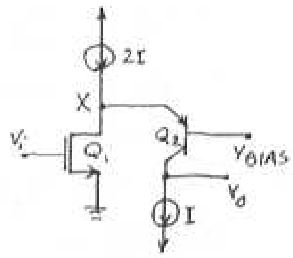
$\frac{v_o}{v_x} = 16.32 V/V$

$\frac{v_o}{v_i} = -262.34 V/V$

In order to reduce the gain by 1% by introducing non-ideal current-source 2I with output resistance R:

$\frac{v_x}{v_i} = -g_{m1} \left(r_{o1} \| R \| (\frac{1}{g_{m2}} + r_{o1})\right) = \frac{90}{100} \times 16.07 = 14.46$

$\Rightarrow (50K \| R \| (51.58^K)) = \frac{14.46}{0.633} \Rightarrow R = 227.5^K$



a) $I_1 = 2I - I = I = 0.1 mA$

b) $V_X = V_{BIAS} + 0.7 = 1.7^V$

c) $g_{m1} = \frac{2I_D}{V_{ov}}$

$V_{ov}^2 = \frac{0.1}{2\times\frac{1}{2}} \Rightarrow V_{ov} = 0.316 V$

$g_{m1} = 0.633\ mA/V$

$r_{o1} = \frac{5}{0.1} = 50 k\Omega$

$g_{m2} = \frac{0.1}{0.025} = 4\ mA/V$, $r_{o2} = \frac{V_A}{I_C} = \frac{100}{0.1} = 1 M\Omega$

d) $V_{omax} = V_X - V_{CEsat} = V_X - 0.2 = 1.5 V$

e) $R_{in} = \infty$

f) $R_{out} = r_{o2}(1+g_{m2}(r_{o1} \| r_\pi))$

$R_{out} = 1^M (1 + 4(50^K \| \frac{100^K}{4})) = 67.67\ M\Omega$

$R_{L2} = R_{out}$

g) $\frac{v_x}{v_i} = -g_{m1} (r_{o1} \| R_{in2}) = -g_{m1} \left(r_{o1} \| \frac{r_{o2}+R_{L2}}{r_{o2}+R_{L2}/\beta+1} r_{e2}\right)$

$r_{e2} = \frac{r_{\pi 2}}{\beta+1} = \frac{\beta}{\beta+1} g_{m2} = 3.96 K\Omega$

$\frac{v_x}{v_i} = -4\left(50^K \| \frac{1^M + 67.67^M}{1 + \frac{67.67}{101}} \times 3.96^K\right) = -152.86 V/V$

$\frac{v_o}{v_x} = (1+g_{m2} r_{o2}) \frac{R_{L2}}{R_{L2}+R_{out}} = (1+g_{m2} r_{o2}) \frac{1}{2}$

$\frac{v_o}{v_x} = (1+4\times 1000)\frac{1}{2} = 2000.5 V/V$

$\frac{v_o}{v_i} = -152.86 \times 2000.5 = -305.8\times 10^3 V/V$

To redue the gain by 1%, the effect of R in $\frac{v_x}{v_i}$ is considered: $0.99 \frac{v_x}{v_i} = 0.99 \times 152.86$

Cont.

$0.99 \times 152.86 = 4 \times \left(50^k \parallel \frac{1+67.67}{1+\frac{67.67}{100}} \parallel R\right)$

$\Rightarrow R = 3.74 M\Omega$

6.108

$g_m = 2 mA/V \Rightarrow g_{mb} = \chi g_m = 0.2 \times 2 = 0.4 mA/V$

$R_{out} = r_o[1 + (g_m + g_{mb})R_s] = 50^k[1 + (2+0.4) \times 0.5^k]$

$\underline{R_{out} = 110 k\Omega}$

$A_{v_o} = -g_m r_o = -2 \times 50 = -100 V/V$

$A_v = -A_{v_o}\frac{R_L}{R_L + R_{out}} = -100 \times \frac{50}{50+110} = \underline{-31.25 V/V}$

using Eq. 6.144:

$G_m = \frac{g_m}{1 + (g_m + g_{mb})R_s} = \frac{2}{1 + (2 + 0.2 \times 2)0.5}$

$\Rightarrow \underline{G_m = 0.91 mA/V}$

$\frac{v_{gs}}{v_i} = \frac{1}{1 + (g_m + g_{mb})R_s}\frac{R_L \parallel R_{out}}{R_L \parallel r_o}$ (Eq. 6.144)

$\frac{v_{gs}}{v_i} = 0.625V \Rightarrow \underline{v_{gs} = 0.625 v_i}$

6.109

$\frac{v_{gs}}{v_i} = \frac{1}{3}$, using Eq. 6.144:

$\frac{v_{gs}}{v_i} = \frac{1}{1 + (g_m + g_{mb})R_s}\frac{R_L \parallel R_{out}}{R_L \parallel r_o}$

$\Rightarrow \frac{1}{3} = \frac{1}{1 + (2 + 0.2 \times 2)R_s} \times \frac{50^k \parallel R_{out}}{50^k \parallel 50^k}$ ①

$R_{out} = r_o[1 + (g_m + g_{mb})R_s] = 50(1 + 2.4R_s)$

$R_{out} = 50(1 + 2.4 \times R_s)$

substitute in ①:

$\frac{1}{3} = \frac{1}{1 + 2.4R_s}\frac{50k \times 50(1 + 2.4R_s)}{50 + 50(1 + 2.4R_s)} \times \frac{1}{25}$

$\Rightarrow \underline{R_s = 1.67 K\Omega}$

$\Rightarrow R_{out} = 50(1 + 2.4 \times 1.67) = 250.4 k\Omega$

$A_v = -A_{v_o}\frac{R_L}{R_L + R_{out}} = -g_m r_o \frac{R_L}{R_L + R_{out}}$

$A_v = -2 \times 50 \times \frac{50}{50 + 250.4} = \underline{-16.64 V/V}$

6.110

a) $A_M = -A_{v_o}\frac{R_L}{R_L + R_{out}} = -g_m r_o \frac{R_L}{R_L + r_o}$

$A_M = -5 \times 40 \times \frac{40}{40+40} = -100 V/V$

$R_L' = R_L \parallel R_{out} = R_L \parallel r_o = 20 k\Omega$

$R_{gd} = R_{sig}(1 + G_m R_L')$ where $G_m = g_m$ (Eq. 6.14

$\Rightarrow R_{gd} = 20^k(1 + 5 \times 20) = 2020 k\Omega = 2.02 M\Omega$

$R_{gd} = 2.02 M\Omega$

$R_s = 0 \Rightarrow R_{gs} = R_{sig} = 20 k\Omega$

$R_{C_L} = R_L' = 20 k\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

$\tau_H = 2 \times 20^k + 0.1 \times 2.02^M + 1 \times 20^k = 262 ns$

$f_H = \frac{1}{2\pi\tau_H} = \underline{607.8 KHz}$

$|A_M| \cdot f_H = +100 \times 607.8 = 60.78 \times 10^3 KHz = \underline{60.78 MHz}$

b) $R_s = 500\Omega$

$R_{out} = r_o[1 + (g_m + g_{mb})R_s] = 40[1 + (5+1)0.5] = 160K$

$A_M = -g_m r_o \frac{R_L}{R_L + R_{out}} = -5 \times 40 \times \frac{40}{40+160} = \underline{-40 V/V}$

$R_L' = R_L \parallel R_{out} = 40k \parallel 160k = 32 k\Omega$

$R_{gd} = R_{sig}(1 + G_m R_L')$

$G_m = \frac{g_m r_o}{r_o[1 + (g_m + g_{mb})R_s]}$ (Eq. 6.144)

$G_m = \frac{5 \times 40}{40[1 + (5+1)0.5]} = 1.25 mA/V$

$R_{gd} = 20k(1 + 1.25 \times 32^k) = 820 k\Omega$

$R_{gs} = \frac{R_{sig} + R_s}{1 + (g_m + g_{mb})R_s \frac{r_o}{r_o + R_L}}$ (Eq. 6.151)

$R_{gs} = \frac{20^k + 0.5^k}{1 + (5+1)0.5\frac{40}{40+40}} = 8.2 k\Omega$

$R_{C_L} = R_L \parallel R_{out} = R_L' = 32 k\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L} = 2 \times 8.2 + 0.1 \times 820 + 32 \times$

$\tau_H = 130.4 ns$

$f_H = \frac{1}{2\pi\tau_H} = \underline{1.22 MHz}$

$\underline{|A_M| \cdot f_H = 48.8 MHz}$

6.111

Using Eq. 6.151: $R_{gs} = \frac{R_{sig} + R_s}{1 + \underbrace{(g_m + g_{mb})R_s}_{K}\frac{r_o}{r_o + R_L}}$

If we define:

$K = (g_m + g_{mb})R_s$ and $R_{sig} \gg R_s$

then: $R_{gs} \cong \frac{R_{sig}}{1 + K\frac{r_o}{r_o + r_o}} = \frac{R_{sig}}{1 + K/2}$

Using Eq. 6.144: $G_m = \frac{g_m}{1 + (g_m + g_{mb})R_s} = g_m/(K+1)$

Cont.

$R'_L = R_L \| R_{out}$

$R_{out} = r_o[1+(g_m+g_{mb})R_S] = r_o(1+K)$

$R'_L = r_o \| r_o(1+K) = r_o \dfrac{(1+K)}{2+K}$

Using Eq. 6.148:

$R_{gd} = R_{sig}(1+G_m R'_L) + R'_L$

$R_{gd} = R_{sig}\left(1+\dfrac{g_m}{1+K} \times r_o \dfrac{1+K}{2+K}\right) + r_o \dfrac{1+K}{2+K}$

$R_{gd} = R_{sig}\left(1+\dfrac{A_o}{2+K}\right) + r_o \dfrac{1+K}{2+K}$

$R_{C_L} = R'_L = r_o \dfrac{1+K}{2+K}$

$\tau_H = R_{gs}C_{gs} + R_{gd}C_{gd} + R_{C_L}C_L$

$\tau_H = \dfrac{R_{sig}}{1+K/2}C_{gs} + R_{sig}\left(1+\dfrac{A_o}{2+K}\right)C_{gd} + (C_L+C_{gd})r_o\dfrac{1+K}{2+K}$

6.112

$R_{out} = r_o[1+(g_m+g_{mb})R_S] = r_o(1+K)$

$R_{out} = 40(1+K)$

$A_M = -g_m r_o \dfrac{R_L}{R_L+R_{out}} = -5 \times 40 \times \dfrac{40}{40+40(1+K)}$

$\boxed{A_M = -\dfrac{200}{2+K}}$

$\tau_H = \dfrac{C_{gs}R_{sig}}{1+K/2} + C_{gd}R_{sig}\left(1+\dfrac{A_o}{2+K}\right) + (C_L+C_{gd})r_o\dfrac{1+K}{2+K}$

(From problem 6.111)

$\tau_H = \dfrac{2^p\, 20^k}{1+K/2} + 0.1^p \times 20^k\left(1+\dfrac{5 \times 40}{2+K}\right) + (1^p+0.1^p)40^k\dfrac{1+K}{2+K}$

$\tau_H = \dfrac{80}{2+K} + 2\left(1+\dfrac{200}{2+K}\right) + 44\dfrac{1+K}{2+K}$

$\tau_H = \dfrac{528+46K}{2+K}$ ns

$f_H = \dfrac{1}{2\pi\tau_H} = \dfrac{(2+K) \times 10^3}{2\pi(528+46K)}$ MHz

$f_T \simeq |A_M| \cdot f_H$.

K	A_M (V/V)	f_H (MHz)	$\|A_M\| \cdot f_H$ (MHz)
0	-100	0.603	60.3
1	-66.67	0.832	55.47
2	-50.00	1.027	51.35
3	-40.00	1.195	47.8
4	-33.33	1.342	44.73
5	-28.57	1.471	42.03
6	-25.00	1.584	39.6
7	-22.22	1.686	37.46
8	-20.00	1.777	35.54
9	-18.18	1.859	33.8
10	-16.67	1.934	32.24
11	-15.38	2.002	30.79
12	-14.28	2.064	29.47
13	-13.33	2.121	28.27
14	-12.5	2.174	26.75
15	-11.76	2.223	26.14

If $f_H = 2$ MHz, then by looking at the table, $K \simeq 11$. Therefore: $K = 11 = (g_m+g_{mb})R_S \Rightarrow$

$R_S = \dfrac{11}{5+1} = 1.83\,k\Omega$

From the table: $A_M = -15.38$

6.113

a) Eq. 6.156: Gain Bandwidth product $= |A_M| \cdot f_H$

$|A_M| \cdot f_H = \dfrac{1}{2\pi C_{gd} R_{sig}}$

For $C_{gd} = 0.1^{pF}$, $R_{sig} = 10\,k\Omega$:

$|A_M| \cdot f_H = 159.2$ MHz

b) If $|A_M| = 20$ V/V, then $f_T = 159.2$ MHz,

$f_H = \dfrac{159.2}{20} = 7.96$ MHz

c) $g_m = 5$ mA/V $\quad \chi = 0.2 \Rightarrow g_{mb} = 1$ mA/V

$A_o = 100$ V/V $\quad R_L = 20\,k\Omega$

$r_o = \dfrac{A_o}{g_m} = \dfrac{100}{5} = 20\,k\Omega = R_L$

$A_V = -A_o \dfrac{R_L}{R_L + R_{out}} \Rightarrow 20 = -100 \dfrac{20}{R_{out}+20}$

$\Rightarrow R_{out} = 80\,k\Omega$

$R_{out} = r_o[1+(g_m+g_{mb})R_S] = 20[1+(5+1)R_S] = 80$

$\Rightarrow 1 + 6R_S = 4 \Rightarrow R_S = 0.5\,k\Omega = 500\,\Omega$

$R_S = 500\,\Omega$

6.114

$R_e = 100\,\Omega$, $I = 0.5$ mA, $\beta = 100$, $V_A = 100$ V, $R_L = r_o$

$R_{in} = (\beta+1)r_e + (\beta+1)R_e \dfrac{1}{1+R_L/r_o}$ when $\dfrac{R_L}{\beta+1} \ll r_o$

(Eq. 6.158)

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{0.5/0.025} = 5\,K\Omega$

$g_m = \dfrac{I}{V_T} = 20$ mA/V

$r_o = \dfrac{V_A}{I_C} = \dfrac{100}{0.5} = 200\,K\Omega$

$r_e = \dfrac{r_\pi}{(\beta+1)} = \dfrac{5^K}{101} = 49.5\,\Omega$

Cont.

$R_{in} = (1+100)\,0.0495 + (1+100)\times 0.1\,\frac{1}{1+1}$

$\underline{R_{in} = 10.05\,K\Omega}$

$R_o \simeq r_o\,(1+g_m R_e')$ where $R_e' = R_e \,||\, r_\pi$ (Eq. 6.160)

$R_o \simeq 200(1+20\times(0.1^K || 5K)) = 592.2K\Omega$

$\underline{R_o = 592.2K\Omega}$

$A_{v_o} \simeq -g_m r_o = -20\times 200 = -4000\ V/V$

$G_m = -\frac{A_{v_o}}{R_o} = 6.75\ mA/V$

$A_v = A_{v_o}\frac{R_L}{R_L+R_o} = -4000\times\frac{200}{200+592.2} = -1009.8V/V$

$\underline{A_v = -1009.8\ V/V}$

$G_v = \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in}+R_{sig}}\,A_{v_o}\,\frac{R_L}{R_L+R_o} = \frac{R_{in}}{R_{in}+R_{sig}}\,A_v$

$G_v = \frac{10.05}{10.05+10}\times(-1009.8) = \underline{-506.2V/V}$

6.115

$g_m = \frac{I}{V_T} = 20\,mA/V$ $\underline{\text{Assume } R_{sig} = 10K\Omega}$

$r_\pi = \frac{\beta}{g_m} = 5K\Omega$

$r_e = \frac{5K}{100+1} = 49.5\Omega$

$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200K\Omega$

$R_{in} = (\beta+1)\,r_e + (\beta+1)R_e\,\frac{1}{1+R_L/r_o}$ (Eq. 6.158)

$R_{in} = 101\times 49.5 + 101\times 100\times\frac{1}{2} = 10049.5\,\Omega = 10.05\,K\Omega$

$\underline{R_{in} = 10.05K\Omega}$

$R_i = (\beta+1)(r_e+R_e) = 101\times(49.5+100) = 15.1K\Omega$

$\underline{R_i = 15.1K\Omega}$

$A_{v_o} = -g_m r_o = -20\times 200 = -4000V/V$

$G_{vo} = \frac{R_i}{R_i+R_{sig}}\,A_{v_o} \simeq \frac{-15.1}{15.1+10}\times 4000 = \underline{-2406.4\ V/V}$

$R_o = r_o\,(1+g_m R_e')$

$R_e' = R_e\,||\,r_\pi = 0.1^K || 5^K = 0.1K\Omega$

$R_o = 200(1+20\times 0.1) = \underline{600K\Omega}$

$G_v = \frac{R_{in}}{R_{in}+R_{sig}}\,A_v = \frac{-10.05}{10.05+10}\times 4000\times\frac{200}{200+600}$

(Note that $A_v = A_{v_o}\,\frac{R_L}{R_L+R_o}$)

$G_v = -501.25\ V/V$

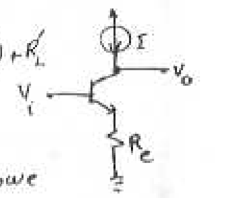
$G_v = G_{vo}\frac{R_L}{R_L+R_{out}} \Rightarrow 501.25 = 2406.4\,\frac{200}{200+R_{out}}$

$\underline{R_{out} = 760.16\,K\Omega}$

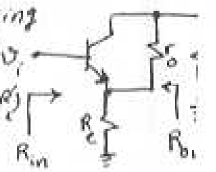
6.116

a) Eq. 6.148: $R_{gd} = R_{sig}(1+G_m R_L') + R_L'$

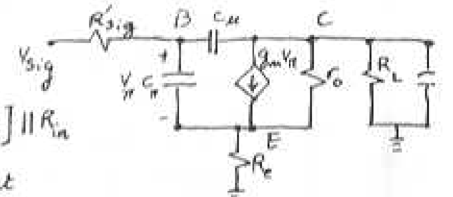
If we try to adapt the above formula for BJTs:



Using Miller's Theorem for C_μ, we would have $C_\mu(1+G_m R_L')$ hanging from the base to the ground. v_i



The resistance seen by $C_\mu(1+g_m R_L')$ is: $R_{sig}' = (R_{sig}+r_x)\,||\,R_{in}$



$R_{sig}' = [R_{sig}+r_x]\,||\,R_{in}$

$R_L' = R_L\,||\,R_{out}$

From the collector side, the resistance seen [by] C_μ is $R_{out}\,||\,R_L$ or R_L'. Therefore the total resistance seen by C_μ, R_μ is:

$$R_\mu = [(R_{sig}+r_x)\,||\,R_{in}]\,(1+G_m R_L') + R_L'$$

The resistance seen by C_L is again $R_L || R_o$ or R_L'. $\Rightarrow R_{C_L} = R_L'$ (Same as Eq. 6.150 fo[r]

Now the resistance seen by C_π is r_π pa[rallel] to

then we can write: $\tau_H = C_\pi R_\pi + C_\mu R_\mu + C_L R_{C_L}$

b) i) $R_e = 0$, $R_{out} \simeq R_o$, $r_\pi = \frac{\beta}{g_m} = 5K\Omega$

$A_M = -\frac{r_\pi}{r_\pi+R_{sig}+r_x}\times g_m R_L'$ Eq. (6.70)

$R_L' = R_L\,||\,R_{out}$, Since $R_{out} \simeq R_o$ the $R_L' = R_L || R$

$R_o = r_o\,(1+g_m R_e') = r_o(1+g_m r_\pi) = (\beta+1)r_o = 101M.$

$R_L' = 5.3^K || 101^M = 5.3^K \Rightarrow A_M = \frac{-5}{5+1+0.2}\times 20\times 5.3$

Now using Formulas given in part(a) we can f[ind]

$f_H = \frac{1}{2\pi\tau_H}$. Since $R_{out} \simeq R_o$, then $R_L' = 5.3K\Omega$

Note that for $R_e = 0$, we have $R_{in} = r_\pi = 5$

and $G_m = g_m = 20\,mA/V$

Cont

$R_\mu = ((R_{sig}+r_x)\,||\,r_\pi)(1+g_m R_L') + R_L'$

$R_\mu = ((1+0.2)\,||\,5)(1+20\times 5.3)+5.3 = 108.85\,k\Omega$

$R_\pi = r_\pi\,||\,(R_{sig}+r_x) = 5^K||1.2^K = 0.97\,k\Omega$

$\tau_H = R_\pi C_\pi + R_\mu C_\mu + R_L C_L = 10\times 0.97 + 0.5\times 108.85 + 2\times 5.3$

$\tau_H = 74.73\,ns \Rightarrow f_H = 2.13\,MHz$, $A_M = -85.5\,V/V$

ii) $R_e = 200\,\Omega$.

$A_{vo} \cong -g_m r_o$ (Eq. 6.159)

$A_{vo} = -20\times 100 = -2000\,V/V$

$R_{in} = (\beta+1)r_e + (\beta+1)R_e \dfrac{1}{1+R_L/r_o}$

$R_{in} = 5^K + 101\times 0.2 \times \dfrac{1}{1+\frac{5.3}{100}} = 24.18\,k\Omega$

$R_o \cong (1+g_m R_e')r_o$ $\quad R_e' = R_e\,||\,r_\pi$

$R_o \cong (1+20(5^K||0.2K))\times 100K = 484.6\,k\Omega$

$G_v = \dfrac{R_{in}}{R_{in}+R_{sig}} A_{vo} \dfrac{R_L}{R_L+R_o} = \dfrac{24.18}{24.18+1}\, 2000\, \dfrac{5.3}{5.3+484.6}$

$G_v = -20.78\,V/V = A_M$

Now to calculate f_H:

$R_L' = R_L\,||\,R_{out}$, $R_{out} \cong R_o = 484.6\,k\Omega$

$R_L' = 5.3\,||\,484.6 = 5.24\,k\Omega = R_{CL}$

$G_m = \dfrac{g_m}{1+g_m R_e} = \dfrac{20}{1+20\times 0.2} = 4\,mA/V$

$R_\mu = ((R_{sig}+r_x)\,||\,R_{in})(1+G_m R_L') + R_L'$

$R_\mu = (1.2^K||24.18^K)(1+4\times 5.24)+5.24 = 30.35\,k\Omega$

$R_\mu = 30.35\,k\Omega$

$R_\pi = r_\pi\,||\,\dfrac{R_{sig}+r_x+R_e}{1+g_m R_e\left(\frac{r_o}{r_o+R_L}\right)} = 5^K\,||\,\dfrac{1+0.2+0.2}{1+20\times 0.2\,\frac{100}{105.3}}$

$R_\pi = 0.276\,k\Omega$

$\tau_H = 0.276\times 10 + 30.35\times 0.5 + 5.24\times 2 = 28.42\,ns$

$f_H = 5.6\,MHz$, $A_M = -20.78\,V/V$

6.117

a) $I = \frac{1}{2}k_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2}\times 160^\mu \times 100\times (0.5)^2 = 2\,mA$

b) $g_m = \dfrac{2I_D}{V_{ov}} = 2\times\dfrac{2}{0.5} = 8\,mA/V$

$g_{mb} = \chi g_m = 0.2\times 8 = 1.6\,mA/V$

$r_o \cong \dfrac{1}{\lambda I_D} = \dfrac{1}{0.05\times 2} = 10\,k\Omega$

c) Using Eq. 6.167: $A_{vo} = \dfrac{g_m r_o}{1+(g_m+g_{mb})r_o}$

$A_{vo} = \dfrac{8\times 10}{1+(8+1.6)\times 10} = 0.82\,V/V$

If we use the approximation formula

Eq. 6.168: $A_{vo} = \dfrac{1}{1+\chi} = 0.83\,V/V$

$R_o = \dfrac{1}{g_m+g_{mb}}\,||\,r_o = \dfrac{1}{8+1.6}^K||10^K = 103\,\Omega$

d) with $R_L = 1\,k\Omega$

$A_v = \dfrac{g_m R_L'}{1+g_m R_L'}$ (Eq. 6.166)

$R_L' = R_L\,||\,r_o\,||\,\dfrac{1}{g_{mb}} = 1^K||10^K||\dfrac{1}{1.6}^K = 370\,\Omega$

$A_v = \dfrac{8^m\times 370\times 10^{-3}}{1+8\times 0.370} = 0.75\,V/V$

6.118

$R_o = r_o\,||\,\dfrac{1}{g_m+g_{mb}} = 20^K + \dfrac{1}{5+1} = 20.17\,k\Omega$

$A_M = \dfrac{v_o}{v_{sig}} = G_v = A_{vo}\dfrac{R_L}{R_L+R_o} = \dfrac{g_m r_o}{1+(g_m+g_{mb})r_o}\,\dfrac{R_L}{R_L+R_o}$

$A_M = \dfrac{5\times 20}{1+(5+1)20}\times\dfrac{20^K}{20^K+20.17^K} = 0.41\,V/V$

$f_Z = \dfrac{g_m}{2\pi C_{gs}}$ (Eq. 6.172) $\Rightarrow f_Z = \dfrac{5}{2\pi\times 2} = 398\,MHz$

$R_{gd} = R_{sig} = 20\,k\Omega$

$R_{gs} = \dfrac{R_{sig}+R_L'}{1+g_m R_L'}$, $R_L' = R_L\,||\,r_o\,||\,\dfrac{1}{g_{mb}} = 20^K||20^K||\dfrac{1^K}{1}$

(Eq. 6.175) $\quad R_L' = 0.91\,k\Omega$

$R_{gs} = \dfrac{20+0.91}{1+5\times 0.91} = 3.77\,k\Omega$

$R_{C_L} = R_L\,||\,R_o = 20K\,||\,20.17 = 10.04\,k\Omega$

$\tau_H = C_{gd}R_{gd} + C_{gs}R_{gs} + C_L R_{C_L}$

$\tau_H = 0.1\times 20^K + 2\times 3.77 + 1\times 10.04 = 19.58\,ns$

$f_H = \dfrac{1}{2\pi\tau_H} = 8.13\,MHz$

Contribution of each capacitor time constant to the overall τ_H is:

10.2% for C_{gd} , 38.5% for C_{gs} , 51.3% for C_L.

6.119

$R_{gd} = R_{sig}$, $R_{gs} = \dfrac{R_{sig}+R_L'}{1+g_m R_L'} \cong \dfrac{R_{sig}}{1+g_m R_L'}$ if $R_{sig} \gg R_L'$

$\tau_H = R_{gd}C_{gd} + R_{gs}C_{gs} + C_L R_{CL}$

$\tau_H = R_{sig}\left(C_{gd} + \dfrac{C_{gs}}{1+g_m R_L'}\right) \Rightarrow f_H = \dfrac{1}{2\pi R_{sig}\left(C_{gd}+\frac{C_{gs}}{1+g_m R_L'}\right)}$

$R_{C_L}C_L$ is ignored and $R_L' = R_L\,||\,r_o\,||\,\dfrac{1}{g_{mb}}$

$f_H \propto (1+g_m R_L')$ Therefore f_H can be increased

Cont.

by increasing $g_m R'_L$. We also know that $R'_L = R_L || r_o || \frac{1}{g_{mb}}$ and hence $R'_L \leq \frac{1}{g_{mb}}$ or $g_m R'_L \leq \frac{g_m}{g_{mb}} = \frac{1}{\chi} \Rightarrow g_m R'_L \leq \frac{1}{\chi}$. Therefore maximum f_H can be achieved when we have $g_m R'_L = \frac{1}{\chi}$:

$$f_H \simeq \frac{1}{2\pi R_{sig}\left(C_{gd} + \frac{C_{gs}}{1+\frac{1}{\chi}}\right)} = \frac{1}{2\pi R_{sig}\left(C_{gd} + \frac{\chi C_{gs}}{\chi + 1}\right)}$$

For the source follower specified in Problem 6.118: $f_{H_{max}} = \frac{1}{2\pi \times 20^K\left(0.1 + \frac{0.2\times 2}{1+0.2}\right)} = \underline{18.37\ MHZ}$

$\chi = \frac{g_{mb}}{g_m} = 0.2$

6.120

Using Eq. 6.178: $f_Z = \frac{1}{2\pi C_\pi r_e}$

$g_m = \frac{I_c}{V_T} = \frac{5}{0.025} = 200 mA/V$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{200} = 0.5 K\Omega$

$r_e = \frac{r_\pi}{\beta+1} = \frac{0.5K}{100+1} = 4.95\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow (C_\pi + C_\mu) = \frac{200^m}{2\pi \times 800^m} = 39.8^{pF}$

$C_\pi = 39.8 - 2 = 37.8 pF$

$f_Z = \frac{1}{2\pi \times 37.8 \times 4.95 \times 10^{-12}} = \underline{851 MHZ}$

From Tabe 5.6: $A_M = \frac{r_o || R_L}{\frac{R_{sig} + r_x + r_\pi}{\beta+1} + r_o || R_L}$

$A_M = \frac{20^K || 1^K}{\frac{10+0.2+0.5}{101} + (20^K || 1^K)}$

$A_M = \frac{0.95}{0.11 + 0.95} = 0.9 V/V$

$R_\mu = R'_{sig} || (r_\pi + (\beta+1)R'_L)$ (Eq. 6.179)

$R'_{sig} = R_{sig} + r_x = 10 + 0.2 = 10.2 K\Omega$

$R'_L = R_L || r_o = 1^K || 20^K = 0.95 K\Omega$

$R_\mu = 10.2K || (0.5 + 101 \times 0.95) = \underline{9.22 K\Omega}$

$R_\pi = \frac{R'_{sig} + R'_L}{1 + \frac{R'_{sig}}{r_\pi} + \frac{R'_L}{r_e}} = \frac{10.2K + 0.95^K}{1 + \frac{10.2}{0.5} + \frac{0.95}{0.5} \times 101} = \underline{52.3^\Omega}$

$f_H = \frac{1}{2\pi(C_\mu R_\mu + C_\pi R_\pi)} = \frac{1}{2\pi(2\times 9.22 + 37.8 \times 0.0523)}$

$\underline{f_H = 7.8 MHZ}$

6.121

$g_m = \frac{I_c}{V_T} = \frac{1}{0.025} = 40 mA/V$, $r_e = \frac{\beta}{(\beta+1) g_m} = 25\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = 2GHZ \Rightarrow C_\pi + C_\mu = 3.18 pF$

$C_\mu = 0.1 pF \Rightarrow C_\pi = 3.08 pF$

$r_\pi = \frac{\beta}{g_m} = 2.5 K\Omega$

$r_o = \frac{V_A}{I_C} = \frac{20}{1} = 20 K\Omega$

r_o is in effect parallel to R_L, so $R'_L = R_L ||$ $R'_L = 1^K || 20^K = 0.95 K\Omega$. From Table 5.6:

$A_M = \frac{R'_L}{\frac{R_{sig} + r_\pi + r_x}{\beta+1} + R'_L} = \frac{0.95}{\frac{1+2.5+0.1}{101} + 0.95} = 0.96$

$R_\mu = R'_{sig} || (r_\pi + (\beta+1) R'_L)$ (Eq. 6.179)

$R'_{sig} = R_{sig} + r_x = 1 + 0.1 = 1.1 K\Omega$

$R_\mu = 1.1^K || (2.5^K + 101 \times 0.95) = 1.08 K\Omega$

$R_\pi = \frac{R'_{sig} + R'_L}{1 + \frac{R'_{sig}}{r_\pi} + \frac{R'_L}{r_e}} = \frac{1.1 + 0.95}{1 + \frac{1.1}{2.5} + \frac{0.95}{0.025}} = 0.052^K\Omega$

$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi(R_\pi C_\pi + R_\mu C_\mu)} = \frac{1}{2\pi(0.052 \times 3.08 + 1.0}$

$\underline{f_H = 593.8 MHZ}$

6.122

$I = 2mA \Rightarrow g_m = \frac{2}{0.025} = 80 mA/V$, $r_\pi = \frac{\beta}{g_m} = 1.$

$r_e = \frac{r_\pi}{\beta+1} = 12.4\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \frac{80^m}{2\pi \times 400 \times 10^6} = 31.85$

$\Rightarrow C_\pi = 31.85 - 2 = 29.85 pF$

$A_M = \frac{R_L}{\frac{R_{sig} + r_\pi}{\beta+1} + R_L} = \frac{1}{\frac{R_{sig}}{101} + 0.0124 + 1} = \underline{\frac{1}{1.0124 +}}$

$R'_L = R_L = 1K\Omega$, $R'_{sig} = R_{sig} + r_x = R_{sig}$

$R_\mu = R'_{sig} || [r_\pi + (\beta+1) R'_L]$ (Eq. 6.179)

$R_\mu = R_{sig} || (1.25 + 101 \times 1) = \underline{R_{sig} || 102.25^K}$

$R_\pi = \frac{R'_{sig} + R'_L}{1 + \frac{R'_{sig}}{r_\pi} + \frac{R'_L}{r_e}}$ (Eq. 6.180)

$R_\pi = \frac{R_{sig} + 1^K}{1 + 0.8 R_{sig} + 80} = \underline{\frac{R_{sig} + 1}{0.8 R_{sig} + 81}}$

$f_H = \frac{1}{2\pi(R_\pi C_\pi + R_\mu C_\mu)} = \frac{1}{2\pi(29.85 R_\pi + 2 R_\mu)}$

a) $R_{sig} = 1^{K\Omega}$: $A_M = 0.978 V/V$

$R_\mu = 0.99^{K\Omega}$, $R_\pi = 24.4\Omega \Rightarrow f_H = 58.8 MHZ$

b) $R_{sig} = 10 K\Omega$: $A_M = 0.9 V/V$

$R_\mu = 9.11 K\Omega$, $R_\pi = 124\Omega \Rightarrow f_H = 7.27 MHZ$

c) $R_{sig} = 100 K\Omega$: $A_M = 0.499 V/V$

$R_\mu = 50.6 K\Omega$, $R_\pi = 627\Omega \Rightarrow f_H = 1.34 MHZ$

6.123

Refer to Fig P6.123:

Each of the transistors is operating at a bias current of approximately 100μA. Thus:

$g_m = \frac{0.1}{0.025} = 4mA/V$, $r_\pi = \frac{100}{4} = 25K\Omega$

$r_e \simeq 250\Omega$, $r_o = \frac{100}{0.1} = 1M\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{4^m}{2\pi \times 400^M} = 1.59pF \Rightarrow C_\pi = 1.39pF$

a) $R_{in} = (\beta+1)[r_{e1} + (r_{\pi2}||r_{o1})]$

$R_{in} = 101[250\times10^{-3} + 25^K || 1M\Omega] \simeq 2.5M\Omega$

$A_M = -\frac{R_{in}}{R_{in}+R_{sig}} \times \frac{r_{\pi2}||r_{o1}}{r_{e1}+(r_{\pi2}||r_{o1})} \times g_{m2} r_{o2}$

$A_M = -\frac{2.5^M}{2.5+0.01} \times \frac{25^K||1^M}{0.25+(25^K||1^M)} \times 4\times1^M$

$A_M = -3943.6 V/V$

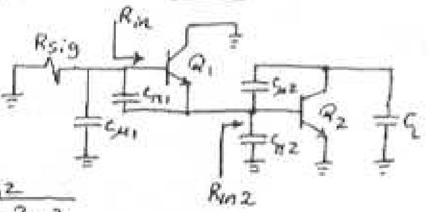
b) To calculate f_H, refer to Example 6.13:

$R_{\mu1} = R_{sig}||R_{in} = 10^K||2.5^M \simeq 10K\Omega$

$R_{in2} = r_{\pi2}||r_{o1}$

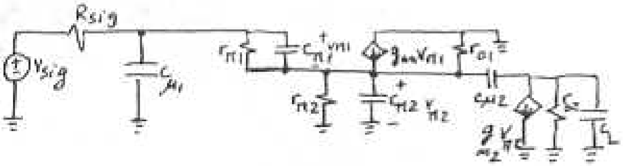
$R_{in2} = 25^K||1^M$

$R_{in2} = 24.4K\Omega$

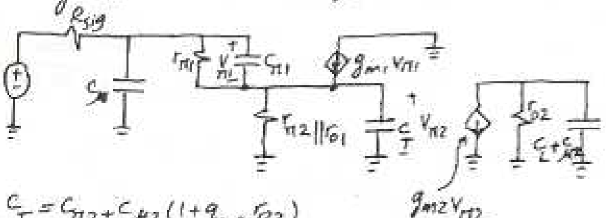


$R_{\pi1} = \frac{R_{sig}+R_{in2}}{1+\frac{R_{sig}}{r_{\pi1}}+\frac{R_{in2}}{r_{e1}}}$

$R_{\pi1} = \frac{10+24.4}{1+\frac{10}{25}+\frac{24.4}{0.25}} = 0.35K\Omega$



Using Miller's Theorem for $C_{\mu2}$:



$C_T = C_{\pi2} + C_{\mu2}(1+g_{m2}r_{o2})$

$C_T = 1.39 + 0.2(1+4\times1000) = 801.6pF$

$R_T = r_{\pi2}||r_{o1}|| \frac{r_{\pi1}+R_{sig}}{\beta+1} = 25^K||1000^K||\frac{25+10}{101}$

$R_T = 342\Omega$

$R_{\mu2} = r_{o2} = 1000K$

$\tau_H = C_{\mu1}R_{\mu1} + C_{\pi1}R_{\pi1} + C_T R_T + (C_{\mu2}+C_L)R_{\mu2}$

$\tau_H = 0.2\times10 + 1.39\times0.35 + 801.6\times0.342 + (0.2+1)\times1000$

$\tau_H = 2 + 0.49 + 274.15 + 1200$ ns

Thus $(C_L + C_{\mu2})R_{\mu2}$ is the dominating term. The second most significant term is $C_T R_T$. So $(C_L+C_{\mu2})$ dominates and then C_T or equivalently $C_{\mu2}$.

$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times1476.6^n} = 107.8MHz$

c) Increasing the bias currents by a factor of 10:

$g_m = 40mA/V$, $r_\pi = 2.5K\Omega$

$r_e \simeq 25\Omega$, $r_o = 100K\Omega$

$C_\pi = C_{je} + C_{de}\times10 = 0.8 + 0.59\times10 = 6.7pF$

$C_\mu = 0.2pF$

$R_{in} = 101[0.025 + (2.5^K||100^K)] = 249K\Omega$

R_{in} is almost decreased by a factor of 10.

$A_M = -\frac{249}{249+10}\times\frac{2.5^K||100^K}{0.025+(2.5^K||100^K)}\times4000$

$A_M = -3807 V/V$

A_M remains almost constant.

$C_T = 6.7 + 0.2(1+40\times100) = 806.9$ (almost Constant)

$R_{\mu1} = R_{sig}||R_{in} = 10^K||249^K = 9.61K\Omega$

$R_{\mu1}$ stays almost the same.

$R_T = 2.5^K||10^K||\frac{2.5+10}{101} = 117.8\Omega$

R_T is almost reduced by a factor of 3.

$R_{in2} = r_{\pi2}||r_{o1} = 2.44K\Omega$

$R_{\pi1} = \frac{10^K+2.44}{1+\frac{10}{2.5}+\frac{2.44}{0.025}} = 120\Omega$

$R_{\pi1}$ is almost decreased by a factor of 3.

$R_{\mu2} = r_{o2} = 100K\Omega$ (decreased by a factor of 10)

$\tau_H = 0.2\times9.61 + 6.7\times0.120 + 806.9\times0.118 + 1.2\times100$

$\tau_H = 1.92+0.8+95.2+120 = 217.92ns$

Thus the dominant effect, that of the output pole, is reduced by a factor of 10. This occurs because $(C_L+C_{\mu2})$ remains constant while r_{o2} decreases by a factor of 10. The second most significant factor (that due to C_T or $C_{\mu2}$ with Miller Effect) also decreases, but only by a factor of 3. The overall result is an increase in f_H.

Cont.

$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 217.9 \times 10^{-9}} = 730.3\ \text{KHz}$

f_H has increased by a factor of nearly 7. Significant increase!

6.124

Note: Although rather long, this is an excellent problem with Considerable educational Value.

a) DC-Bias

For Q_1: $I_{D1} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$

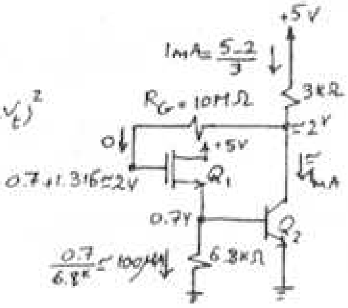
$0.1 = \frac{1}{2} \times 2 (V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 1.316\text{V}$

$I_{D1} \simeq 0.1\text{mA}$

$I_{C2} \simeq 1\text{mA}$

See analysis



b) For Q_1: $g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2 \times 2 \times 0.1} = 0.63\text{mA/V}$

For Q_2: $g_{m2} = 40\text{mA/V}$ $\quad r_{\pi 2} = \frac{200}{40} = 5\text{k}\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6} = 10.6\text{pF}$

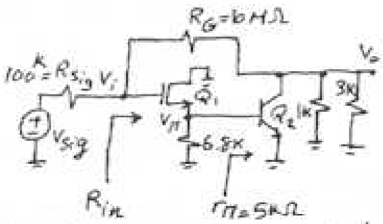
Since $C_\mu = 0.8\text{pF} \Rightarrow \underline{C_\pi = 9.8\text{pF}}$

c) at midband:

$\frac{V_\pi}{V_i} = \frac{6.8 // 5}{(6.8//5) + \frac{1}{g_{m1}}}$

$\frac{V_\pi}{V_i} = \frac{2.88}{2.88 + \frac{1}{0.63}}$

$\frac{V_\pi}{V_i} = 0.64\text{V/V}$



$\frac{V_o}{V_\pi} = -g_{m2} V_\pi (1^K // 3^K)$ where we have neglected the effect of R_G.

$\frac{V_o}{V_\pi} = -40 \times \frac{3}{4} V_\pi = -30 V_\pi \Rightarrow \frac{V_o}{V_i} = 0.64 \times (-30) = -19.2\text{V/V}$

$\underline{\underline{\frac{v_o}{v_i} = -19.2\text{V/V}}}$

$R_{in} = \frac{R_G}{1 - \frac{v_o}{v_i}} = \frac{10^M}{1 - (-19.2)} = \underline{495\text{K}\Omega}$

$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} \times \frac{v_o}{v_i} = \frac{495}{495 + 100} \times 19.2 = \underline{-16\text{V/V}}$

d) At low frequencies:

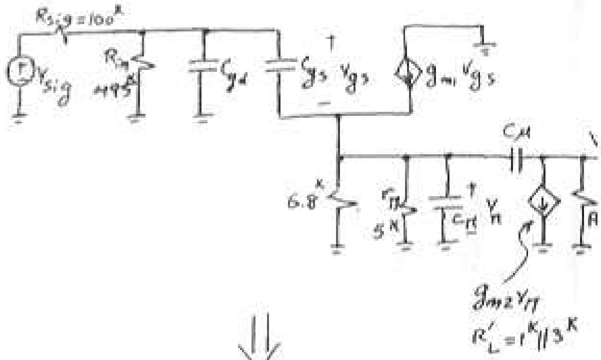
$C_1 \rightarrow f_{P1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times (100 + 495) \times 10^3} = 2.7\text{Hz}$

$C_2 \rightarrow f_{P2} = \frac{1}{2\pi \times 1 \times 10^{-6} (3+1) 10^{-3}} = \underline{40\text{Hz}}$

Thus $f_L \simeq 40\text{Hz}$

e) At high frequencies:

The high frequency equivalent circuit is as follows:



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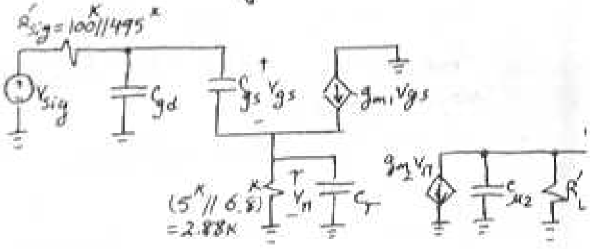


$C_T = C_\pi + C_\mu (1 + g_{m2} R_L') = 9.8 + 0.8(1 + 40 \times \frac{3}{4}) = 34.6\text{p}$

$R_{gd} = R_{sig}' = 100\text{K} // 495\text{K} = 83.2\text{K}\Omega$

$R_{gs} = \frac{R_{sig}' + (6.8^K // 5^K)}{1 + g_{m1}(6.8^K // 5^K)} = \frac{83.2 + 2.88}{1 + 0.63 \times 2.88} = 30.6\text{K}$

$R_T = 6.8 // 5 // \frac{1}{g_m} = 1\text{K}\Omega$

$R_L' = 0.75\text{K}\Omega$

$\tau_H = C_{gd} R_{gd} + C_{gs} R_{gs} + C_T R_T + C_\mu R_L'$

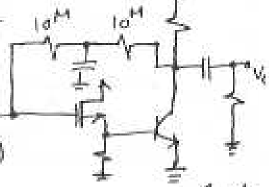
$\tau_H = 1 \times 83.2 + 1 \times 30.6 + 34.6 \times 1 + 0.8 \times 0.75 = 149\text{nS}$

$f_H \simeq \frac{1}{2\pi\tau_H} = \underline{1.07\text{MHz}}$

f) There will no longer be a signal feedback. The left hand side 10MΩ Resistor will in effect appear between the input terminal and ground,

Thus: $R_{in} = 10\text{M}\Omega$

(a factor of 20 increase)



Cont.

and correspondingly A_M becomes:

$A_M = \frac{10}{10.1} \times (-19.2) = \underline{-19\ V/V}$ (an increase from $-16\ V/V$)

Now R'_{sig} becomes approximately $100k\Omega$, as compared to $83.2k\Omega$, and correspondingly R_{gd} becomes $100k\Omega$, and R_{gs} becomes $36.6k\Omega$ while R_T and R'_L remain practically unchanged. Thus τ_H becomes $172.5ns$ and f_H decreases from $1.07MHz$ to $\underline{0.92MHz}$.

6.125

Refer to Fig. P6.125:

$I_{E2} = 10mA \Rightarrow r_{e2} = 2.5\Omega$, $r_{\pi 2} = 253\Omega$

$I_{E1} = \frac{10}{101} \simeq 0.1mA \Rightarrow r_{e1} = 250\Omega$, $r_{\pi 1} = 25.3K\Omega$

$R_{in} = 101 \times [0.25 + 101(0.0025 + 1)] = 10.3M\Omega$

$\underline{R_{in} = 10.3M\Omega}$

$R_{out} = r_{e2} + \frac{1}{\beta_2 + 1}\left[r_{e1} + \frac{R_{sig}}{\beta_1 + 1}\right]$

$R_{out} = 2.5 + \frac{1}{101}\left[250 + \frac{100000}{101}\right] = \underline{14.8\Omega}$

Neglecting r_o:

$A_{vo} = 1000\ V/V$, $A_v = 1 \frac{\times 1000}{14.8 + 1000} = 0.985\ V/V$

6.126

$I_1 = I_2 = I = 1mA \Rightarrow g_m = 40\ mA/V$, $r_\pi = \frac{120}{40} = 3K\Omega$

$r_e = \frac{3}{121} \simeq 25\Omega$, $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40^m}{2\pi \times 700^M} = 9.1pF$ $\Downarrow$ $\underline{C_\pi = 8.6pF}$

Using Eq. 6.185:

$A_M = \frac{v_o}{v_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right) g_m R_L$

$R_{in} = 2r_\pi = 2 \times 3K\Omega = 6K\Omega$

$A_M = \frac{1}{2} \times \frac{6}{6 + 20} \times 40 \times 10^K = 46.15\ V/V$

$f_{P1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)(R_{sig} \| 2r_\pi)} = \frac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right)(20^K \| 6^K)}$

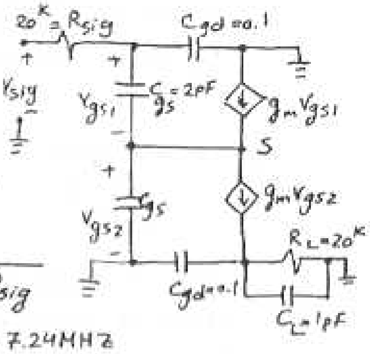
$f_{P1} = 7.19MHz$

$f_{P2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 0.5^P \times 10^K} = 31.8\ MHz$

$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}} = 7.01\ MHz$

(For f_{P1} and f_{P2} Formulas, refer to Eq. 6.186 and 6.187

6.127



$v_i = 2v_{gs}$

$v_o = R_L \times g_m v_{gs}$

$\frac{v_o}{v_i} = \frac{v_o}{v_{sig}} = g_m R_L / 2$

$A_M = \frac{5 \times 20}{2} = \underline{50\ V/V}$

$f_{P1} = \frac{1}{2\pi\left(\frac{C_{gs}}{2} + C_{gd}\right) R_{sig}}$

$f_{P1} = \frac{1}{2\pi(1^P + 0.1)^P 20^K} = 7.24MHz$

$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L) R_L} = \frac{1}{2\pi(0.1 + 1) \times 20^K} = 7.24\ MHz$

$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}} = \underline{5.12MHz}$

6.128

All the transistors in this problem are operating at a bias current of $0.5mA$ and thus have:

$r_e = 50\Omega$, $g_m = 20\ mA/V$, $r_\pi = 5K\Omega$

$C_\pi + C_\mu = \frac{20^m}{2\pi \times 400^M} = 8\ pF$

Since $C_\mu = 2pF \Rightarrow C_\pi = 6pF$, $r_o = \infty$, $r_x = 0$

a) Common-Emitter amplifier:

$R_{sig} = 10k\Omega$, $R_C = 10k\Omega$

$A_M = -\frac{r_\pi}{R_{sig} + r_\pi} g_m R_C = -\frac{5}{10 + 5} 20 \times 10 = \underline{-66.7\ V/V}$

$f_H = \frac{1}{2\pi(R_{sig} \| r_\pi)[C_\pi + (1 + g_m R_C) C_\mu]} \Rightarrow$

$f_H = \frac{1}{2\pi(10^K \| 5^K)[6^P + (1 + 20 \times 10) 2]} = \underline{117\ KHz}$

b) Cascode:

$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig} + r_\pi} = -\frac{100 \times 0.99 \times 10}{10 + 5} = -66\ V/V$

Input pole: $f_{P1} = \frac{1}{2\pi(R_{sig} \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$

$f_{P1} = \frac{1}{2\pi(10^K \| 5^K)(6 + 4)^P} = 4.77\ MHz$

output pole: $f_{P3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^P \times 10^K} = 7.96\ MHz$

pole at midband node:

$f_{P2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^P \times 50} = 530.5\ MHz$

Very high

Cont.

$$f_H = \frac{1}{\sqrt{(\frac{1}{f_{P1}})^2+(\frac{1}{f_{P2}})^2}} = 4.1\,MHz$$

c) CC_CB Cascade (Modified diff. amplifier)

$$A_M = \frac{\beta R_C}{R_{sig}+2r_\pi} = \frac{100\times10}{10+10} = 50\,V/V$$

Input pole: $f_{P1} = \frac{1}{2\pi(R_{sig}\|2r_\pi)(C_\pi/2+C_\mu)}$

$$f_{P1} = \frac{1}{2\pi\,(10^k\|10^k)(3+2)^p} = 6.4\,MHz$$

Output pole: $f_{P2} = \frac{1}{2\pi C_{\mu2}R_C} = \frac{1}{2\pi\times2\times10^k} = 7.96\,MHz$

Thus: $f_H = \frac{1}{\sqrt{(\frac{1}{f_{P1}})^2+(\frac{1}{f_{P2}})^2}} = 5\,MHz$

d) CC_CE Cascade:

$$A_M = -\frac{(\beta_1+1)\beta_2 R_C}{R_{sig}+r_{\pi1}+(\beta_1+1)r_{\pi2}} = -\frac{101\times100\times10}{10+5+101\times5} = -194\,V/V$$

Refer to Example 6.13 in:

$R_{\mu1} = (R_{sig}\|R_{in}) = 10^k\|(\beta+1)[r_{e1}+r_{\pi2}]$

$R_{\mu1} = 10^k\|101\times[0.05+5] = 9.81\,K\Omega$

$R_{\pi1} = r_{\pi1}\|\frac{R_s+r_{\pi2}}{1+g_{m1}r_{\pi2}} = 5\|\frac{10+5}{1+20\times5} = 144\,\Omega$

$R_T = r_{\pi2}\|\frac{r_{\pi1}+R_{sig}}{\beta+1} = 5^k\|\frac{5+10}{101} = 144\,\Omega$

where $C_T = C_{\pi2}+C_{\mu2}(1+g_{m2}R_C) = 6+2(1+200)$

$C_T = 408\,pF$

$R_{\mu2} = R_C = 10\,K\Omega$

$\tau_H = C_{\mu1}R_{\mu1}+C_{\pi1}R_{\pi1}+C_T R_T + C_{\mu2}R_{\mu2}$

$\tau_H = 2\times9.81+6\times0.144+408\times0.144+2\times10$

$\tau_H = 19.62+0.86+58.75+20 = 99.2\,ns$

$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times99.2^n} = 1.6\,MHz$

e) Folded Cascode:

$$A_M = -\frac{\beta_1\alpha_2 R_C}{R_{sig}+r_{\pi1}} = -\frac{100\times0.99\times10}{10+5} = -66\,V/V$$

Input pole:

$f_{P1} = \frac{1}{2\pi(R_{sig}\|r_{\pi1})(C_{\pi1}+2C_{\mu1})} = \frac{1}{2\pi(10\|5)(6+4)}$

$f_{P1} = 4.77\,MHz$

At middle: $f_{P2} = \frac{1}{2\pi C_{\pi2}r_{e2}} = \frac{1}{2\pi\times6\times0.05} = 530\,MHz$ very high!

At output: $f_{P3} = \frac{1}{2\pi C_{\mu2}R_C} = \frac{1}{2\pi\times2\times10} \Rightarrow$

$f_{P3} = 7.96\,MHz$

Thus: $f_H \simeq \frac{1}{\sqrt{\frac{1}{4.77^2}+\frac{1}{7.96^2}}} = 4.1\,MHz$

f) CC_CB Cascade:

$$A_M = \frac{(\beta_1+1)\alpha_2 R_C}{R_{sig}+(\beta_1+1)2r_e} = \frac{101\times0.99\times10}{10+101\times0.1} \simeq 50\,V/V$$

Input pole: $f_{P1} = \frac{1}{2\pi(R_{sig}\|2r_\pi)(C_\pi/2+C_\mu)}$

$f_{P1} = \frac{1}{2\pi(10^k\|10^k)(3^p+2^p)} = 6.4\,MHz$

Output pole: $f_{P2} = \frac{1}{2\pi R_C C_\mu} = \frac{1}{2\pi\times10^k\times2^p} = 7.96\,MHz$

$f_H \simeq \frac{1}{\sqrt{\frac{1}{6.4^2}+\frac{1}{7.96^2}}} = 5\,MHz$

Summary of results:

Configuration	A_M (V/V)	f_H (MHz)	G.B. (MHz)
a) CE	-66.7	0.117	7.8
b) Cascode	-66	4.1	271
c) CC_CB cascade	+50	5.0	250
d) CC_CE cascade	-194	1.6	310
e) Folded cascode	-66	4.1	271
f) CC_CB cascade	+50	5.0	250

6.129

Refer to Fig. 6.58

$I_{REF} = 80\,\mu A = I_R = I_1 = I_2 = I_3$

All transistors have the same g_m, r_o, V_{OV} values.

$I = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 \Rightarrow 0.08 = \frac{1}{2}\times4\times V_{OV}^2 \Rightarrow V_{OV} = 0.2$

$V_{GS} = V_{OV}+V_t = 0.2+0.5 = 0.7\,V$

$V_{G1} = V_{GS} = 0.7\,V = V_{S4} \Rightarrow V_{G4} = 0.7+V_{GS4} = 1.4^V$

$\Rightarrow V_{G3} = 1.4^V \Rightarrow V_{S3} = 1.4^V - V_{GS} = 0.7^V$

$\Rightarrow V_{O\,min} = V_{O3} = V_{S3}+V_{OV} = 0.9\,V$

As explained, the voltage at the gate of Q_3 is $2V_{GS}$ which implies voltage of $V_{GS} = V_{OV}+V_t$ at the source of Q_3. For minimum allowable voltage at the output $V_{DS} = V_{OV}$ or equivalently $V_{o\,min} = V_{OV} + V_{OV} + V_t$

$V_{o\,min} = V_{OV}+V_{OV}+V_t = 2V_{OV}+V_t$.

$g_m = \frac{2I_D}{V_{OV}} = \frac{2\times0.08}{0.2} = 0.8\,mA/V$ $\qquad r_o = \frac{V_A}{I_D} = \frac{8}{0.08} = 100$

Using Eq. 6.189: $R_o = r_{o3}+[1+(g_{m3}+g_{mb3})r_{o3}]r_{o2}$

$R_o = 100K+[1+0.8\times100]\times100 = 8.2\,M\Omega$

6.130

$I_{REF} = 25\mu A$, Refer to Fig. 6.58

$I_4 = 25\mu A = I_1$ $W_1 = W_4 = 2\mu m$ $W_2 = W_3 = 40\mu m$

$I_1 = \frac{1}{2}k_n' \frac{W_1}{L_1} V_{ov_1}^2 \Rightarrow 25 = \frac{1}{2} \times 200 \times \frac{2}{1} V_{ov}^2 \Rightarrow V_{ov_1} = 0.354^V$

$V_{ov_1} = V_{ov_2} \Rightarrow \frac{I_2}{I_1} = \frac{(W/L)_2}{(\frac{W}{L})_1} \Rightarrow I_2 = 25 \times \frac{40}{2} = 500\mu A$

$I_2 = 0.5mA = I_3$

$I_o = 0.5\ mA$

$V_{GS_1} = V_{ov_1} + V_t = 0.354 + 0.6 = 0.954V$

$V_{G_1} = 0.954\ V$

$V_{G4} = V_{GS1} + V_{GS4}$, Since $I_1 = I_4$ and $W_1 = W_4$ then

$V_{GS1} = V_{GS4} \Rightarrow V_{G4} = 2V_{GS1} = 1.91V = V_{G3}$

The lowest possible voltage for the output is when Q_3 has $V_{DS3} = V_{ov3}$ or $V_{omin} = V_{G3} - V_{GS3} + V_{ov_3}$

Since $V_{GS1} = V_{GS2}$ and $I_2 = I_3$ then $V_{GS3} = V_{GS1}$

$\Rightarrow V_{omin} = 1.91 - 0.954 + 0.354 = 1.31V$

$g_{m2} = g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.354} = 2.82 mA/V$

$r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{20}{0.5} = 40K\Omega$

Eq. 6.189 : $R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]r_{o2}$

$R_o = 40 + [1 + 2.82^m \times 40^k] \times 40k = 4.6M\Omega$

6.131

In the gate of Q_3 we see a small incremental resistance of approximately $\frac{1}{g_m}$ and since the incremental voltages across Q_4, Q_5, Q_6 will be small, we can assume that the gates are grounded. Thus the output resistane will be that of the CG transistor Q_3 which has a resisto $R_{o2} = r_{o2} + (1 + (g_{m2} + g_{mb2})r_{o2})r_{o1}$ in its source. We use Eq. 6.101:

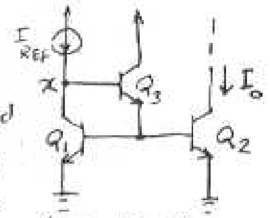
$R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]R_{o2}$

$R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}][r_{o2} + (1 + (g_{m2} + g_{mb2})r_{o2})r_{o1}]$

$R_o \simeq r_{o3} + [(g_{m3} r_{o3})][g_{m2} r_{o2} r_{o1}]$

$R_o \simeq r_{o3} + g_{m2} g_{m3} r_{o1} r_{o2} r_{o3} \simeq g_{m2} g_{m3} r_{o1} r_{o2} r_{o3}$

6.132



$V_x = V_{BE3} + V_{BE1} = 1.4V$

If I_{REF} in is increased to 1mA or equivalently multiplied by 10, then:

$\frac{I_{C2}}{I_{C1}} = \frac{I_s e^{V_{BE2}/V_T}}{I_s e^{V_{BE1}/V_T}} \Rightarrow 10 = e^{(V_{BE2} - V_{BE1})/V_T}$

$V_{BE2} - V_{BE1} = \Delta V_{BE} = V_T \ln 10 = 0.058\ V$

$\Delta V_{BE} = 0.058V \Rightarrow \Delta V_x = 2\Delta V_{BE} = 0.116V$

Now we calculate I_o for $V_o = V_x$:

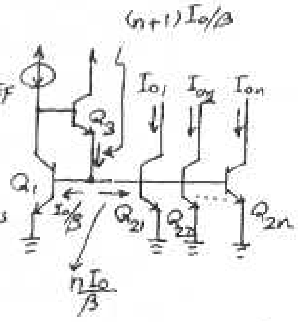
$I_{REF} \simeq I_C = 100\mu A \Rightarrow V_{BE_1} =$

The actual value of $I_o = \frac{I_{REF}}{1 + 2/(\beta^2 + \beta)}$

$\Rightarrow I_o = \frac{100\mu A}{1 + \frac{2}{200 + 200}} = 99.995\mu A$

$\frac{\Delta I_o}{I_o} = \frac{0.005}{100} = 5 \times 10^{-5} = 0.005\%$

6.133



Since $Q_{21}, Q_{22}, \cdots Q_{2n}$ are all matched to Q_1:

$I_{o1} = I_{o2} = \cdots = I_{on} = I_o$

The emitter of Q_3 supplies the basecurrent for all trasistors, so $I_{E3} = \frac{(n+1)I_o}{\beta}$

A node equation at the base of Q_3 yields:

$I_{REF} = I_o + \frac{(n+1)I_o}{\beta(\beta+1)}$, Thus : $\frac{I_o}{I_{REF}} = \frac{1}{1 + \frac{n+1}{\beta^2}}$

For a deviation from unity of less than 0.1% : $\frac{99.9}{100} = \frac{1}{1 + \frac{n+1}{\beta^2}} \Rightarrow \frac{n+1}{\beta^2} = \frac{1}{999}$

$\Rightarrow n = \frac{\beta^2}{999} - 1 \Rightarrow n \simeq 9$

6.134

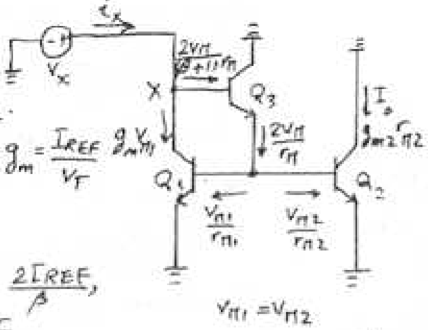
Q_1 and Q_2 are biased at I_{REF}:

$r_{e1} = r_{e2} = \frac{V_T}{I_{REF}} \Rightarrow g_m = \frac{I_{REF}}{V_T}$

$r_{\pi 1} = \frac{\beta V_T}{I_{REF}}$

Q_3 is biased at $\frac{2 I_{REF}}{\beta}$,

Thus $r_{e3} = \frac{\beta V_T}{2 I_{REF}}$



small-signal model

Refer to the small-circuit analysis performed directly on the circuit. Since the current in the emitter of Q_3 is $\frac{2V_{\pi 1}}{r_{\pi 1}}$, the voltage $V_{\pi 3}$ will be: $V_{\pi 3} = \frac{2V_{\pi 1}}{r_{\pi 1}} \times r_{e3}$.

$V_x = V_{\pi 2} + V_{\pi 1} = \frac{2V_{\pi 1} \cdot r_{e3}}{r_{\pi 1}} + V_{\pi 1} = V_{\pi 1}\left(1 + 2\frac{r_{e3}}{r_{\pi 1}}\right)$

$V_x = V_{\pi 1}\left(1 + 2\frac{\beta V_T}{2 I_{REF}} \times \frac{I_{REF}}{\beta V_T}\right) = 2V_{\pi 1}$

and $i_x \simeq g_{m1} V_{\pi 1}$. Thus: $R_{in} = \frac{V_x}{i_x} = \frac{2}{g_{m1}} = \frac{2V_T}{I_{REF}}$

For $I_{REF} = 100\mu A \Rightarrow R_{in} = \frac{2 \times 0.025}{0.1} = 0.5 K\Omega$

6.135

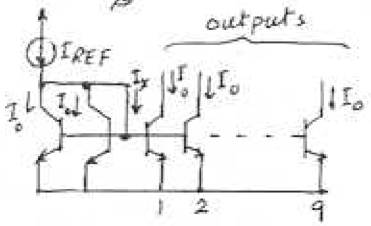
All the output currents are equal to I_o, then we have: $I_{REF} = 2I_o + \frac{11 I_o}{\beta} \Rightarrow \frac{I_o}{I_{REF}} = \frac{1}{2 + 11/\beta}$

I_o is idealy $I_{REF}/2$, For 5% lower I_o:

$$0.95 \times \frac{I_{REF}/2}{I_{REF}} = \frac{1}{2 + \frac{11}{\beta}} \Rightarrow \beta = 104.5 \simeq 105$$

$\beta = 105$

$I_x = 11 \times \frac{I_o}{\beta}$



6.136

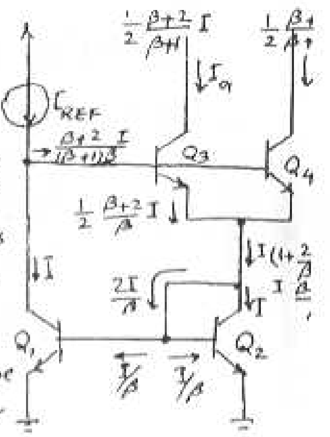
a) See the analysis on the circuit.

$I_{REF} = I + \frac{\beta + 2}{\beta(\beta+1)} I = I \frac{\beta^2 + 2\beta + 2}{\beta(\beta+1)}$

$I_{o1} = I_{o2} = \frac{1}{2}\frac{\beta + 2}{\beta + 1} I$

$\frac{I_{o1}}{I_{REF}} = \frac{I_{o2}}{I_{REF}} = \frac{1}{2}\frac{\beta(\beta+2)}{\beta^2 + 2\beta + 2} = \frac{1}{2} \times \frac{1}{1 + 2/(\beta^2 + 2\beta)}$

$\frac{I_{o1}}{I_{REF}} \simeq \frac{1}{2}\frac{1}{1 + 2/\beta^2}$



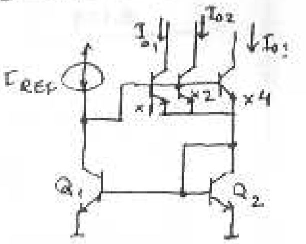
Observe that the deviation factor $\frac{1}{1 + 2/\beta^2}$ is independent of the number of outputs or the value of each output, i.e.: The current I_{REF} can be split into any number of outputs through an appropriate combination of parallel-connected transistors. (Q_3 and Q_4 in this case) The reason the error factor remains unchanged at $\frac{1}{1 + 2/\beta^2}$ is that the base current that need to be supplied by I_{REF} (subst from I_{REF}) remains unchanged.

b) The 7mA reference current can be used to generate three output currents of 1, 2, 4mA by using 3 transistors in parallel having relative area ratios of 1, 2, 4 as shown

$\frac{I_{o1}}{I_{REF}} = \frac{1}{7}\frac{1}{1 + 2/\beta^2} \Rightarrow I_{o1} = 0.998\ mA$ (1mA ideally)

$\frac{I_{o2}}{I_{REF}} = \frac{2}{7}\frac{1}{1 + 2/\beta^2} \Rightarrow I_{o2} = 1.996 mA$ (2mA ideally)

$\frac{I_{o3}}{I_{REF}} = \frac{4}{7}\frac{1}{1 + 2/\beta^2} \Rightarrow I_{o3} = 3.992 mA$ (4mA ideally)

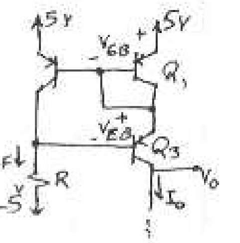


6.137

$I_{REF} = 0.1 mA = \frac{5 - 0.7 - 0.7 - (-5)}{R}$

$\Rightarrow R = 86 K\Omega$

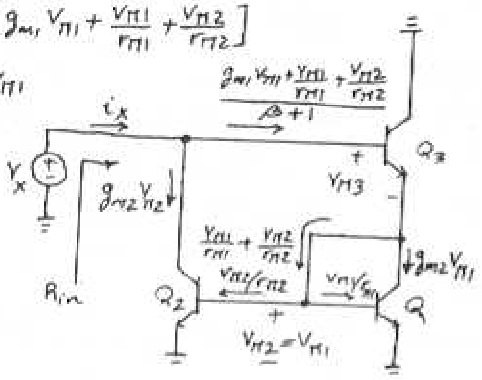
V_{omax} is obtained when Q_3 is saturated: $V_{omax} = 5 - 0.7 - 0.2 = 4.1^V$



6.138

$v_{\pi 3} = r_{e3}\left[g_{m1} v_{\pi 1} + \frac{v_{\pi 1}}{r_{\pi 1}} + \frac{v_{\pi 2}}{r_{\pi 2}}\right]$

$v_x = v_{\pi 3} + v_{\pi 1}$



small-signal analysis

Note that all 3 transistors are biased at I_{REF}

$v_x = v_{\pi 3} + v_{\pi 1} = r_{e3}\left[g_{m1} v_{\pi 1} + \frac{v_{\pi 1}}{r_{\pi 1}} + \frac{v_{\pi 2}}{r_{\pi 2}}\right] + v_{\pi 1}$

$v_x = (r_e g_m + \frac{2r_e}{r_\pi} + 1) v_{\pi 1}$

$v_x = (\frac{\beta}{\beta+1} + \frac{2}{\beta+1} + 1) v_{\pi 1} \simeq 2v_{\pi 1}(1 + \frac{1}{\beta+1}) \simeq 2v_{\pi 1}$

$i_x = g_{m2} v_{\pi 2} + \frac{1}{\beta+1}(g_{m1} v_{\pi 1} + \frac{v_{\pi 1}}{r_{\pi 1}} + \frac{v_{\pi 2}}{r_{\pi 2}})$

$i_x = v_{\pi 1}\left[g_m + \frac{g_m}{\beta+1} + \frac{g_m}{\beta(\beta+1)} + \frac{g_m}{\beta(\beta+1)}\right]$

$i_x \simeq g_m v_{\pi 1}$

Thus: $R_{in} = \frac{v_x}{i_x} = \frac{2v_{\pi 1}}{g_m v_{\pi 1}} = \frac{2}{g_m} = \frac{2V_T}{I_{REF}}$

For $I_{REF} = 100\,\mu A$:

$R_{in} = \frac{2 \times 0.025}{0.1} = 0.5\,k\Omega$

6.139

$R_o = \frac{\beta r_o}{2} = \frac{\beta}{2}\frac{V_A}{I_C} = \frac{100}{2} \times \frac{100}{1} = 5\,M\Omega$

$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{10^V}{5^M} = 2\,\mu A$

$\frac{\Delta I_o}{I_o} = \frac{2^\mu}{1^m} = 0.2\%$

6.140

$I_{REF} = 25\,\mu A$, $I_2 \doteq I_{REF} = 25\,\mu A$

$V_{ov1} = V_{ov2} \Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} \Rightarrow I_1 = 25^\mu \times \frac{2}{40} = 1.25\,\mu A$

$I_o = I_3 = I_1 = 1.25\,\mu A$

Refer to Fig. 6.61a

$V_{ov_2}^2 = \frac{I_2}{\frac{1}{2}k_n' \frac{W}{L}} = \frac{25}{\frac{1}{2} \times 200 \times \frac{40}{1}} = \frac{1}{160} \Rightarrow V_{ov_2} = 0.08^V$

$\Rightarrow V_{GS2} = V_{GS1} = 0.08 + 0.6 = 0.86V$. therefore

$V_{G1} = V_{G2} = 0.86V$

$V_{ov3}^2 = \frac{I_3}{\frac{1}{2}k_n' \frac{W}{L}} = \frac{1.25}{\frac{1}{2} \times 200 \times \frac{40}{1}} = \frac{1}{3200} \Rightarrow V_{ov3} = 0.018^V$

Therefore $V_{GS3} = 0.018 + 0.6 = 0.618V$

$\Rightarrow V_{G3} = V_{S3} + V_{GS3} = 0.86 + 0.618 = 1.48V$

For V_{omin}, we have to have $V_{DS3} = V_{ov3} = 0.018^V$

then: $V_{omin} = V_{GS1} + V_{DS3min} = 0.86 + 0.018 = 0.88^V$

For Q_2: $g_{m2} = \frac{2I}{V_{ov}} = \frac{2 \times 25^\mu}{0.08} = 625\,\mu A/V = 0.625\,mA/V$

$r_{o2} = \frac{V_A}{I} = \frac{20}{0.025} = 800\,k\Omega$

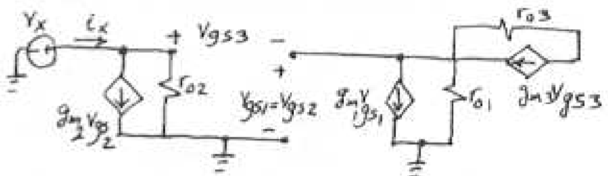
For Q_3: $g_{m3} = \frac{2 \times 1.25}{0.018} = 139\,\mu A/V = 0.139\,mA/V$

$r_{o3} = \frac{20}{0.00125} = 16\,M\Omega$

Refer to section 6.12.4 in textbook:

$R_o = r_{o3}(g_{m3} r_{o2} + 2) = 16^M(0.139 \times 800^k + 2) = 1811.2\,M\Omega$

6.141



$i_x = g_{m2} V_{gs2}$ (1)

$V_{gs2} + V_{gs3} = V_x$.

Since Q_2 and Q_3 have the same parameters and same current, therefore $V_{gs2} = V_{gs3}$

$V_x = 2V_{gs2} \Rightarrow V_{gs2} = \frac{V_x}{2}$

Substitute for V_{gs2} in (1):

$i_x = g_{m2} \times \frac{V_x}{2}$

$R_{in} = \frac{V_x}{i_x} = \frac{2}{g_{m2}}$

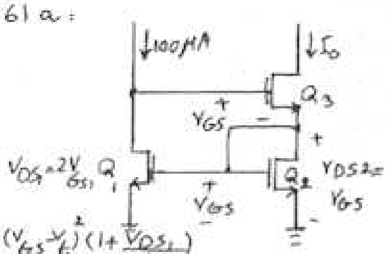
6.142

Refer to Fig. 6.61 a:

$I_{REF} = 100\mu A$

$V_{DS1} = 2V_{GS}$

$V_{DS2} = V_{GS}$



$I_{D1} = I_{REF} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2(1+\frac{V_{DS1}}{V_A})$

$100 = \frac{1}{2}\times 2000\,(V_{GS}-0.6)^2(1+\frac{2V_{GS}}{20})$

$1 = (V_{GS}-0.6)^2(10+V_{GS})$

$V_{GS} \simeq 0.91V$ (by iteration)

$I_o = I_{D2} = \frac{1}{2}\times 2000\,(V_{GS}-0.6)^2(1+\frac{V_{GS}}{20})$

$I_o = 100.47\,\mu A$

Thus there is $\frac{0.47}{100}$ or 0.5% error. Modifying the circuit as Fig. 6.61C ensures that Q_1 and Q_2 have the same V_{DS} and thus eliminate the above error.

6.143

Refer to Fig. 6.62: $I_{REF} = 100\mu A$ $I_o = 10\mu A$

a) $V_{BE1} = 0.7 + V_t \ln\frac{100}{1000} = 0.642V$

$V_{BE2} = 0.7 + V_t \ln\frac{10}{1000} = 0.585V$

$I_o = \frac{V_{BE1}-V_{BE2}}{R_E} = 10\mu A \Rightarrow R_E = 5.7\ k\Omega$

b) $r_{\pi 2} = (\beta+1)\frac{V_T}{10\mu A} = 503k\Omega >> R_E$

$r_{o2} = \frac{V_A}{I_o} = 10M\Omega \Rightarrow R_o = (1+g_{m2}R_E)r_{o2} = 33M\Omega$

$R_o = 33M\Omega$

$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{5}{R_o} = 0.15\mu A$

6.144

a) $\frac{I_o}{I_{REF}} = 0.9 \Rightarrow I_o = 90\mu A$

$V_{RE} = V_T \ln\frac{1}{0.9} = 2.63mV$

$R_E = \frac{2.63mV}{90\mu A} = 29.3\Omega$

$r_o = \frac{V_A}{I_o} = 1.11M\Omega$

$g_m = 3.6\ mA/V$

$R_o = (1+g_m R_E)r_o = \underline{1.23M\Omega}$ Compare to $r_o = 1.11\ M\Omega$

b) $\frac{I_o}{I_{REF}} = 0.1 \Rightarrow I_o = 10\mu A$

$V_{R_E} = V_T \ln 10 = 57.56\ mV$

$R_E = \frac{57.56^m}{10^\mu} = 5.76k\Omega$

$r_o = \frac{100}{10\mu} = 10M\Omega$

$g_m = 0.4mA/V$

$R_o = (1+g_m R_E)r_o = 33M\Omega$ Compare to $r_o = 10$

c) $\frac{I_o}{I_{REF}} = 0.01 \Rightarrow I_o = 1\mu A$

$V_{RE} = V_T \ln 100 = 115mV$

$R_E = \frac{115}{1} = 115k\Omega$

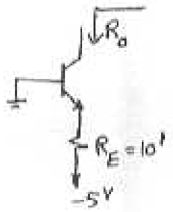
$r_o = \frac{100}{1} = 100M\Omega$

$g_m = 0.04\ mA/V$

$R_o = (1+g_m R_E)r_o = 560M\Omega$ Compare to $r_o = 1$

6.145

$R_o = [1+g_m(R_E \| r_\pi)]\,r_o$



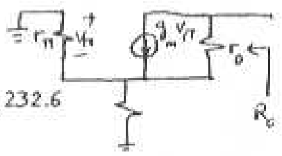
$I_E = \frac{-0.7-(-5)}{R_E} = 0.43mA$

$g_m = \frac{I_C}{V_T} = \frac{0.43}{0.025} = 17.2mA/V$

$r_o = \frac{V_A}{I_C} = \frac{100}{0.43} = 232.6k\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{17.2} = 5.8k\Omega$

$R_E = 10k\Omega$



$R_o = [1+(10^k \| 5.8^k)\times 17.2]\times 232.6$

$\underline{R_o = 14.92M\Omega}$

6.146

a) $I_o = \frac{V_{BE1}+V_{BE2}-V_{BE3}}{R}$

For $I_o = I_{REF}$: $V_{BE1} = V_{BE2} = V_{BE3} \Rightarrow I_o = \frac{V_{BE1}}{R}$

For $I_o = 10\mu A$: $V_{BE} = 0.585$ therefore $\underline{R = 58.5K}$

b) $g_m = \frac{I_C}{V_T} = \frac{10^\mu}{0.025} = 0.4mA/V$, $r_\pi = \frac{\beta}{g_m} = 250k\Omega$

$r_o = \frac{V_A}{I_o} = 10M\Omega$

$R_o = [1+g_m(R \| r_\pi)]\,r_o$

$R_o = [1+0.4(58.5^k \| 250^k)]\,10^M = 199.6M\Omega$

$\underline{R_o \simeq 200M\Omega}$

6.147

Q_1 and Q_2 are matched and $I_{D1} = I_{D2} = I$ therefore $V_{GS1} = V_{GS2}$ and this implies that the source of Q_1 and the source of Q_2 have the same voltage. Hence: $IR = V_{BE6} = V_T \ln \frac{I}{I_S}$

$IR = V_T \ln \frac{I}{I_S}$ (1)

To find the value of R, first we have to calculate I_S.

For $I_E = 1mA$, $V_{BE} = 0.7^V \Rightarrow 1 = I_S e^{0.7/0.025}$

$\Rightarrow I_S = 6.9 \times 10^{-13} mA$

From (1): $R = \frac{V_T}{I} \ln \frac{I}{I_S} = \frac{0.025}{0.010m} \ln \frac{0.010}{6.9 \times 10^{-13}}$

$R = 58.49 K\Omega \approx 58.5 K\Omega$

Chapter 7 - Problems

7.1

$V_{DD} = V_{SS} = 2.5V$

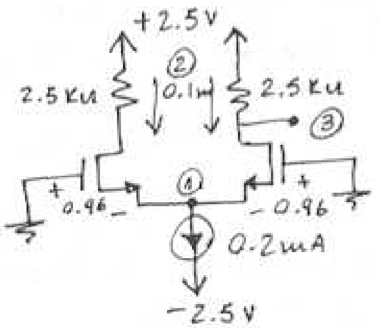
$K_n' \frac{W}{L} = 3 \frac{mA}{V^2}$; $V_{tn} = 0.7V$

$I = 0.2mA$; $R_D = 5K\Omega$

(a) $V_{ov} = \sqrt{I / K_n' W/L}$
$= \sqrt{0.2/3} = \underline{\underline{0.26V}}$

$V_{GS} = V_{ov} + V_t = 0.26 + 0.7$
$= \underline{\underline{0.96V}}$

(b)



① $V_{S1} = V_{S2} = V_{CM} - V_{GS}$
$= 0 - 0.96 = \underline{\underline{-0.96V}}$

② $I_{D1} = I_{D2} = \frac{I}{2} = 0.1mA$

③ $V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} \times R_D$
$= +2.5 - 0.1 \times 2.5 = \underline{\underline{2.25V}}$

(c) If $V_{cm} = +1V$

$V_{S1} = V_{S2} = +1 - 0.96 = \underline{\underline{0.04V}}$

$I_{D1} = I_{D2} = \underline{\underline{0.1mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25V}}$

(d) If $V_{cm} = -1V$

$V_{S1} = V_{S2} = -1 - 0.96 = \underline{\underline{-1.96V}}$

$I_{D1} = I_{D2} = \underline{\underline{0.1mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25V}}$

(e) $V_{CMAX} = V_t + V_{DD} - \frac{I}{2} R_D$
$= 0.7 + 2.5 - 0.1 \times 2.5 = \underline{\underline{+2.95V}}$

(f) $V_{CMIN} = -V_{SS} + V_{CS} + V_t + V_{ov}$
$= -2.5 + 0.3 + 0.7 + 0.26$
$= \underline{\underline{-1.24V}}$

$V_{S\,min} = V_{CMmin} - V_{GS}$
$= -1.24 - 0.96 = \underline{\underline{-2.2V}}$

7.2

(a) $V_{ov} = -\sqrt{I / K_p' (W/L)}$

$= -\sqrt{0.7/3.5} = \underline{\underline{-0.45V}}$

$V_{GS} = V_{ov} + V_t = -0.45 - 0.8$
$= \underline{\underline{-1.25V}}$

$V_{S1} = V_{S2} = V_G - V_{GS}$
$= 0 + 1.25 = \underline{\underline{+1.25V}}$

$V_{D1} = V_{D2} = \frac{I}{2} \times R_D - V_{DD}$

$= \frac{0.7}{2} \times 2 - 2.5 = \underline{\underline{-1.8V}}$

(b) For Q_1 and Q_2 to remain in saturation:

$V_{DS} \leq V_{GS} - V_t$

$\rightarrow V_{CM} \geq \left(\frac{I}{2} R_D - V_{DD}\right) + V_t$

$V_{CM\,min} = \frac{0.7}{2} \times 2 - 2.5 - 0.8$

$= \underline{\underline{-2.6V}}$

To allow sufficient voltage for the current source to operate properly:

$V_{CM} \leq V_{SS} - V_{CS} + (V_t + V_{ov})$

$\rightarrow V_{CM\,max} = 2.5 - 0.5 - 1.25$
$= 0.75V$ //

7.3

(a) $V_{G2} = 0$, $V_{G1} = v_{id}$
if $I_{D1} = I_{D2} = I/2 = 0.1\,mA$
then $V_{G1} = V_{G2}$
thus $v_{id} = 0V$.

(b) $i_{D1} = \frac{I}{2} + \frac{I}{V_{ov}} \cdot \frac{v_{id}}{2}$

$\rightarrow v_{id} = \left(\frac{2 i_{D1}}{I} - 1\right) \cdot V_{ov}$

For $i_{D1} = 0.15\,mA$
$v_{id} = \left(\frac{2 \times 0.15m}{0.2m} - 1\right) \times 0.26$
$= \underline{0.13V}$

(c) $i_{D1} = 0.2\,mA$ and
$i_{D2} = 0$ (when Q_2 just cuts off)
this occurs at
$v_{id} = +\sqrt{2} \times V_{ov}$
$= +\underline{0.367V}$

(d) $i_{D1} = 0.05\,mA$, $i_{D2} = 1.50\,mA$ } Opposite case to (b)
$\longrightarrow v_{id} = -\underline{0.13}V$

(e) $i_{D1} = 0$, $i_{D2} = 0.2\,mA$ when Q_1 just cuts off.
This occurs at $v_{id} = -\sqrt{2} \times V_{ov}$
$= -\underline{0.367}V$

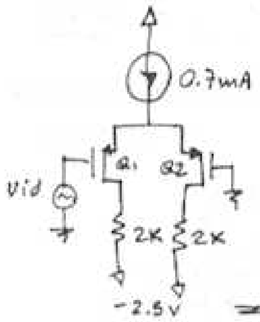
For each case find v_S, v_{D1}, v_{D2}, $v_{D2} - v_{D1}$

$V_{GS} = \sqrt{\frac{2 \times I_D}{K_n W/L}} + V_t$

then $v_S = v_G - v_{GS}$
Solving for each case we obtain the following results

	v_{id} (V)	i_{D1} (mA)	i_{D2} (mA)	V_S (V)	V_{D1} (V)	V_{D2} (V)	$V_{D2}-V_{D1}$ (V)
(c)	+.37	0.2	0	-0.7	1.5	2.5	-1.0
(b)	+.13	0.15	0.05	-0.88	1.75	2.25	-0.5
(a)	0	0.1	0.1	-0.96	2	2	0
(d)	-.13	0.05	0.15	-1.01	2.25	1.75	+0.5
(e)	-.37	0	0.2	-1.07	2.5	1.5	+1.0

7.4



$V_{G2} = 0$
$V_{G1} = v_{id}$

When all the current is on Q_1:
$I = \frac{1}{2}(K_p' \frac{W}{L})(V_{GS1} - V_t)^2$

$\Rightarrow V_{GS1} = V_t + \sqrt{\frac{2I}{K_p' W/L}}$
$= V_t + \sqrt{2}\, V_{ov}$

and V_{GS2} is reduced to V_t
thus $V_S = -V_t$.
Then $v_{id} = v_{GS1} + v_S$
$= V_t + \sqrt{2}\, V_{ov} - V_t = \sqrt{2}\, V_{ov}$
In a similar manner as for the NMOS Differential Amplifier, as v_{id} reaches $-\sqrt{2}\, V_{ov}$ Q_1 turns off and Q_2 on.
Thus the steering range is
$\sqrt{2}\, V_{ov} \leq V_i \leq -(\sqrt{2}\, V_{ov})$

For this particular case
$\sqrt{2} \times -0.45 \leq v_{id} \leq \sqrt{2} \times 0.45$
$-0.63 \leq v_{id} \leq 0.63$
When $v_{id} = -0.63V$,
$i_{D1} = 0.7mA$, $i_{D2} = 0$
$V_S = -V_{t2} = +0.8V$
$V_{D1} = 2K \times 0.7m - 2.5 = -1.1V$

CONT.

$V_{D2} = 0 - 2.5V = \underline{\underline{-2.5V}}$

When $v_{id} = +0.63$

$i_{D1} = 0$; $i_{D2} = 0.7mA$

$V_S = v_{id} - V_{GS1} = v_{id} - V_{t1}$

$= 0.63 + 0.8 = \underline{\underline{1.43}}$

$V_{D1} = -2.5V$ //

$V_{D2} = -1.1V$ //

7.5

$V_{G1} = v_{id}$ $\quad i_{D1} = 0.11mA$

$V_{G2} = 0$ $\quad i_{D2} = 0.09mA$

$I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2$

For Q_1:

$0.11m = \frac{1}{2} 3m (V_{GS1} - 0.7)^2$

$\longrightarrow V_{GS1} = \underline{\underline{0.97V}}$

For Q_2:

$0.09m = \frac{1}{2} 3m (V_{GS2} - 0.7)^2$

$\longrightarrow V_{GS2} = \underline{\underline{0.94V}}$

$V_S = -V_{GS2} = \underline{\underline{-0.94V}}$

$v_{id} = V_S + V_{GS1} = -0.94 + 0.97$

$= \underline{\underline{0.03V}}$

$V_{D2} - V_{D1} = 5K(i_{D1} - i_{D2})$

$= 5K(0.11 - 0.09)m$

$= \underline{\underline{0.1V}}$

thus

$\frac{V_{D2} - V_{D1}}{v_{id}} = \frac{0.1}{0.03} = \underline{\underline{3.33}}$

When $i_{D1} = 0.09mA$ and $i_{D2} = 0.11mA$ is the reverse condition from the case we just studied, thus $v_{id} = \underline{\underline{-0.03V}}$

7.6

We know that there is a linear relationship between V_{ov} & v_{id} since:

$$V_{ov} = \frac{v_{id}/2}{\sqrt{0.1}}$$

Then from the data in table 7.3 we can tell that for $v_{imax} = 150mV$

$$V_{ov} = 0.2 \times \frac{150}{126} = \underline{\underline{0.238V}}$$

For W/L: $\frac{W}{L} = \frac{1}{(V_{ov})^2} \cdot \frac{I}{K}$

where I and K are constant thus, for W/L:

$$\left(\frac{W}{L}\right)_2 = \frac{50}{\left(\frac{150}{126}\right)^2} = 35.3$$

For g_m: $g_m = \frac{I}{V_{ov}}$ where I is constant

$$\longrightarrow g_{m2} = \frac{g_{m1}}{\left(\frac{150}{126}\right)} = \frac{2}{\frac{150}{126}} = \underline{\underline{1.68}}$$

7.7

$$\left(\frac{v_{idmax}/2}{V_{ov}}\right)^2 = K$$

$$\Rightarrow \underline{\underline{2V_{ov}\sqrt{K} = v_{idmax}}}$$

Q.E.D

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{ov}}\right)\frac{v_{id}}{2}\sqrt{1-K}$$

CONT.

$$i_{D1} = \frac{I}{2} \pm \frac{I}{V_{ov}} \cdot \frac{2V_{ov}\sqrt{K}}{2} \cdot \sqrt{1-K}$$

$$\rightarrow i_{D1} = \frac{I}{2} \pm I\sqrt{K(1-K)}$$

thus $\Delta I = 2I\sqrt{K(1-K)}$

Q.E.D.

For $K = 0.01$

$$\Delta I = 2I\sqrt{0.01(1-0.01)} = 0.198 \times I$$

$$v_{id\,max} = 2V_{ov}\sqrt{0.01} = 0.2\,V_{ov}$$

For $K = 0.1$

$$\Delta I = 2I\sqrt{0.1(1-0.1)} = 0.6I$$

$$v_{id\,max} = 2V_{ov}\sqrt{0.2} = 0.894\,V_{ov}$$

7.8

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2$$

$$\frac{200}{2} = \frac{1}{2} \times 90 \times \frac{100}{1.6}(V_{GS}-0.8)^2$$

$$\Rightarrow V_{GS} = 1.19V$$

$$g_m = \frac{2I_D}{V_{GS}-V_t} = \frac{2 \times 100}{(1.19-1)} = 1.06\,\frac{mA}{V}$$

$$v_{id}\Big|_{\substack{\text{full current}\\ \text{switching}}} = \sqrt{2}(V_{GS}-V_t) = 0.27V$$

To double this value, $V_{GS}-V_t$ must be doubled which means that I_D should be quadrupled. i.e. I changed to: $800\mu A$

7.9

$$g_m = \frac{2I_D}{V_{ov}} \rightarrow 1m = \frac{I}{0.2}$$

$$\rightarrow I = 0.2mA$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2$$

$$100 = \frac{1}{2} \times 90 \times \frac{W}{L} \times (0.2)^2$$

$$\Rightarrow \frac{W}{L} = 55.6$$

7.10

$$i_D = \frac{1}{2}k_n\frac{W}{L}(V_{GS}-V_t)^2$$

$$50 = \frac{1}{2} \times 400(V_{GS}-1)^2$$

$$\Rightarrow V_{GS} = 1.5V$$

For $V_{G1} = V_{G2} = 0$, $V_S = -1.5V$

For $V_{G1} = V_{G2} = 2V$, $V_S = +0.5V$

The drain currents are equal in both cases.

For $V_{G2} = 0$:

To reduce i_{D2} by 10%,

$$i_{D2} = 0.9 \times 50 = 45\mu A$$

$$i_{D1} = 55\mu A$$

$$V_{GS2} = \sqrt{\frac{2\,i_{D2}}{400}} + 1 = 1.47V$$

$$V_{GS1} = \sqrt{\frac{2 \times 55}{400}} + 1 = 1.52V$$

Thus, $V_{G1} = V_{GS1} - V_{GS2} = 0.05V$

To increase i_{D2} by 10%

$$i_{D2} = 55\mu A$$

$$i_{D1} = 45\mu A$$

CONT.

$V_{GS2} = 1.52V$
$V_{GS1} = 1.47V$
$\Longrightarrow V_{G1} = \underline{\underline{-0.05V}}$

i_{D2}/i_{D1}	i_{D2} (µA)	i_{D1} (µA)	V_{GS2} (V)	V_{GS1} (V)	$V_{G} - V_{G1}$ (V)
1	50	50	1.5	1.5	0
0.5	33.3	66.7	1.408	1.577	-0.17
0.9	47.4	52.6	1.487	1.513	-0.026
0.99	47.75	50.25	1.4886	1.5012	-0.013

For $i_{D1}/i_{D2} = 20 \Rightarrow i_{D2} = 4.76\mu A$
$i_{D1} = 95.24\mu A$
$V_{GS2} = 1.154V, \; V_{GS1} = 1.690$
Thus $V_{G1} - V_{G2} = \underline{\underline{0.536V}}$

7.11

$$V_{ov} = \sqrt{I / k_n' \frac{W}{L}} = \sqrt{\frac{0.5}{0.25 \times 50}} = \underline{\underline{0.2V}}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.5mA}{0.2V} = \underline{\underline{2.5\frac{mA}{V}}}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{(0.5m/2)} = \underline{\underline{40K\Omega}}$$

$$A_d = g_m \times (R_D \parallel r_o) = 2.5\frac{mA}{V}(4K\Omega \parallel 40K\Omega)$$

$$= \underline{\underline{9.09}}\ V/V$$

7.12

From Equ. (7.23)

$$\left(\frac{V_{id}/2}{V_{ov}}\right)^2 = 0.1 \rightarrow \left(\frac{0.2/2}{V_{ov}}\right)^2 = 0.1$$

$$\rightarrow V_{ov} = \sqrt{0.1} = \underline{\underline{0.316V}}$$

$$g_m = \frac{I}{V_{ov}} \rightarrow 3\frac{mA}{V} = \frac{I}{0.316}$$

$$\rightarrow I = \underline{0.95mA}$$

also: $V_{ov} = \sqrt{\dfrac{I}{k_n' W/L}}$

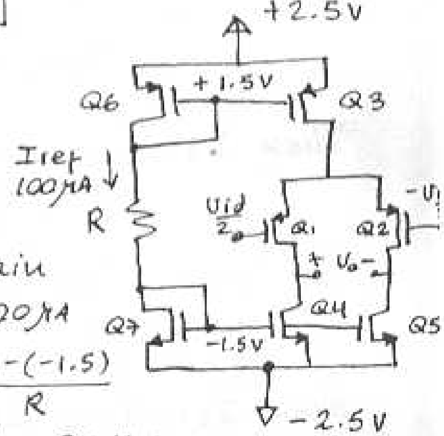
$$\Rightarrow (0.316)^2 = \frac{0.95mA}{0.1\frac{mA}{V^2} \times \left(\frac{W}{L}\right)}$$

$$\rightarrow \frac{W}{L} = \underline{\underline{95}}$$

If $R_D = 5K\Omega \Rightarrow$
$A_d = g_m R_D = 3\frac{mA}{V} \times 5K\Omega = \underline{\underline{15\frac{V}{V}}}$

if $V_{id} = 0.2 \Rightarrow V_{od} = V_{id} \times A_d$
$= 0.2 \times 15 = \underline{\underline{3V}}$

7.13



To obtain
$I_{ref} = 100\mu A$
$100\mu = \dfrac{1.5 - (-1.5)}{R}$
$\Longrightarrow R = \underline{\underline{30K\Omega}}$

$V_{GS\,7,4,5} = -1.5 + 2.5 = \underline{1V}$
$V_{ov\,7,4,5} = V_{GS} - V_{tn} = 1 - 0.7 = 0.3V$

$V_{GS\,6,3} = 1.5 - 2.5 = \underline{-1V}$
$V_{ov\,6,3} = -1 - (-0.7) = -0.3V$

The differential half circuit is an active-loaded common-source amplifier

CONT

thus, for Q_1, Q_4:

$$v_{o+} = \frac{v_{id}}{2} \times g_{m_1} (r_{o1} \| r_{o4})$$

For Q_2, Q_5:

$$v_{o-} = -\frac{v_{id}}{2} \times g_{m_2} (r_{o2} \| r_{o5})$$

Since $r_{o1} = r_{o2} = r_{o4} = r_{o5} \equiv r_o$

$$v_{o+} - v_{o-} = v_{id} \times g_{m_{1,2}} \times \frac{r_o}{2}$$

$$\rightarrow \frac{v_{o+} - v_{o-}}{v_{id}} = A_d = g_{m_{1,2}} \times \frac{r_o}{2}$$

$$= \frac{g_{m_{1,2}}}{2} \times \frac{V_{An}}{I_{D_{1,2}}}$$

$$= \frac{1}{2} \times \frac{2 I_{D_{1,2}}}{V_{ov}} \times \frac{V_{An}}{I_{D_{1,2}}} = \frac{V_{An}}{V_{ov_{1,2}}}$$

thus:

$$80 = \frac{20}{|V_{ov_{1,2}}|} \rightarrow V_{ov_{1,2}} = -0.25$$

PMOS.

Then $V_{GS_{1,2}} = -0.25 - 0.7$
$= \underline{\underline{-0.95\,V}}$

We have: $I_{D7} = I_{D6} = \underline{\underline{100\,\mu A}}$

If we choose:

$I_{D3} = I_{D6} = 100\,\mu A$ //

then $I_{D1} = I_{D2} = I_{D4} = I_{D5} = \underline{\underline{50\,\mu A}}$

To obtain W/L ratios:

$$I_D = \frac{1}{2} \mu C_{ox} (W/L) V_{ov}^2$$

$$\Longrightarrow \frac{W}{L} = \frac{2 I_D}{\mu C_{ox} V_{ov}^2}$$

where:

$\mu_n C_{ox} = 90\,\mu A/V^2$

$\mu_p C_{ox} = 30\,\mu A/V^2$

For Q_7:

$$\left(\frac{W}{L}\right)_7 = \frac{2 \times 100\mu}{90\mu \times (0.3)^2} = \underline{\underline{24.7}}$$

For Q_4 and Q_5:

$$\left(\frac{W}{L}\right)_{4,5} = \frac{2 \times (100\mu/2)}{90\mu \times (0.3)^2} = \underline{\underline{12.3}}$$

For Q_1 and Q_2:

$$\left(\frac{W}{L}\right)_{1,2} = \frac{2 \times (100\mu/2)}{30\mu \times (0.25)^2} = \underline{\underline{53.3}}$$

For Q_6 and Q_3:

$$\left(\frac{W}{L}\right)_{6,3} = \frac{2 \times 100\mu}{30\mu \times (0.3)^2} = \underline{\underline{74.1}}$$

In summary, the results are:

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	
$\mu_n C_{ox}$	30	30	30	90	90	30	90	$\mu A/V^2$
I_D	50	50	100	50	50	100	100	μA
V_{ov}	-0.25	-0.25	-0.3	0.3	0.3	-0.3	0.3	V
W/L	53.3	53.3	74.1	12.3	12.3	74.1	24.7	
V_{GS}	-0.95	-0.95	-1	1	1	-1	1	

7.14

(a) $I_{D1} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS_1} - V_t)^2$

$I_{D2} = \frac{1}{2} k_n' \left(2 \times \frac{W}{L}\right) (V_{GS_2} - V_t)^2$

Since $V_{GS} - V_t$ is equal for both transistors:

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1}{2}$$

but $I = I_{D1} + I_{D2}$

$\Longrightarrow 2 I_{D1} = I - I_{D2}$

$I_{D1} = \underline{\underline{I/3}}$

$I_{D2} = \underline{\underline{2I/3}}$

(b) $V_{ov} = V_{GS} - V_t$

CONT.

$V_{ov_1} = V_{ov_2} = V_{ov}$

For Q_1: $\frac{I}{3} = \frac{1}{2} k_n'\left(\frac{W}{L}\right) V_{ov}^2$

$\Rightarrow V_{ov} = \sqrt{\frac{2}{3}\frac{I}{k_n' W/L}}$

(c) $g_m = \frac{2I_D}{V_{ov}} \rightarrow g_{m1} = \frac{2I}{3V_{ov}}$

$g_{m2} = \frac{4}{3}\frac{I}{V_{ov}}$

$v_{o1} = -g_{m1} \times \frac{v_{id}}{2} \cdot R_D$

$= -\frac{2}{3}\frac{I}{V_{ov}} \cdot R_D \cdot v_{id}$

$v_{o2} = +g_{m2} \times \frac{v_{id}}{2} \cdot R_D$

$= \frac{4}{3}\frac{I}{V_{ov}} \cdot R_D \cdot v_{id}$

$\Rightarrow \frac{v_{o2} - v_{o1}}{v_{id}} = \left(\frac{4}{3} + \frac{2}{3}\right)\frac{I}{V_{ov}} \cdot R_D$

$= 2 \times \frac{I}{V_{ov}} \cdot R_D$

7.15

$V_{ov} = \sqrt{\frac{I}{k_n' W/L}} = \sqrt{\frac{0.2}{3}} = 0.26\text{ V}$

$g_m = \frac{I}{V_{ov}} = \frac{0.2\text{ mA}}{0.26\text{ V}} = 0.77\frac{\text{mA}}{\text{V}}$

(a) Single-ended output:

From Eqn. (7.42)

$|A_d| = \frac{1}{2} g_m \times R_D = \frac{0.77 \times 10}{2}$

$= 3.85\text{ V/V}$

From Eqn. (7.41)

$|A_{cm}| = \frac{R_D}{2R_{SS}} = \frac{10}{2 \times 100} = 0.05\text{ V/V}$

$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{3.85}{0.05} = 77$

i.e 37.7 dB

(b) Differential output, and 1% mismatch in R_D's:

Eqn (7.52) $|A_d| = g_m R_D$

$= 0.77 \times 10 = 7.7\text{ V/V}$

Eqn. (7.51) $|A_{cm}| = \frac{R_D}{2R_{SS}} \times \left(\frac{\Delta R_D}{R_D}\right)$

$= \frac{10}{2 \times 100} \times 0.01 = 0.5\text{ mV/V}$

$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{7.7}{0.5 \times 10^{-3}} = 15{,}400$

i.e 83.7 dB

7.16

$V_{ov} = -\sqrt{\frac{I}{k_p' W/L}} = -\sqrt{\frac{0.7\text{ mA}}{3.5\frac{\text{mA}}{\text{V}^2}}}$

$= -0.45\text{ V}$

$g_m = \frac{I}{|V_{ov}|} = \frac{0.7\text{ mA}}{0.45\text{ V}} = 1.56\frac{\text{mA}}{\text{V}}$

$|A_d| = g_m R_D = 1.56 \times 2 = 3.12\text{ V/V}$

$|A_{cm}| = \frac{R_D}{2R_{SS}} \cdot \left(\frac{\Delta R_D}{R_D}\right) = \frac{2}{2 \times 30} \times 0.02$

$= 6.7 \times 10^{-4}$

$CMRR = \frac{3.12}{6.7 \times 10^{-4}} = 4680 \rightarrow 73.4\text{ dB}$

7.17

(a) $I_{D1} = I_{D2} = \frac{1mA}{2} = 0.5mA$

$I_D = \frac{1}{2} K_n' \frac{W}{L} \cdot V_{ov}^2$

$\Rightarrow 0.5m = \frac{1}{2} \times 2.5m \times V_{ov}^2$

$\longrightarrow V_{ov} = 0.632V$

$V_{ov} = V_{GS} - V_t = V_{GS} - 0.7$

$\longrightarrow V_{GS} = 0.632 + 0.7 = 1.332V$

To obtain 1mA over $R_{SS} = 1K\Omega$

$V_S = 1m \times 1K = 1V.$

$\rightarrow V_{CM} = V_S + V_{GS} = 1 + 1.332 = \underline{2.332V}$

(b) $g_m = \frac{I}{V_{ov}} = \frac{1mA}{0.632V} = 1.6 \frac{mA}{V}$

Eqn. (7.45): $A_d = g_m \cdot R_D$

for $A_d = 8V/V$ $\quad R_D = \frac{8}{1.6m} = \underline{\underline{5K\Omega}}$

(c) At the drains:

$V_{D1} = V_{D2} = 5V - \frac{1mA}{2} \times 5K\Omega = \underline{\underline{+2.5V}}$

(d) Eqn. (7.39):

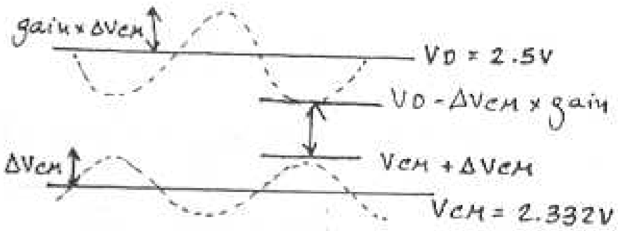
$\frac{V_{o1}}{V_{icm}} = \frac{-R_D}{\frac{1}{g_m} + 2R_{SS}}$

$\Rightarrow |A_{cm}| = \left| \frac{\Delta V_{o1}}{\Delta V_{cm}} \right| = \frac{5K}{\frac{1}{1.6m} + 2 \times 1K}$

$= \underline{\underline{1.9}} V/V$

(e) On the edge of the triode region:

$V_G - V_D = V_t$



If: $V_G - V_D = V_t$

$\Rightarrow V_{CM} + \Delta V_{CM} - V_D + \Delta V_{CM} |A_{cm}| = V_t.$

$\rightarrow 2.332 + \Delta V_{CM} - 2.5 + \Delta V_{CM} \cdot 1.9 = 0.7$

$2.9 \Delta V_{CM} = 0.868$

$\Delta V_{CM} = \underline{\underline{0.3V}}$

7.18

(a) $R_{D1} = R_D + \Delta R/2$

$R_{D2} = R_D - \Delta R/2$

$g_{m1} = g_m + \Delta g_m/2$

$g_{m2} = g_m - \Delta g_m/2$

The following Eqns are still valid:

Eqn. (7.62) $i_{d1} = \frac{g_{m1} v_{icm}}{2 g_m R_{SS}}$

Eqn. (7.63) $i_{d2} = \frac{g_{m2} \cdot v_{icm}}{2 g_m \cdot R_{SS}}$

From which:

$i_{d1} - i_{d2} = (g_{m1} - g_{m2}) \frac{v_{icm}}{2 g_m R_{SS}}$

$= \Delta g_m \frac{v_{icm}}{2 g_m R_{SS}}$ (1)

$i_{d1} + i_{d2} = (g_{m1} + g_{m2}) \frac{v_{icm}}{2 g_m R_{SS}}$

Assuming that Δg_m is small, then

$g_{m1} + g_{m2} \simeq 2 g_m.$

$\rightarrow i_{d1} + i_{d2} = \frac{2 g_m}{2 g_m R_{SS}} \cdot v_{icm} \rightarrow$

CONT.

$$i_{d1} + i_{d2} = \frac{v_{icm}}{R_{SS}} \quad (2)$$

The differential output is:

$$v_{o2} - v_{o1} = -i_{d2}.R_{D2} + i_{d1}.R_{D1}$$
$$= -i_{d2}\left(R_D - \frac{\Delta R_D}{2}\right) + i_{d1}\left(R_D + \frac{\Delta R_D}{2}\right)$$
$$= R_D.(i_{d1} - i_{d2}) + \frac{\Delta R_D}{2}(i_{d1} + i_{d2})$$

Substituting (1) & (2):

$$= \frac{R_D . \Delta g_m}{2\, g_m R_{SS}} . v_{icm} + \frac{\Delta R_D}{2} \frac{v_{icm}}{R_{SS}}$$

From which the common-mode gain is:

$$A_{cm} = \frac{v_{o2} - v_{o1}}{v_{icm}} = \frac{R_D}{2R_{SS}} . \frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{2R_{SS}}$$
$$\simeq \frac{R_D}{2R_{SS}}\left[\frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D}\right]$$

Q.E.D.

(b) For $R_D = 5 K\Omega$,
$R_{SS} = 25 K\Omega$

$\rightarrow A_{cm} = 0.002\, V/V$

For identical R_D's, this common-mode gain is caused by a mismatch in g_m.

Thus, from Eqn. (7.64)

$$A_{cm} = \left(\frac{R_D}{2R_{SS}}\right) . \left(\frac{\Delta g_m}{g_m}\right)$$
$$0.002 = \frac{5}{2 \times 25} . \frac{\Delta g_m}{g_m}.$$
$$\rightarrow \frac{\Delta g_m}{g_m} = 0.02$$

To reduce A_{cm} to zero, create a mismatch in R_D such as:

$$0 = \frac{R_D}{2R_{SS}}\left(\frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D}\right)$$
$$\Longrightarrow \frac{\Delta R_D}{R_D} = -\frac{\Delta g_m}{g_m} = -0.02$$

i.e $-\underline{\underline{2\%}}$ of R_D

7.19

$$g_m = K_n'\left(\frac{W}{L}\right) . (V_{GS} - V_t)$$

Recalling from calculus that for a function $u = f(x,y)$ the total derivative is:

$$du = \frac{\partial u}{\partial x} . dx + \frac{\partial u}{\partial y} . dy$$

Thus,

$$\Delta g_m = \frac{\partial g_m}{\partial (W/L)} \Delta(W/L) + \frac{\partial g_m}{\partial V_t} . \Delta V_t$$
$$= K_n'(V_{GS} - V_t)(\Delta W/L) - K_n'(W/L) . \Delta V_t$$

thus,

$$\frac{\Delta g_m}{g_m} = \frac{K_n'(V_{GS} - V_t)\Delta(W/L)}{K_n'(W/L)(V_{GS} - V_t)} - \frac{K_n'(W/L) . \Delta V_t}{K_n'(W/L)(V_{GS} - V_t)}$$
$$\frac{\Delta g_m}{g_m} = \frac{\Delta(W/L)}{(W/L)} + \frac{-\Delta V_t}{(V_{GS} - V_t)}$$

but: $V_{GS} - V_t = V_{OV}$

$$\Rightarrow \frac{\Delta g_m}{g_m} = \frac{\Delta(W/L)}{(W/L)} + \frac{-\Delta V_t}{V_{OV}}.$$

Q.E.D

CONT.

If $\frac{\Delta(W/L)}{W/L} = \pm 0.01$; $\Delta V_t = \pm 5mV$

$V_{ov} = 0.25V$.

then, the worst-case fractional mismatch in g_m is:

$$\frac{\Delta g_m}{g_m} = \left|\frac{\Delta W/L}{W/L}\right| + \left|\frac{\Delta V_t}{V_{ov}}\right|$$

$$= 0.01 + \frac{0.005}{0.25} = \underline{\underline{0.03}}$$

If: $R_D = 5K\Omega$, $R_{SS} = 25K\Omega$

$I = 1mA$

From Eqn. (7.64)

$$A_{cm} = \frac{R_D}{2R_{SS}} \cdot \frac{\Delta g_m}{g_m}$$

$$A_{cm} = \frac{5}{2\times 25} \times 0.03 = \underline{\underline{0.003}}\ V/V$$

From Eqn. (7.66)

$$CMRR = \frac{2 g_m R_{SS}}{\left(\frac{\Delta g_m}{g_m}\right)}$$

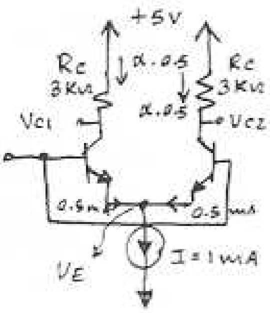
where: $g_m = I / V_{ov}$

$= 1mA / 0.25V = 4 \frac{mA}{V}$

$$CMRR = \frac{2\times 4\times 25}{0.03} = 6,666.7$$

$\rightarrow$ i.e $\underline{\underline{76.5dB}}$

7.20



$V_{BE} = 0.7$ @ $i_c = 1mA$

$\rightarrow$ at $i_c = 0.5mA$

$V_{BE} = 0.7 + 25m \ln\left(\frac{0.5}{1}\right)$

$= 0.683V$

Thus,

$V_E = V_{CM} - V_{BE}$

$= -2 - 0.683 = \underline{\underline{-2.683V}}$

$$i_{C1} = i_{C2} = \alpha \times 0.5 = \frac{100}{101} \times 0.5$$

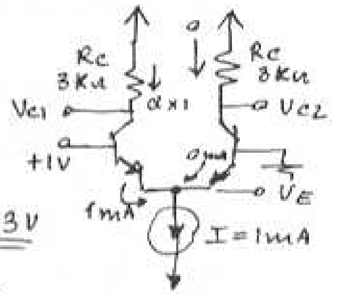
$= 0.495mA$

$V_{C1} = V_{C2} = V_{CC} - i_C R_C$

$= 5 - 0.495 \times 3$

$= +\underline{\underline{3.515V}}$

7.21



$i_{C1} = \alpha I$

$= 1mA$

$V_{BE1} = 0.7V$

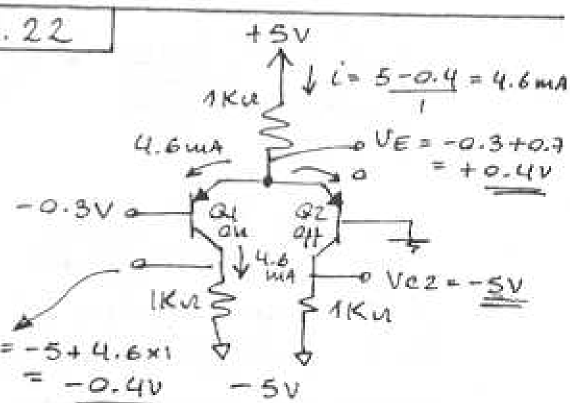
$V_E = 1 - 0.7 = +\underline{\underline{0.3V}}$

$V_{C1} = V_{CC} - i_{C1} R_C$

$\simeq 5 - 1\times 3 = \underline{\underline{2V}}$

$V_{C2} = V_{CC} - 0\times R_C = \underline{\underline{5V}}$

7.22



$V_{C1} = -5 + 4.6\times 1$

$= \underline{\underline{-0.4V}}$

7.23

$$i_{E1} = \frac{I}{1 + e^{(V_{B2} - V_{B1})/V_T}} = 0.8I$$

$\rightarrow V_{B2} - V_{B1} = V_T \ln 0.25$

$= -34.6mV$

Thus, $V_{B1} - V_{B2} = \underline{\underline{34.6mV}}$

7.24

(a) $v_{CMmax} = v_{C1,2} = V_{CC} - \frac{I}{2} \cdot R_C$

(b) If the current is steered to Q_1, then
$v_{C1} = V_{CC} - IR_C$, a change of: $-\frac{I}{2} R_C$

$v_{C2} = V_{CC}$, a change of $+\frac{I}{2} R_C$

(c) $v_{CMmax} = 3 = 5 - \frac{I}{2} R_C$
$\Rightarrow IR_C = 4V$

(d) $\frac{I/2}{\beta+1} \leq 2\mu A$
$\Rightarrow I \leq 4(\beta+1)\mu A$
Thus, $I = 4 \times 101 \mu A = 0.404 \mu A$
Select $I = 0.4mA$

$$R_C = \frac{4V}{I} = \frac{4V}{0.4mA} = 10K\Omega$$

7.25

$$i_{E1} = \frac{I}{1+e^{-v_d/V_T}}, \quad v_d = v_{B1} - v_{B2}$$

$$\frac{\Delta i_{E1}}{I} = \frac{i_{E1} - I/2}{I} = \frac{i_{E1}}{I} - 0.5$$

Define normalized Gain
$$G_n = \frac{\Delta i_{E1}/I}{v_d}$$

v_d (mV)	5	10	20	30	40
G_n	9.97	9.87	9.50	8.95	8.30

Observe that the gain stays relatively constant up to v_d neerly 20 mV. Then it decreases significantly with the increase in signal level. Whenever gain depends on signal level, nonlinear distortion occurs.

7.26

With:
$V_{B1} - V_{B2} = 10mV$
$$i_{E1} = \frac{I}{1+e^{-10/25}} = 0.598 I$$
Since $i_{E1} + i_{E2} = I$
$i_{E2} = 0.402 I$
For a collector resistance R_C
$$\begin{aligned} v_o = v_{C1} - v_{C2} &= (V_{CC} - i_{C1} R_C) - (V_{CC} - i_{C2} R_C) \\ &= -(i_{C2} - i_{C1}) R_C \\ &= -\alpha (i_{E2} - i_{E1}) R_C \\ &\simeq -0.196 I R_C \end{aligned}$$
Thus, for
$V_o = 1V$; $0.196 I R_C = 1$
$I R_C = 5.102$
Now $I = 2mA$, thus
$R_C = 2.5K\Omega$

DC (bias) voltage at each collector
$= V_{CC} - \frac{I}{2} R_C = 10 - 1 \times 2.5$
$= 7.5V$
For a $-1V$ output swing, the minimum voltage at each collector is:

CONT.

$7.5 - 0.5 = 7.0V$

Thus, $V_{Icm}|_{max} = \underline{\underline{7V}}$

$\boxed{7.27}$

(a) $i_{E1} = \dfrac{I}{1 + e^{(V_{B2}-V_{B1})/V_T}}$

$i_{E2} = \dfrac{I}{1 + e^{(V_{B1}-V_{B2})/V_T}}$

For $v_{B1} - v_{B2} = 5mV$, $V_T = 25mV$

$i_{E1} = 0.550I$, $i_{E2} = 0.450I$ } Note that $i_{E1}+i_{E2} = I$ as should be the case.

$\alpha \simeq 1$, thus $i_{C1} = 0.55I$, $i_{C2} = 0.45I$

$v_O = v_{C2} - v_{C1} \Rightarrow$

$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$

$= (i_{C1} - i_{C2})R_C$

$= 0.10\, IR_C$

Voltage gain $A_v = \dfrac{v_O}{v_{B1}-v_{B2}} = \dfrac{0.1IR_C}{0.005}$

$= \underline{20\,IR_C}\ V/V$

(b) Bias voltage at each collector is: $V_{CC} - \dfrac{I}{2}R_C$.

For an output differential voltage of $0.1R_C$, each collector should be allowed to fall by $(0.05IR_C)$ below its bias value, thus

$v_{Cmin} = V_{CC} - 0.5IR_C - 0.05IR_C$

$= V_{CC} - 0.55IR_C$

Thus for operation in saturation with

$v_{CB} = 0$ (minimum)

$v_{Icm}|_{max} = v_{Cmin} = V_{CC} - 0.55IR_C$

$= V_{CC} - 0.55\dfrac{A_v}{20}$

$= \underline{V_{CC} - 0.0275A_v}$

Obviously for a given V_{CC}, increasing A_v reduces the maximum allowable v_{Icm}

A_v (V/V)	100	200	300	400
$V_{ICM\,max}$ (V)	$V_{CC}-2.75$	$V_{CC}-5.5$	$V_{CC}-8.25$	$V_{CC}-11$
IR_C (V)	5	10	15	20
R_C (kΩ)	5	10	15	20

Example: For $V_{CC} = 10V$, a gain of 200 can be achieved using $R_C = 10k\Omega$. The corresponding maximum input common-mode voltage is 4.5V. If a gain of 300 is required, it can be achieved by increasing R_C to $15k\Omega$, however the maximum permitted input common-mode voltage then becomes 1.75V only.

$\boxed{7.28}$

$i_{C1} = \alpha i_{E1} \simeq i_{E1} = I/(1+e^{-v_d/V_T})$

$i_{C2} = \alpha i_{E2} \simeq i_{E2} = I/(1+e^{v_d/V_T})$

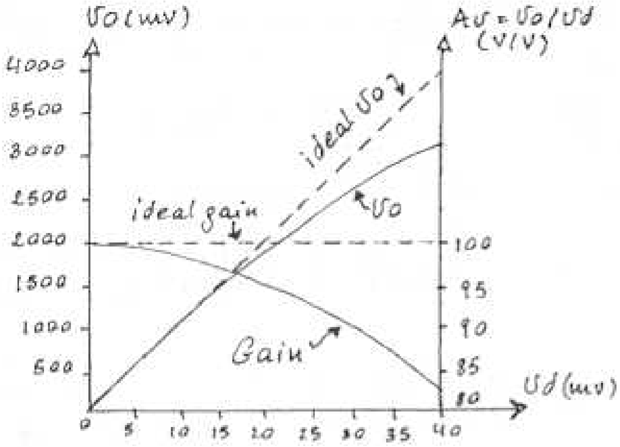
$v_O = v_{C2} - v_{C1} = (V_{CC} - i_{C2}R_C) \ldots$
$- (V_{CC} - i_{C1}R_C)$

$= IR_C\left(\dfrac{1}{1+e^{-v_d/V_T}} - \dfrac{1}{1+e^{v_d/V_T}}\right)$

For $IR_C = 5V$:

v_d (mV)	5	10	15	20	25	30	35	40
v_O (mV)	498	987	1457	1900	2311	2685	3022	3320
$\frac{v_O}{v_d}\left(\frac{V}{V}\right)$	99.7	98.7	97.1	95.0	92.4	89.5	86.3	83.0

CONT.



7.29

$I = 6mA$

The current will divide between the two transistors in proportion to their emitter areas. Thus with no input,

$I_{E1} = 1.5 I_{E2}$

$I_{E1} + I_{E2} = 2.5 I_{E2} = 6mA$

$I_{E2} = 2.4mA$

$I_{E1} = 3.6mA$

For $\alpha \simeq 1$

$I_{C1} = \underline{\underline{3.6mA}}$

$I_{C2} = \underline{\underline{2.4mA}}$

To equalize the collector currents we apply a difference signal $v_d = v_{B2} - v_{B1}$ whose value can be determined as follows:

$i_{E1} = I_{SE1}\, e^{((v_{B1} - v_E)/V_T)}$

$i_{E2} = I_{SE2}\, e^{((v_{B2} - v_E)/V_T)}$

where $I_{SE1}/I_{SE2} = 1.5$

Now, $i_{E1} = i_{E2}$ when

$1 = 1.5\, e^{(v_{B1} - v_{B2})/V_T}$

$v_d = v_{B2} - v_{B1} = V_T \ln 1.5 = \underline{\underline{10.1\,mV}}$

7.30

Refer to Fig. P7.30

(a) For v_I sufficiently low so that Q_1 is cut off:

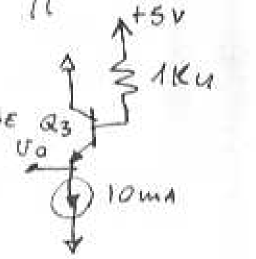
$V_{BE}|_{Q3} = 0.7 + V_T \ln \frac{10}{1}$

$= 0.76V$

$V_{OH} = v_O = 5 - \frac{10}{101} \times 1 - V_{BE}$

$= 5 - \frac{10}{101} - 0.76$

$= \underline{\underline{4.14V}}$



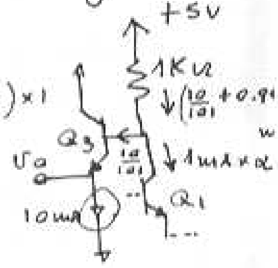
(b) For v_I sufficiently high so that Q_1 is conducting all the current I:

$V_{BE}|_{Q3} = 0.76V$

$V_{OL} = v_O = 5 - (\frac{10}{101} + 0.99) \times 1$

$- 0.76$

$= \underline{\underline{3.15V}}$



(c) $i_{E1} = I/(1 + e^{(v_{B2} - v_{B1})/V_T})$

$= I/100$

$e^{(3.64 - v_I)/V_T} = 99$

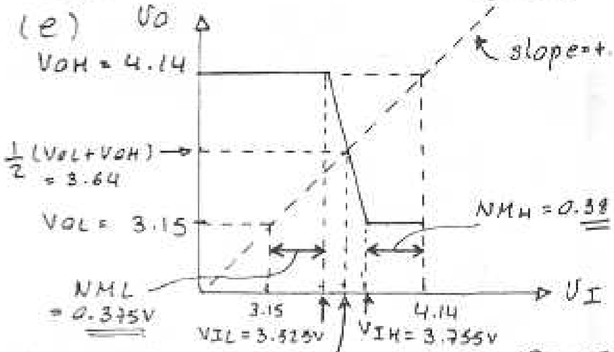
$v_I = 3.64 - V_T \ln 99 = \underline{\underline{3.525V}} = V_{IL}$

(d) $i_{E1} = 0.99I = \dfrac{I}{1 + e^{(3.64 - V_{IH})/V_T}}$

$1 + e^{(3.64 - V_{IH})/V_T} = 1.01$

$V_{IH} = 3.64 + V_T \ln 100 = \underline{\underline{3.755V}}$

(e)



Midpoint: $(V_{IL} + V_{IH})/2 = 3.64V$

CONT

Since the mid-point of the output voltage swing is equal to 3.64V, the output voltage swing is centred on the mid-point of the input range; an ideal choice for it equalizes the noise margins.

7.31

Each device is operating at a current of 150μA = 0.15mA. Thus,

$$g_m = \frac{0.15\,mA}{25\,mV} = \underline{6\,\frac{mA}{V}}$$

$$R_{id} = 2(\beta+1)r_e = 2r_\pi$$
$$= 2 \times \frac{150}{4} = \underline{\underline{75\,K\Omega}}$$

7.32

$R_{id} \geq 10K\Omega$; $A_d = 200\ V/V$; $\beta \geq 100$; $V_{CC} = 10V$

$$R_{id} = 10^4 = 2r_\pi = 2 \times \frac{100}{g_m}$$

$\Rightarrow g_m = 20\,mA/V$

Thus each device is operating at 0.5mA and $I = \underline{\underline{1mA}}$

Voltage gain = $g_m . R_C$

$200 = 20 R_C$

$\Rightarrow R_C = \underline{\underline{10K\Omega}}$

7.33

$$\underline{5mV}$$

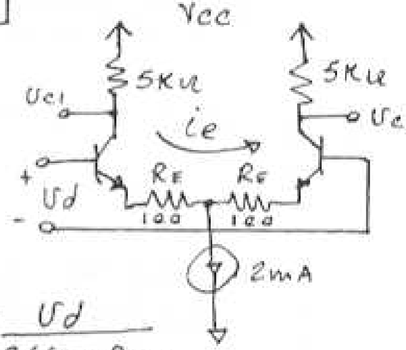
$$r_e = \frac{V_T}{I/2} = \frac{25mV}{50\mu A} = \underline{500\Omega}$$

Half-circuit gain = $\frac{\alpha R_C}{r_e} \simeq \frac{R_C}{r_e}$

$$= \frac{10K}{500} = \underline{20}\ V/V$$

At one collector we expect a signal of (+100mV) and at the other a signal of (−100mV)

7.34



$$r_e = \frac{V_T}{I_E} = \frac{25m}{1m} = 25\Omega$$

(a) $i_e = \frac{V_d}{2(r_e + R_E)}$

$$= \frac{0.1\,V}{2(25+100)\Omega} = \underline{0.4mA}$$

(b) $i_{E1} = 1 + 0.4 = \underline{\underline{1.4}}\ mA$

$i_{E2} = 1 - 0.4 = \underline{\underline{0.6mA}}$

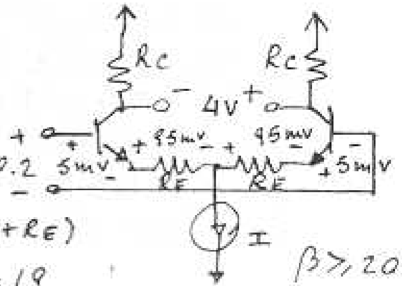
(c) $V_{C1} = -i_e R_C \simeq -0.4 \times 5 = \underline{\underline{-2V}}$

$V_{C2} = \underline{\underline{+2V}}$

(d) $V_{od} = 4V$

$$A_d = V_{od}/V_{id} = \frac{4}{0.1} = \underline{\underline{40}}\ V/V$$

7.35



$$\frac{2R_C}{2(r_e+R_E)} \simeq \frac{4}{0.2}$$

$R_C = 20(r_e + R_E)$

$$\frac{R_E}{r_e} = \frac{95}{5} = 19$$

$\beta \geq 200$

$R_{in} \geq 80K$

CONT.

$R_{in} = 2(\beta+1)(r_e + R_E)$

$= 2 \times 201 \times 20 r_e = 80 K\Omega$

$\Rightarrow r_e \simeq \frac{80000}{8000} = 10\Omega$

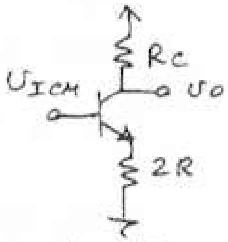
Thus each device is operating at a current of $\frac{25mV}{10\Omega} = 2.5mA$

$\Rightarrow I = \underline{\underline{5mA}}$

$R_E = 19 \times 10 = \underline{\underline{190\Omega}}$

$R_C = 20 \times 200 = \underline{\underline{4K\Omega}}$

7.36



$\underline{\underline{2M\Omega}}$

$\frac{V_o}{V_{ICM}} \simeq \frac{R_C}{2R} = \frac{20K\Omega}{2M\Omega}$

$= \underline{\underline{0.01}}$ V/V or $\underline{\underline{-40dB}}$

7.37

Refer to Fig. P7.37

$\frac{V_o}{V_i} = \frac{\alpha \times 20K\Omega}{(2r_e + 2 \times 200)\Omega}$

Where $r_e = \frac{V_T}{0.5/2} = \frac{0.05V}{0.5mA} = 100\Omega$

$\frac{V_o}{V_i} \simeq \frac{20\,000}{500} = \underline{\underline{33.3V/V}}$

$R_i = (\beta+1)(2r_e + 2 \times 200)$

$= 101 \times 2 \times 300 \simeq \underline{\underline{60K\Omega}}$

7.38

Refer to Fig P.7.38

Each transistor is operating at $I_E = 1mA$, thus

$r_e = 25\Omega$ and $r_\pi = 101 \times 25$
$= 2525\Omega$

$\frac{V_o}{V_i} = \frac{\alpha \times 7.5K\Omega}{(2r_e + 200)\Omega} \simeq \frac{7500}{250} = \underline{\underline{30}}$

$R_i = (\beta+1)(r_e + 200 + r_e) \simeq \underline{\underline{25K\Omega}}$

7.39

Refer to Fig P.7.39

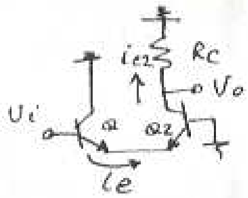
(a) As a differential amplifier the gain is given by

$\frac{V_o}{V_i} = \frac{\alpha R_c}{2r_e}$ (1)

(b) Transistor Q_1 can be considered as a common-collector stage. It is biased at $I/2$ and has a resistance r_{e2} in its emitter, thus.

$i_e = \frac{V_i}{r_{e1} + r_{e2}}$

$= \frac{V_i}{2r_e}$ (2)



Now, Q_2 is connected in the common-base configuration. It has an input signal current i_e. Thus its collector signal current (in the direction indicated) will be $i_{c2} = \alpha i_e$ (3)

The output voltage will be

$V_o = i_{c2} R_c$ (4)

Combining equations (2) (3) and (4) provides:

CONT.

$\frac{v_o}{v_i} = \frac{\alpha R_C}{2r_e}$, which is identical to the result found above in part (a)

7.40

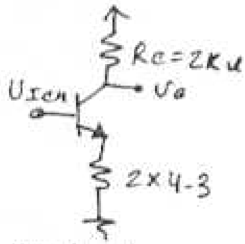
(a) $V_E = -0.7V$

Thus, $I = \frac{-0.7-(-5)}{4.3} = 1mA$ $r_e = 50\Omega$

$A_d = \frac{v_o}{v_d} = \frac{\alpha \times 2}{2r_e} \simeq \frac{2000}{2\times 50} = \underline{\underline{20}}\ V/V$

(b) $A_{cm} \simeq \frac{\alpha R_C}{8.6 + r_e}$

$= \frac{2}{8.65} = \underline{\underline{0.23}}\ V/V$



(c) CMRR

CM half circuit

$= 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log\frac{20}{0.23}$

$= \underline{38.8}\ dB$

(d) $v_d = 2\times 0.005 \sin(2\pi\times 1000t)$

$v_{ICM} = 0.1 \sin(2\pi\times 60t)$

Thus, $v_o = A_d \cdot v_d + A_{cm}\cdot v_{Icm}$

$= 20\times 0.01 \sin 2\pi\times 1000t$

$+ 0.23\times 0.1 \sin(2\pi\times 60t)$

$= \underline{0.2 \sin 2\pi\times 1000t + 0.023 \sin 2\pi 60t}$

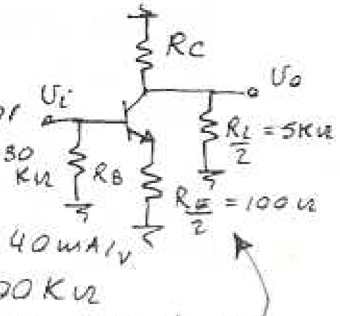
7.41

Each transistor is biased at 1mA. Thus,

$r_e = 25\Omega$, $g_m = 40mA/V$

$r_o = 100/1 = 100K\Omega$

The differential half-circuit is



$A_d = \frac{v_o}{v_i} = \frac{\alpha[R_C \| (R_L/2)]}{r_e + R_E/2}$

$\simeq \frac{10\|5}{0.025+0.100} = \underline{26.7}\ V/V$

$R_{id} = 2[R_B \| (\beta+1)(r_e + R_E/2)]$

$= 2[30 \| 101(0.025+0.100)]$

$= \underline{17.8}\ K\Omega$

The common-mode half circuit

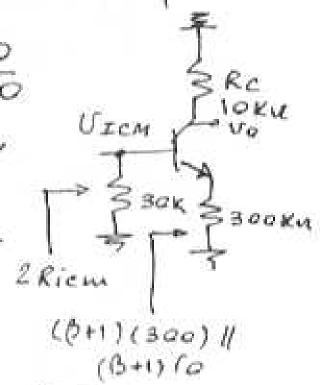
$A_{cm} = \frac{v_o}{v_{Icm}} \simeq \frac{10}{300}$

$= \frac{1}{30} = \underline{\underline{0.033}}\ V/V$

$2R_{icm} = 30K \| 7.5M\Omega$

$= 30K\Omega$

$R_{icm} = \underline{\underline{15K\Omega}}$



$\simeq 30\|10 = 7.5M\Omega$

Without the R_B resistors $R_{icm} = \underline{\underline{3.75M\Omega}}$

7.42

(a) $A_d\big|_{\text{single-ended output}} = \frac{\alpha(R_C \| r_o)}{2r_e}$

where $r_e = \frac{0.025V}{0.25mA} = 100\Omega$

$r_o = \frac{200V}{0.25mA} = 800K\Omega$

$A_d\big|_{\text{single ended}} \simeq \frac{20}{2\times 0.1} = \underline{\underline{100}}\ V/V$

(b) $A_d\big|_{\text{diff output}} = 2\times A_d\big|_{\text{single ended}}$

$= \underline{\underline{200}}\ V/V$

(c) $R_{id} = 2r_\pi = 2\times 201\times 100$

$= \underline{\underline{40.2K\Omega}}$

CONT.

(d) $A_{cm}\Big|_{\text{single-ended output}} = \dfrac{R_C}{2R}$

$= \dfrac{20}{2000} = \underline{\underline{0.1\ V/V}}$

(e) $A_{cm}\Big|_{\text{diff out}} = 0$

7.43

The output resistance of the current source $(R) = r_o = \dfrac{V_A}{0.1\,mA}$

$= \dfrac{200}{0.1} = 2M\Omega$

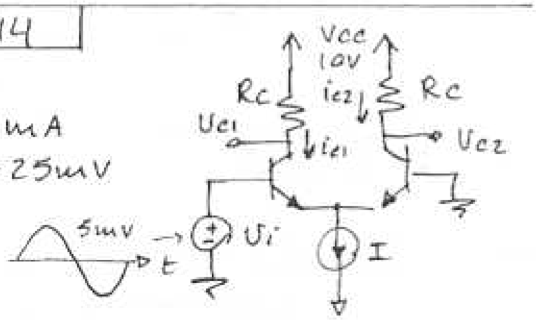
r_o for each transistor is the pair $= 200/0.05 = 4M\Omega$

Thus,

$R_{icm} = (\beta+1)\dfrac{r_o}{2} \,//\, (\beta+1)R$

$= \dfrac{51\times 4}{2} \,//\, 51\times 2 = \underline{\underline{51\ M\Omega}}$

7.44



I in mA

$V_T = 25mV$

$i_{c1} = \dfrac{I}{2} + \left(\dfrac{I/2}{V_T}\right)\left(\dfrac{5}{2}\right)\sin\omega t$

$i_{c2} = \dfrac{I}{2} - \left(\dfrac{I/2}{V_T}\right)\left(\dfrac{5}{2}\right)\sin\omega t$

$v_{c1} = V_{cc} - \dfrac{I}{2}R_C - \dfrac{I/2}{V_T}R_C\dfrac{5}{2}\sin\omega t$

$v_{c2} = V_{cc} - \dfrac{I}{2}R_C - \dfrac{I/2}{V_T}R_C\dfrac{5}{2}\sin\omega t$

$v_{c1}, v_{c2} \geq 0$

$\Rightarrow 10 - 5I - 0.5I = 0$

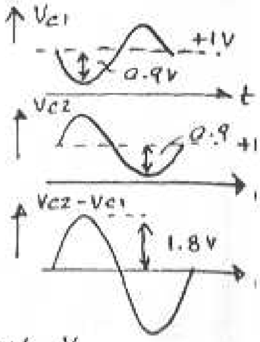
$I = \underline{\underline{1.8\,mA}}$

$V_{C1} = V_{C2} = 1V$

$A_d = \dfrac{20K\Omega}{2r_e}$, where

$r_e = \dfrac{25}{0.9} = 27.8\Omega$

Thus, $A_d = \underline{\underline{360\,V/V}}$



$v_{c2} - v_{c1} = \underline{\underline{1.8\sin\omega t}}, V$

7.45

At $I_C = 1mA$, $r_e = 25\Omega$

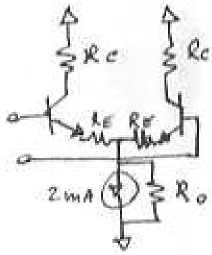
$R_{id} = (\beta+1)2r_e = 5.05K\Omega < 10$

$\Rightarrow$ need emitter resistors

In this case:

$R_{id} = (\beta+1)(2r_e + 2R_E) = 10K$

Thus, $R_E = \underline{\underline{25\Omega}}$



$A_d = 100 = \dfrac{R_C}{2(R_E+r_e)}$

$\Rightarrow R_C = \underline{\underline{10K}}$

$A_{cm} = 0.1 \geq \dfrac{R_C}{2R_o + R_E + r_e}$

$\Rightarrow R_o \geq \underline{\underline{50K\Omega}}$

For ± 2 swing $V_{C1} = V_{C2}$
$= V_{cc} - \dfrac{I}{2}R_C = 2$

$\Rightarrow V_{cc} = 2 + 10^{-3}\times 10^{4} = 12V$

Choose $\underline{\underline{V_{cc} = \pm 15V}}$ although 12V is OK.

$2R_{icm} = (\beta+1)(2R_o + R_E + r_e)$

$\Rightarrow R_{icm} = \underline{\underline{5M\Omega}}$

7.46

Taken single-endedly

$$A_{cm_s} = \frac{\alpha R_C}{2R_O}$$

Let collector resistors be R_C & $R_C + \Delta R_C$, then

$$A_{cm} = \frac{\alpha}{2R_O}(R_C + \Delta R_C - R_C)$$

$$= \frac{\alpha \Delta R_C}{2R_O}$$

Which can be written as

$$A_{cm_d} = \frac{\alpha R_C}{2R_O} \cdot \frac{\Delta R_C}{R_C} = A_{cm_s}\frac{\Delta R_C}{R_C}$$

$$CMRR = \frac{A_d}{A_{cmd}} = \frac{2 \cdot A_s}{A_{cm_s}\frac{\Delta R_C}{R_C}}$$

$$= \frac{A_s}{A_{cms}} \cdot \frac{2}{\frac{\Delta R_C}{R_C}}$$

Thus, $20 \log \frac{2}{\frac{\Delta R_C}{R_C}} = 40\,dB$

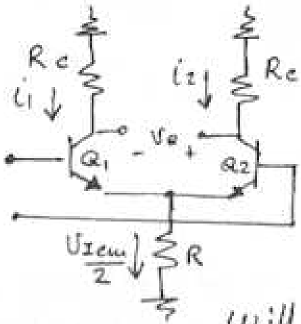
$$\rightarrow \Delta R_C / R_C = \underline{\underline{2\%}}$$

7.47

The bias current will split between the two transistors according to their area ratio. Thus the large-area device will carry twice the current of the other device.

That is, the bias currents will be $2I/3$ and $I/3$.

Now with v_{Icm} applied, the CM signal current will $\simeq v_{Icm}/R$ split between Q_1 and Q_2 in the same ratio. This is because their r_e values will be related in the same way. Thus, if Q_1 is the large device r_{e1} will be half the value of r_{e2}.



The result will be that

$$i_1 = \frac{2}{3}\frac{v_{Icm}}{R} \quad \text{and} \quad i_2 = \frac{1}{3}\frac{v_{Icm}}{R}$$

Thus the differential output voltage v_o will be

$$v_o = 0 - i_2 R_C - (-i_1 R_C) = (i_1 - i_2)R_C$$

$$= \frac{1}{3}\frac{v_{Icm}}{R} \cdot R_C$$

$$A_{cm} = \frac{1}{3}\frac{R_C}{R} = \frac{1}{3} \times \frac{12}{1000} = \underline{\underline{0.004}}\ \frac{V}{V}$$

7.48

For $I = 200\mu A$:

$$g_m = \sqrt{2K_n' W/L\, I_D} = \sqrt{2 \times 4 \times 0.1}$$

$$= 0.89\,mA/V$$

$R_D = 10K\Omega$

Thus, $A_d = g_m R_D = 10 \times 0.89 = \underline{8.9}\ V/V$

$$V_{os} = \frac{(V_{GS} - V_t)}{2} \cdot \frac{\Delta R_D}{R_D}$$

where $\frac{\Delta R_D}{R_D} = 0.02$ (worst case)

and $V_{GS} - V_t = \sqrt{\frac{2I_D}{K_n' W/L}} = \sqrt{\frac{2 \times 0.1}{4}}$

$$= \underline{0.223V}$$

CONT.

Thus, $V_{os} = \frac{1}{2} \times 0.223 \times 0.02$

$= \underline{2.23 mV}$

For $I = 400 \mu A$:

$g_m = \sqrt{2 \times 4 \times 0.2} = 1.265 mA/V$

$A_d = 12.65 V/V$

$V_{ov} = V_{GS} - V_t = 0.316 V$

$V_{os} = \frac{1}{2} \times 0.316 \times 0.02 = \underline{3.16 mV}$

Thus both Ad and Vos increase by the same ratio since both are proportional to $\sqrt{I}$

To find the required mismat[ch] ΔR_D that can correct for V_{os}

$13.11 mV = \frac{V_{ov}}{2} \cdot \frac{\Delta R_D}{R_D}$

$\Rightarrow \frac{\Delta R_D}{R_D} = \frac{2 \times 13.11 mV}{0.3 V}$

$= 0.087$ or $\underline{8.7\%}$

If ΔV_t is reduced by a facto[r] of 10 to 1mV, V_{os} reduces to:

$\sqrt{6^2 + 6^2 + 1^2} = 8.54 mV$

and $\frac{\Delta R_D}{R_D} = \frac{2 \times 8.54 mV}{0.3 V} = \underline{5.6}$

7.49

Worst cases: $\Delta V_t = 10 mV$

$\frac{\Delta R_D}{R_D} = 0.04$; $\frac{\Delta (W/L)}{(W/L)} = 0.04$

V_{os1} (due to ΔR_D) $= \frac{V_{ov}}{2} \frac{\Delta R_D}{R_D} = \frac{0.3 \times 0.04}{2}$

$= 6 mV$

V_{os2} (due to $\Delta W/L$) $= \frac{V_{ov}}{2} \frac{\Delta W/L}{W/L} = \frac{0.3 \times 0.04}{2}$

$= 6 mV$

V_{os3} (due to ΔV_t) $= \Delta V_t = \underline{10 mV}$

Since these offsets are not correlated

$V_{os} = \sqrt{V_{os1}^2 + V_{os2}^2 + V_{os3}^2}$

$V_{os} = \sqrt{6^2 + 6^2 + 10^2} = \underline{13.11 mV}$

The major contribution is due to the the threshold mismatch ΔV_t.

7.50

$V_{ov} = \sqrt{\frac{I}{k_n' W/L}} = \sqrt{\frac{100}{100 \times 20}} = 0.22[3]$

Using Eqn. (7.112) we obtain V_{os} due to $\Delta R_D / R_D$ as:

$V_{os} = \frac{V_{ov}}{2} \frac{\Delta R_D}{R_D} = \frac{0.223 \times 0.0[5]}{2}$

$= 5.57 mV$

From Eqn. (7.117), V_{os} due to $\Delta(W/L)/(W/L)$ is:

$V_{os} = \left(\frac{V_{ov}}{2}\right) \frac{\Delta W/L}{W/L} = \frac{0.223 \times 0.[05]}{2}$

$= \underline{5.57 mV}$

The offset arising from ΔV_t i[s] (Eqn. (7.120)):

$V_{os} = \Delta V_t = \underline{5 mV}$

Worst case offset is:

$5.57 + 5.57 + 5 = 16.15 mV$

Applying the root-sum-of-squares (Eq. (7.121))

$V_{os} = \sqrt{2(5.57m)^2 + 5m^2} = \underline{9.33 m[V]}$

7.51

$\Delta V_C = \Delta R_C \cdot \frac{I}{2}$

$A_d = \frac{R_C}{r_e} = \frac{R_C}{V_T/\frac{I}{2}} = \frac{I R_C}{2 V_T}$

$\Rightarrow V_{OS} = \frac{\Delta V_C}{A_d} = \frac{\Delta R_C}{R_C} \cdot V_T$

$= 0.1 \times 25 = \underline{\underline{2.5 mV}}$

7.52

Eq (7.130) $V_{OS} = V_T \cdot \frac{\Delta I_S}{I_S}$

$= 25 \times 0.1 = \underline{\underline{2.5 mV}}$

7.53

$\Delta V_C = \Delta R_C \frac{I}{2}$

$A_d = \frac{R_C}{r_e + R_E} = \frac{R_C}{\frac{2V_T}{I} + R_E} = \frac{I R_C}{2V_T + I R_E}$

$V_{OS} = \frac{\Delta V_C}{A_d} = \underline{\underline{\frac{\Delta R_C}{R_C}\left(V_T + \frac{I R_E}{2}\right)}}$

7.54

$\Delta V_C = \alpha_1 \frac{I}{2} R_C - \alpha_2 \frac{I}{2} R_C$

$= \frac{I}{2} R_C (\alpha_1 - \alpha_2)$

$= \frac{I}{2} R_C \left(\frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1}\right)$

For $\beta_1, \beta_2 \gg 1$

$\Delta V_C = \frac{I}{2} R_C \cdot \frac{\beta_1 - \beta_2}{\beta_1 \cdot \beta_2}$

$= \frac{I}{2} R_C \left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)$

$A_d = \frac{R_C}{r_e} = \frac{I R_C}{2 V_T}$

$V_{OS} = \frac{\Delta V_C}{A_d} = V_T \left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)$ Q.E.D.

For $\beta_1 = 100$ and $\beta_2 = 200$

$V_{OS} = 25\left(\frac{1}{200} - \frac{1}{100}\right)$

$= \underline{\underline{-125 \mu V}}$

7.55

Since the two transistors are matched except for their V_A value, we can express the collector currents when the input terminals are grounded as,

$I_{C1} = I_C \left(1 + \frac{V_{CE}}{V_{A1}}\right)$

$I_{C2} = I_C \left(1 + \frac{V_{CE}}{V_{A2}}\right)$

where I_C can be determined from

$I_{C1} + I_{C2} = I$

$\Rightarrow I_C = \frac{I}{2 + \frac{V_{CE}}{V_{A1}} + \frac{V_{CE}}{V_{A2}}}$

Note that for $V_{CE} \ll V_{A1}, V_{A2}$, $I_C \simeq \frac{I}{2}$. Thus, the differential gain A_d can still be written as

$A_d \simeq \frac{R_C}{r_e} = \frac{I R_C}{2 V_T}$

The offset voltage at the output can be found from

$\Delta V_C = V_{C2} - V_{C1} = (I_{C1} - I_{C2}) R_C$

CONT.

$$= I_C R_C \left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}} \right)$$

$$\simeq \frac{I}{2} R_C \left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}} \right)$$

Thus, $V_{OS} = \dfrac{\Delta V_C}{A_d}$

$$V_{OS} = V_T \left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}} \right)$$

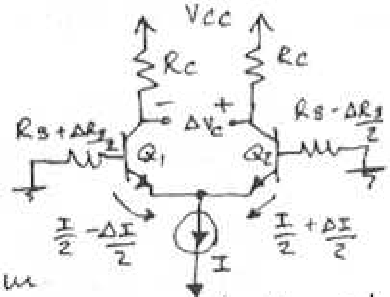
For $V_{CE} = 10V$, $V_{A1} = 100V$ and $V_{A2} = 300V$

$$V_{OS} = 25 \left(\frac{10}{100} - \frac{10}{300} \right)$$

$$= \underline{\underline{1.7mV}}$$

7.56

Equating the incremental changes in voltage from ground to emitter on both sides of the pair (and neglecting second-order terms i.e. $\Delta \times \Delta$ terms):



$$\frac{I}{2(\beta+1)} \cdot \frac{\Delta R_S}{2} - \frac{\Delta I}{2(\beta+1)} \cdot R_S - \frac{\Delta I}{2} \cdot r_e$$

$$\simeq \frac{-I}{2(\beta+1)} \frac{\Delta R_S}{2} + \frac{\Delta I}{2(\beta+1)} R_S + \frac{\Delta I}{2} r_e$$

$$\Delta I \left[r_e + \frac{R_S}{\beta+1} \right] = \frac{I}{2(\beta+1)} \cdot \Delta R_S$$

$$\Delta I = \frac{I \, \Delta R_S}{2(\beta+1)} \cdot \frac{1}{r_e + \frac{R_S}{\beta+1}}$$

$$\Delta V_C = -\Delta I \cdot R_C$$

$$= \frac{-I R_C \Delta R_S}{2(\beta+1)} \cdot \frac{1}{r_e + R_S/(\beta+1)}$$

$A_d = R_C / r_e$

Thus, $V_{OS} \equiv \Delta V_C / A_d$

$$= \frac{-I \Delta R_S}{2(\beta+1)} \cdot \frac{r_e}{r_e + \frac{R_S}{\beta+1}}$$

For $\dfrac{R_S}{\beta+1} \ll r_e$ and $\beta \gg 1$,

$$|V_{OS}| \simeq \frac{I}{2\beta} (\Delta R_S) \qquad \text{Q.E.D.}$$

7.57

Refer to Fig. P7.57.

(a) $R_{C1} = 5 \times 1.05 = 5.25 K\Omega$

$R_{C2} = 5 \times 0.95 = 4.75 K\Omega$

Perfect offset nulling will be achieved when x is such that

$R_{C1} + (x \times 1K\Omega) = R_{C2} + (1-x) \times 1K\Omega$

$\Rightarrow 5.25 + x = 4.75 + 1 - x$

$\Rightarrow x = \underline{\underline{0.25}}$

(b) $I_{C1} = 1.05 mA$

$I_{C2} = 0.95 mA$

Offset nulling is achieved when x is such that

$1.05 (x + 5) = 0.95 ((1-x) + 5)$

$x = \underline{\underline{0.225}}$

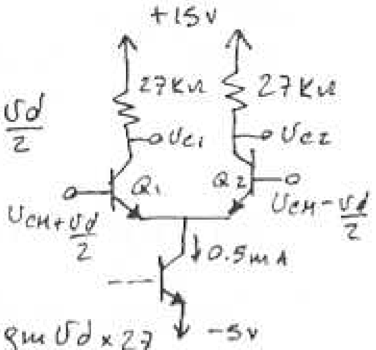
7.58

$$I_{B max} = \frac{I/2}{\beta_{min}+1} = \frac{300}{80+1} = \underline{\underline{3.7 \mu A}}$$

$$I_{B min} = \frac{I/2}{\beta_{max}+1} = \frac{300}{200+1} \simeq \underline{\underline{1.5 \mu A}}$$

$$I_{OS} = I_{B max} - I_{B min} = \underline{\underline{2.2 \mu A}}$$

7.59



$v_{C1} \geq v_{CM} + \frac{v_d}{2}$

$15 - 0.25\times 27 - g_m \frac{v_d}{2}\times 27 \geq v_{CM} + \frac{v_d}{2}$

Thus $v_{CM}\big|_{max} = 15 - 0.25\times 27 + -100\times 0.01\times 27 - 0.01$

$= \underline{+5.54\,V}$

To find v_{CM} min we observe that to keep the current-source transistor in the active region, the voltage at its collector should exceed the collector-emitter saturation voltage (0.2-0.3V). Thus, $v_{E\,min} = -4.7V$ and correspondingly $v_{C\,min} = -\underline{4V}$

7.60

$I_{E1} = \frac{2}{3}I$ and $I_{E2} = \frac{1}{3}I$

(Q_1 twice the area of Q_2)

$\Delta v_C = v_{C2} - v_{C1} \simeq \underline{\frac{1}{3}IR_C}$

Nominally,

$A_d = \frac{R_C}{r_e} = \frac{IR_C}{2V_T}$

$V_{OS} = \frac{\Delta v_C}{A_d} = \frac{2}{3}V_T = \underline{16.7\,mV}$

Thus, small-signal analysis predicts that a 16.7mV DC voltage applied as $v_{B2} - v_{B1} = 16.7$ would restore the current balance in the pair and reduce Δv_C to zero.

Using large-signal analysis:

$i_{E1} = I_{S1}\cdot e^{\frac{v_{B1}-v_E}{V_T}}$

$i_{E2} = I_{S2}\cdot e^{\frac{v_{B2}-v_E}{V_T}}$

Thus,

$\frac{i_{E1}}{i_{E2}} = \frac{I_{S1}}{I_{S2}}\cdot e^{\frac{v_{B1}-v_{B2}}{V_T}}$

To restore balance, $i_{E1} = i_{E2}$, thus

$1 = 2e^{\frac{V_{B1}-V_{B2}}{V_T}}$

$\Rightarrow V_{B1} - V_{B2} = -V_T \ln 2$

$V_{B2} - V_{B1} = \underline{17.3\,mV}$

Nominally

$I_B = \frac{I/2}{\beta+1} \simeq \frac{100}{2\times 100} = \underline{0.5\,\mu A}$

But with the imbalance,

$I_{B1} \simeq \frac{2I/3}{\beta} = \frac{2\times 100}{300} = 0.67\,\mu A$

$I_{B2} = \frac{I/3}{\beta} = \frac{100}{300} = 0.33\,\mu A$

$I_B = \frac{I_{B1}+I_{B2}}{2} = \underline{0.5\,\mu A}$

$I_{OS} = |I_{B1} - I_{B2}| = \underline{0.34\,\mu A}$

7.61

$R_C = 20K\Omega$; $A_d = 90\,V/V$

$V_{OS} = \pm 3mV$

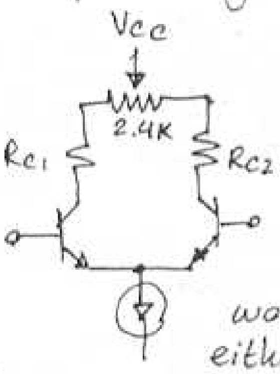
Worst case $|V_{OS}|$ is 3mV

$|V_{OS}| = V_T\left(\frac{\Delta R_C}{R_C}\right)$ $|V_{OS}| = 3mV$

$\Rightarrow \frac{3m\times 20K}{25m} = 2.4K$, $\Delta R_C = \underline{2.4K\Omega}$

CONT.

This is the maximum mismatch that occurs in R_C.
Thus, if the lowest collector resistor is adjusted from $R_{Cmin} + \Delta R$ with ΔR varying between zero and 2.4KΩ, then the offset would be eliminated!
This can be achieved with the following circuit:



When R_{C1} x R_{C2} are equal the potentiometer is tuned to the middle point. In the worst case, when either R_C is higher by 2.4KΩ, the potentiometer is adjusted to one extreme such as to increase the lowest R_C by 2.4KΩ. In all other cases when ΔR_C is distributed between R_{C1} and R_{C2} the potentiometer is adjusted as in Problem 7.57 above.

7.62

For each transistor $I_D = I/2$.
From Eqn. (7.147): $r_{o2} = r_{o4} = r_o$

$$A_d = \frac{1}{2} g_m r_o$$

but $g_m = \frac{2I_D}{V_{ov}}$ and $r_o = \frac{V_A}{I_D}$

$$\Rightarrow A_d = \frac{1}{2}\left(\frac{2I_D}{V_{ov}}\right)\frac{V_A}{I_D} = \frac{V_A}{V_{ov}}$$

$$\rightarrow 80\ V/V = 20V/V_{ov}$$

$$\rightarrow V_{ov} = 20/80 = 0.25V$$

Finally,

$$I = 2I_D = K\frac{W}{L} V_{ov}^2$$

$$= 3.2\frac{mA}{V^2}(0.25V)^2 = \underline{\underline{0.2}}$$

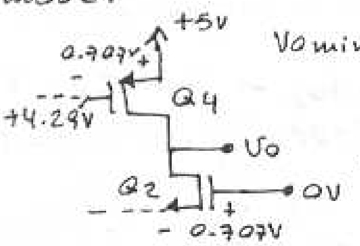
7.63

For all transistors $I_D = I/2$ and all r_o's are equal.

$$V_{ov_{100}} = \sqrt{\frac{2I_D}{K'W/L}} = \sqrt{\frac{100\mu A}{0.2mA/V^2}} = 0.7$$

$$V_{ov_{200}} = \sqrt{\frac{400\mu A}{0.2mA/V^2}} = 1.414V$$

(a) For $I = 100\mu A$:
Range of the differential mo is $-\sqrt{2}\,V_{ov} \leq V_{id} \leq \sqrt{2}\,V_o$ (as in Eqn. (7.10))
But the range of V_o is limite by the requirement of keeping the transistors in saturatio mode.



$$V_{omin} = V_{G2} - V_t = 0 - V_t = \underline{-V_t}$$

$$V_{omax} = V_{G4} + |V_t| = V_{DD} - |V_{GS}| = 5 - 0.707 = \underline{4.29}$$

$$g_m = \frac{2I_D}{V_{ov}} \rightarrow g_{m1} = g_{m2} = \frac{100\mu}{0.707} = \underline{0.1414mA/V}$$

$$r_o = \frac{V_A}{I_D} \Rightarrow r_{o2} = r_{o4} = \frac{20}{50\mu} = \underline{\underline{400K}}$$

$$R_o = r_{o2} \| r_{o4} = \frac{1}{2} \times 400K = \underline{\underline{200K}}$$

$$A_d = \frac{1}{2} g_m r_o = \frac{1}{2} \times (0.1414m)(400K$$

CONT

$A_d = \underline{\underline{28.28\ V/V}}$

(b) For $I = 400\mu A$:
Linear range of v_o

$v_{o\,min} = -V_t$ //
$v_{o\,max} = (5V - |V_{GS}|) + |V_t|$
$= \underline{\underline{5 - 1 = 4V}}$

$g_m = \dfrac{2 \times 200\mu}{1.414} = \underline{0.28\ mA/V}$

$r_o = 20/(400/2) = \underline{100K\Omega}$

$R_o = 1/2\ 100K\Omega = \underline{\underline{50}}\,K\Omega$

$A_d = \dfrac{1}{2}(0.28m \times 100K) = \underline{\underline{14}}\ V/V$

7.64

Recall from Equ. (7.155)

$CMRR = (g_m r_o)(g_m R_{SS})$

(a) For a simple current mirror

$R_{SS} = r_{o5} \Rightarrow$ (for $I_D = I/2$)

$CMRR = (g_m r_o)(g_m r_{o5})$

$= \left(\dfrac{2I_D}{V_{OV}} \cdot \dfrac{V_A}{I_D}\right) \cdot \left(\dfrac{2I_D}{V_{OV}} \cdot \dfrac{V_A}{2I_D}\right)$

$= 2 \cdot \dfrac{V_A}{V_{OV}} \cdot \dfrac{V_A}{V_{OV}}$

$= 2\left(\dfrac{V_A}{V_{OV}}\right)^2$ // Q.E.D.

(b) for the modified Wilson current source of Fig. P7.64

$R_{SS} \simeq g_{m7} \cdot r_{o7} \cdot r_{o5}$

$\Rightarrow CMRR = (g_m r_o)(g_m \cdot g_{m7} r_{o7} r_{o5})$

For $Q_{5,6,7,8}$:

$V_{OV_5} = \sqrt{\dfrac{2I}{K'W/L}}$

while for $Q_{1,2,3,4}$:

$V_{OV} = \sqrt{\dfrac{I}{K'W/L}}$

$\Rightarrow V_{OV_5} = \sqrt{2}\,V_{OV}$.

Thus, (for $I = 2I_D$)

$CMRR = \dfrac{I}{V_{OV}} \cdot \dfrac{V_A}{(I/2)} \cdot \dfrac{I}{V_{OV}} \cdot \dfrac{2I}{\sqrt{2}V_{OV}} \cdot \dfrac{V_A}{I} \cdot \dfrac{V_A}{I}$

$= \dfrac{4}{\sqrt{2}}\dfrac{V_A{}^3}{V_{OV}{}^3} = \underline{\underline{2 \cdot \sqrt{2}\,\dfrac{V_A{}^3}{V_{OV}{}^3}}}$

For $K'W/L = 10mA/V^2$
$I = 1mA$
$|V_A| = 10V$

$V_{OV} = \sqrt{\dfrac{1mA}{10mA/V^2}} = 0.316\ V$

$\Rightarrow$ For the simple current mirror case:

$CMRR = 2\left(\dfrac{10}{0.316}\right)^2 = 2000$
$\rightarrow \underline{66\,dB}$

For the Wilson source:

$CMRR = 2\sqrt{2}\,\dfrac{(10)^3}{(0.316)^3} = 89442$
$\rightarrow \underline{\underline{99\,dB}}$

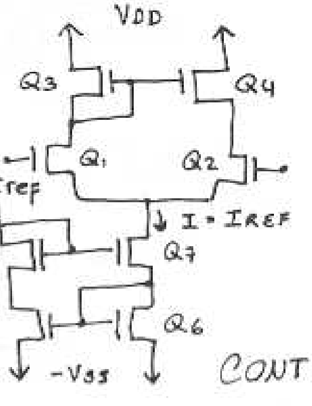
7.65

$V_{GS_{1-4}} = \sqrt{\dfrac{50\mu \cdot 2}{800\mu}} + 1$

$= 1.35V$

$V_{GS_{5-8}} = \sqrt{\dfrac{2 \times 100}{800}} + 1$

$= 1.5V$



CONT.

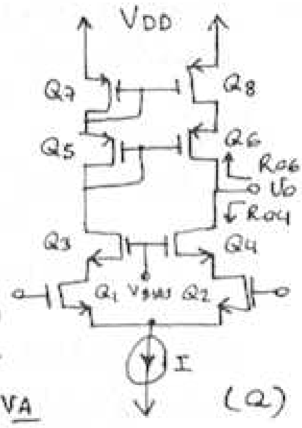
For $V_{DS} = V_{GS}$

$$-V_{SS} + 2V_{GS_{5-8}} + 2V_{GS_{1-4}} = V_{DD}$$

Thus,

$$V_{DD} + V_{SS} = 2(1.5) + 2(1.35) = \underline{\underline{5.7V}}$$

7.66



(b)

$$R_{o4} = (g_{m4} r_{o4}) r_{o2} = g_m r_o^2$$

$$R_{o6} = (g_{m6} r_{o6}) r_{o8} = g_m r_o^2$$

$$A_d = g_m (R_{o4} \parallel R_{o6}) = g_m \cdot \tfrac{1}{2} g_m^2 r_o^2$$

$$g_m = \frac{2I_D}{V_{OV}} \qquad r_o = \frac{V_A}{I_D}$$

thus, $g_m r_o = 2V_A / V_{OV}$

$$\Rightarrow \underline{\underline{A_d = 2(V_A/V_{OV})^2}}$$

Q.E.D.

For $V_{OV} = 0.25V$ & $V_A = 20V$

$$A_d = 2(20/0.25)^2 = \underline{\underline{12800 \tfrac{V}{V}}}$$

7.67

$$R_{id} = (\beta+1)\, 2r_e \; ; \; r_e = \frac{25mV}{50\mu A} = 500\Omega$$

$$\rightarrow R_{id} = 101 \times 1000 = \underline{101K\Omega}$$

$$R_o = r_{o4} \parallel r_{o2} = \frac{r_o}{2} \; ; \; r_o = \frac{V_A}{I_C}$$

$$\rightarrow r_o = \frac{160V}{50\mu A} = 3.2M\Omega$$

Thus, $R_o = \underline{\underline{1.6M\Omega}}$

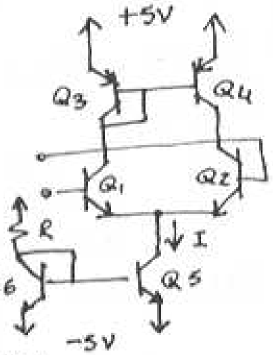
$$G_m = g_{m1} = g_{m2} = \frac{50\mu A}{25mV} = \underline{\underline{2mA/V}}$$

$$A_d = G_m R_o = 2 \times 1600 = \underline{\underline{3200\ V/V}}$$

With a subsequent stage having a $100K\Omega$ input resistance,

$$A_d = G_m (R_o \parallel 100K\Omega) = \underline{\underline{188.2\ V/V}}$$

7.68



$$G_m = \frac{I/2}{V_T} = 5\frac{mA}{V}$$

$$I = \underline{\underline{250\mu A}}$$

$$R = \frac{5-(-5)-V_{BE}}{I} = \frac{9.3}{0.25} = \underline{\underline{37.2K\Omega}}$$

$R_{id} = (\beta+1)\, 2r_e$ where,

$$r_e = \frac{V_T}{I/2} = \frac{25mV}{0.125mA} = 200\Omega$$

$$\Rightarrow R_{id} = 151 \times 2 \times 0.2 = \underline{\underline{60.4K\Omega}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.125} = 800K\Omega$$

$$R_o = \frac{r_o}{2} = \underline{\underline{400K\Omega}}$$

$$A_d = g_m R_o = 5 \times 400 = \underline{\underline{2000\ V/V}}$$

$$I_B = \frac{I/2}{\beta+1} = \frac{125}{151} = \underline{\underline{0.83\mu A}}$$

$$v_{ICM}|_{max} \simeq v_{C1} + 0.4V = 5 - 0.7 + 0.4 = \underline{\underline{4.7V}}$$

$$v_{ICM}|_{min} = V_{B5} - 0.4 + 0.7 = -5 - 0.4 + 0.7 = -4V$$

Thus, the input common-mode range is $-4V$ to $+4.7V$ (where we have assumed that a transistor remains active

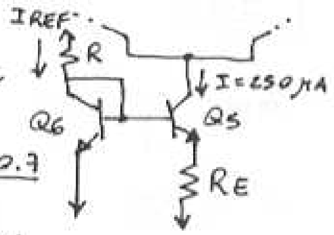
CONT.

even through its base-collector junction is forward biased by 0.4V)

$R_{icm} \simeq (\beta+1)\,[r_{o5} \,\|\, r_{o1} \,\|\, r_{o2}]$

$= 151 \times (400 \,\|\, 800 \,\|\, 800) = \underline{30M\Omega}$

7.69



Use $R = 2K\Omega$

Thus,

$I_{REF} = \frac{5-(-5)-0.7}{2}$

$= 4.65mA$

$V_{BE6} - V_{BE5} = V_T \ln(4.65/0.25)$

$= 73.1mV$

$R_E = 73.1mV / 0.25mA$

$= \underline{292\Omega}$

The only amplifier parameter that changes is R_{icm}. This is because now R_{o5} is:

$R_{o5} \simeq (1+g_{m5}R_E)\, r_{o5}$

$= (1+10\times 0.292)\times 400$

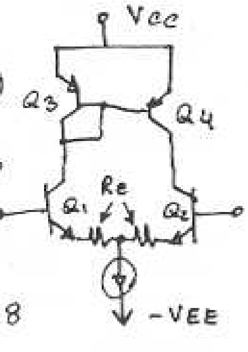
$= \underline{1.57M\Omega}$

Thus, $R_{icm} = (\beta+1)\,[R_{o5} \,\|\, r_{o1} \,\|\, r_{o2}]$

$= 151\,[1.57 \,\|\, 0.80 \,\|\, 0.8]$

$= \underline{48M\Omega}$

7.70



$R_{id} = (\beta+1)\,2\,(r_e+R_E)$

G_m is still equal to: $g_m = 5mA/V$

$\Rightarrow I = \underline{250\mu A}$

(From Problem 7.68 above)

and $r_e = \frac{V_T}{I/2} = 200\Omega$, $r_o = 800K\Omega$

If $R_{id} = 100K\Omega \Rightarrow$

$100K = 151 \times 2 \times (200 + R_E)$

$\rightarrow R_E = 131\Omega$

To obtain A_d:

$A_d = G_m . R_o$ (Eqn. 7.165)

As in the derivation of R_{o2} in Eqn. (7.162), R_{o2} can be found using Eqn. (6.159), but this time noting that R_e at the emitter of Q_2 is:

$r_{e1} + 2R_E$

Thus,

$R_{o2} = r_{o2}\left[1 + g_m\left((r_{e1}+2R_E) \,\|\, r_{\pi 2}\right)\right]$

$R_{o2} = 800K\left[1 + 5m\left((200+2\times 131) \,\|\, \underbrace{30.2K}_{(\beta+1)r_e}\right)\right]$

$R_{o2} = 2620K\Omega$

$R_o = R_{o2} \,\|\, r_{o4}$

$= (2620 \,\|\, 800)K = 613K\Omega$

$\Rightarrow A_d = 5m \times 613K = \underline{3065}\ V/V$

7.71

$G_m = g_m = \frac{I/2}{V_T} = \frac{0.5m/2}{25m}$

$g_m = \underline{10mA/V}$

$R_o = r_{o2} \,\|\, r_{o4} = \frac{V_A}{I_{C2}} \,\|\, \frac{V_A}{I_{C4}} = \frac{1}{2}\frac{V_A}{I/2}$

$= \frac{120}{0.5m} = \underline{240K\Omega}$

$A_d = G_m R_o = 10 \times 240 = \underline{2400\ \frac{V}{V}}$

$R_{id} = 2r_\pi \simeq 2\frac{V_T}{I/2}\beta = \frac{25m \times 150}{0.5m}$

$R_{id} = \underline{7.5K\Omega}$

For a simple current mirror

CONT.

the output resistance (thus R_{EE}) is r_o

$$\Rightarrow R_{EE} = \frac{V_A}{I} = \frac{120}{0.5m} = \underline{\underline{240K\Omega}}$$

$$A_{cm} = \frac{-r_{o4}}{\beta_3 R_{EE}} = \frac{-(2\times 240K)}{150\times 240K} = -13.3\,mV/V$$

and, $CMRR = \left|\frac{2400}{-13.3m}\right| = 180,451$

i.e. $\underline{\underline{105\ dB}}$

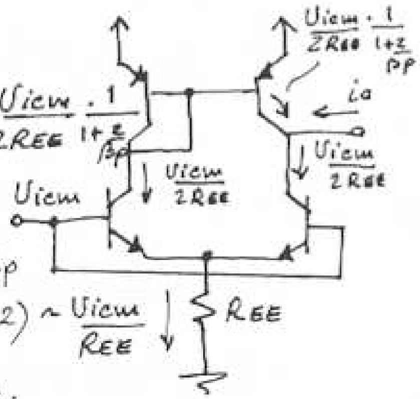
$$\frac{V_i}{V_s} = \frac{R_{id}}{R_{id}+R_s} = \frac{7.5K}{7.5K+10K} = 0.43\,\frac{V}{V}$$

$\Rightarrow$ Overall gain A:

$$A = \frac{v_i}{v_s}\cdot\frac{v_o}{v_i} = 0.43\times 2400$$

$$= \underline{\underline{1032\ V/V}}$$

7.72



$$i_o = \frac{V_{icm}}{2R_{EE}} - \frac{V_{icm}}{2R_{EE}}\cdot\frac{1}{1+\frac{2}{\beta_P}}$$

$$= \frac{V_{icm}}{2R_{EE}}\cdot\frac{2/\beta_P}{1+2/\beta_P}$$

$$\simeq \frac{V_{icm}}{\beta_P R_{EE}}\ (\beta_P \gg 2) \sim \frac{V_{icm}}{R_{EE}}$$

Q.E.D.

Thus, $G_{mcm} = \frac{i_o}{V_{icm}} = \frac{1}{\beta_P R_{EE}}$

Q.E.D.

$$CMRR = \frac{G_{md}}{G_{mcm}} = \frac{g_m}{1/\beta_P R_{EE}}$$

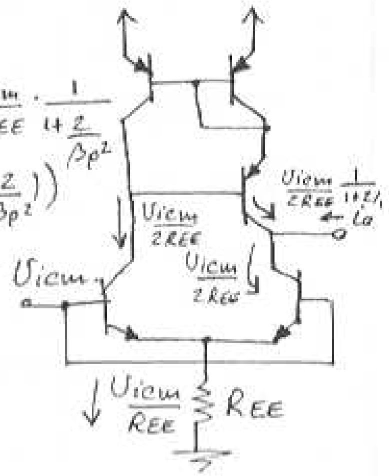
$$= g_m \beta_P R_{EE} = \underline{\underline{\frac{\beta_P\cdot I R_{EE}}{2V_T}}}$$

For $I = 0.2mA$, $R_{EE} = 1M\Omega$ and $\beta_P = 25$:

$$CMRR = \frac{\beta_P\cdot I R_{EE}}{2V_T} = \frac{25\times 0.2\times 1000}{2\times 0.025}$$

$$= \underline{\underline{10^5}}\ \text{i.e.}\ \underline{\underline{100dB}}$$

7.73



$$i_o = \frac{V_{icm}}{2R_{EE}} - \frac{V_{icm}}{2R_{EE}}\cdot\frac{1}{1+\frac{2}{\beta_P^2}}$$

$$\simeq \frac{V_{icm}}{2R_{EE}}\left(1-\left(1-\frac{2}{\beta_P^2}\right)\right)$$

$$= \underline{\underline{\frac{V_{icm}}{\beta_P^2 R_{EE}}}}\ \text{Q.E.D.}$$

$$G_{mcm} = \frac{i_o}{V_{icm}} = \underline{\underline{\frac{1}{\beta_P^2\cdot R}}}$$

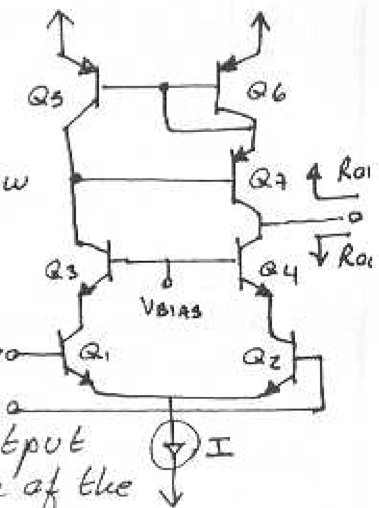
$$CMRR = \frac{G_{md}}{G_{mcm}} = \beta_P^2 g_m R_{EE} = \underline{\underline{\frac{\beta_P^2 I R_{EE}}{2V_T}}}$$

Thus,

$$CMRR = \frac{(25)^2\times 0.2\times 1000}{2\times 0.025} = 25\times 10^5$$

or $\underline{\underline{128dB}}$

7.74



$A_{vo} = G_m R_o$

$G_m = g_m$.

$R_o = R_{oc} \parallel R_{ow}$

where R_{oc} is the output resistance of the cascode stage & R_{ow} the output resistance of the wilson stage

CONT.

$R_{oc} = \beta r_o$ & $R_{ow} = \frac{\beta r_o}{2}$

$\Rightarrow R_o = \beta r_o \,\|\, \frac{\beta r_o}{2} = \frac{\frac{\beta r_o}{2}\cdot \beta r_o}{\beta r_o (1+\frac{1}{2})}$

$= \frac{\beta r_o}{2\times\frac{3}{2}} = \frac{\beta r_o}{3}$

$\Rightarrow A_{vo} = G_m R_o = g_m \frac{\beta r_o}{3}$ Q.E.D

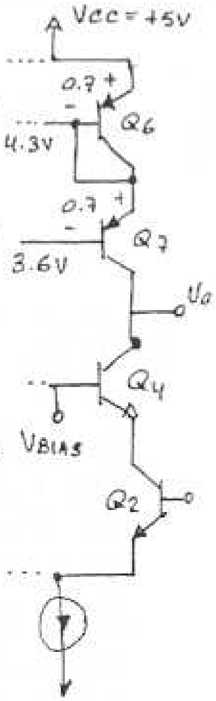
For: $I = 0.4mA$, $\beta = 100$, $V_A = 120V$

$$A_{vo} = \frac{I/2}{V_T}\cdot\frac{\beta}{3}\cdot\frac{V_A}{I/2} = \frac{\beta}{3}\,\frac{V_A}{V_T}$$

$$= \frac{100}{3}\times\frac{120V}{25mV} = 160000$$

i.e. 104 dB

7.75



(a) $V_{omax} - V_{B7} = 0.4V$

but: $V_{B7} = 5 - 2\times 0.7V = 3.6V$

$\Rightarrow V_{omax} = 3.6 + 0.4 = 4.0V$

(b) For a 1.5V max. positive swing the DC bias at the output should be:

$V_{oDC} = 4 - 1.5 = 2.5V$

(c) In the edge of saturating Q_4

$V_{BIAS} - V_{omin} = 0.4V$

$\rightarrow V_{omin} = V_{BIAS} - 0.4V$

if V_{omin} is: $V_{oDC} - 1.5$

$2.5 - 1.5 = 1V$

$\Rightarrow V_{BIAS} = 1.4V.$

(d) If $V_{BIAS} = 1.4V$

$\Rightarrow V_{C2} = 1.4 - 0.7 = 0.7V$

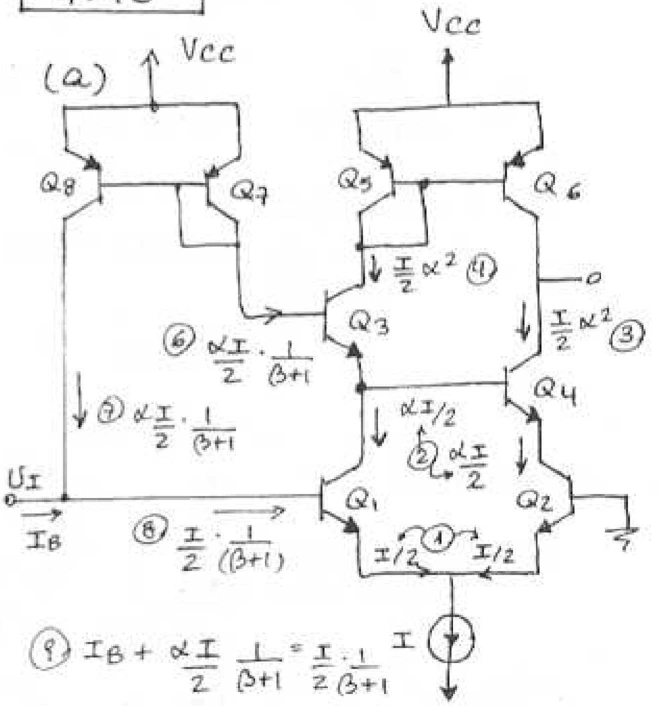
The upper limit of V_{icm} is such that brings Q_2 to the edge of saturating.

$V_{icm\,min} - V_{C2} = 0.4V$

$\Rightarrow V_{icm\,min} = 0.4 + 0.7 = 1.1V$

7.76

(a)



(9) $I_B + \frac{\alpha I}{2}\,\frac{1}{\beta+1} = \frac{I}{2}\cdot\frac{1}{\beta+1}$

$\Rightarrow I_B = \frac{I}{2}\,\frac{1}{\beta+1}(1-\alpha) = \frac{I}{2}\,\frac{1}{(\beta+1)^2}$

Without Q_8 & Q_7

$I_B = \frac{I}{2}\cdot\frac{1}{(\beta+1)}$

thus, the input DC current is reduced by $\times 1/(\beta+1)$

(b) Without Q_8 & Q_7:

$R_{in} = 2r_{\pi 1,2}$; $r_\pi = \frac{V_T}{I_B} = \frac{V_T\, 2(\beta+1)}{I}$

$\Rightarrow R_{in} = \frac{4V_T(\beta+1)}{I}$

CONT.

With Q_8 & Q_7: $R_{in} = 2(r_\pi \| r_{o8})$

where $r_{o8} = \frac{V_A}{\alpha I} \cdot 2(\beta+1)$

$$\rightarrow R_{in} = \frac{4V_T}{I}(\beta+1) \;\|\; \frac{V_A}{\alpha I}\,2(\beta+1)$$

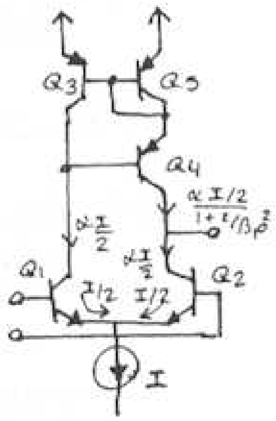
Solving, yields:

$$R_{in} = \frac{4V_T}{I}(\beta+1) \cdot \frac{V_A}{2\alpha V_T + V_A}$$

but $\frac{V_A}{2\alpha V_T + V_A} \simeq 1 \Rightarrow R_{in} = \underline{\underline{\frac{4V_T(\beta+1)}{I}}}$

Just like when Q_8 & Q_7 are not present.

7.77



For the Wilson current source

$$\frac{I_4}{I_3} = \frac{1}{1+2/\beta_p^2}$$

Since $I_3 = \alpha I/2$

$$\rightarrow I_4 = \frac{\alpha I/2}{1+2/\beta_p^2}$$

$$\Delta i = \frac{\alpha I}{2} - \frac{\alpha I/2}{1+2/\beta_p^2}$$

$$= \frac{\alpha I}{2} \cdot \frac{2/\beta_p^2}{1+2/\beta_p^2} = \alpha I \cdot \frac{1}{\beta_p^2+2}$$

$$\simeq \frac{\alpha I}{\beta_p^2}$$

thus. $V_{os} = \frac{-\Delta i}{G_m}$ with $G_m = g_m = \frac{\alpha I/2}{V_T}$

$$\Rightarrow V_{os} = -\frac{\alpha I/\beta_p^2}{\alpha I/2V_T} = -\frac{2V_T}{\beta_p^2}$$

For $\beta = 50$:

$$V_{os} = \frac{-2 \times 25\,mV}{(50)^2} = \underline{\underline{-20\,\mu V}}$$

7.78

To obtain maximum positive swing V_{bias} must be as low as possible To keep the top current sources out of saturation:

$$V_{CC} - 0.2 - 0.7 = V_{bias\,max}$$

$$V_{bias\,max} = 4.1\,V$$

And: $V_O - V_{bias\,min} = +0.4\,V$

Since $V_O \sim 0 \Rightarrow V_{bias\,min} = -0.4$

$\Rightarrow$ Range of V_{bias} is:

$$(-0.4 \ll V_{bias} \leq 4.1)\,V$$

For: $I = 0.4\,mA$, $\beta_P = 50$, $\beta_N = 150$ & $V_A = 120$

$$G_m = g_{m1} = \frac{0.2\,mA}{25\,mV} = \underline{\underline{8\,\frac{mA}{V}}}$$

For the folded cascode: $R_{o4} = \beta_4 r_o$

For the Wilson mirror: $R_{o5} = \beta_5 \frac{r_o}{2}$

$$\Rightarrow R_o = [\beta_4 \cdot r_{o4} \,\|\, \beta_5 \cdot \frac{r_{o5}}{2}]$$

$$r_{o4} = r_{o5} = 120/0.2\,mA = 600\,K\Omega$$

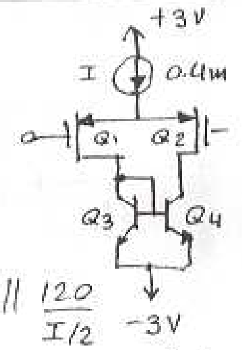
$$\rightarrow R_o = [50 \times 600K \,\|\, 150 \times \frac{600K}{2}]$$

$$= [30M \,\|\, 45M]$$

$$= \underline{18\,M\Omega}$$

$$A_d = G_m R_o = 8\,\frac{mA}{V} \times 18\,M\Omega = \underline{\underline{144\,0}}$$

7.79



$K_p' W/L = 6.4\,mA/V^2$

$|V_{Ap}| = 10\,V$

$V_{A\,NPN} = 120$.

$$R_o = r_{o2} \| r_{o4} = \frac{V_{Ap}}{I/2} \,\|\, \frac{120}{I/2}$$

$$R_o = (10/0.2m) \,\|\, (120/0.2m) = \underline{\underline{4}}$$

$$G_m = g_{m1} = \sqrt{I \times K_p' W/L}$$

$$= \sqrt{0.4\,mA \times 6.4\,mA/V^2}$$

$$\Rightarrow G_m = \underline{\underline{1.6\,\frac{mA}{V}}}$$

$$A_d = G_m \times R_o = 1.6\,\frac{mA}{V} \times 46\,K\Omega$$

$$\longrightarrow A_d = \underline{\underline{73.6\,V/V}}$$

7.80

$K_n'(W/L) = 128 \frac{\mu A}{V^2} \times 25 = 3.2 mA/V^2$

(a) $V_{ov} = \sqrt{\frac{I}{K_n' W/L}} = \sqrt{\frac{0.2}{3.2}} = \underline{\underline{0.25V}}$

$g_m = \frac{I}{V_{ov}} = \frac{0.2mA}{0.25V} = \underline{\underline{0.8\frac{mA}{V}}}$

(b) $A_d = g_m(R_D \| r_o)$

where $r_o = \frac{V_A}{I/2} = \frac{20}{0.2/2} = 200K\Omega$

$\Rightarrow A_d = 0.8m \times (20K \| 20K)$

$= \underline{\underline{14.54}}\ V/V$

(c) For a CS amplifier when R_{sig} is low:

$f_H = \frac{1}{2\pi(C_L + C_{gd})R_L'}$

where $R_L' = R_D \| r_o$

$= 20K \| 200K$

$= 18.18K\Omega$

and $C_L' = C_L + C_{db}$

Since for a grounded source terminal C_{db} is in parallel with the load.

$\rightarrow C_L' = 90 + 5 = 95 pF$

thus,

$f_H = \frac{1}{2\pi(95+5)10^{-12} \times 18.18K}$

$= \underline{\underline{87.54}}\ MHz$

(d) Using the open-circuit time-constants method for $R_S = 20K\Omega$

$f_H = \frac{1}{2\pi \tau_H}$

where $\tau_H = C_{gs}.R_S$
$+ C_{gd}[R_S(1+g_m R_L') + R_L']$
$+ C_L.R_L'$

thus.

$\tau_H = 30f \times 20K$
$+ 5f[20K(1+0.8\times18.18)+18.18K]$
$+ (90f + 5f) \times 18.18K$

$\tau_H = 0.6ns$
$+ 1.64ns$
$+ 1.72ns = 3.96ns$

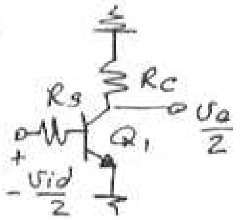
$\Rightarrow f_H = \frac{1}{2\pi\, 3.96ns} = \underline{\underline{40.2 MHz}}$

7.81

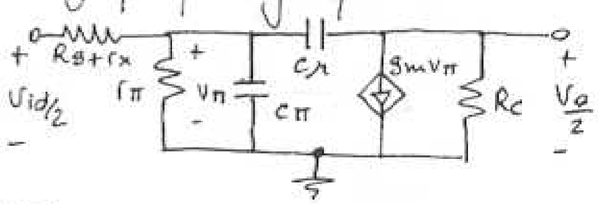
$f_Z = \frac{1}{2\pi C_{ss} R_{ss}} = \frac{1}{2\pi(0.2p)(100K)} = 7.\underline{95 MHz}$

7.82

(a) Differential half-circuit



High-frequency equivalent circuit



(b) $I_E = 0.5mA \rightarrow g_m = \frac{0.5}{25}$

$g_m = 20mA/V$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20m} = \underline{5K\Omega}$

$C_\pi + C_\mu = \frac{20\times10^{-3}}{2\pi\times600\times10^6} = 5.3pF$

$C_\mu = 0.5p \Rightarrow C_\pi = 4.8pF$

Now, $R_S = 10K\Omega$, $R_C = 10K\Omega$, $r_x = 100$

For this common-emitter amplifier (Refer to Section..

CONT.

5.9 Equ. (5.164))

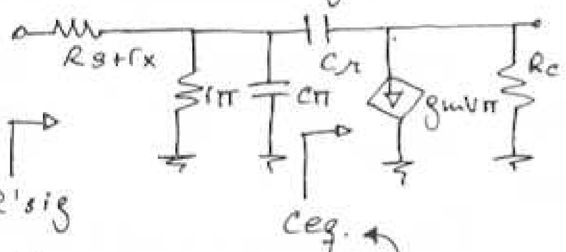
$$\frac{V_o}{V_i} = \frac{-r_\pi}{r_\pi + (R_s + r_x)} \cdot g_m R_c$$

$$= \frac{-5K}{5K + (10K + 100)} \cdot 20m \times 10K$$

$$= \underline{-66.22} \text{ V/V}$$

(c) Refer to Section 5.9, Equ's (5.176), (5.173), (5.168)
From Equ. (5.176):

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$



where: $C_{in} = C_\pi + C_{eq}$
Using Miller's theorem
$C_{eq} = C_\mu (1 + g_m R_c)$
and
$R'_{sig} = (R_s + r_x) \parallel r_\pi$

Thus,

$$f_H = \frac{1}{2\pi \left[(R_s + r_x) \parallel r_\pi\right]\left[C_\pi + C_\mu (1 + g_m R_c)\right]}$$

$$= \frac{1}{2\pi((10K + 100) \parallel 5K)(4.8p + 0.5p(1 + 20 \times 10))}$$

$$= \underline{452 \text{ KHz}}$$

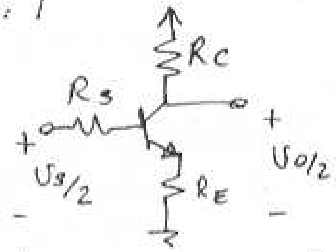
$GBW = 66.22 \times 452K = \underline{30 MHz}$

7.83

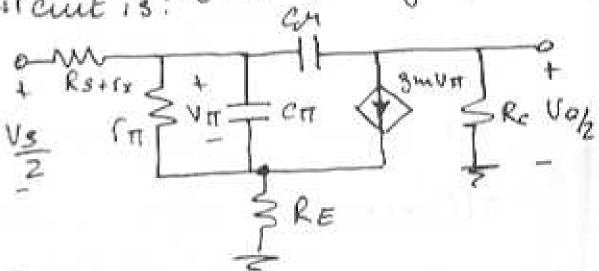
From problem 7.82 above:

$g_m = 20mA/V$ $r_\pi = 5K\Omega$ $C_\pi = 4.8pF$
$C_\mu = 0.5pF$ $R_s = 5K\Omega$ $R_c = 10K\Omega$
$r_x = 100\Omega$

If we add an emitter resistor, then the equivalent half-circuit is:



And the high-frequency equivalent circuit is:



For this circuit the low-frequency gain is:

$$A_o = \frac{V_o}{V_s} = \underbrace{\frac{-(\beta+1) \cdot (r_e + R_E)}{(R_s + r_x) + (\beta+1)(r_e + R_E)}}_{\text{obtained by reflecting emitter resistance to the base}} \times \underbrace{\frac{\alpha R_c}{R_E + r_e}}_{\text{gain } V_o/V_i \text{ as in Eq 5.13}}$$

where $r_e = \frac{V_T}{I_E} = \frac{25}{0.5} = 50\Omega$

$$A_o = \frac{-101 \times (50 + 100)}{(10K + 100) + 101 \times (50 + 100)} \times \frac{100}{101} \cdot \frac{10K}{(100 + }$$

$$= \underline{-39.6} \text{ V/V}$$

Using the method of open-circuit time constants:
$f_H = 1/(2\pi (C_\pi R_\pi + C_\mu R_\mu))$

CONT.

Refering to the high-frequency equivalent circuit:

$$R_\pi = r_\pi \,//\, \frac{R_s + r_x + R_E}{1 + g_m R_E}$$
$$= 5K \,//\, \frac{10K + 100 + 100}{1 + 20m \times 100}$$
$$= 5K \,||\, 3.4K = 2.02K\Omega$$

To obtain R_μ:

$R_{in} = (\beta+1)(R_e + r_e)$
$= 101 \times (100 + 50) = 15.15K\Omega$

$$G_m = \frac{g_m}{1 + g_m R_e} = \frac{20m}{1 + 20m \times 100} = 6.67m.$$

Thus,

$R_\mu = [(R_s + r_x) \,||\, R_{in}](1 + G_m R_c) + R_c$

$= [(10K + 100) \,||\, 15.15K] \cdot (1 + 6.67 \times 10) + 10K$
$= 420.26K\Omega$

Thus, $C_\pi R_\pi + C_\mu R_\mu =$
$= 4.8pF \times 2.02K\Omega + 0.5pF \times 420.26K$
$= 219.8ns \simeq 220ns.$

$$\Rightarrow f_H = \frac{1}{2\pi \times 220ns} = \underline{\underline{723.4KHz}}$$

$GBW = 39.6 \times 723.4K = \underline{28.6MHz}$

Approx. the same as in problem 7.82 above.

7.84

From the solution of problem 7.81:

$g_m = 20mA/V \quad r_e = 50\Omega \quad r_\pi = 5K\Omega$
$C_\pi = 4.8pF \quad C_\mu = 0.5pF \quad R_s = 10K\Omega$
$R_c = 10K\Omega \quad r_x = 100$

$f_H = 1/(2\pi \tau_H) = 1 \times 10^6 Hz$
$\Rightarrow \tau_H = 1/(2\pi \times 10^6)$
$= 159.15ns$

But: $\tau = C_\pi R_\pi + C_\mu R_\mu$
where

$$R_\pi = r_\pi \,||\, \left(\frac{R_s + r_x + R_e}{1 + g_m R_e}\right)$$
$$= \frac{1}{\frac{1}{r_\pi} + \frac{1 + g_m R_e}{R_s + r_x + R_e}}$$
$$= \frac{1}{\frac{1}{5K} + \frac{1 + g_m R_e}{10.1K + R_e}}$$

$R_\mu = [(R_s + r_x) \,||\, R_{in}](1 + G_m R_c) + R_c$

with: $R_{in} = (\beta+1)(R_e + r_e)$
$G_m = g_m / (1 + g_m R_e)$

thus,

$$R_\mu = \frac{1}{\frac{1}{R_s + r_x} + \frac{1}{(\beta+1)(R_e + r_e)}} \cdot \left(\frac{1 + g_m R_c}{1 + g_m R_e}\right) + R_c$$

$$= \frac{1 + g_m R_c + g_m R_e}{\frac{1 + g_m R_e}{R_s + r_x} + \underbrace{\frac{1 + g_m R_e}{(\beta+1)(R_e + r_e)}}} + R_c$$

$$= \frac{1 + g_m R_e}{(\beta+1)(R_e + \frac{1}{g_m})}$$

$$= \frac{g_m}{\beta+1} \cdot \frac{(1 + g_m R_e)}{(1 + g_m R_e)}$$

and, since $r_\pi = \beta / g_m$

$$\Rightarrow \frac{g_m}{\beta+1} \simeq \frac{1}{r_\pi}$$

thus,

$$R_\mu = \frac{1 + g_m R_c + g_m R_e}{\frac{1 + g_m R_e}{R_s + r_x} + \frac{1}{r_\pi}} + R_c$$

CONT.

$$159.15ns = \frac{4.8pF}{\frac{1}{5K} + \frac{1+g_mR_e}{10.1K+R_e}} + \frac{0.5pF(1+200+g_mR_e)}{\frac{1+g_mR_e}{10.1K} + \frac{1}{5K}} + \underbrace{0.5pF \times 10K\Omega}_{5ns}$$

Assuming that $R_e \ll 10.1K\Omega$

$$154.15ns = \frac{4.8pF + 0.5pF(1+200+g_mR_e)}{\frac{1}{5K} + \frac{1+g_mR_e}{10.1K}}$$

$$(154.15n)\left(\frac{1}{5K} + \frac{1+g_mR_e}{10.1K}\right) =$$

$$= 4.8p + 0.5p(1+g_mR_e) + 100p$$

Solving for $1+g_mR_e$ yields:

$1+g_mR_e = 5 \Rightarrow R_e = \underline{\underline{200\Omega}}$

The DC gain is:

$$A_o = \frac{-101\times(50+200)}{(10K+100)+101\times(50+200)} \times \frac{100}{101} \times \frac{10K}{(200+50)}$$

$$= -\underline{\underline{28.28}}\ V/V$$

$R_{in} = 101\times(200+50) = 25.25K\Omega$

$G_m = 20m/(1+20m\times200) = 4m$

$R_\mu = ((10K+100)\|25.25K)(1+4\times10)+10K$

$= 305,785\Omega$

$R_\pi = 5K \| \frac{10K+100+200}{1+20m\times200} = 1459\Omega$

$\tau_H = 1459\times4.8p + 305,785\times0.5p$

$= 159.8ns$

$f_H = \frac{1}{2\pi\tau_H} = 0.99MHz \sim \underline{\underline{1MHz}}$

$GBW = 28.28\times1 = \underline{\underline{28.28}}\ MHz$

$\boxed{7.85}$

$I = 0.6mA$

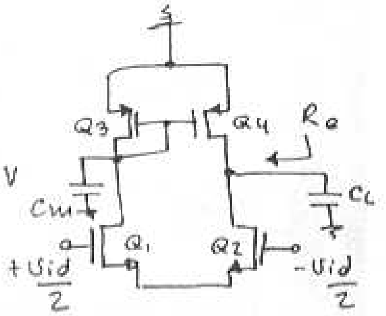
$V_{ovn} = 0.3V$

$V_{ovp} = 0.5V$

$V_{An} = |V_{Ap}| = 9V$

$C_m = 0.1pF$

$C_L = 0.2pF$



All r_o's are identical:

$$r_o = \frac{V_A}{I_D} = \frac{9}{0.3m} = 30K\Omega.$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow g_{m1,2} = \frac{0.6m}{0.3} = 2\frac{mA}{V}$$

$$g_{m3,4} = \frac{0.6m}{0.5} = 1.2\frac{mA}{V}$$

The low frequency differential gain is:

$$A_d = g_{m1,2}(r_{o2}\|r_{o4})$$

$$= 2\frac{mA}{V}(30K\|30K) = \underline{\underline{30}}$$

From Eqn. (7.192)

$f_{p1} = 1/(2\pi C_L R_o)$

where $R_o = r_{o2}\|r_{o4} = 15K\Omega$

$$f_{p1} = \frac{1}{2\pi\times0.2p\times15K} = \underline{\underline{53MHz}}$$

(7.193) $f_{p2} = \frac{g_{m3}}{2\pi C_m} = \frac{1.2mA/V}{2\pi\times0.1pF}$

$= \underline{\underline{1.9GHz}}$

(7.194) $f_z = \frac{2g_{m3}}{2\pi C_m} = \frac{2\times1.2m}{2\pi\times0.1p}$

$= \underline{\underline{3.8GHz}}$

$\boxed{7.86}$

The CMRR will have.. CONT

poles at $\underline{500KHz}$ and at

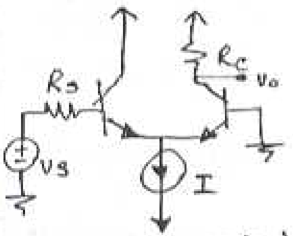
$$\frac{1}{2\pi \times 10^6 \times 10 \times 10^{-12}} = \underline{15.9\,KHz}$$

7.87

From the solution to Problem 7.82 above:

$g_m = 20mA/V$ $\quad r_e = 50\Omega$ $\quad r_\pi = 5K\Omega$
$C_\pi = 4.8pF$ $\quad C_\mu = 0.5pF$ $\quad R_s = 10K\Omega$
$R_C = 10K\Omega$

Here we neglect r_x, i.e $r_x = 0$.



$$V_{b1} = V_s \cdot \frac{2r_\pi}{R_s + 2r_\pi}$$

$$V_o = \frac{V_{b1}}{2r_e}\alpha \cdot R_C$$

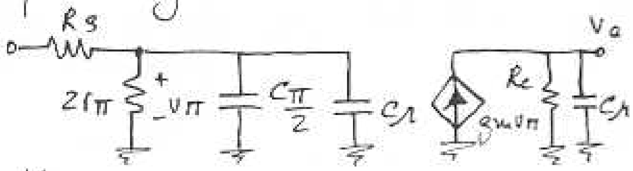
$$\Rightarrow \frac{V_o}{V_s} = \frac{\alpha R_C}{2r_e} \cdot \frac{2r_\pi}{R_s + 2r_\pi}$$

$$= \frac{\alpha \cdot R_C \cdot r_e(\beta+1)}{r_e(R_s + 2r_\pi)} = \frac{\alpha R_C}{\frac{R_s}{\beta+1} + 2r_e}$$

$$= \frac{\beta R_C}{R_s + 2r_\pi}$$

$$A_o = \frac{V_o}{V_s} = \frac{100 \times 10K}{10K + 2\times 5K} = \underline{50}\,V/V$$

Using the equivalent circuit of Fig 6.57. i.e.



thus,

$$f_{P1} = \frac{1}{2\pi(R_s \| 2r_\pi)(\frac{C_\pi}{2} + C_\mu)}$$

$$= \frac{1}{2\pi(10K \| 10K)(\frac{4.8}{2}p + 0.5p)}$$

$$= \underline{11MHz}$$

and, $$f_{P2} = \frac{1}{2\pi R_C C_\mu}$$

$$= \frac{1}{2\pi \times 10K \times 0.5p} = 31.8 \sim \underline{32MHz}$$

thus,

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{11}\right)^2 + \left(\frac{1}{32}\right)^2}} = \underline{10.4\,MHz}$$

7.88

$$V_{G1} = V_s \cdot \frac{2/g_m}{2/g_m + R_s} \qquad I = \frac{V_{G1}}{2/g_m}$$

$$V_o = IR_D = \frac{V_{G1}}{2/g_m} \times R_D$$

$$= V_s \times \frac{2/g_m}{2/g_m + R_s} \cdot \frac{R_D}{2/g_m}$$

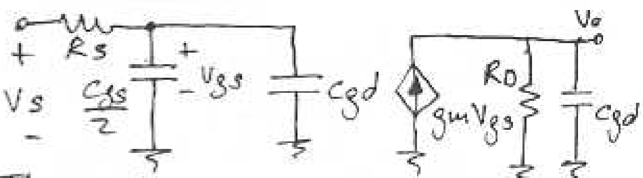
$$= \frac{V_s \cdot R_D}{2/g_m + R_s}$$

$$\Rightarrow A_o = \frac{V_o}{V_s} = \frac{g_m R_D}{2 + g_m R_s}$$

$$g_m = \frac{200\mu A}{0.25V} = 0.8\frac{mA}{V}$$

$$\Rightarrow A_o = \frac{0.8 \times 50}{2 + 0.8 \times 200} = 0.24\,V/V$$

The high-frequency equivalent circuit is:



Thus, the pole at the input has a frequency f_{P1}:

CONT.

$$f_{P1} = \frac{1}{2\pi R_s \times \left(\frac{C_{gs}}{2} + C_{gd}\right)}$$

$$= \frac{1}{2\pi \times 200K \times (1/2 + 1)p}$$

$$= \underline{530\ KHz}$$

and the pole at the output has a frequency f_{P2}:

$$f_{P2} = \frac{1}{2\pi R_o C_{gd}} = \frac{1}{2\pi \times 50K \times 1p}$$

$$= \underline{3.18\ MHz}$$

Thus $f_H \simeq \frac{1}{\sqrt{\left(\frac{1}{530K}\right)^2 + \left(\frac{1}{3.18M}\right)^2}}$

$$= \underline{523\ KHz}$$

Notice that this low value of f_H is due to the large value of R_s.

$$2\pi f_T = \frac{g_m}{C_\pi + C_\mu} \Rightarrow C_\pi + C_\mu = \frac{4m}{2\pi\, 600}$$

$C_\pi + C_\mu = 1.06\ pF$

Since $C_\mu = 0.2\ pF \rightarrow C_\pi = 0.86\ p$

$$f_{P1} = \frac{1}{2\pi (R_s \| 2r_\pi)\left(\frac{C_\pi}{2} + C_\mu\right)}$$

$$= \frac{1}{2\pi (50K \| 2 \times 50K)\left(\frac{0.86}{2} + 0.2\right)p}$$

$$= \underline{7.58\ MHz}$$

$$f_{P2} = \frac{1}{2\pi R_C C_\mu} = \frac{1}{2\pi\, 50K \times 0.2p}$$

$$= \underline{15.9\ MHz}$$

Thus the estimate of f_H is:

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{7.58}\right)^2 + \left(\frac{1}{15.9}\right)^2}}$$

$$= \underline{6.84\ MHz}$$

7.89

Refer to Fig. P7.89.
It can be shown that this circuit is equivalent to that of Problem 7.87 above.

$$g_m = \frac{0.1mA}{25mV} = 4\frac{mA}{V} \qquad r_e \simeq 250\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{4mA/V} = 50K\Omega$$

$$A_O = \frac{\alpha R_C}{\frac{R_s}{\beta+1} + 2r_e} = \frac{\frac{200}{201} \times 50K}{\frac{50K}{201} + 2 \times 250}$$

$$= \underline{66.44\ V/V}$$

7.90

To ensure that the op amp will not have a systematic offset voltage, use Eqn. (7.201)

$$\frac{(W/L)_6}{(W/L)_4} = 2\frac{(W/L)_7}{(W/L)_5}$$

$$\Rightarrow (W/L)_6 = 2 \times \frac{(60/0.5)(10/0.5)}{(60/0.5)}$$

$$\Rightarrow (W/L)_6 = \underline{(20/0.5)}$$

Since Q_8 and Q_5 are matched $I = I_{REF} = 225\mu A$
Thus Q_1, Q_2, Q_3 and Q_4

CONT.

each conducts a current equal to $I/2 = 112.5\mu A$.

Since Q_7 is matched to Q_5 and Q_8, the current in Q_7 is equal to $I_{REF} = 225\mu A$.

Finally Q_6 conducts an equal current of $225\mu A$.

Thus,

$I_{D1} = 112.5\mu A$ $I_{D5} = 225\mu A$

$I_{D2} = 112.5\mu A$ $I_{D6} = 225\mu A$

$I_{D3} = 112.5\mu A$ $I_{D7} = 225\mu A$

$I_{D4} = 112.5\mu A$ $I_{D8} = 225\mu A$

To find $|V_{ov}|$ we use:

$$I_D = \frac{1}{2}\mu C_{ox}(W/L)V_{ov}^2$$

then we find $|V_{GS}|$ from:

$$|V_{GS}| = |V_t| + |V_{ov}|$$

this results in:

$|V_{ov1}| = 0.25V$ $|V_{GS1}| = 1V$

$|V_{ov2}| = 0.25V$ $|V_{GS2}| = 1V$

$|V_{ov3}| = 0.25V$ $|V_{GS3}| = 1V$

$|V_{ov4}| = 0.25V$ $|V_{GS4}| = 1V$

$|V_{ov5}| = 0.25V$ $|V_{GS5}| = 1V$

$|V_{ov6}| = 0.25V$ $|V_{GS6}| = 1V$

$|V_{ov7}| = 0.25V$ $|V_{GS7}| = 1V$

$|V_{ov8}| = 0.25V$ $|V_{GS8}| = 1V$

To find g_m: $g_m = 2I_D/|V_{ov}|$
and r_o: $r_o = |V_A|/I_D$

Thus:

$g_{m1} = 0.9\,mA/V$ $r_{o1} = 80K\Omega$

$g_{m2} = 0.9\,mA/V$ $r_{o2} = 80K\Omega$

$g_{m3} = 0.9\,mA/V$ $r_{o3} = 80K\Omega$

$g_{m4} = 0.9\,mA/V$ $r_{o4} = 80K\Omega$

$g_{m5} = 1.8\,mA/V$ $r_{o5} = 40K\Omega$

$g_{m6} = 1.8\,mA/V$ $r_{o6} = 40K\Omega$

$g_{m7} = 1.8\,mA/V$ $r_{o7} = 40K\Omega$

$g_{m8} = 1.8\,mA/V$ $r_{o8} = 40K\Omega$

$$A_1 = -g_{m1}(r_{o2} \| r_{o4}) = -0.9\frac{mA}{V}(80 \| 80)K\Omega = -\underline{36}\,V/V$$

$$A_2 = -g_{m6}(r_{o6} \| r_{o7}) = -1.8\frac{mA}{V}(40 \| 40)K\Omega = -\underline{36}\,V/V$$

Thus, the DC open-loop gain is:

$$A_0 = A_1A_2 = (-36)\times(-36) = \underline{\underline{1296}}\ V/V$$

$\rightarrow \underline{\underline{62.25dB}}$

Input common-mode range:
lower limit is when the input is such that Q_1 & Q_2 leave the saturation region.

$$V_{D1} = -V_{SS} + V_{GS3} = -1.5 + 1 = -0.5V$$

$$V_{in_{CM}min} = -0.5 - |V_{tp}| = -0.5 - 0.75 = -\underline{1.25V}$$

The upper limit is the values of V_{inCM} at which Q_5 leaves the saturation region.

$$V_{DS_{CM}}max = +1.5 - |V_{ov5}| = 1.25V$$

$\rightarrow V_{in_{CM}}max = 1.25 - 1 = +\underline{\underline{0.25}}V$

Input range is: -1.25 to $0.25V$

Output voltage range:
V_{omax} is the value at which Q_7 leaves saturation.

$$V_{DD} - |V_{ov7}| = +1.5 - 0.25 = 1.\underline{25}V$$

The lowest V_o is the value at which Q_6 leaves saturation
i.e. $-V_{SS} + V_{ov6} = -1.5 + 0.25 = -\underline{\underline{1.25}}V$

The output range is
-1.25 to $+1.25V$

7.91

$I = \frac{1}{2} K V_{ov}^2$

(a) $V_{ov} = \sqrt{\frac{2I}{K}}$

If K increases by 4 → V_{ov} decreases by $\underline{\underline{1/2}}$

$g_m = 2I/V_{ov} = K.V_{ov}$
→ If K increases by 4
g_m increases by $\underline{\underline{\times 2}}$

(b) $A_1 = g_m R_{o1}$
⟹ A_1 increases ×2 an so A_o

(c) Offsets due to Vt mismatches are unaffected. Others reduced $\times\frac{1}{2}$ since A_o increases ×2

(d) $A_o f_P = f_T \Rightarrow f_P = \frac{f_T}{A_o}$
and is reduced $\times\frac{1}{2}$
⟹ Cc must be doubled.

7.92.

$I_{D7} = \frac{W_7}{W_8} I_{REF} = \frac{50}{40} \times 90\mu A$
$= 112.5\mu A$ //

Output offset current $= I_{D7} - I_{D6}$
$= 112.5 - 90 = 22.5\mu A$

$\Rightarrow V_o = 22.5\mu\,(r_{o6} \,\|\, r_{o7})$

$r_{o7} = \frac{10}{112.5\mu} = 88.9K\Omega$

$\Rightarrow V_o = 22.5\mu\,(111K \,\|\, 88.9K)$
$= \underline{1.11V}$

$V_{os} = \frac{V_o}{A_o} = \frac{1.11V}{1109} = \underline{\underline{1mV}}$

7.93

Offset current $= I_{D2} - I_{D4}$
$= I_{D3} - I_{D4}$

$I_{D3} = \frac{K}{2}(V_{GS} - V_t)^2$

$I_{D4} = \frac{K}{2}(V_{GS} - (V_t + \Delta V_t))^2$

$I_O = I_{D3} - I_{D4}$
$= \frac{K}{2}\left[(V_{GS} - V_t - V_{GS} + V_t + \Delta V_t) \times (V_{GS} - V_t + V_{GS} - V_t - \Delta V_t)\right]$
$= \Delta V_t . \frac{K}{2}(2V_{GS} - 2V_t - \Delta V_t)$

$\simeq K(V_{GS} - V_t) . \Delta V_t$

$I_o = \underline{\underline{g_{m3} \Delta V_t}}$

Recall $I_o = G_{m1} . V_{os}$
and $G_{m1} = g_{m1}$
$\Rightarrow V_{os} = \frac{g_{m3}}{g_{m1}} . \Delta V_t$

For $\Delta V_t = 2mV$
$V_{os} = \frac{0.3m}{0.3m} \times 2m = \underline{\underline{2mV}}$

7.94

From Eqn. (7.211) $W_t = \frac{G_{m1}}{C_c}$

$G_{m1} = g_{m1} = 1mA/V$
For $f_T = 50MHz$

$C_c = \frac{1mA/V}{2\pi \times 50MHz} = \underline{\underline{3.18pF}}$

From Eqn. (7.206)
$f_Z = \frac{G_{m2}}{2\pi C_c}$

$G_{m2} = g_{m6} = 3mA/V$

CON'

$$f_z = \frac{3mA/V}{2\pi \times 3.18pF} = \underline{\underline{150.14\,MHz}}$$

From Eqn. (7.210)

$$f_{P2} = \frac{G_{m2}}{2\pi C_2} = \frac{3mA/V}{2\pi \times 3pF} = \underline{\underline{160\,MHz}}$$

7.95

(a) $\underline{\underline{I_{E1} = I_{E2} = 0.1mA \simeq I_{E3}, I_{E4}}}$

$\underline{\underline{I_{E5} \simeq 1mA}}$ and since the output is held at 0V $\underline{\underline{I_{E6} = 2mA}}$

(b) $r_{e1} = r_{e2} = \frac{25mV}{0.1mA} = 250\Omega$

$$r_{e5} = \frac{25mV}{1mA} = 25\Omega$$

$$r_{e6} = \frac{25mV}{2mA} = 12.5\Omega$$

For the active loaded differential pair; Recall from Eqn. (7.161) $G_{m1} = g_{m1}$

$$\simeq \frac{1}{r_{e1}} = \frac{1}{250} = 4\frac{mA}{V}$$

$R_{o1} = (\beta+1) r_{e5}$ Since all r_o's $= \infty$

$R_{o1} = 101 \times 25 = 2525\Omega$

$\Rightarrow A_1 = G_{m1} R_{o1} = 4\frac{mA}{V} \times 2525\Omega$

$$= 10.1\,V/V$$

For the common-emitter:

$$A_5 = -g_{m5} \cdot R_{c5}$$

$$\simeq -\frac{\beta R_L}{r_{e5}} = -\frac{100 \times 10K}{25}$$

$$= -40,000\,V/V$$

For the emitter follower:

$A_6 \simeq 1$

$A_{2nd\_stage} = A_5 \cdot A_6 = -40,000\,V/V$

$A = A_1 \cdot A_{2nd\text{-}stage} = 10.1 \times -40,000$

$$= \underline{\underline{-404,000\,V/V}}$$

(c) Since the dominant low-frequency pole is set by C_c & $r_{\pi 5}$

$$f_P = \frac{1}{\underbrace{2\pi \cdot R_{o1}(A_5+1)C_c}_{\text{by Miller effect}}} = 100Hz$$

$\Rightarrow C \simeq 1/(2\pi \times 2525 \times 40K \times 100)$

$$= \underline{\underline{15.76pF}}$$

7.96

$I_B = 225\mu A$

$\mu_n C_{ox} = 180\mu A/V^2$

$\mu_p C_{ox} = 60\mu A/V^2$

For Q_8 & Q_9: $W/L = 60/0.5$

$\Rightarrow |V_{ov}| = \sqrt{\frac{2I_D}{K_p'(W/L)}}$

$$|V_{ov}|_{8,9} = \sqrt{\frac{2 \times 225\mu}{60\mu \times 120}} = 0.25V$$

then $g_{m8,9} = \frac{2I_D}{|V_{ov}|} = \frac{2 \times 225\mu}{0.25V}$

$$= 1.8\,mA/V$$

Since g_m of Q_{10}, Q_{11} & Q_{13} are identical to g_m of Q_8 & Q_9 then $V_{ov13} = 0.25V$

Thus for Q_{13}

$$(0.25)^2 = \frac{2 \times 225\mu}{180\mu \times (W/L)_{13}}$$

$\rightarrow (W/L)_{13} = 40$ i.e $(20/0.5)$

Since Q_{12} is 4 times as wide as Q_{13}, then

CONT.

$$(W/L)_{12} = \frac{4 \times 20}{0.5} = 80/0.5$$

$$R_B = \frac{2}{\sqrt{2K_n'(W/L)_{12}\,I_B}} \cdot \left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{\sqrt{2 \times 180\mu \times \frac{80}{0.5} \times 225\mu}} \cdot \underbrace{\left(\sqrt{\frac{80/0.5}{20/0.5}} - 1\right)}_{\sqrt{4} - 1}$$

$$\longrightarrow R_B = \underline{555.6\,\Omega}$$

The voltage drop on R_B is:

$$555.6 \times 225\mu = \underline{0.125\,V}.$$

To obtain the gate voltages:
(assume $|V_{tn}| = |V_{tp}| = 0.7V$)

$$V_{OV12} = \sqrt{\frac{2 \times 225\mu}{180\mu \times \frac{80}{0.5}}} = 0.125\,V$$

$$V_{OV12} = V_{GS12} - V_{tn}$$

$$\rightarrow V_{GS12} = 0.125 + 0.7 = 0.825V$$

thus,

$$V_{G12,13} = V_{GS12} + I_B R_B - V_{SS}$$
$$= 0.825 + 0.125 - 1.5$$
$$= -\underline{0.55}\,V$$

$$V_{OV11} = |V_{OV8}| = 0.25V$$

$$\Rightarrow V_{GS11} = 0.25 + 0.7 = 0.95V$$

$$V_{G11} = -0.55 + 0.95$$

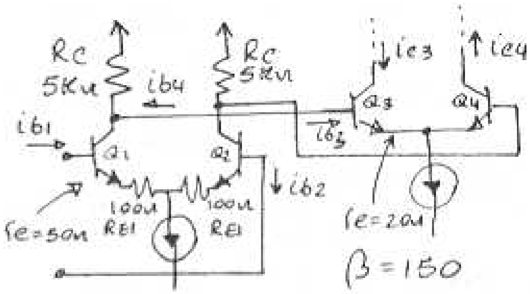
$$V_{G11} = V_{G10} = \underline{0.4V}$$

$$V_{G8} = V_{DD} - V_{SG8} = 1.5 + (-0.25 - 0.7)$$
$$= +\underline{0.55V}$$

Finally, from the results above:

$(W/L)_{10} = 20/0.5$
$(W/L)_{11} = 20/0.5$
$(W/L)_{12} = 80/0.5$
$(W/L)_{13} = 20/0.5$

7.97



$$A_1 = \frac{2R_C \parallel R_{id2}}{2(R_{E1} + r_{e1})}$$

$$R_{id2} = (\beta+1)(2r_{e2}) = 6.04K\Omega$$

$$\Rightarrow A_1 = \underline{12.5}\,V/V$$

$$A_i = \frac{i_{c4}}{i_{b1}} = \beta_1 \cdot \frac{2R_C}{R_{id2} + 2R_C}\,\beta_4$$

$$= 1.4 \times 10^4\ \underline{A/A}$$

7.98

$$R_{id1} = (\beta+1)(R_{E1} + r_{e1})2$$
$$= 101 \times (100 + 100)2 = \underline{40.4K\Omega}$$

increase!

$$R_{id2} = (\beta+1)(2)(R_{E2} + r_{e2})$$
$$= 101 \times (25 + 25)2 = 10.1K\Omega$$

$$\therefore A_1 = \frac{2(20K) \parallel R_{id2}}{2(R_{E1} + r_{e1})} = \underline{20.2}\,V/V$$

decrease

$$A_2 = -\frac{R_3 \parallel R_{i3}}{2(R_{E2} + r_{e2})} = -29.6\ V/V$$

observe that A_3 and A_4 are unchanged.

$$A = A_1 A_2 A_3 A_4$$
$$= (20.2)(29.6)(6.42)(0.998)$$
$$= \underline{3823}\ V/V \quad \text{decrease.}$$

7.99

$$R_o \simeq \frac{R_5}{\beta+1} + r_{e8} = R_C'$$

CONT.

Thus R_5 affects R_o

We want $R_o' \,//\, 3K = 76$

$\Rightarrow R_o' = 78\Omega$

$\Rightarrow R_5 = (78 - r_{e8})(\beta+1)$

$= \underline{\underline{7.37\,k\Omega}}$

$A_3 = -\dfrac{R_5 // R_{i4}}{r_{e4} + R_4}$; $R_{i4} \cong 304\,k\Omega$

and $A_3 = -3.09$ V/V

and $A = 8513 \cdot \dfrac{3.09}{6.42} = \underline{\underline{4104}}$ V/V

The gain has been reduced by a factor of 2.07 and can be restored by reducing R_4 by this same factor to increase A_3, Thus $R_4 = \underline{1.11\,k\Omega}$

(Note that this is a first order approximation).

7.100

(a) $A_3 = \dfrac{-R_{i4}}{2.325\,k\Omega} = -\dfrac{303.5}{2.325}$

$= -130.5$ V/V

i.e A_3 is increased by $\dfrac{130.5}{6.42}$

$= 20.33$

$\Rightarrow A = 8513 \times 20.33$

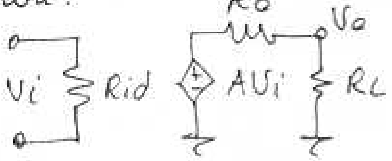
$= \underline{\underline{173.1 \times 10^3 \text{ V/V}}}$

(b) Let the output resistance of the current source be $R \to \infty$

$R_o' = 3K \,\|\, \left(\dfrac{R}{\beta+1} + r_e\right)$

$= 3K$

The amplifier can be modelled as shown:



Thus,

$A_{LOAD} = \dfrac{A \cdot R_L}{R_L + R_o}$

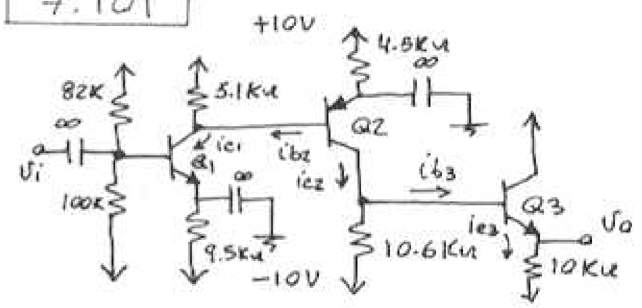
$= 173.1 \times 10^3 \dfrac{100}{100 + 3000}$

$= \underline{\underline{5583}}$ V/V

For the original amplifier:

$A_{LOAD} = 8513 \times \dfrac{100}{100 + 152} = \underline{\underline{3378}}$ V/V

7.101



(a) $I_{E1} = \dfrac{\dfrac{20V \times 100K}{82K + 100K} - 0.7}{9.5K + \dfrac{(82K \| 100K)}{\beta+1}}$

$\beta = 100 \Rightarrow I_{E1} = 1.03$ mA

$\alpha = \dfrac{100}{101} \Rightarrow I_{C1} = \underline{\underline{1.02\text{ mA}}}$

$V_{C1} \simeq 10V - 1.02\text{mA} \times 5.1\,k\Omega = 4.8V$

$I_{E2} = \dfrac{(10 - 0.7 - 4.8)V}{4.5\,k\Omega} = 1$ mA

$\rightarrow I_{C2} = \underline{0.99\text{ mA}}$

$V_{C2} \simeq 0.99\text{m} \times 10.6K - 10 = 0.5V$

$\Rightarrow V_{O\,DC} = 0.5 - 0.7 = \underline{-0.2V}$

$I_{E3} = \dfrac{-0.2 - (-10)}{10K} = 0.98$ mA

$\rightarrow I_{C3} = \underline{0.97\text{ mA}}$

Thus all transistors are operating at $I_C \simeq \underline{\underline{1\text{ mA}}}$

CONT.

(b) $R_{in} = 82K \parallel 100K \parallel r_{\pi 1}$

where $r_{\pi 1} = \dfrac{\beta}{g_{m1}} = \dfrac{100}{40m} = 2.5K\Omega$

$\Rightarrow R_{in} = (82 \parallel 100 \parallel 2.5)K = \underline{\underline{2.37 K\Omega}}$

$R_{out} = 10K \parallel \left[r_{e3} + \dfrac{10.6K}{\beta+1} \right]$

$= 10K \parallel \left[25 + \dfrac{10.6K}{101} \right]$

$= \underline{\underline{128\,\Omega}}$

(c) $\dfrac{i_{c1}}{v_i} = g_{m1} = 40 mA/V$

$\dfrac{i_{b2}}{i_{c1}} = \dfrac{5.1K}{5.1K + r_{\pi 2}} = \dfrac{5.1}{5.1 + 2.5} = 0.671 \dfrac{A}{A}$

$\dfrac{i_{e2}}{i_{b2}} = \beta_2 = 100\ A/A$

$\dfrac{i_{b3}}{i_{c2}} = \dfrac{10.6K}{10.6K + (\beta+1)(r_{e3} + 10K)}$

$= 0.01036\ A/A$

$\dfrac{i_{e3}}{i_{b3}} = \beta_3 + 1 = 101$

$v_o = i_{e3} \times 10K$

Thus,

$\dfrac{v_o}{v_i} = 10 \times 101 \times 0.01036 \times 100 \times 0.671 \times 40$

$= \underline{\underline{2.81 \times 10^4\ V/V}}$

(d) $f_{P2} = 1/(2\pi C_2 \cdot R_2)$

where: $R_2 = 5.1K \parallel r_{\pi 2}$

$= 5.1K \parallel 2.5K = 1.68 K\Omega$

$C_2 = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_{L2})$

with:

$R_{L2} = 10.6K \parallel (\beta+1)(r_{e3} + 10K)$

$= 10.6K \parallel 101 \times (25 + 10K)$

$= 10.5K\Omega.$

$\Rightarrow C_2 = 10p + 2p(1 + 40m \times 10.5K)$

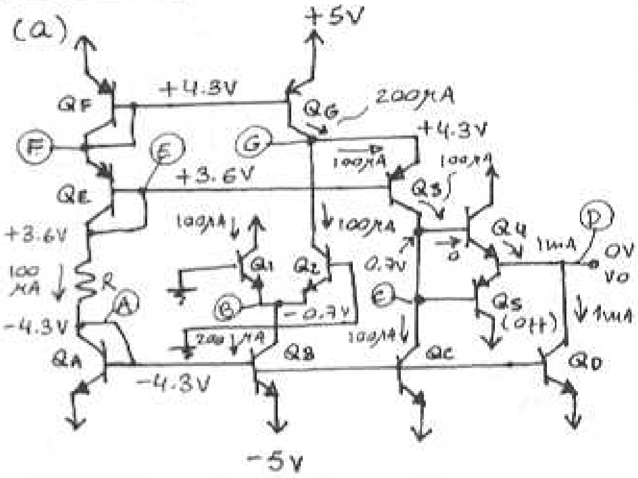
$= 852 pF$

$\Rightarrow f_{P2} = \dfrac{1}{2\pi \times 852p \times 10.5K}$

$= \underline{\underline{17.8}}\ KHz$

$\boxed{7.102}$

(a)



DC Analysis

$R = \dfrac{3.6 - (-4.3)}{100\mu A} = \underline{\underline{79 K\Omega}}$

Node voltages:

$V_A = \underline{\underline{-4.3V}}$, $V_B = \underline{\underline{-0.7V}}$

$V_C = \underline{\underline{+0.7V}}$, $V_D = \underline{\underline{0V}}$

$V_E = \underline{\underline{+3.6V}}$, $V_F = \underline{\underline{+4.3V}}$

$V_G = \underline{\underline{+4.3V}}$

(b)

Transistor	I_C (mA)	g_m (mA/V)	r_o (MΩ)
Q_1	0.1	4	2
Q_2	0.1	4	2
Q_3	0.1	4	2
Q_4	1.0	40	0.2
Q_5	0	0	∞
Q_A	0.1		
Q_B	0.2		
Q_C	0.1		2
Q_D	1.0		0.2
Q_E	0.1		
Q_F	0.1		
Q_G	0.2		1

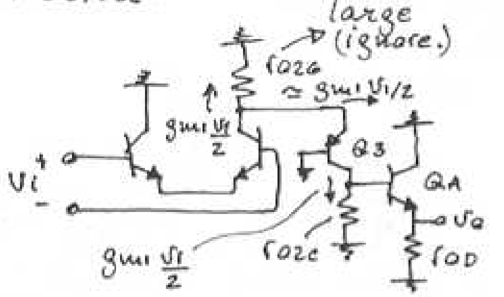
CONT.

(c) Total resistance at collector Q_3 is

$\approx \beta_3 r_{o3} \,//\, r_{oC} \,//\, (\beta_4+1)(r_{o4} \,//\, r_{oD})$

$= 100\times 2 \,//\, 2 \,//\, 101\,(0.2 \,//\, 0.2)$

$= 1.65\,M\Omega$



$\frac{v_{c3}}{v_i} = +g_{m1} \times \frac{1}{2} \times 1.65\times 10^3 = 3300\ \frac{V}{V}$

$\frac{v_o}{v_{c3}} \simeq 1$

Thus, $\frac{v_o}{v_i} \simeq \underline{3300}$ V/v (polarity correct)

(d) $R_{in} = 2 r_\pi$

$= 2\times\frac{100}{4} = \underline{50\,K\Omega}$

$R_{out} = r_{oD} \,//\, r_{o4} \,//\, \left[r_{e4} + \frac{r_{o2C} \,//\, \beta_3 r_{o3}}{\beta+1}\right]$

$= 0.2 \,//\, 0.2 \,//\, \left[25.10^{-6} + \frac{2 \,//\, 100\times 2}{101}\right]$

$\simeq \underline{16.4\,K\Omega}$

(e) $v_{ICM}|_{min} = -4.3 - 0.4 + 0.7$
$= \underline{-4V}$

$v_{ICM}|_{max} = V_G + 0.4 = \underline{+4.7V}$

(f) The voltage at the base of Q4 can rise to V_{B3}:
$(V_E) + 0.4 = +4V$
before Q3 saturating. Thus v_o can go up to $\underline{+3.3V}$

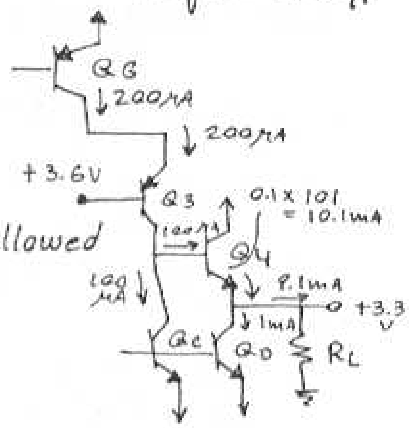
The voltage at the output can go down to $V_{\text{base of } Q_D} + 0.4$
$= V_A - 0.4 = -4.3 - 0.4 = \underline{-4.7V}$

Thus the linear range at the output is $\underline{-4.7V \text{ to } +3.3V}$

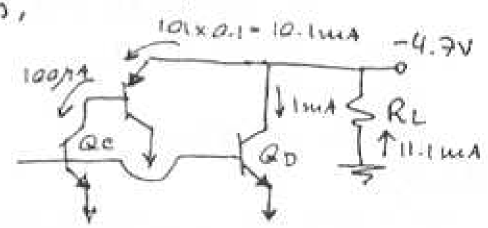
(g) At the positive limit of v_o, i.e. $v_o = +3.3V$ and Q_2 just cut off

$R_L = \frac{3.3V}{9.1mA}$
$= \underline{363\,\Omega}$

(this is the minimum allowed R_L for +3.3V output)



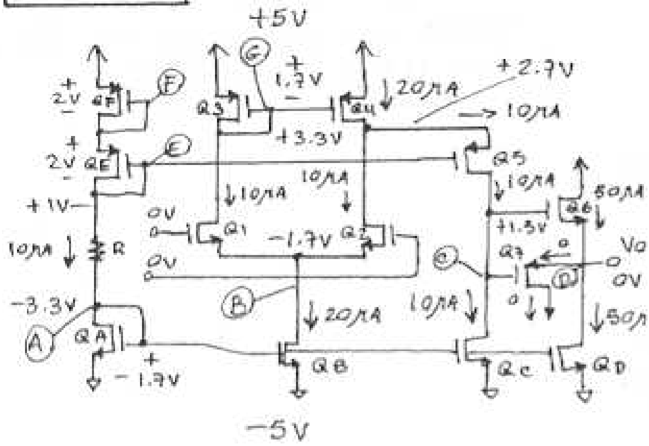
At the negative limit of v_o, i.e. $v_o = -3.3V$ and Q_1 has cut-off. Q_3 will also be cut-off, and Q_4 will cut-off.
Thus,



$R_L = \frac{4.7}{11.1\,mA} = \underline{423\,\Omega}$

This is the minimum allowed R_L for a -4.7V output.

7.103



DC analysis.

(a) $I_{REF} = 10\mu A = \frac{1}{2} \times 40 \times \frac{5}{5}(V_{GS_A} - V_t)^2$

$\Rightarrow V_{GS_A} = 1.71V \simeq \underline{1.7V}$

$10 = \frac{1}{2} \times 20 \times \frac{5}{5}(V_{GS_{E,F}} - 1)^2$

$\Rightarrow V_{GS_{E,F}} = 2V$

$R = \frac{3 - (-3.3)}{10\mu A} = \underline{\underline{660 K\Omega}}$

(b) See figure above

$V_{GS1} = V_{GS2} = V_{GSA} \simeq 1.7V$

$V_{GS3} = \sqrt{\frac{2 \times 10}{20 \times \frac{10}{3}}} + 1 = 1.71V \simeq 1.7V$

$V_{GS5} = V_{GS3} = 1.7V$

For Q_6: $50 = \frac{1}{2} \times 40 \times \frac{50}{5}(V_{GS_6} - V_t)^2$

$\Rightarrow V_{GS6} = 1.50V$

$V_A = \underline{\underline{-3.3V}}$ $V_B = \underline{\underline{-1.7V}}$

$V_C = +\underline{\underline{1.5V}}$ $V_D = \underline{\underline{0V}}$

$V_E = +\underline{\underline{1V}}$ $V_F = \underline{\underline{+3V}}$

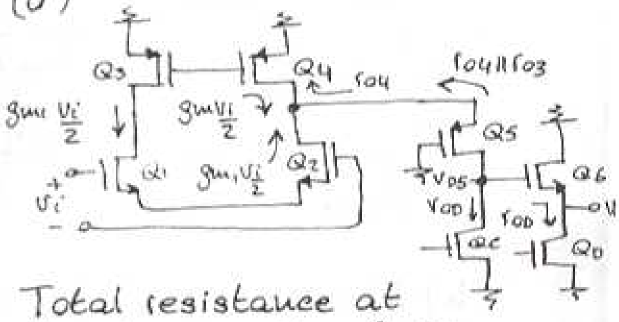
$V_G = +\underline{\underline{3.3V}}$ $V_H = \underline{\underline{+2.7V}}$

(c)

Transistor	I_D (μA)	V_{GS} (V)	g_m (μA/V)	r_o (MΩ)
Q_1	10	1.7	28.3	5
Q_2	10	1.7	28.3	5
Q_3	10	1.7	28.3	5
Q_4	20	1.7	56.6	2.5
Q_5	10	1.7	28.3	5
Q_6	50	1.5	200	1
Q_7	0	-1.5 (*)	0	∞
Q_A	10	1.7	28.3	5
Q_B	20	1.7	56.6	2.5
Q_C	10	1.7	28.3	5
Q_D	50	1.7	141.4	1
Q_E	10	2	20	5
Q_F	10	2	20	5

(*) cut-off.

(d)



Total resistance at the drain of Q_5, R, is:

$R = (g_{m5} r_{o5})(r_{o4} \| r_{o2}) \| r_{oC}$

$= [(28.3 \times 5)(2.5 \| 2)] \| 5$

$= 4.9 M\Omega$

Thus, $\frac{V_{ds}}{V_i} = g_{m1} R$

$= 28.3 \times 4.9 = 138.7 \ V/V$

and $\frac{V_o}{V_{ds}} = \frac{(r_{oD} \| r_{o6})}{(r_{oD} \| r_{o6}) + \frac{1}{g_{m6}}}$

$= \frac{(1 \| 1)}{(1 \| 1) + \frac{1}{200}} \simeq 1$

$\frac{V_o}{V_i} = \underline{\underline{138.7}} V/V$

$R_{in} = \infty$

$R_{out} = r_{oD} \| r_{o6} \| 1/g_{m6}$

$= 1 \| 1 \| 1/200 \ M\Omega$

$\simeq \underline{\underline{5K\Omega}}$

CONT.

(e) $v_{ICM}\big|_{max} = V_G + V_t$
$= +\underline{\underline{4.3V}}$

$v_{ICM}\big|_{min} = V_{GS1} + V_{Bmin}$
$= V_{GS1} + V_A - V_t$
$= 1.7 - 3.3 - 1 = \underline{\underline{-2.6V}}$

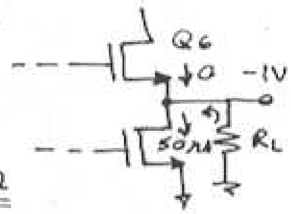
(f) $v_{O\,max} = v_{C\,max} - V_{GS6}$
$= V_E + |V_t| - V_{GS6}$
$= +1 + 1 - 1.5 = \underline{\underline{+0.5V}}$

$v_{O\,min} = V_A - V_t = -3.3 - 1 = \underline{\underline{-4.3V}}$

(g) Q_6 cuts off
thus,

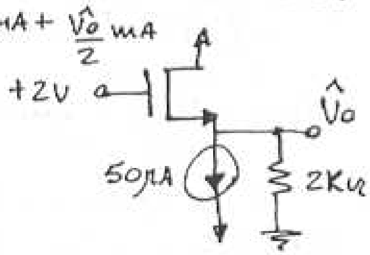
$\frac{1V}{R_L} = 50\mu A$

$R_L = \frac{1V}{50\mu A} = \underline{\underline{20\,K\Omega}}$



(h) Maximum possible voltage at drain of Q_5 is +2V.
At this value we have:

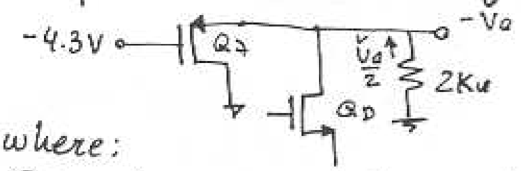
$I_D = 50\mu A + \frac{\hat{V}_o}{2}\,mA$



$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\,(2 - \hat{V}_o - V_t)^2$

$\Rightarrow \hat{V}_o \simeq 0.17V$

For the lowest possible output, the circuit becomes



where:
Q_6 cuts off and Q_7 conducts

$I_D = \frac{\check{V}_o}{2} - 0.05mA$

$= \frac{1}{2}\mu_p C_{ox}\,\underbrace{\left(\frac{100}{5}\right)}_{(W/L)_7}\,(-\hat{V}_o + 4.3 - 1)^2$

$\Rightarrow \hat{V}_o = 1.45V$

That is, the range of v_O is $\underline{\underline{-1.45V}}$ to $+\underline{\underline{0.17V}}$

Chapter 8 – Problems

8.1

$$A_F = \frac{A}{1+A\beta} = 100$$

$$A\beta = \frac{10^5}{100} - 1 = 999$$

$$\Rightarrow \beta = \frac{999}{10^5} = 9.99 \times 10^{-3}$$

$$A = 10^3, \quad A_F = \frac{10^3}{1+10^3(9.99\times10^{-3})}$$

$$= 90.99$$

$$\frac{\Delta A_f}{A_f} = \frac{90.99-100}{100} \Rightarrow -9\%$$

8.2

(b) $A_f = 10 = \dfrac{100}{1+100\beta} \Rightarrow \beta = 90\times10^{-3}$

$\beta = \dfrac{R_1}{R_1+R_2} \Rightarrow \dfrac{R_2}{R_1} = \dfrac{1}{\beta} - 1 = 10.11$

(c) amount of feedback $= 1+A\beta$

$= 1 + 100(90\times10^{-3}) = 10 \equiv 20\,dB$

(d) $V_o = 10V_s = 10V$

$V_f = \beta V_o = 90\times10^{-3}\times10 = 0.9V$

$V_i = V_s - V_f = 1-0.9 = 0.1V$

(e) $A_f = \dfrac{80}{1+80(90\times10^{-3})} = 9.756$

$\dfrac{\Delta A_f}{A_f} = \dfrac{9.756-10}{10} \Rightarrow -2.44\%$

8.3

(b) $A_f = 10^3 = 10^3/(1+10^3\beta)$

$\Rightarrow \beta = 900\times10^{-6}$

$\dfrac{R_2}{R_1} = \dfrac{1}{\beta} - 1 = 1110$

(c) $1+A\beta = 1+10^3(900\times10^{-6}) = 10 \equiv 20\,dB$

(d) $V_s = 0.01V$

$V_o = 10^3 V_s = 10V$

$V_f = \beta V_o = 900\times10^{-6}\times10 = 9mV$

$V_i = V_s - V_f = 1mV$

(e) $A = 8000$

$A_f = \dfrac{8000}{1+8000(900\times10^{-6})} = 975.6$

$\dfrac{\Delta A_f}{A_f} = \dfrac{975.6-1000}{1000} \equiv -2.44\%$

8.4

All output voltage is fed back $\therefore \beta = 1$

$A_f = \dfrac{100}{1+100\times1} = 0.99$

$1+A\beta = 1+100\times1 = 101 \equiv 40.1\,dB$

$V_o = 0.99V_s = 0.99V$

$V_i = V_s - V_o\beta = 1-0.99 = 10mV$

$A = 90 \Rightarrow A_f = \dfrac{90}{1+90\times1} \approx 0.989$

$\dfrac{\Delta A_f}{A_f} = \dfrac{0.989-0.99}{0.99} \equiv -0.1\%$

8.5

$V_s = 1V, \quad V_i = 10mV, \quad V_o = 10V$

$A_o = (V_o/V_i) = 10V/10mA = 1000$

$V_i = V_s - V_f = V_s - \beta V_o = 1-\beta 10$

$\Rightarrow \beta = (1-10^{-2})10 = 0.099$

8.6

$$A_F = \frac{A_o}{1+A_o\beta} = \frac{1}{1/A_o + \beta} = \frac{1}{\beta(1+1/A_o\beta)}$$

so $A_f \to 1/\beta$ will be within $x\%$ when $1/(A_o\beta) = 0.01 \times x$

(a) For 1%: $A_o\beta = 1/0.01 = 100$

Many possible solutions.

Let $A_o = 10^5 \to A_o\beta = 100 \Rightarrow \beta = 10^{-3}$

(b) For 5%: $A_o\beta = 1/0.05 = 20$

Let $A_v = 10^5 \to A_o\beta = 20 \Rightarrow \beta = 2\times10^{-4}$

(c) For 10%: $A_o\beta = 1/0.1 = 10$

Let $A_o = 10^5 \to A_o\beta = 10 \Rightarrow \beta = 10^{-4}$

(d) For 50%: $A_o\beta = 1/0.5 = 2$

Let $A_o = 10^5$: $A_o\beta = 2 \Rightarrow \beta = 2\times10^{-5}$

% error	A_o	$A_o\beta$	$1+A_o\beta$
1	10^5	100	101
5	10^5	20	21
10	10^5	10	11
50	10^5	2	3

8.7.

$0 \le \beta \le 1$ Linear

(a) For $A_o = 1$: $A_{f_1} = \dfrac{A_o}{1+A_o\beta} = \dfrac{1}{1+0} = 1\,V/V$

$A_{f_2} = \dfrac{1}{1+1\times0.5} = 0.667\,V/V$

$A_{f_3} = \dfrac{1}{1+1\times1} = 0.5\,V/V$

(b) For $A_o = 10$: $A_{f_1} = \dfrac{10}{1+0} = 10\,V/V$

$A_{f_2} = \dfrac{10}{1+10/2} = 1.\dot{6}\,V/V$

$A_{f_3} = \dfrac{10}{1+10\times1} = 0.909\,V/V$

(c) For $A_o = 100$: $A_{f_1} = \dfrac{100}{1+0} = 100\,V/V$

$A_{f_2} = \dfrac{100}{1+100/2} = 1.96\,V/V$

$A_{f_3} = \dfrac{100}{1+100} = 0.99\,V/V$

(d) For $A_o = 10^4$: $A_{f_1} = \dfrac{10^4}{1+0} = 10^4\,V/V$

$A_{f_2} = \dfrac{10^4}{1+10^4/2} = 1.99\,V/V$

$A_{f_3} = \dfrac{10^4}{1+10^4} = 0.9999\,V/V$

8.8

A_o : 2mV $\rightarrow$ 10V

$A_o = 10V/(2\times10^{-3})V = 5000 \doteq 74\,dB$

A_F : 200mV $\rightarrow$ 10V

$A_F = (10^4/(200) = 500 \doteq 54dB$

$A_F = \dfrac{A_o}{1+\beta A_o} = \dfrac{5000}{1+5000\beta} = 500$

$\Rightarrow 1+5000\beta = 10$

$\Rightarrow \beta = 9/5000 = 0.0018 \equiv -54dB$

$(1+A_o\beta) = 10 \equiv 20dB$

$A_o\beta = 5000(9/5000) = 9 \equiv 19.08dB.$

8.9

$\dfrac{dA_f}{A_f} = \dfrac{1}{1+A\beta}\cdot\dfrac{dA}{A}$

$\dfrac{dA_f/A_f}{dA/A} = \dfrac{1}{1+A\beta} \equiv -20dB$

$\Rightarrow 1+A\beta = +20dB \equiv 10$

$\therefore A\beta = 9$

Require $\dfrac{1}{1+A\beta} = \dfrac{1}{2} \Rightarrow A\beta = 1$

8.10

$A_o \equiv 1000 \pm 30\%$ want $A_f = 100 \pm 1\%$

To reduce % change in A_o we need

$\dfrac{1}{1+A\beta} \doteq \dfrac{1}{30} \Rightarrow A_f = \dfrac{1000}{30} < 100$

For single stage $A_F = \dfrac{A}{1+A\beta_1} \Rightarrow \dfrac{1}{1+A\beta_1} = \dfrac{100}{1000} = \dfrac{1}{10}$

For two stages (identical) $A_2 = \dfrac{A}{1+A\beta_2} = 10$

$\Rightarrow (1+A\beta_2) = 1000/10 = 100$

Thus each stage has $\pm 30/100\,\% = \pm0.3\%$

But two such stages may give $\pm 0.6\%$ OK

[3 stages $A_3 = A_f^{1/3} = 100^{1/3}$

$(1+A_o\beta_3) = 1000/100^{1/3} \doteq 215$

Now each stage has $\pm 0.14\%$]

8.11

Gain desensitivity factor $= [1+A_o\beta]$

$A_F = \dfrac{A_o}{1+A_o\beta} = \dfrac{10^5}{1+10^5\beta} = 10^3$

$\Rightarrow \beta = 0.99 \times 10^{-3}$

From $\dfrac{dA_F}{A_f} = \dfrac{1}{1+A\beta} \cdot \dfrac{dA}{A}$

(a) When A drops 10%

then A_F drops by $\dfrac{10\%}{1+A_0\beta} = \dfrac{10\%}{10^5/10^3} \cong 0.1\%$

But using $A_F = \dfrac{A}{1+A\beta} = \dfrac{0.9\times10^5}{1+(0.9\times10^5)(99/10^5)}$

$= 998.89$

Corresponding % $\dfrac{1000-998.89}{1000} \cong 0.11\%$ (close)

(b) When A drops 30%

then A_F drops by $\dfrac{30\%}{100} = 0.3\%$

while $A_F = \dfrac{0.7\times10^5}{1+(0.7\times10^5)(99/10^5)} = 995.7$

and $\dfrac{1000-995.7}{1000} \cong 0.43\%$ (not so close)

8.12

$$A(s) = A_m \frac{s}{s+\omega_L}$$

$$A_f(s) = \frac{A_m \frac{s}{s+\omega_L}}{1 + \frac{A_m s}{s+\omega_L}\beta} = \frac{A_m\, s}{s+\omega_L+A_m\beta s}$$

$$= \frac{A_m}{1+A_m\beta} \cdot \frac{s}{s+\frac{\omega_L}{1+A_m\beta}}$$

Thus

$$A_{mf} = \frac{A_m}{1+A_m\beta}$$

$$\omega_{Lf} = \frac{\omega_L}{1+A_m\beta}$$

Both decreased by same amount

8.13

Worst case: $A_{F1} = \dfrac{A_0}{1+A_0\beta} = 9.8$ (down 2%)

full battery: $A_{F2} = \dfrac{2A_0}{1+2A_0\beta} = 10$

from A_{F1}: $1+A_0\beta = A_0/9.8$

$\therefore \beta = \dfrac{1}{9.8} - \dfrac{1}{A_0}$

Then $A_{F2} = \dfrac{2A_0}{1+2A_0\left[\frac{1}{9.8}-\frac{1}{A_0}\right]} = 10$

$\Rightarrow 1+2A_0\left[\frac{1}{9.8}-\frac{1}{A_0}\right] = \dfrac{2A_0}{10}$

$\Rightarrow 2A_0\left[\frac{1}{9.8}-\frac{1}{A_0}\right] = 2-1$

$\therefore 2A_0 = 490$

[Check $\dfrac{2A_0}{1+2A_0\left[\frac{1}{9.8}-\frac{2}{2A_0}\right]} = 10$ ← β const

$\dfrac{A_0}{1+A_0\left[\frac{1}{9.8}-\frac{1}{A_0}\right]} = 9.8$]

8.14

$A_f = \dfrac{A_0}{1+A_0\beta} = 10 = \dfrac{100}{1+100\beta}$

$\therefore (1+A_0\beta) = 100/10 = 10$

$f_L' = f_L/(1+A_0\beta) = 100/10 = 10\text{ Hz}$

$f_h' = f_H(1+A_0\beta) = 10K\times10 = 100\text{ KHz}$

8.15

Need pole at 0.5MHz. any others should be well above for stability.

$f_H = 10\text{KHz}$ and $f_{Hf} = 500\text{KHz}$

$\therefore$ need $(1+A\beta) = 500/10 = 50$

Thus $A_{f1} = \dfrac{A}{1+A\beta} = \dfrac{1000}{50} = 20$

and $\beta = (50-1)/1000 = 4.9\times10^{-3}$

Subsequent stages must provide gain

$A_{Total}/A_{f1} = 1000/20 = 50$

Let $f_{H2} = 3\times f_{H1} = 3\times500 = 1500\text{KHz}$

needs $(1+A\beta_2) = \dfrac{1500}{10} = 150$

Then $A_{f2} = \dfrac{1000}{150} = 6.67$

and $\beta_2 = \dfrac{150-1}{1000} = 0.149$

This leaves stage 3 with $A_{f3} = \frac{50}{6.67} = 7.5$

This gain needs $1 + A\beta = \frac{1000}{7.5} = 133.3$

and $f_{Hf3} = 10K(1+A\beta) = 1.33\text{ MHz}$

this is safely well above 500 KHz

and $\beta_3 = 0.132$

The actual f_{Hf} is given by

$$f_{Hf} = \frac{1}{\left[\left(\frac{1}{0.5}\right)^2 + \left(\frac{1}{1.5}\right)^2 + \left(\frac{1}{1.33}\right)^2\right]}$$

$= 0.447\text{ MHz}$ close!

Using a 4 stage amplifier would allow lower stage gains and higher f_H's

8.16

$$V_o = V_S \frac{A_1 A_2}{1 + A_1 A_2 \beta} + V_N \frac{A_1}{1 + A_1 A_2 \beta}$$

and $\frac{S}{N} = \frac{V_S}{V_N} A_2$

if we raise V_S by factor A_2 then S/N is improved by A_2 also

To reduce V_N from $\pm 1V$ to $\pm 1mV$ we must have $A_2 = 1000$, then

$$V_{os} = \frac{A_1 A_2}{1 + A_1 A_2 \beta} = \frac{1000 \times 0.9}{1 + 1000 \times 0.9\beta} = 10$$

$\Rightarrow \beta = 0.099$

if $A_2 = 100$; $V_n = \pm 10mV$

$$V_{os} = \frac{100 \times 0.9}{1 + 100 \times 0.9\beta} = 10$$

$\Rightarrow \beta = 0.089$

if $A_2 = 10$; $V_n = \pm 100mV$

$A_1 A_2 = 0.9 \times 10$ $\therefore V_o = 9V < 10V$!!

8.17

$$A_F = \frac{A_0}{1 + A_0\beta}$$

$$\frac{\Delta A_F / A_F}{\Delta A_0 / A_0} = \frac{1}{1 + A_0\beta}$$

Then for 1% change in A_F for 90% change in A_0

$1 + A_0\beta = 90 \Rightarrow A_0\beta = 89$

Then $A_F = 100 = \frac{A_0}{90} \Rightarrow A_0 = 9000$

Let $A_0^* \to 10 \times A_0$ then for $\beta = 89/A_0$

$$A_F = \frac{9000 \times 10}{1 + (9000 \times 10)(89/9000)} = 101.12$$

Therefore, select $A_0 \geq 9 \times 10^4$

and $\beta = 89/A_0$

if $A_0^* \to 100 A_0$ + same β.

$$A_F = \frac{9000 \times 100}{1 + (9000 \times 100)(89/9000)}$$

$\approx 9000 \times 100 / 8900$

≈ 101.12

if $A_0^* \to \infty$

$A_F \Rightarrow A_0/(A_0\beta) \Rightarrow 1/\beta$.

$\approx 9000/89 = 101.12$

Ideally select A_0 as high as possible

8.18

A_1 has f_{H1} high, A_2 has $A_m = 10V/V$ with $f_L = 80Hz$, $f_H = 8KHz$.

$$A_F = \frac{A_1 A_2}{1 + A_1 A_2 \beta} = 100$$

Require $f_{Hf} = 40KHz = 8(1 + A_1 A_2 \beta)$

$\therefore 1 + A_1 A_2 \beta = 40/8 = 5$

and $A_F = \frac{A_1 A_2}{5} = 100 \Rightarrow A_1 A_2 = 500$

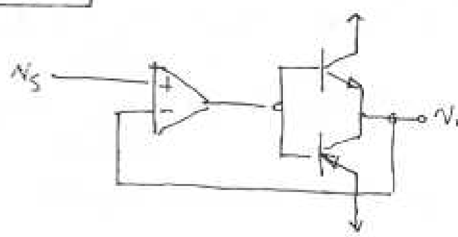
$\Rightarrow A_1 = 500/A_2 = 500/10 = 50$

$1 + A_1 A_2 \beta = 5 \Rightarrow \beta = 4/A_1 A_2 = 4/500$

$\therefore \beta = 0.08$

$f_{Lf} = f_L/(1 + A_1 A_2 \beta) = 80/5 = 16Hz$

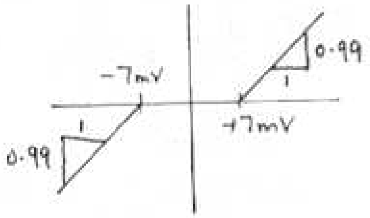
8.19



Dead band with be narrowed by the factor $1 + A\beta = 1 + A$ since $\beta = 1$ and since $A \gg 1$, $1 + A \to A$

∴ new limits are $\pm \frac{0.7}{A} = \pm \frac{0.7}{100}$

$= \pm 7\,mV$



New slope ≡ gain = $A_f = \frac{A}{1+A}$

$\Rightarrow \frac{100}{1+100} = 0.99$

8.20

For $A = v_o/v_I = 10^2$ (select lowest A_o) to reduce % change in gain by factor of 10

$1 + A\beta = 10 \Rightarrow \beta = 9/10^2$

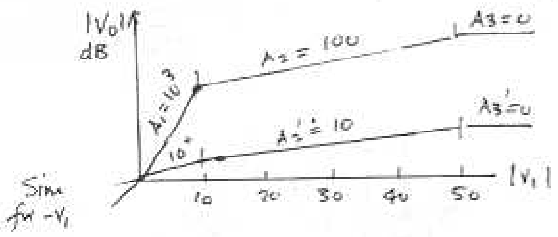
For $A_2 = 10^2$: $A_{2F} = 10^2/10 = 10$

For $A_1 = 10^3$: $A_{1F} = \frac{A}{1+A\beta}$

$\therefore A_{2F} = \frac{10^3}{1+10^3(9/10^2)} = \frac{10^3}{91} = 10.98$

For $A_3 = 0$: stays saturated

[For 10mV in and $A_1 = 10^3$, $v_o = 10\,V$?
For 10mV in and $A_2 = 10^2$, $v_o = 1\,V$]



8.21

$V_s = 100\,mV$, $V_f = 95\,mV$, $V_o = 10\,V$

$V_i = V_s - V_f = 100 - 95 = 5\,mV$

$A_v = \frac{V_o}{V_i} = \frac{10\,V}{5\,mV} = 2\times10^3\ V/V$

$\beta = \frac{V_f}{V_o} = \frac{95\times10^{-3}}{10} = 9.5\times10^{-3}\ V/V = 0.0095$

8.22

$I_s = 100\,\mu A$, $I_F = 90\,\mu A$, $I_o = 10\,mA$

$A \equiv \frac{I_o}{I_s - I_F} = \frac{10\times10^{-3}}{(100-90)\times10^{-6}} = 10^3\ A/A = 1\,mA/\mu A$

$\beta \equiv \frac{I_F}{I_o} = \frac{90\,\mu A}{10\,mA} = 9\,\mu A/mA$

8.23

For r_{o1}, r_{o2}, R_s very large in Fig 8.5

At G1: $I_s = I_F$ since $I_{G1} = 0$

$V_{G1} = [R_1 + R_2]I_s + I_o R$

$V_{D1} = -g_{m1} V_{g1} R_{L1}$
$= -g_{m1} R_L [I_s(R_1+R_2) + I_o R_1]$

$I_o = g_{m2}(V_{D1} - V_{S2})$
$= g_{m2}[(-g_{m1}R_{L1})[I_s(R_1+R_2) - I_s R_1 - I_o R_1$

$\therefore \frac{I_o}{I_s} = -\frac{g_{m2}[g_{m1}R_{L1}(R_1+R_2)]}{1 + g_{m2}g_{m1}R_{L1}R_1 + g_{m2}R_1}$

$A_f = -\frac{R_1 + g_{m1}R_{L1}(R_1+R_2)}{R_1 + 1/g_{m2} + g_{m1}R_{L1}R_1}$

QED

$R_{IN} \equiv \frac{V_s}{I_s} = I_s[R_1+R_2] + I_o R_1$
$= I_s[R_1 + R_2 + A_f R_1]$

$\therefore R_{IN} = R_1 + R_2 + A_f R_1$

QED

(b) when $g_{m1}R_{L1} = 100$, $R_L = 10K$, $R_2 = 90K$ and $g_{m2} = 5\,mA/V$

$A_f = -\frac{R_1 + g_{m1}R_{L1}(R_1+R_2)}{R_1 + 1/g_{m2} + g_{m1}R_{L1}R_1}$

$= -\frac{10^4 + 100(100\times10^3)}{10^4 + 1000/5 + 100\times10^4}$

$= -9.91\ A/A \approx -10\ A/A$

$R_{IN} = 10K + 90K - 10K \times 10K \approx 0$

or $10K + 90K - 9.91 \times 10K$

$= 100 - 99.1K = 0.9K$

(c) $\beta \equiv \frac{I_F}{I_o} = \frac{R_1}{R_1+R_2} \Rightarrow \beta = \frac{10}{10+90} = \frac{1}{10}$

from above $A_F \approx -10 \equiv -1/\beta$

8.24

series - series

$V_s = 100mV$, $V_f = 95mV$, $I_o = 10mA$

$V_i = V_s - V_f = 100 - 95 = 5mV$

$V_f = \beta I_o \Rightarrow \beta = \frac{95mV}{10mA} = 9.5V/A$

$A \equiv \frac{I_o}{V_i} = \frac{10mA}{5mV} = 2\,mA/V$

$A_F \equiv \frac{I_o}{V_s} = \frac{A}{1+A\beta} = \frac{2}{1+2(9.5)} = 0.1mA/V$

8.25

shunt- shunt

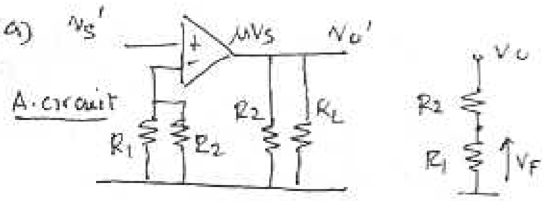
$I_s = 100\mu A$, $I_F = 95mA$, $V_o = 10V$

$A \equiv \frac{V_o}{I_i} = \frac{V_o}{I_s - I_F} = \frac{10}{(100-95)\times 10^{6}} = 2\times 10^6\,V/A$

$\beta \equiv \frac{I_F}{V_o} = \frac{95}{10}\frac{\mu A}{V} = 9.5\,\mu A/V$

8.26

(a)



series - shunt; sample V, return V_F

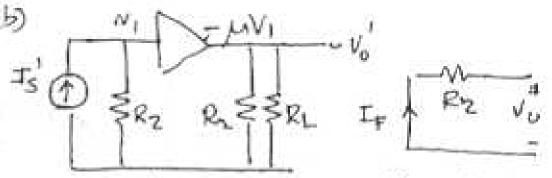
$\beta \equiv \frac{V_F}{V_o} = \frac{R_1}{R_1+R_2}$

$A \equiv \frac{V_o'}{V_s'} = \mu$

$A_F \equiv \frac{V_o}{V_s} = \frac{A}{1+A\beta} = \frac{\mu}{1+\mu\frac{R_1}{R_1+R_2}}$

$\Rightarrow \frac{R_1+R_2}{R_1} = 1/\beta$

(b)



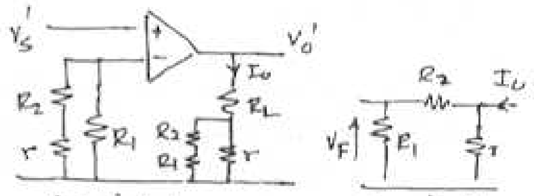
A-circuit B-circuit

$\beta \equiv \frac{I_F}{V_o} = -\frac{1}{R_2}$ $A \equiv \frac{V_o'}{I_s'} = -\mu R_2$

$A_F \equiv \frac{V_o}{I_s} = \frac{A}{1+A\beta} = \frac{-\mu R_2}{1+\mu R_2/R_2} = -\frac{\mu R_2}{1+\mu}$

$\Rightarrow -R_2 = -1/\beta$

(c)



A circuit B-circuit

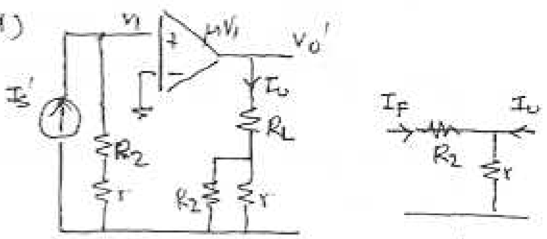
series - series.

$\beta \equiv \frac{V_F}{I_o} = \frac{R_1 r}{R_1+R_2+r}$ let $r^* = r \| (R_1+R_2)$

$A \equiv \frac{I_o}{I_s} = \frac{\mu}{R_L + r\|(R_1+R_2)} = \frac{\mu}{R_L+r^*}$

$A_F \equiv \frac{A}{1+A\beta} = \frac{\frac{\mu}{R_L+r^*}}{1+\frac{\mu}{R+r^*}\cdot\frac{R_1 r}{R_1+R_2+r}}$

(d)



A-Circuit B-Circuit

shunt-series : return I_F sample I_o

$\beta \equiv \frac{I_F}{I_o} = \frac{r}{R_2+r}$, $A \equiv \frac{V_o'}{I_s'} = \frac{\mu(R_2+r)}{R_L + r\|R_2}$

$$\frac{I_o}{I_s} = \frac{\dfrac{\mu(R_2+r)}{R_L + r\|R_2}}{1 + \dfrac{\mu(R_2+r)}{R_L+r\|R_2}\cdot\dfrac{r}{r+R_2}}$$

$$\underset{\mu \gg 1}{\Rightarrow} \frac{1}{r/(r+R_2)} \equiv \frac{1}{\beta}$$

Section 8.4 The Series-Shunt Feedback Amplifier

8.27

$$A_F = \frac{A}{1+A\beta} = \frac{10^3 \times 2}{1 + 2.10^3 \times 0.1} = 9.95\ V$$

$$R_{if} = R_1(1+A\beta) = 1(201) = 201\,K\Omega$$

$$R_{of} = R_o/(1+A\beta) = 1/201 = 4.975\,K\Omega$$

8.28

Here R_o is lowered by amount of feedback

i.e. $(1+A\beta) = 80$

$\Rightarrow A\beta = 79$

$R_o = R_{of}(1+A\beta) = 100 \times 80 = 8\,K\Omega$

8.29

$A_o = 10^4\ V/V \qquad f_t = 1\,MHz$

20 dB/dec roll off $\therefore f_P = f_t/A_o = 100\,Hz$

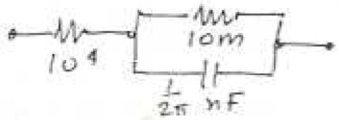
$$A(f) = \frac{A_o}{1 + jf/f_P} = \frac{10^4}{1+jf/100}$$

$$\text{loop gain} = A(f)\beta = \frac{10^4(0.1)}{1+jf/100} = \frac{10^5}{jf+100}$$

$$Z_{if} = R_1(1+A\beta) = 10^4\left(1 + \frac{10^3}{1+jf/100}\right)$$

$$= 10^4 + \frac{10^7}{1+jf/100} \equiv R_A + \frac{R_B}{1+sR_BC}$$

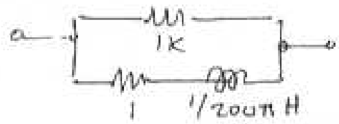
Thus



$$Z_{of} = \frac{R_o}{(1+A\beta)} = \frac{10^3}{1 + 10^3/(1+jf/100)}$$

$$\therefore Z_{of} = \frac{1}{\dfrac{1}{10^3} + \dfrac{1}{1+jf/100}}$$

$$= \frac{1}{\dfrac{1}{R_A} + \dfrac{1}{R_B + j\omega L}}$$



At $f = 10^3$

$$|Z_{if}(10^3)| = \left|10^4 + \frac{10^7}{1 + j\,10^3/100}\right| = 1\,M\Omega$$

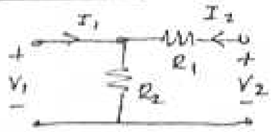
$$|Z_{of}(10^3)| = \left|\frac{1}{\frac{1}{10^3} + \frac{1}{1+j10^2}}\right| \approx 10\,\Omega$$

At $f = 10^5$

$$|Z_{if}(10^5)| = \left|10^4 + \frac{10^7}{1 + j\,10^5/100}\right| = 14.1\,K$$

$$|Z_{of}(10^5)| = \left|\frac{1}{\frac{1}{10^3} + \frac{1}{1+j10^3}}\right| = 700\,\Omega$$

8.30



$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

(a) $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0} = R_1 \| R_2 = \dfrac{R_1R_2}{R_1+R_2}\ \Omega$

$h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0} = \dfrac{R_2}{R_1+R_2}\ V/V$

$h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0} = -\dfrac{R_2}{R_1+R_2}\ A/A$

$h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0} = \dfrac{1}{R_1+R_2}\ \mho$

(b) $\beta = \left.\dfrac{V_1}{V_2}\right|_{I_1=0} = h_{12} = \dfrac{R_2}{R_1+R_2}$

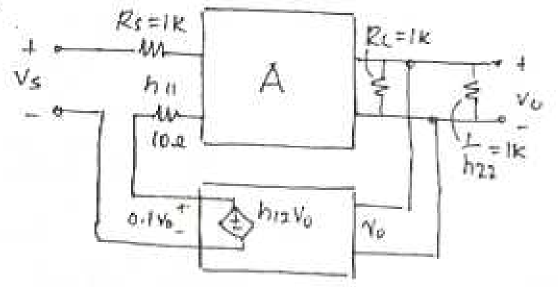
$\Rightarrow \dfrac{R_1}{R_2} = \dfrac{1}{\beta} - 1 = 99$

$\Rightarrow R_2 = R_1/99 = 10.1\,\Omega$

Thus $h_{11} = 10\,\Omega$; $h_{12} = 0.01\ V/V$

$h_{21} = -0.01\ A/V$; $h_{22} = 0.99 \times 10^3$

(c)



8.31

$R_{of} = \frac{R_o}{1+A\beta} = \frac{1000}{1+A\beta} = 100\Omega$

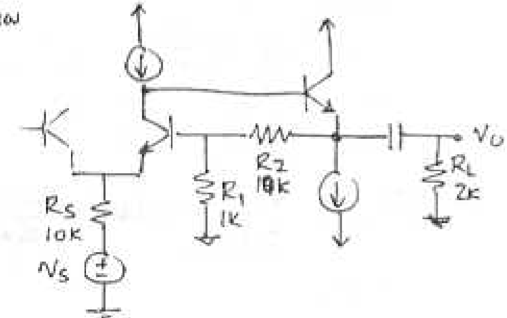
$\Rightarrow 1 + A\beta = 1000/100 = 10$

$\Rightarrow A_F = \frac{A}{1+A\beta} = \frac{100}{10} = 10\,V/V$

if $\beta = 1$: $R_{of} \Rightarrow \frac{R_o}{1+A} = \frac{1000}{1+10^4} = 9.9\Omega$

8.32

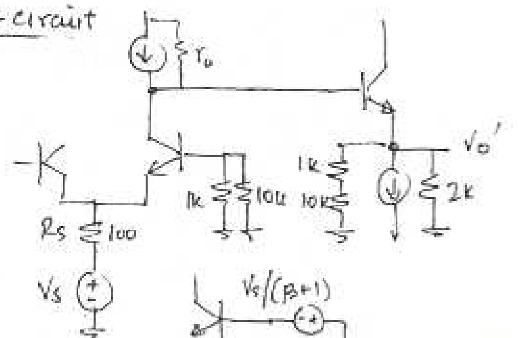
Redraw

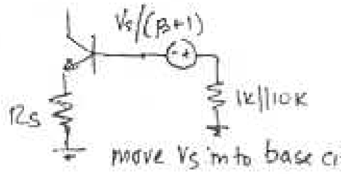


$r_{e1} = V_T/I_E = 25mV/0.1mA \approx 250\Omega$

$r_{e2} = 25/1 = 25\Omega$

A-circuit





move Vs into base circuit

$R_i = R_S + r_{e_1} + \frac{(R_1\|R_2)}{1+\beta}$

$= 100 + 250 + 909/101 = 700\Omega$

$R_o = R_L\|(R_1+R_2)\|(r_e + r_{o1}/\beta+1)$

$= 2K\|10K\|\infty = 1.67\,K\Omega$

$A = \frac{r_o\|(\beta+1)(r_{e3} + 10K\|2K)}{R_S + r_{e1}} \times \frac{2K\|10K}{2K\|10K + r_{e3}}$

$\beta = \frac{R_1}{R_1+R_2} = \frac{1}{1+10} = 0.0909\,V/V$

$A_F = \frac{A}{1+A\beta} \doteq \frac{\frac{16.4}{0.7}\cdot\frac{1.67}{1.69}}{1 + 23.15\times 0.0909} = 7.45\,V/V$

$R_{if} = R_{IN}(1+A\beta) = 700(3.1) = 2170\Omega$

Neglecting base currents since $\beta >> 1$

DC voltage at input $= I_{B1}R_S = 0.01V$

DC voltage at output $= 0V$

8.33

$Q_3 + Q_4$ form current multiplier $\times 120/40 = \times 3$

$g_{m1} = 2\sqrt{\frac{1}{2}120(20/1)\,100} \doteq 693\,\mu A/V$

$g_{m5} = 2\sqrt{\frac{1}{2}60\,(20/1)\,1000} \doteq 1550\,\mu A/V$

$g_{m3} = 2\sqrt{\frac{1}{2}60\,(40/1)\,100} \doteq 693\,\mu A/V$

$g_{m4} = 2\sqrt{\frac{1}{2}60\,(120/1)\,300} \doteq 2078\,\mu A/V$

$g_{m2} = g_{m1} = 693\,\mu A/V$

$r_{o1} = 24/100 \Rightarrow 240\,K\Omega$

$r_{o2} = 24/100 \Rightarrow 240\,K\Omega$

$r_{o3} = 24/100 \Rightarrow 240\,K\Omega$

$r_{o4} = 24/300 \Rightarrow 80\,K\Omega$

$r_{o5} = 24/1000 \Rightarrow 24\,K$

(c) Open loop gain $A\beta \simeq g_{m1}(r_{o1}\|r_{o3}) \times 1$

$\left[(3\times g_{m1})\left(\frac{r_{o4}}{3}\right) \equiv g_{m1}r_{o1} \right]$

$\beta = 1$: $\therefore A = g_{m1}(r_{o2}\|r_{o3})$

$\Rightarrow A \simeq 693 \times 120 \times 10^{-3} = 84$

(d) $A_F = \frac{A}{1+A\beta} = \frac{84}{1+84} = 0.988$ V/V

$R_o = r_{o5} \| r_{o5} = 12K$

$R_{of} = R_o/(1+A\beta) = 12/85 = 140\Omega$

(e) To obtain $V_o/V_s = 5$ we could change direct connection from Q55 to Q26 by voltage divider $R_1/(R_1+R_2)$ to change β from 1 to 1/5.3 then

$$A_F = \frac{84}{1+84\times 1/5.3} = \frac{84}{16.8} = 5$$

Now $1+A\beta = 16.8$

$R''_{of} = R_o/(1+A\beta') = 12/16.8 = 714\Omega$

8.34

Since $V_{G1} = 0 = V_{G2}$

$\Rightarrow V_{E3} = V_o = 0$ and $V_{B3} = +0.7V$

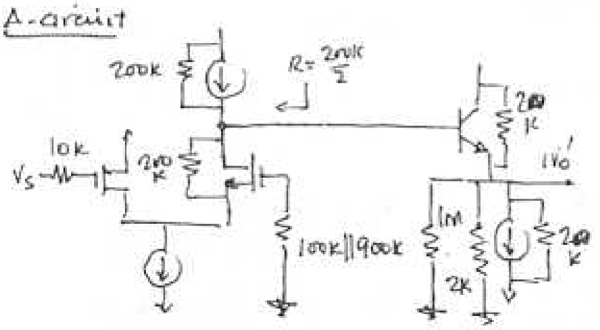
$g_{m1} = g_{m2} = 2\sqrt{\frac{1}{2}k'(W/L)I}$

$= 2\sqrt{\frac{1}{2}1(0.5)} = 1mA/V$

$r_{e3} = V_T/5mA = 5\Omega$

$r_o = V_A/I = 100/0.5 = 200K$

A-circuit



$R_i = \infty$

$R_o = 1M\|2K\|\frac{20K}{2}\|(r_{e3} + \frac{200K}{\beta+1}) = 622.2\Omega$

$$A = \frac{100\|(\beta+1)(r_{e3} + 1M\|10K\|2K)}{2/g_m} \times \frac{1.66K}{r_{e3}+1.66K} = 31.3 V/V$$

$\beta = \frac{100}{100+900} = 0.1$ V/V

$A_F = \frac{A}{1+A\beta} = \frac{31.3}{1+31.3\times0.1} = 7.58$ V/V

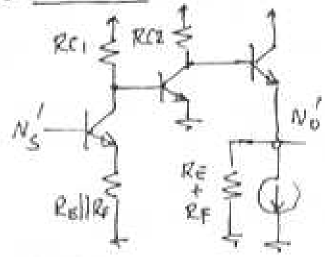
$R_{if} = \infty \quad \therefore R_{IN} = \infty$

$R_{of} = \frac{R_o}{1+A\beta} = \frac{622.2}{1+3.13} = 150.6\Omega$

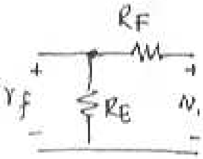
$R_{of} = R_{out}\|R_L \Rightarrow R_{out} = 163\Omega$

8.35

(a) A-circuit



β-circuit



(b) For $A\beta \gg 1$: $A_F = \frac{A}{1+A\beta} \Rightarrow \frac{1}{\beta}$

$\beta = \frac{R_E}{R_E+R_F} \Rightarrow A_f \approx \frac{R_E+R_F}{R_E} = 1 + \frac{R_F}{R_E}$

(c) $R_E = 50\Omega$

$\Rightarrow A_F = 1 + R_F/R_E = 25$ V/V

$\Rightarrow R_F/R_E = 24$ and $R_F = 24R_E = 1.$

(d) $I_{Q1} = 1mA$, $I_{Q2} = 2mA$, $I_3 = 5mA$

$\beta = 100$

$r_{e1} = \frac{25mV}{1mA} = 25\Omega$, $r_{e2} = 12.5\Omega$, $r_{e3} =$

$A_1 = \frac{-R_{C1}\|r_{\pi2}}{r_{e1} + R_E\|R_F} = -10$

$\Rightarrow R_{C1}\|r_{\pi2} = 10(25 + 50\|1.2K) = 730$

$\Rightarrow R_{C1} = 1.75K\Omega$

$A_2 = \frac{-R_{C2}\|[\beta(r_{e1} + R_E + R_F)]}{r_{e2}} = -$

$\Rightarrow R_{C2}\|125.5K = 5\times 12.5 = 625.$

$\Rightarrow R_{C2} = 628.1\Omega$

$A_3 = \frac{R_E+R_F}{r_{e3}+R_E+R_F} = \frac{1.25}{1.255} = 0.996$

(e) $\therefore A_1A_2A_3 = 10\times50\times0.996 = 498$ V

$A\beta = 498(50/1250) = 19.92$

$A_F = \frac{A}{1+A\beta} = 23.8$ V/V

(f) $R_i = (\beta+1)(r_{e1} + R_E\|R_F)$

$\therefore R_i = 101(25+4.8) = 7.37K\Omega$

$R_{if} = R_1(1+19.92) = 154\,K\Omega$

$R_o = 1.25K \,\|\, (r_{e3} + R_{c2}\|101) = 11.12\Omega$

$R_{OF} = \dfrac{R_o}{1+19.92} = 0.53\Omega$

Section 8.5
Series-Series Feedback

8.36

For $A\beta \gg 1$: $A_F = \dfrac{A}{1+A\beta} \Rightarrow \dfrac{1}{\beta} = A_F^*$

Here, $\beta = \dfrac{V_f}{I_o} = \dfrac{R_{E2} \times R_{E1}}{R_{E2}+R_F+R_{E1}} = \dfrac{100\times100}{100+640+100} = 11.9\Omega$

$\dfrac{I_o}{V_s} \approx A_F^* \equiv \dfrac{1}{\beta} = 84\,\mu\dfrac{A}{V}$ (cf. 83.7)

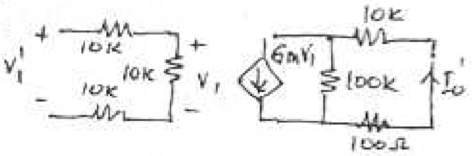
$\dfrac{V_o}{V_s} = -\dfrac{I_oR_{C3}}{V_s} = -84\times0.6 = -50.4\,V/V$ (cf. 50.2)

8.37

$A = G_m = 100mA/V \qquad \beta = 0.1\,V/mA$

$r_{in} = 10K$, $r_{out} = 100K$

A.circuit



$V_i = V_1' \dfrac{10}{10+10+10} = V_1'/3$

$I_o' = \dfrac{G_mV_1\,100}{100+10+0.1} = 30.28V_1'\,mA$

$A = \dfrac{I_o'}{V_1'} = 30.28\,mA/V$

$A_F = \dfrac{A}{1+A\beta} = \dfrac{30.28}{1+30.28(0.1)} = 7.52\,\frac{mA}{V}$

$R_i = R_S + R_{id} + R_1 = 30K\Omega$

$R_{if} = R_1(1+A\beta) = 120.8\,K\Omega$

$R_{in} = R_{if} - R_s = 110.8\,K\Omega$

$R_o = R_L + R_{op} + R_2 = 110.1\,K\Omega$

$R_{of} = R_o(1+A\beta) = 443.4K\Omega$

$R_{out} = R_{of} - R_L = 433.4K\Omega$

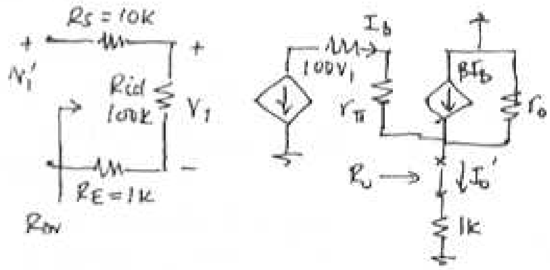
8.38

For $A\beta \gg 1$: $I_o/V_s \approx 1/\beta$

R_E samples I_o/α and $V_E = I_uR_E/\alpha$

$\beta = \dfrac{V_F}{I_u} = R_E \;\therefore\; A_F \rightarrow \dfrac{1}{\beta} = \dfrac{1}{R_E}$ QED

A. circuit



For $\dfrac{I_o}{V_s} = \dfrac{1mA}{V} \approx \dfrac{1}{\beta} \approx \dfrac{1}{R_E} \Rightarrow R_E = 1K\Omega$

$V_1 = V_1' \dfrac{R_{id}}{R_S+R_{id}+R_E} = \dfrac{100V_1'}{100+10+1} = 0.9V_1'$

$I_b = \dfrac{G_{m1}V_1}{1+r_\pi+(1+\beta)(r_o\|R_E)}$

$I_o' = \dfrac{(\beta+1)I_b(R_E\|r_o)}{R_E}$

$\dfrac{I_o'}{V_1'} = \dfrac{1}{R_E}\cdot\dfrac{(\beta+1)(R_E\|r_o)\,100\times0.9}{1+r_\pi+(1+\beta)(R_E\|r_o)} \longrightarrow \dfrac{1}{R_E}$

$A = \dfrac{I_o'}{V_1'} = \dfrac{1}{1}\cdot\dfrac{101(1\times100)\times90}{1+2.5+101(1\|100)} = 87\,\frac{mA}{V}$

$A_F = \dfrac{I_E}{V_S} = \dfrac{87}{1+87\times1} = 0.989\,mA/V$

Thus $\dfrac{I_o}{V_s} = \dfrac{\beta}{\beta+1}\cdot\dfrac{I_E}{V_s} = 0.989\times0.99$

$= 0.98\,mA/V$

$R_i = 10+100+1 = 111K\Omega$

$R_{if} = 111 \times 88 = 9768K\Omega$

$R_{IN} = R_{if} - R_s = 9758K \simeq 9.8M\Omega$

R_o (looking into xx in A circuit)

$= R_E \| r_o + \underline{\dfrac{1K + r_\pi}{\beta+1}}$

$= (1\|100) + \dfrac{1+2.5}{101} = 1.025K\Omega$

R_{if} (in amplifier circuit) $= 1.025 \times 88$

$= 90.2K\Omega$

$R_{OUT} = r_o\,[1 + g_m(90.2\|r_\pi)] = 9.83K\Omega$

8.39

$I_1 = 0.2mA/2 = 100\mu A$

$I_5 = 0.8mA = 800\mu A$

$g_{m1} = 2\sqrt{\tfrac{1}{2}\,20\,(36/10)\,100} = 120\mu A/V$

$g_{m5} = 2\sqrt{\tfrac{1}{2}\,20\,(200/10)\,800} = 800\mu A/V$

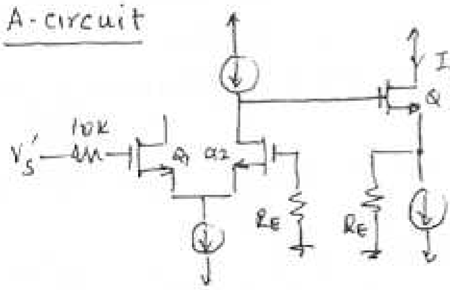
$r_{o1} = V_A/I_1 = 100V/100\mu A = 1M\Omega$

$r_{o5} = V_A/I_5 = 100V/800\mu A = 125K\Omega$

$r_{e1} = V_T/I_1 = 25mV/100\mu A = 250\Omega$

$r_{e5} = V_T/I_5 = 25mV/800\mu A = 31.25\Omega$

A-circuit



$\beta = \dfrac{V_F}{I_o} = \dfrac{I_o R_E}{I_o} = R_E\ V/A$

$A \equiv \dfrac{I_o'}{V_s'} \simeq \dfrac{g_{m1} r_{o2}}{2} \cdot \dfrac{1}{R_E \| r_{o5}}$

$= \dfrac{120 \times 1000}{2} \times \dfrac{1}{9.25} = 6.48 mA/V$

$\Rightarrow \dfrac{I_o}{V_s} = \dfrac{6.48 \times 10^{-3}}{1 + 6.48\,(10)} \simeq 98.5\mu A/V$

$\dfrac{V_o}{V_{os}} = \dfrac{I_o(R_E \| R_{o5})}{V_{os}} = 98.5\dfrac{\mu A}{V}\ 9.25\dfrac{V}{mA}$

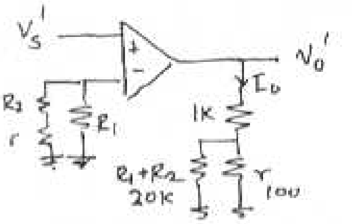
$= 0.911\ V/V \approx 1\ V/V$

B.40

$A_f \equiv \dfrac{I_o}{V_s}$ (Assume $(R_1+R_2) \gg r$)

$\beta \equiv \dfrac{V_F}{I_o} = R_1 \dfrac{r}{r+R_1+R_2}\ V/A$

A-circuit



$\dfrac{I_o'}{V_s'} = \dfrac{A}{r+R_L}$

$A_F = \dfrac{I_o}{V_s} = \dfrac{A}{1+A\beta} \doteq \dfrac{A/(r+R_L)}{1 + \dfrac{A}{r+R_L} \cdot \dfrac{rR_1}{r+R_1+R_2}}$

$= \dfrac{A}{(R_L+r) + \dfrac{A r R_1}{r+R_1+R_2}}$

$\underset{A \gg 1}{\Longrightarrow} \dfrac{r+R_1+R_2}{rR_1} \equiv \dfrac{1}{\beta}$

(a) when $\mu = 10^5 V/V$ and $R_1 = 100\Omega$

$A_F = \dfrac{\mu = 10^5}{(1K+100) + \dfrac{10^5 \cdot 100 \cdot 100}{100+100+1K}}$

$= \dfrac{10^5}{1100 + \dfrac{10^5 \cdot 10^4}{1200}} = 0.12\ \dfrac{mA}{V}$

$\dfrac{1}{\beta} = \dfrac{100 + 100 + 1000}{100 \times 100} = \dfrac{1200}{10^4} = 0.12\dfrac{mA}{V}$

$R_i' = R_{id} + R_1 = 10K + 100 = 10.1K\Omega$

$R_{of} = R_i'(1+A\beta) = R_i'(1 + \tfrac{1}{\beta}\cdot\beta) = 20.2K$

$R_o = r_o \| (R_L + (r\|R_2))$

$= 100 \| (1050) = 91.3\Omega$

$R_{of} = R_o'(1+A\beta) = 2 \times 91.3 = 182.6\Omega$

(b) $\beta = V_F/I_o = I_o r/I_o = \cancel{x}\ A/V = 100\Omega$

$A_F = 10^4/[(1100) + 10^4 \cdot 100] \approx 1/100$

$A_F\beta = 1$ and $(1+A\beta) = 2$

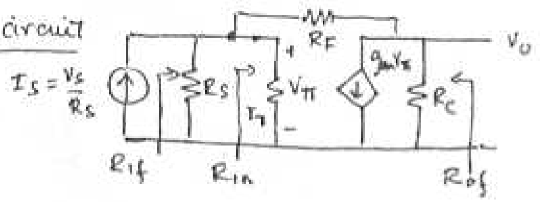
$R_1 = R_i'(1+A\beta) \to \infty$

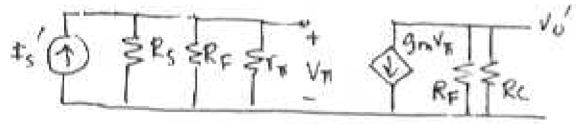
$R_{of} = 2(100\|1025) = 182.2\Omega$

Section 8.6 The Shunt-shunt and Shunt Series Feedback Amplifiers

8.41

A-circuit

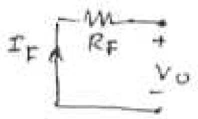




$$V_\pi = I_S(R_S \| R_F \| r_\pi)$$

$$A \equiv \frac{V_O'}{I_S'} = -g_m(R_S \| R_F \| r_\pi)(R_F \| R_C)$$

B-circuit



$$\beta \equiv \frac{I_F}{V_O} = -\frac{1}{R_F}$$

$$A_F \equiv \frac{V_O}{I_S} = \frac{A}{1+A\beta} = \frac{-g_m R^*}{1+g_m R^X/R_F}$$

$$\rightarrow \frac{-g_m R^*}{g_m R^X} \cdot R_F = -R_F$$

$$A_V = \frac{V_O}{I_O R_S} \simeq \frac{-R_F}{R_S} \qquad \text{QED}$$

For $R_S = 10K$, $R_F = 47K$, $R_C = 4.7K$

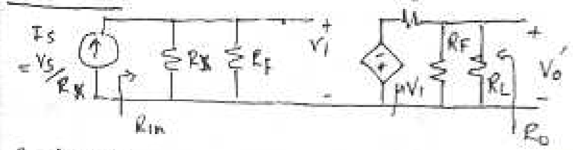
$$A_V \approx -\frac{47K}{10} = -4.7 \text{ V/V}$$

For $A_V = -7.5 \text{ V/V}$

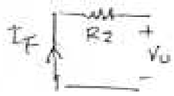
$$R_F = A_V R_S = 7.5 \times 10 = 75K$$

8.42

A-Circuit



B-circuit



$$\beta \equiv \frac{I_F}{V_O} = -\frac{1}{R_F}$$

Here let $R_X = R_S + R_1$ and $\mu = g_m r_o$

$$A \equiv \frac{V_O'}{I_S'} = -[R_X \| R_2][R_2 \| r_o] g_m$$

$$A_F = \frac{V_O}{I_S} = \frac{A}{1+A\beta} = \frac{-g_m[R_X\|R_2][R_2\|r_o]}{1+(A)(-1/R_2)}$$

$$= \frac{-g_m \frac{R_X R_2}{R_X+R_2} \cdot \frac{R_2 r_o}{R_2+r_o}}{1+g_m \frac{R_X R_2}{R_X+R_2} \cdot \frac{R_2 r_o}{R_2+r_o}\left[\frac{-1}{R_2}\right]}$$

$$\xrightarrow{A \text{ large}} -\frac{R_2}{} = -\frac{1}{\beta}$$

Thus $\frac{V_O}{I_S R_X} = \frac{V_O}{V_1} = \frac{1}{\beta R_X} = \frac{-R_2}{R_1+R_S}$

$$R'_{IN} = (R_S+R_1) \| R_2 = (10K + 1M) \| 4.7M$$
$$= 1.01M \| 4.7M = 0.83 M\Omega$$

Now $g_m = \frac{2 I_D}{V_{OV}} = \frac{2 \times 1}{0.8-0.6} = \frac{2}{0.2} = 10 \text{ mA/V}$

$$r_o = \frac{V_A}{I} = \frac{30V}{1mA} = 30K\Omega$$

Hence $A = -[(R_S+R_1) \| R_2][R_2 \| r_o] g_m$

$$= -10[1.01M \| 4.7M][4.7M \| 0.03M]$$
$$= -10[831K][29K] \text{ V/mA}$$
$$= -24100$$

$$1 + A\beta = 1 + \left(24100 \times \frac{1}{4700}\right) = 6.13$$

$$A_F = \frac{-24100}{6.13} = -3931 \text{ K}\Omega$$

Hence $\frac{V_O}{V_1} = \frac{V_O}{I_S(R_S+R_1)} = \frac{-3931K}{1010} \simeq -3.89 \text{ V/V}$

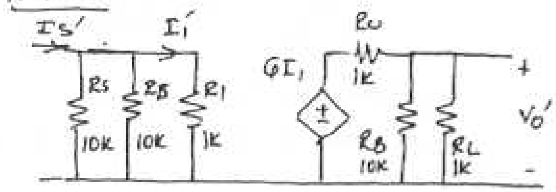
$$R_O' = R_2 \| r_o = 4.7M \| 30K = 29.8K\Omega$$
$$R_{of} = R_O'/(1+A\beta) = 29.8/6.13 = 4.86K$$
$$R_i' = (R_S+R_1) \| R_2 = 1010 \| 4.7 = 0.83M\Omega$$
$$R_{if} = R_i/(1+A\beta) = 0.83/6.13 = 136K\Omega$$
$$R_{IN} = R_{if} - R_1 = 136K - 10K = 126K$$

8.43

$G_m = 100 \text{ mA/V}$, $R_{input} = 1K$, $R_{output} = 1K$

$R_1 = 10K$, $R_2 = 100$, $R_S = 10K$, $R_L = 1K$

A-circuit



$\beta = 0.1$

$$A \equiv \frac{V_o'}{I_s'} = \frac{(R_S\|R_B)G}{(R_S\|R_B)+R_1} \cdot \frac{(R_B\|R_L)}{(R_B\|R_L)+R_o}$$

$$= \frac{(10\|10)100}{(10\|10)+1} \cdot \frac{(10\|1)}{(10\|1)+1} = 75.75 \text{ V/mA}$$

$$A_F \equiv \frac{A}{1+AB} = \frac{75.75}{1+75.75\times 0.1} = 8.83 \text{ V/mA}$$

$(1+AB) = 8.575$

$R_1 = R_S\|R_B\|R_1 = 833\,\Omega$

$R_{if} = R_1/(1+AB) = 97.1\,\Omega$

$R_{IN}\|R_S = R_{if} \rightarrow R_{IN} \approx 89.7\,\Omega$

$R_o' = (R_o\|R_B\|R_L) = 476\,\Omega$

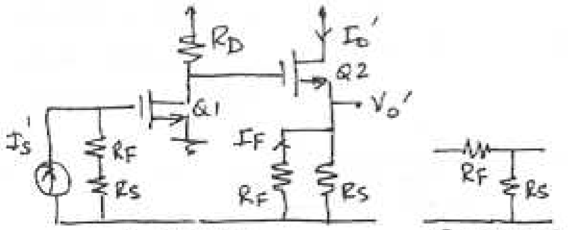
$R_{of} = R_o'/(1+AB) = 55.5\,\Omega$

$R_{OUT}\|R_L = R_{of} \Rightarrow R_{out} \approx 58.76\,\Omega$

8.44

Here shunt-series: sample I_o, return I_F

Because Q_2 is voltage-driven device, unlike BJT we find $I_o = V_o/R$ as voltage follower.



A. circuit B-network

$$A \equiv \frac{I_o'}{I_s'} = \frac{-g_{m1}R_D(R_S+R_F)}{(R_S\|R_F)} = -555.5$$

$$\beta \equiv \frac{I_F}{I_o} = \frac{R_S}{R_F+R_S} = \frac{10}{100} = 0.1$$

$$A_F \equiv \frac{I_o}{I_s} = \frac{\dfrac{-g_{m1}R_D(R_F+R_S)}{(R_S\|R_F)}}{1+\dfrac{g_{m1}R_D(R_F+R_S)}{(R_S\|R_F)}\cdot\dfrac{R_S}{(R_F+R_S)}}$$

$$\longrightarrow \frac{R_F+R_S}{R_S} = 1+R_F/R_S$$

$A\beta = 56 \Rightarrow 1+A\beta = 57$

$$A_F = \frac{-555.5}{1+555.5\times 0.1} = -9.8 \text{ A/A}$$

$R_1' = R_F + R_S = 100K$

$R_{if} = R_1'/(1+AB) = 1.58\,K\Omega$

$R_{of} = (R_F\|R_S) + r_{o2}$

$= 9K + 20K = 29K\Omega$

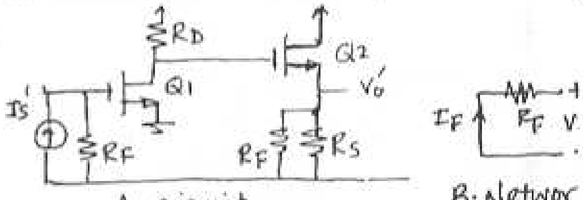
$R_o \approx r_o(1+g_m R_{of}) =$

$= 20K(1+5\times 29)$

$\approx 2.92\,M\Omega$

8.45

Here shunt-shunt: sample V_o return I_f



A. circuit B. Networ

$$A = \frac{V_o'}{I_s'} = -g_{m1} R_D . R_F \quad \text{(neglecting } r_i$$

$$\beta \equiv \frac{I_F}{V_o} = -1/R_F, \quad AB = g_m R_D$$

$$A_F \equiv \frac{V_o}{I_s} = \frac{A}{1+AB} = \frac{-g_{m1}R_D R_F}{1+g_{m1}R_D R_F/R}$$

$$= \frac{-5\times 10\times 90}{1+5\times 10}$$

$$= -4500/51 = -88.$$

$R_1' = R_F = 90K \rightarrow R_{if} = R'/(1+AB) = 1.76$

$R_o' = r_o\|R_F\|R_S = 20\|90\|20 = 6.2K\Omega$

$\Rightarrow R_{of} = R_o'/(1+AB) = 121.6\,\Omega$

8.46

A circuit

$\beta = -1/R_F$

$V_o = -\mu V_1 R_F$

$V_1 = I_s(R_s \| R_F)$

$\therefore V_o = -\mu I_s (R_s\|R_F) R_F$

$A \equiv \dfrac{V_o}{I_s} = -\mu \dfrac{R_S R_F}{(R_S+R_F)} . R_F$

$A_F = \dfrac{A}{1+A\beta} = \dfrac{-\mu (R_S\|R_F) R_F}{1+\mu(R_S\|R_F)R_F/R_F}$

$= -\dfrac{\mu (R_S\|R_F) R_F}{1+\mu(R_S\|R_F)}$

$\dfrac{V_o}{V_S} = \dfrac{V_o}{I_S R_S} = \dfrac{-\mu(R_S . R_F) R_F}{[(R_S+R_F)+\mu(R_S R_F)]R_S}$

$\underset{\mu>>1}{\Longrightarrow} -\dfrac{\mu(R_S R_F)}{\mu(R_S R_F)} . \dfrac{R_F}{R_S} = -\dfrac{R_F}{R_S}$

QED

(b) For circuit of Fig P8.46(b)

$I_B = \dfrac{V_{cc}\frac{10}{10+15} - V_{BE}}{(15\|10)+(4.7\times 101)}$

$= \dfrac{15(10/25)-0.7}{6+474.7} = \dfrac{5.3V}{480.7K} \approx 0.011$ mA

$I_C = 100 I_B = 1.1$ mA

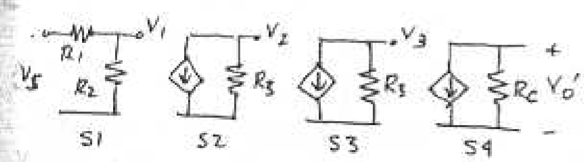
$r_e = V_T/I = 22.6\Omega$

$r_\pi = (\beta+1)r_e = 2.286K$

$g_m = \beta/r_\pi = 100/2.286K \rightarrow 43.7$ mA/V

$R_{IN} = (15\|10\|2.286)K \doteq 1.5K$

$R_C \| R_B \| r_\pi = (7.5\|6\|2.286) \doteq 1.35K$



For S_1: $R_1 = R_S$, $R_2 = R_{IN}$

$\therefore V_1 = 1.5/11.5\, V_S = 0.13 V_S$

For S_2: $R_3 = R_C\|R_B\|r_\pi$

$\therefore \dfrac{V_2}{V_1} = -g_m R_3 \doteq -43.7\times 1.35$

$\doteq -59$ V/V

For S3: Same as S2 $\therefore \dfrac{V_3}{V_2} = -59$ V/V

For S4: $\dfrac{V_o}{V_3} = -g_m R_C = -43.7\times 7.5$

$= -327.75$ V/V

$\dfrac{V_o}{V_S} = -0.13\times 59\times 59\times 327.75$

$\rightarrow \dfrac{V_o}{V_S} = -1.488\times 10^5$

Because we have ignored r_o etc let us estimate $V_o/V_S = -1\times 10^5$ which is quite large.

Then $A_F = \dfrac{A}{1+A\beta} \approx 100$ needed

$= \dfrac{10^5}{1+10^5\beta} \approx \dfrac{1}{\beta} = 100$

Select R_F so that $R_F/R_S = 100$

$\rightarrow R_F = 100\times 10K = 1M\Omega$

We can ignore loading effect of R_F in A-circuit. R_L will cause loading of R_C & $V_L = (R_L/(R_C+R_L))V_o$

$= (1/8.5) = 0.11 V_o$

Now $A_o \approx 10^4$.

$A_F' = \dfrac{10^4}{1+10^4/100} \doteq 99.00$.

8.47

(a) To lower R_{IN} and raise R_{out}

SHUNT - SERIES

(b) To raise R_{IN} and R_{out}

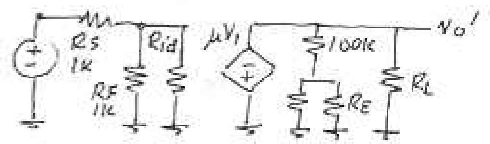
SERIES - SERIES

(c) To lower R_{IN} and R_{out}

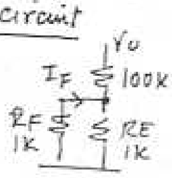
SHUNT - SHUNT

8.48

A-circuit



β-circuit



$\beta = \dfrac{I_F}{V_o} = -\dfrac{100K\|1K}{100K\|1K+1K} \cdot \dfrac{1}{100K}$

$\doteq -4.98\ \mu$A/V

$V_1 = (1\|100\|1.99)\times 10^3 I_s' = 661.2 I_s'$

$\frac{V_o'}{I_s'} = -661.2\,(10)^4 \frac{2k\|100.5k}{2k\|100.5k + 1k}$

$= -4.38\times 10^6 = -4.38\,M\Omega$

$\Rightarrow \frac{V_o}{V_s} = \frac{A}{1+A\beta} = \frac{-4.38\times 10^6}{1+4.38\times 4.98}$

$= -191.86\times 10^3\ V/A$

$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = -191.9\ V/V$

$R_1 = (1\|100\|1.99)k = 661.2k$

$R_{if} = R_i/22.8 = 29 = R_{in}\|R_s$

$\Rightarrow R_{in} = 29.4\,\Omega$

$R_o = 1k\|100.5k\|2k = 662.2\,\Omega$

$R_{of} = R_{out}\|R_L = R_o/22.8 = 29\,\Omega$

$\Rightarrow R_{out} = 29.5\,\Omega$

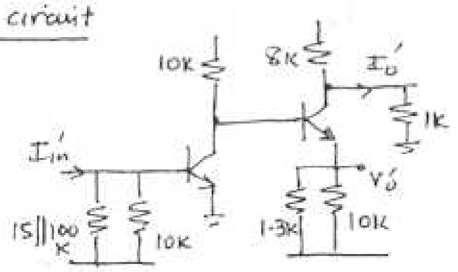
8.49

$\beta = -1/R_F = -10^{-4}$

$I_{E1} = I_{E2} = 1mA \rightarrow r_{e1} = r_{e2} = 25\Omega$

$r_{\pi 1} = r_{\pi 2} = 2.5k\Omega,\ r_o = 100K$

A - circuit



$\mu = \frac{V_{E2}}{V_{B1}} = \frac{-10k\|r_o\|\beta(1.3k\|100k)}{25} \times \frac{1.3k\|10k}{25 + 1.3k\|10k}$

$= -337(0.979) = -330\ V/V$

$\frac{V_o'}{I_{in}'} = (15k\|100k\|10k\|2.5k)(-330) = -572\ k\Omega$

$\Rightarrow \frac{V_o}{I_{IN}} = \frac{-572}{1+572(0.1)} = -9.83\ k\Omega$

$R_{if} = 15K\|100K\|10K\|2.5K = 1.73K$

$R_{in} = R_{if} = 1.73k/58.2 = 29.7\,\Omega$

$I_{out} = \frac{8k}{1k+8k}\cdot\frac{V_o}{1.3k\|10k} = 0.773\times 10^{-3} V_o$

$\Rightarrow \frac{I_o}{I_{IN}} = -9.83\times 0.773 = -7.6\ A/A$

cf. Ex 8.4 $\frac{I_o}{I_{in}} = -7.57\,A/A,\ R_{in} = 30\,\Omega$

8.50

$I_o/I_s = 100\,A/A,\ R_{in} = 1K,\ R_{out} = 10K$

$\beta = 0.1$ shunt-series topology

$A_p = \frac{I_o'}{I_s'} = 100$

$A_F = \frac{A_o}{1+A_o\beta} = \frac{100}{1+100(0.1)} = 9.09\ A/A$

$R_i = 1K\Omega$

$\Rightarrow R_{if} = R_i/(1+A\beta) = 90.9\,\Omega$

$R_o = 10K\Omega$

$\Rightarrow R_{of} = R_o(1+A\beta) = 110K\Omega$

8.51

Neglect I_{B2}; $I_{B1} \approx \frac{200}{100} = 2\mu A$

$V_{BE} = 0.7V \quad \therefore V_{B1} = +0.7V$

But no d.c. component in V_s

$\therefore I_{RS}$ (into V_s) $= 0.7/10K = 0.07\,mA$

Thus $I_F = I_{RS} + I_{B1} = 0.07 + 0.002 = 0.072\,mA$

$V_{E2} = 0.7 + 10\times 0.072 = 0.7 + 0.72 = 1.42$

$I_{C2} = 1.42/140 + 0.072 = 10.2\,mA$

$I_{B2} = I_{E2}/(\beta+1) = 0.1\,mA \approx \frac{1}{2}\,200\mu$

Iterate:

$I_{B1} = \frac{200\mu A - 100\mu A}{100} = 0.001\,\mu A$

$V_{E2} = 0.7 + 10\times 0.073 = 1.41V$

$I_{E2} = 1.41/0.140 + 0.071 = 10.1\,mA$

$I_{B2} = 10.1/101 = 100\mu A \quad \therefore I_{C2} = 10m$

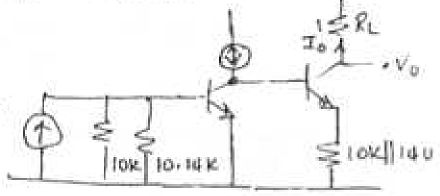
$V_{B2} = V_{E2} + V_{BE} = 1.41 + 0.7 = 2.11V$

$V_o = 10 - 10\times 500\Omega = +5V$

$r_{e1} = \frac{V_T}{I_E} = \frac{25mV}{0.1mA} = 250\Omega$, $r_{e2} = 25\Omega$

$\beta = \frac{R_E}{R_F + R_E} = \frac{140}{10K + 140} \approx 0.0138$

A-circuit



$V_{B1} = 10.14k \| 10k \| \beta(250) I_s'$

$= 4.2 \times 10^3 \; I_s'$

$\Rightarrow \frac{I_o'}{I_s'} = 4.2 \times 10^3 \frac{(\beta+1)(r_{e2} + 10k\|140)}{250} \times \frac{1}{(r_{e2} + 10k\|140)}$

$= 1.69 \times 10^3$ A/A

$A_F \equiv \frac{I_o}{I_s} = \frac{A}{1+A\beta} = \frac{1.69\times10^3}{1 + 1.69\times10^3 \times (0.0138)}$

$= 69.6$

$\Rightarrow \frac{V_o}{V_s} = \frac{I_o R_L}{I_s R_s} = \frac{500}{10^4} \cdot 69.6 = 3.5$ V/V

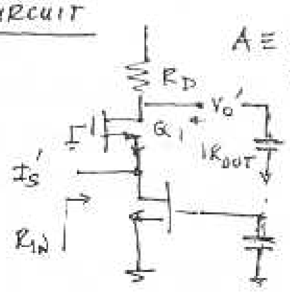
$R_i = 10K \| 10.14K \| 25K = 4.2$ KΩ

$R_{if} = R_i/(1+A\beta) = \frac{4.2}{1+23.3} = 172.8\Omega$

$\Rightarrow \; = R_{IN} \| R_s \quad \Rightarrow R_{IN} = 175.8\Omega$

8.52

A-CIRCUIT

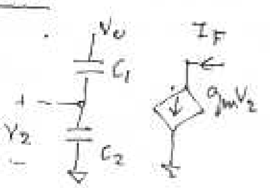


$A \equiv \frac{V_o'}{I_s'} = [R_D \| X_C] = R_D$ KΩ

Neglecting loading of C's

$A_F = \frac{A}{1+A\beta}$

B-CIRCUIT



$\beta = \frac{C_1}{C_1 + C_2} g_{mf}$

$= \frac{1}{10} \times 1 \frac{mA}{V}$

Here. $g_{m1} = 5$mA/V $g_{mf} = 1$mA/V

$R_D = 10K$

Thus $A = 10K\Omega$

$A_F = \frac{10K}{1 + 10K(0.1)} = 5K\Omega$

$R_{IN} = (R_D \| r_o) \rightarrow R_D$

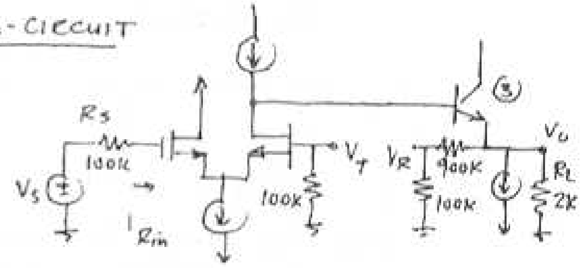
$R_{if} = R_D/(1+A\beta)$ shunt $= \frac{R_D}{2} = 5K\Omega$

$R_{out} = R_D/(1+A\beta) = \frac{R_D}{2} = 5K\Omega$

Section 8.7
Determining the Loop Gain

8.53

A-CIRCUIT



$V_R = -\frac{[100k \| (\beta+1)[r_{e2} + (1M \| \frac{20k}{2} \| R_L)]]}{2/g_m} \times \frac{r_{e2}}{[r_{e2} + (1M \| \frac{20k}{2} \| R_L)]}$

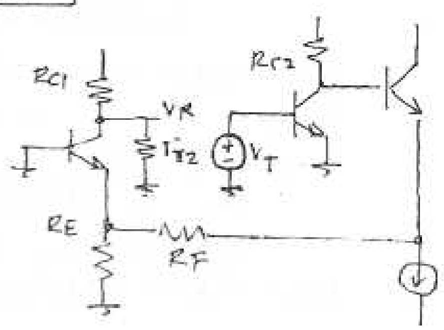
$= -\frac{[100k \| 168k]}{2} \times \frac{5}{1665} = -31.25$

$R_o = [1M \| \frac{20k}{2} \| R_L \| [\frac{200k}{2}/\beta + r_{e3}]$

$= 623\Omega$

which agree with previous solution

8.54



$v_R = v_{c1}$

$v_{c1} = -I_{c1}(R_C \| r_\pi)$

$I_{c1} = (\beta/(\beta+1)) I_{E1} = I_F R_E/(R_E + r_{e1})$

$I_F = I_2 = (\beta+1) I_{B1}$

$$I_{B1} = \frac{R_C I_{C2}}{R_C + (\beta+1)[r_{e3} + R_F + (R_E \| r_{e1})]}$$

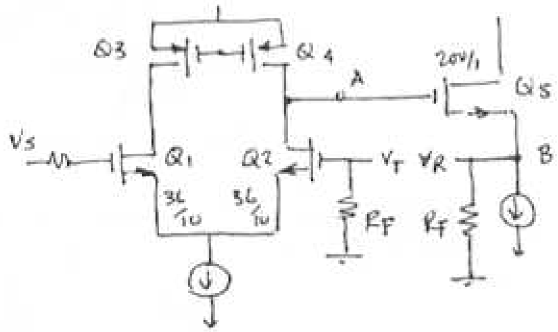
$I_{C2} = g_m V_T$

Combining

$$\frac{V_R}{V_T} = \frac{g_{m2} R_C(\beta+1)}{R_C + (\beta+1)[r_{e3} + R_F + (R_E \| r_{e1})]} \times \frac{\alpha R_E (R_C \| r_2)}{R_E + r_{e1}}$$

QED

8.55



$V_T = V_{G2}$ and $V_S \to 0$

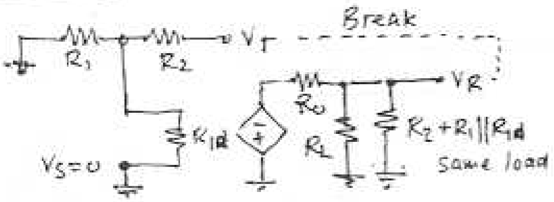
$V_A = -g_{m2}(r_{o2} \| r_{o4})$

$$V_B = V_A \frac{(R_F \| r_{o5})}{(R_F \| r_{o5}) + 1/g_{m5}}$$

$$A\beta = -\frac{V_T}{V_R} = +\frac{g_{m2}(r_{o2} \| r_{o4})(R_F \| r_{o5})}{(R_F \| r_{o5}) + 1/g_{m5}}$$

QED

8.56



$$\frac{V_R}{V_T} = \frac{-\mu\, R_L \| (R_2 + R_1 \| R_{id})}{R_o + R_L \| (R_2 + R_1 \| R_{id})} \cdot \frac{R_1 \| R_{id}}{R_2 + R_1 \| R_{id}}$$

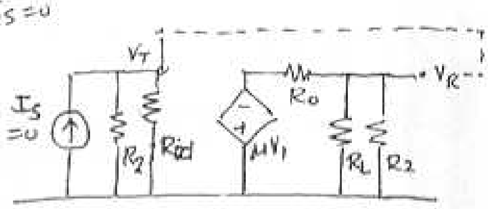
For $(R_1 + R_2) \gg R_L$ and $R_{id} \gg R_1$

and $R_o \ll R_L$

$$\frac{V_R}{V_T} \approx -\mu \cdot 1 \cdot \frac{R_1}{R_1 + R_2} = \frac{-\mu R_1}{R_1 + R_2}$$

$= -A\beta.$

(b) $I_S = 0$

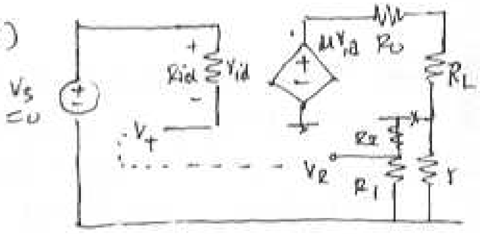


$$\frac{V_R}{V_T} = -\mu \frac{R_L \| R_2}{R_o + R_L \| R_2}$$

For $R_o \ll R_L$

$$\frac{V_R}{V_T} \approx -\mu = -A\beta.$$

(c)



$$\frac{V_R}{V_T} = \frac{-\mu \cdot [r \| (R_1 + R_2)]}{R_o + R_L + [r \| (R_1 + R_2)]} \cdot \frac{R_1}{R_1 + R_2}$$

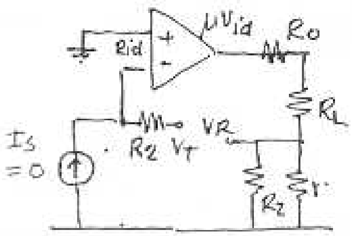
For $r \ll (R_1 + R_2)$ and $R_o \ll R_L$

$$\frac{V_R}{V_T} \approx \frac{-\mu r}{R_L + r} \cdot \frac{R_1}{R_1 + R_2} = -A\beta$$

[Note: breaking at R_2-r is simpler

$$\frac{V_R}{V_T} = \frac{-\mu R_1}{R_{id} \| R_1 + R_2} \cdot \frac{r}{R_o + R_L + r}$$

(d)

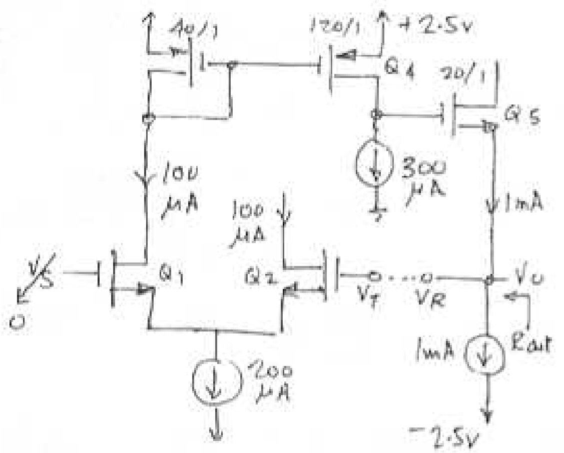


$$V_{id} = V_T \frac{R_{id}}{R_2 + R_{id}}$$

$$\frac{V_R}{V_T} = -\frac{R_{id}}{R_{id}+R_2}\,\mu\,\frac{(r\|R_2)}{(r\|R_2)+R_L+R_o}$$

$$A\beta = \mu\,\frac{r}{r+R_L} \quad \text{for } R_{id} \gg R_1, R_2,\ R_o \ll R_L$$

8.57



From P8.33 $g_{m1} = g_{m2} = 693\,\mu A/V$

$g_{m3} = 693\,\mu A/V$, $g_{m4} = 2078\,\mu A/V$

$g_{m5} = 1550\,\mu A/V$. $r_{o5} = 24\,K\Omega$

$r_{o1} = r_{o2} = r_{o3} = 240\,K\Omega$, $r_{o4} = 80\,K\Omega$

$R_{out} = (r_{o5}\|r_{o5}) = 12\,K\Omega$

$i_{c1} = -g_{m1} V_T$

$i_{c4} = -3 g_{m1} V_T$

$v_{c4} = -3 g_{m1} V_T (r_{o4}\|r_{o4})$

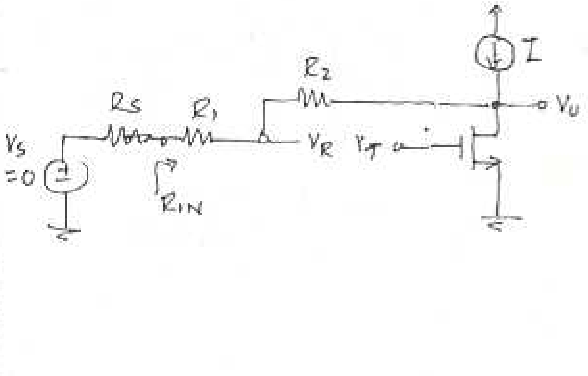
$v_{s5} = v_{c4} = V_R$

$$\therefore \frac{V_R}{V_T} = -3 g_{m1}\left(\frac{r_{o1}\|r_{o1}}{3}\right) = -\frac{g_{m1} r_{o1}}{2}$$

$$A\beta = -\frac{V_R}{V_T} = \frac{g_{m1} r_{o1}}{2} = 83.16$$

$R_o = (r_{o5}\|r_{o5}) = 12\,K\Omega$

8.58



Apply V_T with $V_S = 0$

$$\frac{V_O}{V_T} = -g_m [R_2 + R_1 + R_S] \| r_o$$

$$\frac{V_R}{V_T} = -g_m \left[[R_2+R_1+R_S]\|r_o\right] \frac{[R_1+R_S]}{[R_2+R_1+R_S]}$$

$$= -\frac{g_m (R_1+R_S)\, r_o}{R_1+R_2+R_S+r_o}$$

$$= -A\beta$$

For $I = 1mA$, $V_{GS} = 0.8V$, $V_t = 0.6V$

$V_A = 30V$, $R_S = 10K$, $R_1 = 1M$, $R_2 = 4.7M$.

$$g_m = \frac{2I}{V_{ov}} = \frac{2}{0.8-0.6} = \frac{2}{0.2} = 10\,mA/V$$

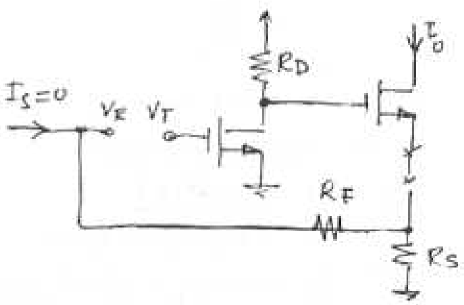
$r_o = V_A/I = 30V/1mA = 30K\Omega$

$$\text{then } A\beta = \frac{10\times10^{-3}(10K+10^3K)\,30K}{1M+4.7M+10K+30K}$$

$$= \frac{10\times1010\times30\times10^{-3}}{5740}$$

$$= 52.8\times10^{-3}$$

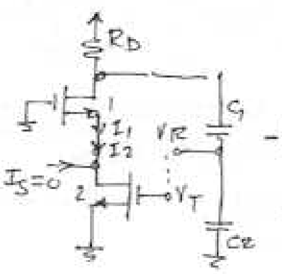
8.59



$$\frac{V_R}{V_T} = (g_{m2}R_S)(-g_{m1}R_D)$$

$$= -g_{m1} g_{m2} R_D R_S = -A\beta.$$

8.60



$$\frac{V_R}{V_T} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_2}+\frac{1}{sC_1}} \cdot g_{m2} R_D$$

$$-A\beta = \frac{C_1}{C_1+C_2} \cdot g_{m2} R_D$$

since $I_1 = I_2$

Section 8.8
The Stability Problem

8.61

$$A(s) = \frac{10^5}{1 + s/100}$$

$$\text{Ang}(A) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4}$$

at ω_{180}: $\text{Ang}(A) = -180°$ for $\omega_{180} \gg 100$

$$\Rightarrow 18^\circ = 90° + 2\tan^{-1}\left[\frac{\omega_{180}}{10^4}\right]$$

hence $\tan^{-1}\frac{\omega_{180}}{10^4} = \frac{90°}{2}$

i.e. $\frac{\omega_{180}}{10^4} = \tan(45°) = 1$

$\therefore \omega_{180} = 10^4$ rad/s

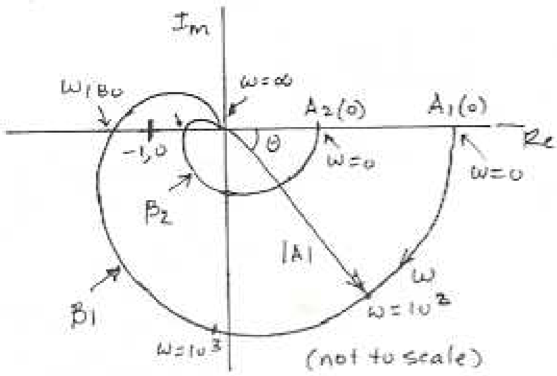
$$|A\beta| = \frac{10^5\beta}{\sqrt{1+(10^4/10^2)^2}} \cdot \frac{1}{(\sqrt{1+1})^2} = 1$$

$\Rightarrow \beta = 0.002$

$$A_f(0) = \frac{10^5}{1 + 10^5(0.002)} \approx 500\text{V/V}$$

8.62

ω	Ang(A)	$\lvert A\beta_1\rvert$	$\lvert A\beta_2\rvert$
0	0	10^5	10^2
10^2	45	7.07×10^4	70.7
10^3	95.7	9.85×10^3	9.85
10^4	180	500	0.5
∞	0	0	0



8.63

$$A(s) = \frac{10^3}{1+s/10^4}$$

$$\beta(s) = \frac{K}{(1+s/10^4)^2}$$

$$\text{Ang}(A\beta) = -\tan^{-1}\frac{\omega}{10^4} - 2\tan^{-1}\frac{\omega}{10^4}$$

$$= -3\tan^{-1}\frac{\omega}{10^4}$$

For $180°$ $\omega_{180} = \sqrt{3}\times 10^4$ rad/s

For $|A\beta(\omega_{180})| < 1$

$$\frac{10^3}{\sqrt{1+(\sqrt{3})^2}} \cdot \frac{K}{1+(\sqrt{3})^2} < 1$$

$\Rightarrow K < 0.008$

8.64

$$A(s) = \frac{1000}{(1+s/10^4)(1+s/10^5)^2}$$

and β is independent of frequency

$$\text{Ang}(A) = -\tan^{-1}\frac{\omega}{10^4} - 2\tan^{-1}\frac{\omega}{10^5}$$

try $\omega = 10^4$: $\theta = 45° + 2\times5.7 = 56.$

try $\omega = 10^5$: $\theta = 84.2° + 2\times 45 = 174$

Iteration yields $\omega \approx 1.1\times10^5$ rad/s

For oscillations: $|A\beta(\omega_{180})| \geq 1$

$$\frac{\beta\, 10^3}{(\sqrt{1+11^2})(\sqrt{1+1.1^2})} \geq 1$$

$\Rightarrow \beta \geq 0.0244$

Section 8.9
Effect of Feedback on Amplifier Poles

8.65

$$A(jf) = \frac{(10\times10^6)/10^4}{1 + jf/10^4}$$

$\therefore A(jf) = \dfrac{10^3}{1 + jf/10^4}$

$\beta = 0.1$ independent of frequency

$A_f(jf) = \dfrac{10^3}{1+10^3(0.1)} \cdot \dfrac{1}{1 + \dfrac{jf}{10^4(1+10^3(0.1))}}$

$= \dfrac{9.9}{1+jf/(101\times10^4)}$

$A_f(0) = 9.9$ V/V

$f_{pf} = 10^4(101) = 1.01\,MHz$

for $\dfrac{f}{f_{pf}} \gg 1$: $A_f \approx 9.9\,\dfrac{10^4(101)}{f}$

for $A_f = 1$: $f = f_t = 10\,MHz$

Pole is shifted by $(1 + A(0)\beta) = 101$

8.66

$A(jf) = \dfrac{10^3}{(1+jf/10^4)(1+jf/10^5)}$

(a) closed-loop poles given by

$1 + A(f)B = 0$

using $p = jf$

$p^2 + p(10^4 + 10^5) + (1+10^3\beta)10^4.10^5 = 0$

i.e. $p^2 + (1.1\times10^5)p + 10^9(1+10^3\beta) = 0$

compare terms with

$(p + f_{pf})^2 = p^2 + 2f_{pf}.p + f_{pf}^2$

$2f_{pf} = (1.1\times10^5)$

$(1+10^3\beta)\times10^9 = f_{pf}^2$

$\Rightarrow f_{pf} = 5.5\times10^5$

and $(1+10^3\beta) = 3.025 \Rightarrow \beta = 2.025\times10^{-3}$

(b) $A_f(0) = \dfrac{10^3}{1 + 10^3(2.025\times10^3)} = 330.6\,V/V$

(c) 55KHz : $|A(f)B| = 1$

$\Rightarrow A_f(55KHz) = \dfrac{A_f(0)}{1+|AB|} = 165.3\,V/V$

(c) from $s^2 + (\omega_0/Q)s + \omega_0^2$ cf. above

$Q = \dfrac{f_{pf}}{2f_{pf}} = \dfrac{1}{2}$

(d) $p^2 + 1.1\times10^5 p + (1+10^3\beta) = 0$

$\Rightarrow p^2 + 1.1\times10^5 p + 21.25\times10^9 = 0$

$\therefore p = \dfrac{-1.1\times10^5 \pm \sqrt{(1.1\times10^5)^2 - 4(1)(21.25\times10^9)}}{2}$

$= \dfrac{-1.1\times10^5 \pm j\,2.7\times10^5}{2}$

$= -5.5\times10^4 \pm j1.35\times10^5\,Hz$

$Q = \dfrac{|P|}{2(5.5\times10^4)} = \dfrac{\sqrt{(5.5\times10^4)^2 + (1.35\times10^5)^2}}{1.1\times10^5}$

$= 1.33$

8.67

$A(jf) = \dfrac{10^3}{(1+jf/10)(1+jf/f_p)}$

$A_f(0) = \dfrac{10^3}{1+10^3B} = 100$

$\Rightarrow \beta = 9\times10^{-3}$ V/V

Maximally flat when $Q = 0.707 = 1/\sqrt{2}$

from

$p^2 + p(f_1+f_2) + (1+A_0\beta)(f_1\times f_2) = 0$

$Q = \dfrac{\sqrt{(1+A_0\beta)f_1f_2}}{f_1+f_2}$

$\Rightarrow \dfrac{\sqrt{(1+10^3\beta)10^3 f_{pf}}}{10^3 + f_{pf}} = \dfrac{1}{\sqrt{2}}$

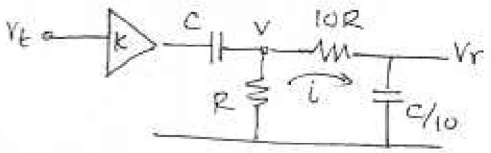
$\Rightarrow f_{pf}^2 + (2\times10^3)f_{pf} + 10^6$

$= 2(1+10^3\beta)^2 10^6$

$\Rightarrow f_{pf} = \dfrac{18\times10^3 \pm \sqrt{(18\times10^3)^2 - 4(10^6)}}{2}$

$= 17.94\,KHz.$

8.68



$i = \dfrac{sCV_r}{10} \Rightarrow V = V_r + sCRV_r$

$= V_r(1+sCR)$

By KCL: $\Sigma i = 0$ at V

$$\Rightarrow \frac{(KV_t - V)}{1/sC} = \frac{V}{R} + i = \frac{V}{R} + \frac{sCV_r}{10}$$

$$KsCV_t - sCV_r(1+sCR) =$$

$$= V_r\left(\frac{1+sCR}{R} + \frac{sC}{10}\right)$$

Collecting terms:

$$KsCV_tR = V_r[(sCR)^2 + 2.1\,sCR + 1]$$

Thus $L(s) \triangleq -V_r/V_t$

$$= \frac{-K/CR \;\; s}{s^2 + \frac{2.1s}{CR} + \frac{1}{CR^2}}$$

$$\equiv \frac{-K/CR \;\; s}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

from which $\omega_0 = 1/CR$

$$Q = \frac{1}{2.1 - K}$$

poles coincide when $Q = 1/2$

$$\Rightarrow K = 2.1 - 2 = 0.1$$

maximally flat when $Q = 1/\sqrt{2}$

$$\Rightarrow K = 2.1 - 1.414 = 0.686$$

Oscillates when $Q \Rightarrow \infty$

$$\Rightarrow K = 2.1$$

8.69

$$A(jf) = \frac{K}{1 + jf/10^7}$$

for $\beta = 1$: $A\beta = \frac{K^3\beta}{(1+jf/10^7)^3} = \frac{K^3}{(1+jf/10^7)^3}$

For oscillations:

$|A\beta| \geq 1$ at $\text{Ang}(A\beta) = 180°$

i.e. $+3\tan^{-1}\left(\frac{f_{180}}{10^7}\right) = 60°$

$$\Rightarrow f_{180} = \sqrt{3} \times 10^7 \text{ Hz}$$

$$\therefore \left[\frac{K}{\sqrt{1+(\sqrt{3})^2}}\right]^3 \geq 1 \Rightarrow K \geq 2$$

$$f_{osc} = f_{180} = 17.3 \text{ MHz}.$$

Section 8.10
Stability Study using Bode Plots

8.70

$$A(f) = \frac{10^5}{1 + jf/10}$$

for $\beta = 1$: $A(f)\beta = \frac{10^5}{1+jf/10}$

for $f \gg 10$: $|A\beta| \approx 10^5 \cdot \frac{10}{f_1}$,

$$\Rightarrow f_1 = 1 \text{MHz}$$

at f_1: phase margin $= 180° - \tan^{-1}\frac{10^6}{10}$

$$= 90°$$

8.71

$$A(f) = \frac{10^5}{(1+jf/10)(1+jf/10^4)}$$

$$A\beta(0) = 10^5\beta$$

$$A_F(0) = 100 = \frac{10^5}{1+10^5\beta} \Rightarrow \beta \simeq 0.01$$

$$|A\beta| = 1 \Rightarrow |1 + jf/10| \cdot |1 + jf/10^4|$$

$$= 10^5\beta = 10^3$$

$$\left(1 + \frac{f^2}{10^2}\right)(1 + f^2/10^8) = 10^6$$

$$f^4 + f^2(10^8 + 10^2) - (10^8)(10^2)10^6 = 0$$

$$f^2 \approx \frac{-10^8 + \sqrt{10^{16} + 4 \times 10^6}}{2}$$

$$\Rightarrow 61.8 \times 10^6 \Rightarrow f = 7.86 \text{ kHz}$$

Phase margin

$$= 180 - \left(\tan^{-1}\frac{7.86 \times 10^3}{10} + \tan^{-1}\frac{7.86}{10}\right)$$

$$\approx 180 - 90° - 38.16° = 51.8°$$

For PM $\geq 45°$: $\tan^{-1} f_1/10^4 \leq 45°$

$$\Rightarrow f_1 \leq 10^4$$

thus

$$|A\beta| = 1 = \frac{10^5\beta}{\sqrt{(1+(10^3)^2} \cdot \sqrt{2}}$$

$$\Rightarrow \beta = \sqrt{2}/100 = 0.0141$$

8.72

$$|1+e^{-j\theta}| = |1+\cos\theta - j\sin\theta|$$
$$= [(1+\cos\theta)^2 + (\sin\theta)^2]$$
$$= [1+2\cos\theta+\cos^2\theta+1-\cos^2\theta]^{\frac{1}{2}}$$
$$= \sqrt{2}(1+\cos\theta)^{1/2}$$

for 5%: $1+\cos\theta = \dfrac{1}{1.05^2(2)} = 0.4535$

$\theta = 123.13^\circ$ and $PM = 180-\theta = 56.87^\circ$

for 10%: $1+\cos\theta = \dfrac{1}{1.1^2(2)} = -0.586$

$\theta = 125.93^\circ$ and $PM = 54.07^\circ$

for 0.1dB $\equiv 10^{0.1/20} = 1.0116$

$\cos\theta = \dfrac{1}{2(1.0116)^2} - 1 = -0.5114$

$\theta = 120.76^\circ$ and $PM = 59.24^\circ$

for 1dB $\equiv 10^{1/20} = 1.122$

$\cos\theta = \dfrac{1}{2(1.122)^2} - 1 = -0.6028$

$\theta = 127.07^\circ$ and $PM = 52.93^\circ$

8.73

$$A(jf) = \frac{10^5}{\left(1+\frac{jf}{10^5}\right)\left(1+\frac{jf}{3.16\times10^5}\right)\left(1+\frac{jf}{10^6}\right)}$$

Assume β independent of frequency

For 45° PM: $\theta = 180-45$

$$\tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{3.16\times10^5} + \tan^{-1}\frac{f_1}{10^6} = 135^\circ$$

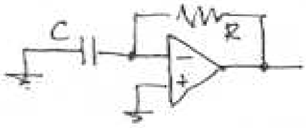
Solve $\Rightarrow f_1 = 3.16\times10^5$ Hz

$$|A\beta(f_1)| = 1 = \frac{10^5\beta}{\sqrt{1+(3.16)^2}\cdot\sqrt{2}\cdot\sqrt{1+(0.316)^2}}$$

$\Rightarrow \beta = 49\times10^{-6}$

$$A_f(0) = \frac{10^5}{1+10^5(4.9\times10^{-6})} = 16.9\times10^3$$

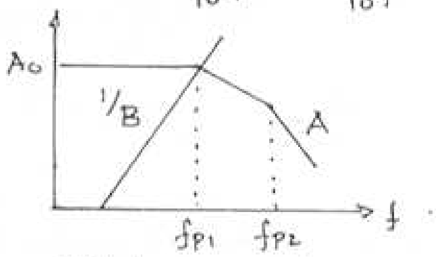
8.74



$$\beta = \frac{1/sC}{R+1/sC} = \frac{1}{1+sCR}$$

$$\beta(f) = \frac{1}{1+j2\pi fCR}$$

$$A(jf) = \frac{10^3}{\left(1+\frac{jf}{10^6}\right)\left(1+\frac{jf}{10^7}\right)}$$



From sketch, we need

$$A_0\frac{1}{2\pi f_{p1}CR} = 1 = A\beta$$

$$\Rightarrow RC = \frac{A_0}{2\pi f_{p1}} = \frac{10^3}{2\pi\times10^6} = 159.2\,\mu s$$

At 1MHz $\quad Ang(\beta) = -90^\circ$

$Ang(A) = -\tan^{-1}1 - \tan^{-1}0.1$

$= -45 - 5.7 = -50.7^\circ$

$\therefore PM = 180-(90+50.7) = 39.3^\circ$

Gain margin exists at ω_{180}

then $\tan^{-1}\dfrac{f_1}{10^6} + \tan^{-1}\dfrac{f}{10^7} = 90^\circ$

$\therefore f_{180} = \sqrt{10^6\cdot10^7}$ = geometric mean

$= 3.16$ MHz

$|A\beta(f_{180}) = 20\log|A| - 20\log|1/\beta|$

$|A|$ has fallen 10db, $|\beta|$ has risen 10dB

thus $GM = +10-(-10) = 20$ dB

8.75

For 90° PM:

$$\tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{10^6} + \tan^{-1}\frac{f_1}{10^7} = 90^\circ$$

From graph $f_1 = 3\times10^5$ Hz

thus $71.6 + 16.7 + 1.72 = 89.9°$ (close)

$$|A(f_1)| = \frac{10^5}{\sqrt{1+3^2}\cdot\sqrt{1\cdot0.3^2}\cdot\sqrt{1+0.03^2}} = 30.28\times10^3$$

$|AB| = 1 \Rightarrow \beta = 33.0\times10^{-6}$

$$\therefore A_f(0) = \frac{10^5}{1+10^5\beta} = 2.32\times10^4$$

For PM $=45°$ $f_1 \approx 10^6$ Hz from graph

thus $84.3 + 45 + 5.7 = 135°$ (ok)

$$|A(f_2)| = \frac{10^5}{\sqrt{1+10^2}\cdot\sqrt{2}\sqrt{1+0.1^2}} = 7\times10^3$$

$|A\beta| = 1 \Rightarrow \beta = 1.43\times10^{-4}$

$$\therefore A_f(0) = \frac{10^5}{1+10^5\beta} = 6.54\times10^3$$

Section 8.11
Frequency Compensation

8.76

$f_1 = 2$ MHz

$A_0 = 80$ dB $\equiv 10^4$

$\Rightarrow f_p = f_1/A = (2\times10^6)/10^4 = 200$ Hz

8.77

$f_{p1} = 2$ MHz, $f_{p2} = 10$ MHz

$A_0 = 80$ dB $\equiv 10^4$

$$f_D = \frac{f_P}{A_0} = \frac{10\times10^6}{10^4} = 10^3 \text{ Hz}$$

$f'_D = 1/(C_x + C_c)2\pi R_x \rightarrow C\times\frac{2\times10^6}{10^3} = 2000C$

8.78

$$w_{p1} = \frac{1}{10CR} \quad . \quad w_{p2} = \frac{1}{CR}$$

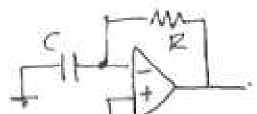
$$w'_{p1} \approx \frac{1}{g_m R_2 C_f R_1} = \frac{1}{\frac{100}{R}\cdot R\cdot C_f R} = \frac{1}{100 C_f R}$$

$$w'_{p2} \approx \frac{g_m C_f}{C_1C_2 + C_f(C_1+C_2)} = \frac{g_m C_f}{C_1(C_2+C_f)+C_fC_2}$$

for $C_f \gg C_2 = C$

$$w'_{p2} \approx \frac{g_m}{C_1+C_2} = \frac{100}{R(C_1+C_2)} = \frac{9.1}{CR}$$

8.79



$A_0 = 10^4$

poles at $10^5, 10^6, 10^7$ Hz

For $\beta = 1$, f_P must be kept $\times10^4$ lower than lowest amplifier pole at 10^5 Hz

$$\Rightarrow f_P = \frac{10^5}{10^4} = 10 \text{ Hz}$$

$$f_P = \frac{1}{2\pi CR} \quad \text{and } R = 1\text{M}\Omega$$

$$\Rightarrow C = \frac{1}{2\pi\, 10^6(10)} = 15.9\text{nF}$$

8.80

$A_0 = 80$ dB $\equiv 10^4$

$$f_{P1} = 10^5 = \frac{1}{2\pi C_1R_1} \Rightarrow R_1 = \frac{1}{2\pi f_{P1}C_1}$$

$$\Rightarrow R_1 = \frac{1}{2\pi\cdot10^5\cdot(150\times10^{-12})} = 10.62\text{K}\Omega$$

$$f_{P2} = 10^6 = \frac{1}{2\pi C_2R_2}$$

$$\Rightarrow R_2 = \frac{1}{2\pi\,10^6(5\times10^{-12})} = 31.85\text{K}\Omega$$

Assuming $f'_{P2} \gg f_{P3}$

$$f'_{P1} = \frac{f_{P3}}{10^4} = \frac{2\times10^6}{10^4} = 200 \text{ Hz}$$

$$\text{and } f'_{P1} = \frac{1}{2\pi g_m R_1R_2C_f} \Rightarrow C_f = \frac{1}{2\pi g_m R_1R_2 f'_{P1}}$$

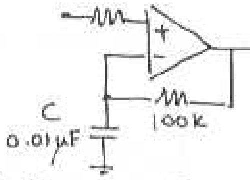
$$\therefore C_f = \frac{1}{2\pi(40\times10^{-3})(10.62\times10^3)(31.85\times10^3)200}$$

$= 58.8$ pF

$$f'_{P2} = \frac{1}{2\pi}\frac{g_m C_f}{C_1C_2 + C_f(C_1+C_2)}$$

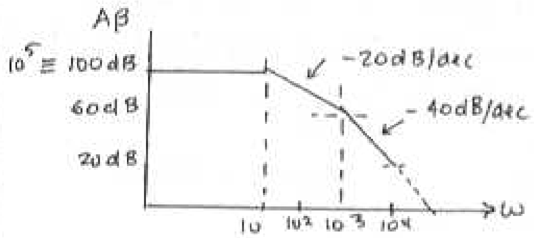
$$= \frac{1}{2\pi}\frac{40\times10^{-3}(58.8\times10^{-12})}{(150\times5)10^{-24} + 58.8(155)10^{-24}} = 38.8\text{MHz}$$

8.81



$$\beta(s) = \frac{1}{1+sCR} = \frac{1}{1+10^{-3}s}$$

(a) $$A\beta(s) = \frac{10^5}{1+s/10} \cdot \frac{1}{1+s/10^3}$$



(b) From plot $|A\beta| = 20\,dB$ at $10^4 = \omega$

Hence $|A\beta| = 1$ at 31.6 krad/s ($\frac{1}{2}$ dec)

(c) $$A_f(s) = \frac{\dfrac{10^5}{1+s/10}}{1+\dfrac{10^5}{1+s/10}\cdot\dfrac{1}{1+s/10^3}}$$

$$= \frac{10^5(1+s/10^3)}{(1+s/10)(1+s/10^3)+10^5}$$

$$\therefore A_f(s) = \frac{1+s/10^3}{1+s/10^6+s^2/10^9} = \frac{10^6 s + 10^9}{s^2+10^3 s+10^9}$$

Zero at $s = -10^3$ rad/s

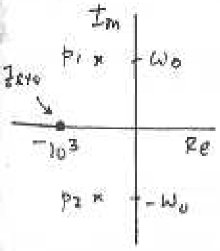
Poles at $$\frac{-10^3 \pm \sqrt{10^6 - 4\times10^9}}{2}$$

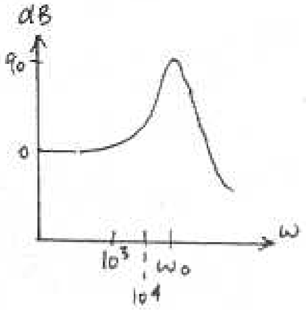
$$= \frac{-10^3 \pm j\,63.2\times10^3}{2}$$

$$= -500 \pm j\,31.6\times10^3 \text{ rad/s}$$

$\omega_0 = 31.6$ Krad/s

$Q = 31.6$





Chapter 9 – Problems

Section 9.1: The two-stage CMOS op-Amp

9.1

$V_{ICM(max)} \leq V_{DD} - |V_{tp}| - |V_{OV1}| - |V_{OV5}|$
$\leq +2.5 - 0.7 - 0.3 - 0.3$
$\leq +1.2V$

$V_{ICM(min)} \geq -V_{SS} + V_{OV3} + V_{tn} - |V_{tp}|$
$\geq -2.5 + 0.3(+0.7 - 0.7)$
$\geq -2.2V$

$-V_{SS} + V_{OV6} \leq V_O \leq V_{DD} - |V_{OV7}|$
$-2.5 + 0.3 \leq V_O \leq +2.5 - 0.3$
$-2.2V \leq V_O \leq +2.2V$

9.2

$V_A' = 25V/\mu m$, $|V_p'| = 20V/\mu m$, $L = 0.8\mu m$
Hence $V_A = 20V$ and $|V_p| = 16V$
For all devices $V_{OV} = 0.25V$

$A = A_1A_2 = G_{m1}(r_{o2}\|r_{o4})\,G_{m2}(r_{o6}\|r_{o7})$
$[r_{op}\|r_{on}] = \left[\frac{V_A}{I} \times \frac{V_p}{I}\right] \times \frac{I}{V_A + V_p} = \left[\frac{V_A\|V_p}{I}\right]$

For A_2: $R_o = \frac{8.89}{I} \rightarrow \frac{8.89V}{0.4\ mA} = 22.2K\Omega$

To avoid systematic output d.c. offset
$$\frac{(W/L)_6}{(W/L)_4} = \frac{2(W/L)_7}{(W/L)_5}$$

Since Q_5, Q_6, Q_7 carry I and Q_4 only $I/2$
satisfy requirement by making Q_4 have $(W/L)/2$
Since $g_m = \sqrt{2(\mu C_{ox})(W/L)I} = 2K(V_{OV})$
$g_{m1} = 2I_1/V_{OV} = 0.4mA/0.25V = 1.6mA/V$
$g_{m6} = 2I_6/V_{OV} = 3.2mA/V$
$\therefore A = (1.6)(44.4)(3.2)(22.2) = 5047\ V/V$
For unity gain amplifier
$$A_F = \frac{A}{1+A\beta} = \frac{5047}{1+5047\beta} = 1$$

Thus $(1 + A\beta) = 5047$
Then $R_{of} = R_o/(1+A\beta)$
$= 22.2K/5047 \approx 4.4\Omega$

9.3

$A = A_1A_2 = G_{m1}(r_{o2}\|r_{o4})\,G_{m2}(r_{o6}\|r_{o7})$
$= \frac{2I_1}{V_{OV}} \cdot \frac{1}{2}\frac{V_A}{I_1} \cdot \frac{2I_2}{V_{OV}} \cdot \frac{1}{2}\frac{V_A}{I_2}$
$= \left[\frac{V_A}{V_{OV}}\right]^2 = 2500$

where $V_A = 10V/\mu m \times 1\mu m = 10V$
Hence $V_{OV} = V_A/50 = 10/50 = 0.2V$

9.4

From $\frac{(W/L)_6}{(W/L)_4} = \frac{2(W/L)_7}{(W/L)_5}$
$(W/L)_6 = \frac{2(60/0.5)(10/0.5)}{(60/0.5)} = (20/0.5)$

$I_{D_p} = \frac{1}{2}[60](W/L)(V_{GS} - V_t)^2$
Given $I_{ref} = 225\mu A = \frac{1}{2}60(60/0.5)[V_{OV}]^2$
$\Rightarrow V_{OV} = 15/60 = 0.25V$
Given $V_T = 0.75V$
$\Rightarrow V_{GS} = V_T + V_{OV} = 0.25 + 0.75 = 1V$
Since Q_5, Q_7, Q_8 have same (W/L)
$\Rightarrow I_1 = I_2 = I_3 = I_4 = \frac{1}{2}I_5 = \frac{1}{2}I_8$
For Q_1Q_2: $I_1 = \frac{1}{2}60(30/0.5)(V_{OV})^2 = 112.5\mu A$
$\Rightarrow V_{OV_1} = 0.25V$
$\Rightarrow V_{GS_1} = V_T + V_{OV_1} = 1.0V$
For Q_3Q_4: $I_3 = \frac{1}{2}180(10/0.5)(V_{OV})^2 = 112.5\ mA$
$\Rightarrow V_{OV3} = 0.25V$
$\Rightarrow V_{GS3} = 0.25 + 0.75 = 1V$
For Q_6: $I_6 = \frac{1}{2}180(20/0.5)(V_{OV})^2 = 225\mu A$
$\Rightarrow V_{OV6} = 0.35V$
$\Rightarrow V_{GS6} = 0.35 + 0.75 = 1.1V$

$r_o = \frac{V_A\ V}{I\ mA} = r_o\ K\Omega$ and $|V_A| = 9V$

$g_m = \frac{2I}{V_{OV}}$ and $|V_{OV}|$ vary

Q	K ($\mu A/V^2$)	I (μA)	V_{ov} (V)	V_{GS} (V)	g_m (mA/V)	r_o (kΩ)
Q1	60	112.5	0.25	0.81	0.9	80
Q2	60	112.5	0.25	0.81	0.9	80
Q3	180	180	0.25	1.0	0.9	80
Q4	180	180	0.25	1.0	0.9	80
Q5	60	225	0.25	1.0	1.8	40
Q6	180	225	0.35	1.1	1.285	40
Q7	60	60	0.25	1.0	1.8	40
Q8	60	60	0.25	1.0	1.8	40

$A_1 = G_{m1}(r_{o2} \| r_{o4}) = 0.9 \times 40 = 36$ V/V

$A_2 = G_{m6}(r_{o6} \| r_{o7}) = 1.285 \times 20 = 25.7$ V/V

$A = A_1 A_2 = 36 \times 25.7 = 925$ V/V

$V_{icm(max)} \le V_{DD} - |V_{tp}| - |V_{ov1}| - |V_{ov5}|$

$\le 1.5 - 0.75 - 0.25 - 0.25$

$\le +0.25$ V

$V_{icm(min)} \ge -V_{SS} + V_{ov3} + V_{tn} - |V_{tp}|$

$\ge -1.5 + 0.25 = -1.25$V

$-V_{SS} + V_{ov6} \le V_O \le +V_{DD} - |V_{ov7}|$

$-1.5 + 0.35 \le V_O \le +1.5 - 0.25$

$-1.15\text{V} \le V_O \le +1.25\text{V}$

9.5

$G_{m1} = 0.3$ mA/V $\quad G_{m2} = 0.6$ mA/V $\quad C_2 = 1$p

$r_{o2} = r_{o4} = 222$ kΩ $\quad r_{o6} = r_{o7} = 111$ kΩ

(a) $f_{P2} = \dfrac{G_{m2}}{2\pi C_2} = \dfrac{0.6 \times 10^{-3}}{2\pi\, 10^{-12}} = 95.5$ MHz

(b) $R = \dfrac{1}{G_{m2}} = \dfrac{1}{0.6} = 1.66$ kΩ

(c) For PM = 80°: $\tan^{-1}\dfrac{f_t}{f_{P2}} = 10°$

$f_t = f_{P2} \tan 10°$

$= 95.5 \times 0.176 = 16.84$ MHz

$C_c = \dfrac{G_{m1}}{2\pi f_t} = \dfrac{0.3 \times 10^{-3}}{2\pi\, 16.84 \times 10^6} \Rightarrow 2.83$ pF

$A = A_1 A_2 = G_{m1}(r_{o2} \| r_{o4}) G_{m2}(r_{o6} \| r_{o7})$

$= 0.3 \times 111 \times 0.6 \times 55.5$

$= 1109 \equiv 60.8$ dB

Dominant pole $f_{P1} = f_t / |A|$

Thus f_{P1} is approx 3 decades below f_t i.e at 16.84 KHz providing uniform 20 dB/dec slope down to f_t.

(d) $f_t = \dfrac{G_{m1}}{2\pi C_c}$ $\quad \therefore$ to double f_t, halve C_c

$C_{c(new)} = 1.4$ pF

$\tan^{-1}\dfrac{f_t}{f_P} = \tan^{-1}\dfrac{33.7}{95.5} = 19.4°$

The zero must be moved to reduce the $19.1 - 10 = 9.4°$

$\tan^{-1}\dfrac{f_t}{f_Z} = 9.4° \rightarrow \dfrac{f_t}{f_Z} = 0.16$

$\Rightarrow f_Z = 0.16 f_t = 0.16 \times 33.7$

$= 5.6$ MHz

$f_Z = \dfrac{1}{2\pi C_c [R - \frac{1}{G_m}]} \rightarrow [R - \frac{1}{G_m}] = \dfrac{1}{2\pi f_Z C_c}$

Hence $[R - 1/G_m] = \dfrac{10^{12} \cdot 10^{-6}}{2\pi\, 5.6 \times 1.4}$

$R = 1.67 + 20.3 = 21.97$ KΩ

9.6

Two-stage amp with $C_2 = 1$pF

$f_t = 100$ MHz, PM = 75°

For PM = 75°: $\tan^{-1}\dfrac{f_t}{f_{P2}} = 15°$

$\therefore f_{P2} = f_t \tan 15° = 3.73 f_t = 373$ MHz

$f_{P2} = \dfrac{G_{m6}}{2\pi C_2} = \dfrac{1}{2\pi R_2\, 10^{-12}} = 373$ MHz

$\Rightarrow R_2 = \dfrac{10^{12}}{2\pi (373 \times 10^6)} = 426\,\Omega$

$\Rightarrow G_{m6} = \dfrac{1}{R_2} = 2.35 \times 10^{-3}$ mA/V

To move zero to infinity $R = \dfrac{1}{G_{m6}} = 426\,\Omega$

$SR = \dfrac{I}{C_c} = \dfrac{200\,\mu A}{C_c}$

$SR = 2\pi f_t V_{ov1} = 2\pi\, 10^8 \times 0.2 = 1.26 \times 10^8$ V/S

$\Rightarrow C_c = \dfrac{200 \times 10^{-6}}{1.26 \times 10^8} \Rightarrow 1.6$ pF

9.7

$G_{m1} = 1 mA/V$, $G_{m2} = 2 mA/V$, $R = 500 \Omega$

$$f_t \approx \frac{G_{m1}}{2\pi C_c} \Rightarrow C_c = \frac{G_{m1}}{2\pi f_t} = \frac{1\times10^{-3}}{2\pi\, 100\times10^6} \Rightarrow 1.59 pF$$

For $\frac{1}{G_{m2}} - R = \frac{10^3}{2} - 500 = 0$

Zero has been moved to ∞

For PM = 60: $f_t = f_{P2} \tan(90-30)^\circ$

$\Rightarrow f_{P2} = f_t / \tan 30^\circ = 173 MHz$

$$C_2 \approx \frac{G_{m2}}{2\pi f_{P2}} = \frac{2\times10^{-3}}{2\pi\, 173\times10^6} = 1.84 pF$$

9.8

$SR = 60 V/\mu s$ $\quad f_t = 50 MHz$

(a) $SR = 2\pi f_t V_{ov1}$

$\Rightarrow V_{ov1} = SR/2\pi f_t$

$$= \frac{60\times10^6}{2\pi(50\times10^6)} \approx 0.2V$$

(b) $SR = \frac{I}{C_c} \Rightarrow C_c = \frac{100\mu A}{60\times10^6} = 1.67 pF$

(c) $I = \frac{1}{2}\mu C_{ox}(W/L)[V_{ov}]^2$

$$\Rightarrow \left(\frac{W}{L}\right) = \frac{2I}{50[0.2]^2} = \frac{100}{1}$$

9.9

Invert circuit leaving V_{DD} + V_{SS} and reverse all arrows on FETs.

Section 9.2
The Folded-Cascode OP AMP

9.10

From circuit: Current drawn from V_{DD} rail $= 2 I_B$ = Current returned to V_{SS} rail

$\therefore$ Power $= (V_{DD}+V_{SS}) \times 2I_B \Rightarrow$

$1 mW = (1.65+1.65)\times 2I_B$

$$\therefore I_B = \frac{1 mW}{4\times1.65 V} = 151.5\mu A$$

$\Rightarrow I = I_B/1.2 = 126.3\mu A$

9.11

V_{BIAS1}: V_S can rise to $V_{DD} + V_T - V_{ov}$

V_{D3} can rise to $V_{S3} - V_{ov}$

$\therefore V_{BIAS1} = V_{DD} - V_{ov10} - V_{ov4} + V_T$

$= 1.65 - 0.2 - 0.2 + 0.5$

$= 1.75 V$

$V_{BIAS2} = V_{DD} - V_{ov10}$

$= +1.65 - 0.2 = +1.45V$

$V_{BIAS3} = -V_{SS} + V_{ov11}$

$= -1.65 + 0.2 = -1.45V$

$V_{ICM(max)} = V_{DD} - |V_{ova}| + V_{tn}$

$= +1.65 - 0.2 + 0.5$

$= +1.75V$

$V_{ICM(min)} = -V_{SS} + V_{ov11} + V_{ov1} + V_T$

$= -1.65 + 0.2 + 0.2 + 0.5$

$= -0.75V$

$-V_{SS} + 2V_{ov} + V_t \le V_o \le +V_{DD} - 2V_{ov}$

$-1.65 + 0.4 + 0.5 \le V_o \le +1.65 - 0.4$

$-0.75V \le V_o \le +1.25V$

9.12

$I = 125\mu A$, $I_B = 150\mu A$. $V_t = 0.2 V$

For Q_9, Q_{10}: $I_B = 150\mu A$

$I = \frac{1}{2}(\mu C_{ox})(W/L)(V_{ov})^2$

$150 = \frac{1}{2}\, 90\, (W/L)(0.2)^2$

$\Rightarrow (W/L)_{9,10} = (83.33/1)$

For Q_1, Q_2: $I = 125\mu A/2$

$\frac{1}{2}.125 = \frac{1}{2}\, 250 (W/L)(0.2)^2$

$\Rightarrow (W/L)_{1,2} = (12.5/1)$

For Q_{11}: $I = 125\mu A$

$125 = \frac{1}{2}\, 250 (W/L)(0.2)^2$

$\Rightarrow (W/L)_{11} = (25/1)$

For $Q_3 Q_4$: $I = 125\mu A/2$

$\frac{1}{2} 125 = \frac{1}{2}\, 60\, (W/L)(0.2)^2$

$\Rightarrow (W/L)_{3,4} = (52/1)$

For $Q5, Q6, Q7, Q8$: $I = 125\mu A/2$

$\frac{1}{2} 125 = \frac{1}{2}\, 250 (W/L)(0.2)^2$

$\Rightarrow (W/L)_{5,6,7,8} = (12.5/1)$

9.13

(1) $SR = \frac{I}{C_L} \Rightarrow I = SR \times C_L$

$= 10\times10^6 \times 10\times10^{-12}$

$= 100\mu A$

(2) $I_B = 1.2\, I = 120\mu A$

(3) $f_p = \frac{1}{2\pi C_L R_o}$. $f_t = \frac{G_m}{2\pi C_L}$

$G_m = \frac{2I/2}{V_{ov}} = \frac{100\mu A}{0.2V} = 0.5 mA/V$

$f_t = \frac{0.5\times10^{-3}}{2\pi\ 10\times10^{-12}} = 7.96\ MHz$

(4) $R_{of} \approx \frac{1}{G_m} = \frac{1000}{0.5} = 2K\Omega$

$A_V = f_t/f_p = G_m R_o$

But $R_{of} = \frac{R_o}{1+G_m R_o} \Rightarrow R_o = 2M\Omega$

$\therefore A_V = 0.5\times10^{-3} \times 2\times10^6 = 1000$

$f_p = f_t/1000 = 7.96\ KHZ$

(5) $\theta = -\tan^{-1}\frac{f_t}{f_p} - 2(\tan^{-1}\frac{f_t}{f_{p2}})$

$PM\ @\ f_{p2} = 90^\circ - \tan^{-1}\frac{f_t}{f_{p2}}$

$= 90^\circ - \tan^{-1}\left[\frac{7.96 MHz}{25\ MHz}\right]$

$= 90^\circ - 17.7^\circ = 72.3^\circ$

(6) For $PM = 75^\circ$: $\tan^{-1}\frac{f_t}{f_{p2}} = 15^\circ$

Thus $f_t^* = f_{p2} \tan 15^\circ$

$= 25 MHz \times 0.27 = 6.7 MHz$

$\frac{f_t^*}{f_t} = \frac{C_L}{C_L^*} \Rightarrow \frac{6.7}{7.96} = \frac{10pF}{C_L^*}$

$\Rightarrow C_L^* = C_L \frac{7.96}{6.7} = C_L \times 1.19$

$\therefore$ Increase C_L by 19%.

(7) $SR^* = \frac{I}{C_L^*} \Rightarrow \frac{SR}{1.19} = 8.4 V/\mu s$

9.14

$G_m = \frac{I}{V_{ov}}$ $\quad r_o = \frac{V_A}{I}$

$R_o = R_{o4} \| R_{o6}$

$R_{o4} = g_{m4} r_{o4} (r_{o2} \| r_{o10})$

$R_{o6} = g_{m6} (r_{o6} \| r_{o8})$

$A = G_{m1} R_o$

Q_{10}: $I = I_B \Rightarrow g_{m10} = 0.75\ mA/V$

$r_{o10} = 66.6\ K\Omega$

Q_x: $I = 125\mu A \Rightarrow g_x = 0.31\ mA/V$

$r_{ox} = 160\ K\Omega$

$\therefore R_{o4} = 0.31 \times 160 \times 47K = 2359K$

$R_{o6} = 0.31 \times 160 \times 160K = 8000K$

$R_o = 2.35 \| 8.0 = 1.8 M\Omega$

$A = g_{m1} R_o = 0.31 \times 1800 = 558$

$\frac{V_o}{V_i} = 1 + \frac{(1/sC)}{(1/s9C)} = 1 + \frac{9sC}{sC} = 10$

$\therefore \beta = 1/10 = 0.1$

$A_F = \frac{A}{1+A\beta} = \frac{558}{1+558\times0.1} = 9.8$

$R_{of} = \frac{R_o}{1+A\beta} = \frac{1.8}{56.8} = 31.69\ K\Omega$

9.15

Rail-to-Rail OP Amp

All $V_{OV} = 0.2V$ All $V_T = 0.5V$

$V_{DD} = V_{SS} = 1.65V$

$V_{ICM}(max) = V_{DD} - |V_{OV9}| + V_T$
$= 1.65 - 0.2 + 0.5 = +1.95V$

$V_{ICM}(min) = -V_{SS} + V_{OV11} + V_{OV1} + V_T$
$= -1.65 + 0.2 + 0.2 + 0.5$
$= -0.75V$

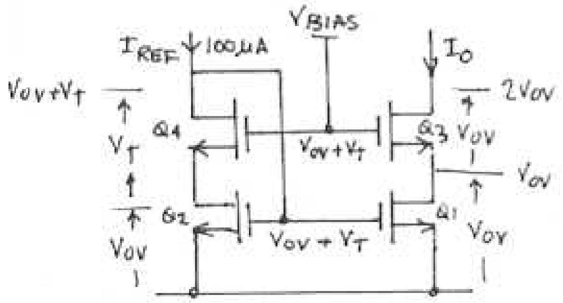
(a) For NMOS stage
$-0.75V \le V_{ICM} \le +1.95V$

(b) For PMOS stage (symetrical)
$-1.95V \le V_{ICM} \le +0.75V$

(c) For both active
$-0.75V \le V_{ICM} \le +0.75V$

(d) overall
$-1.95V \le V_{ICM} \le +1.95V$

9.16



All same $k(W/L)$ $\therefore I_O \simeq I_{REF}$

All same $r_o = V_A/I = 10V/100\mu A = 100K\Omega$

$I = \frac{1}{2}k[W/L][V_{OV}]^2$

$V_{D3} = V_O$: $V_O(min) = 2V_{OV}$

R_O (looking into Q3D and assuming I_O current source is ideal $r_o' = \infty$)

$= r_{o3}(1 + g_{m3} r_{o1}) \approx g_m r_o^2$

$g_m = \frac{2I}{V_{OV}} = \frac{2 \times 100}{0.2} \Rightarrow 1mA/V$

Then $R_O \approx g_m r_o^2 = 1 \times 10^5 \times 10^5$
$\approx 10^4 M\Omega$

9.17

$A = 80dB$ $f_t = 10MHz$ $C_L = 10p$

$I_B = I$ All same $|V_{OV}|$, $L = \phi\mu m$

$|V_A| = 20V$

$g_m = \frac{2I}{V_{OV}}$ and $f_t = \frac{g_m}{2\pi C_L}$

$A = g_{m1}\left[g_{m4} r_{o4}(r_{o2}||r_{o10})\right] || \left[g_{m6} r_{o6} r_o\right]$

Consider Q_1: $I_1 = \frac{1}{2}k_n[W/L]_1[V_{OV}]^2$
$= \frac{1}{2}200[W/L]_1[V_{OV}]^2$

$Q_1 Q_2 Q_5 Q_6 Q_7 Q_8$ are same

$g_{m1} = \frac{2I_1}{V_{OV}}$ and $r_{o1} = \frac{V_A}{I_1}$

Consider Q_3, Q_4: $I_3 = \frac{1}{2}k_p[W/L]_3[V_{OV}]^2$
$I_1 = \frac{1}{2}\frac{200}{2.5}[W/L]_3[V_{OV}]^2$

$\Rightarrow [W/L]_3 = 2.5[W/L]_1$

$g_{m3,4} = g_{m1}$ and $r_{o3,4} = r_{o1}$

Consider $Q_9 Q_{10}$: $I_{10} = \frac{1}{2}k_p[W/L]_{10}[V_{OV}]^2$
$2I_1 = \frac{1}{2}\frac{200}{2.5}[W/L]_{10}[V_{OV}]^2$

$\Rightarrow [W/L]_{10} = 5[W/L]_1$

$g_{m9,10} = 2g_{m1}$ and $r_{o9,10} = r_{o1}/2$

Consider Q_{11}: $I_{11} = \frac{1}{2}k_n[W/L]_{11}[V_{OV}]^2$
$2I_1 = \frac{1}{2}200[W/L]_{11}[V_{OV}]^2$

$\Rightarrow [W/L]_{11} = 2[W/L]_1$

$g_{m11} = 2g_{m1}$ and $r_{o11} = r_{o1}/2$

Thus $A = g_{m1}\left[g_{m1} r_{o1}(r_{o1} || \frac{r_{o1}}{2})\right] || \left[g_{m1} r_{o1} r_{o1}\right]$
$= g_{m1}\left[g_{m1} r_{o1}\right](r_{o1} || \frac{r_{o1}}{2} || r_{o1})$

$10^4 = \frac{1}{4} g_{m1} g_{m1} r_{o1} r_{o1}$

$\Rightarrow g_{m1} r_{o1} = 200$

Now $g_{m1} r_{o1} = \frac{2I}{V_{OV}} \cdot \frac{V_A}{I}$

$\Rightarrow V_{OV} = 2V_A/200 = 2(20)/200 = 0.2$

Hence $g_{m1} = \frac{1}{2\pi f_t C_L} = 0.628\ mA/V$

$\rightarrow r_{o1} = 200/g_{m1} = 318K\Omega$

$$g_m = \frac{2I}{V_{ov}} \Rightarrow I = \frac{g_m V_{ov}}{2}$$

$$\Rightarrow I_1 = \frac{g_{m1} V_{ov}}{2} = \frac{0.628\ mA/V \times 0.2V}{2}$$

$$= 62.8\ \mu A$$

$$SR = 2\pi f_t V_{ov} = 2\pi\ 10\times10^6 \times 0.2 = 12.5\ V/\mu s$$

$Q_1 Q_2 Q_5 Q_6 Q_7 Q_8$:

$$I = \tfrac{1}{2} k_n [W/L][V_{ov}]^2$$

$$62.8 = \tfrac{1}{2}\, 200\, [W/L][V_{ov}]^2$$

$$\Rightarrow [W/L]_1 = 15.7$$

For Q_3, Q_4: $I = \frac{1}{2}\frac{200}{2.5}[W/L][V_{ov}]^2$

$$62.8 = \frac{1}{2}\frac{200}{2.5}[W/L][0.2]^2$$

$$\Rightarrow [W/L]_3 = 2.5[W/L]_1 = 39.25$$

For Q_9, Q_{10}:

$$[W/L]_9 = 5[W/L]_1 = 78.5$$

For Q_{11}:

$$[W/L]_{11} = 2[W/L]_1 = 31.4$$

For $L = 1\mu m$: $W_x = [W/L]_x\ \mu m$

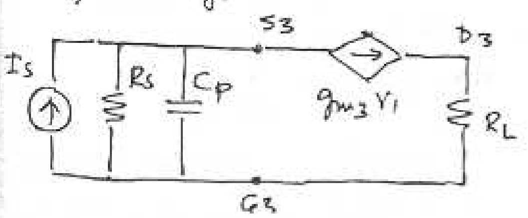
∴ Width for $Q_1, Q_2, Q_5, Q_6, Q_7, Q_8$ $= 15.7\ \mu m$
for Q_3, Q_4 $= 39.25\ \mu m$
for Q_9, Q_{10} $= 78.5\ \mu m$
for Q_{11} $= 31.4\ \mu m$

9.18

Simply invert circuit relative to V_{DD}, V_{SS} and reverse all arrows on FETs

9.19

Model for CG stage:



at S_3: $I_S - g_{m3}V_1 - V_1\frac{(1+sC_pR_S)}{R_S} = 0$

$$\Rightarrow \frac{V_1}{I_S} = \frac{1}{g_{m3} + (1+sC_pR_S)/R_S}$$

$$= \frac{1}{(g_{m3} + 1/R_S) + sC_pR_S/R_S}$$

3dB down when

$$sC_p = (g_{m3} + 1/R_S)$$

$$\Rightarrow f_{P3} = \frac{(g_{m3} + 1/R_S)}{2\pi C_p} \approx \frac{g_{m3}}{2\pi C_p}$$

QED.

Also $f_t \approx \frac{g_{m1}}{2\pi C_c}$

$$PM = 180 - \phi_{total} = 90° - \tan^{-1}\frac{f_t}{f_2}$$

For $PM = 75°$: $\frac{f_t}{f_2} = \tan 15° = 0.27$

$$\Rightarrow \frac{f_t}{f_2} = \frac{C_p}{C_L} = 0.27$$

$$\Rightarrow C_p = 0.27\,C_L$$

$\boxed{20}$ $\boxed{9}$

$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$g_m = \frac{I_C}{V_T}$$

$$r_e = \frac{1}{g_m}$$

$$r_\pi = \beta r_e$$

$$r_o = \frac{V_A}{I_C}$$

$$r_\mu = \beta \frac{10 V_A}{I_C}$$

DEVICE	V_{BE} (mV)	g_m (mA/V)	r_e (Ω)	r_π (kΩ)	r_o (MΩ)	r_μ (GΩ)
$Q_1 - Q_4$, $Q_5 - Q_6$	517	0.38	2632	526	13.2	26.3
Q_{16}	530	0.648	1543	309	7.72	15.4
Q_{17}	618	22	45.5	9.09	0.227	0.46

$\boxed{21}$ $\boxed{9}$

$$I_3 = I_1 \sqrt{\frac{I_{S3}\, I_{S4}}{I_{S1}\, I_{S2}}}$$

$$= 154 \sqrt{\frac{10^{-14} \cdot 10^{-14}}{3\times10^{-14} \cdot 6\times10^{-14}}}$$

$$= \underline{\underline{36.3\,\mu A}}$$

$\boxed{22}$ $\boxed{9}$

$$I_{E_{TOT}} = 0.73\,mA$$

$$I_{EA} = 0.25(0.73) = 0.1825\,mA$$

$$I_{EB} = 0.75(0.73) = 0.5475\,mA$$

$$V_{EB_A} = V_T \ln \frac{0.1825\times10^{-3}}{0.25\times10^{-14}} = \underline{\underline{0.625\,V}}$$

$$g_{mA} = \frac{I_C}{V_T} \simeq \frac{I_E}{V_T} = \underline{\underline{7.3\,mA/V}}$$

$$r_{eA} = \frac{\alpha}{g_{mA}} = \underline{\underline{134.3\,\Omega}}$$

$$r_{\pi A} = (\beta+1) r_{eA} = \underline{\underline{6.85\,k\Omega}}$$

$$r_{oA} = \frac{V_A}{I_{CA}} = \underline{\underline{274\,k\Omega}}$$

$$V_{EB_B} = V_{EB_A} = \underline{\underline{0.625\,V}}$$

$$g_{mB} = \frac{0.5475}{25} = \underline{\underline{21.9\,mA/V}}$$

$$r_{e_B} = \frac{\alpha}{g_{mB}} = \underline{\underline{44.7\,\Omega}}$$

$$r_{\pi B} = (\beta+1) r_{eB} = \underline{\underline{2.28\,k\Omega}}$$

$$r_{oB} = \underline{\underline{91.3\,k\Omega}}$$

$\boxed{23}$ $\boxed{9}$

Let $V_{B2} = 0$

For breakdown $v_{ID} = v_{B1} - v_{B2}$
$> V_{BE1} + V_{BE2} + 7 + 50$

or $v_{ID} \geq \underline{\underline{58.4\,V}}$

$\boxed{24}$ $\boxed{9}$

$$V_{SG1} + V_{GS2} = V_{SG4} + V_{GS3}$$

Since V_t's are equal :

$$\sqrt{\frac{I_1}{K_1}} + \sqrt{\frac{I_1}{K_2}} = \sqrt{\frac{I_4}{K_4}} + \sqrt{\frac{I_3}{K_3}}$$

$$\sqrt{I_1}\left[\frac{1}{\sqrt{K_1}} + \frac{1}{\sqrt{K_2}}\right] = \sqrt{I_3}\left[\frac{1}{\sqrt{K_4}} + \frac{1}{\sqrt{K_3}}\right]$$

or

$$\underline{\underline{\sqrt{\frac{I_1}{I_3}} = \frac{\sqrt{\frac{1}{K_3}} + \sqrt{\frac{1}{K_4}}}{\sqrt{\frac{1}{K_1}} + \sqrt{\frac{1}{K_2}}}}}$$

$K_1 = K_2$, $K_3 = K_4 = 16 K_1$

$$\sqrt{I_1} = \sqrt{I_3}\sqrt{\frac{K_2}{K_4}}$$

or $I_1 = I_3 \dfrac{K_2}{K_4}$

$$= \frac{I_3}{16} = \underline{\underline{100\,\mu A}}$$

$\boxed{25}$ $\boxed{9}$

As $V_{BE} = 0.7$

$$I_{ref} = \frac{5 - 1.4 - (-5)}{R_5}$$

$$= \underline{\underline{220.5\,\mu A}}$$

At this current level

$$V_{BE} = V_T \ln \frac{220.5\times10^{-3}}{10^{-14}} = 595\,mV$$

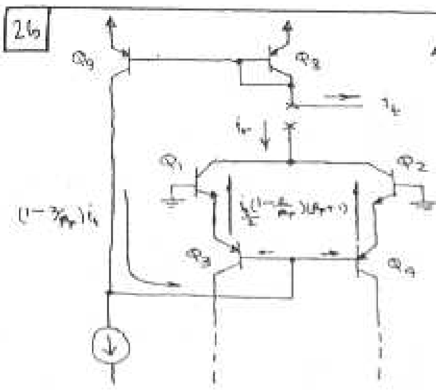
$$\Rightarrow I_{ref} = \frac{10 - 2(0.595)}{39k} = \underline{\underline{226\,\mu A}}$$

For $I_{ref} = 0.73\,mA$, $V_{BE} = 0.625\,V$

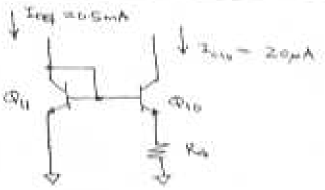
$$R_5 = \frac{10 - 2(0.625)}{0.73\times10^{-3}} = \underline{\underline{12\,k\Omega}}$$

26 $\boxed{9}$

Assume $\beta_P >> 1$



27 $\boxed{9}$



$I_{C10} R_4 = V_{BE11} - V_{BE10}$

$= V_T \ln \frac{I_{ref}}{I_{C10}}$

$\therefore R_4 = \frac{25\times10^{-3}}{20\times10^{-6}} \ln \frac{0.5\times10^{-3}}{20\times10^{-6}} = \underline{\underline{4.02\ k\Omega}}$

$V_{BE11} = V_T \ln \frac{0.5\times10^{-3}}{10^{-14}} = \underline{\underline{616 mV}}$

$V_{BE10} = V_T \ln \frac{20\times10^{-6}}{10^{-14}} = \underline{\underline{535 mV}}$

28 $\boxed{9}$

Assume $\beta_P >> 1$

$I_{C10} \doteq \frac{2I}{1+2/\beta_P} + \frac{2I}{\beta_P}$

$\simeq 2I\left(1 - \frac{2}{\beta_P} + \frac{1}{\beta_P}\right)$

$= 2I(1 - 1/\beta_P)$

$\Rightarrow I \simeq \frac{I_{C10}}{2}\left(1 + \frac{1}{\beta_P}\right)$

Thus $\frac{1}{\beta_P} = 0.1 \Rightarrow \underline{\underline{\beta_P = 10}}$

Without the above assumption and using the exact relationship $\beta_P = 7.74$.

29 $\boxed{9}$

In this case

$\frac{4I}{1+2/\beta_P} + \frac{2I}{\beta_P} = I_{C10}$

For $\beta_P >> 1$

$I_{C10} \simeq 4I$ or $I = \underline{\underline{4.75\mu A}}$

To correct we need $I_{C10} = 38\mu A$

$\Rightarrow R_4 = \frac{V_T}{I_{C10}} \ln \frac{0.73 mA}{I_{C10}} = \underline{\underline{1.94\ k\Omega}}$

30 $\boxed{9}$

At $I = 9.5\mu A$

$V_{BE5} = V_{BE6} = 517 mV$

and $V_{B6} = V_{BE6} + IR_2$

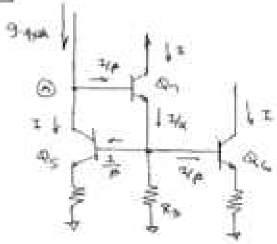
$= 526.5 mV$

If R_2 is shorted $V_{BE6} = V_{B6} = 526.5 mV$

and $I_{C6} = I_S e^{V_{BE6}/V_T}$

$= \underline{\underline{14\mu A}}$

31 $\boxed{9}$



ΣI @ A $\quad I + I/\beta = 9.4\mu A$

$\Rightarrow I = \frac{9.4}{1+1/\beta} = \underline{\underline{9.353\mu A}}$

$I_{R3} = \frac{I}{\alpha} - \frac{2I}{\beta} = 9.347\mu A$

$V_{B5} = I_{R3} R_3 = V_{BE5} + \frac{I}{\alpha}R$

$V_{BE5} = V_T \ln \frac{9.353\mu}{10^{-14}} = 516.4 mV$

Thus $V_{B5} = 525.8 mV$

and $R_3 = \frac{V_{B5}}{I_{R3}} = \underline{\underline{56.5\ k\Omega}}$

32 $\boxed{9}$

Assume equal collector current

$I_{C1} = I_{C2} = 9.5\mu A$

$I_{OS} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2}$

$= \underline{\underline{15.7 nA}}$

$I_B = \frac{1}{2}(I_{B1} + I_{B2})$

$= \underline{\underline{55.4 nA}}$

33

$I_B = 40\,nA$, $I_{os} = 4\,nA$

Thus, base currents are

$I_{B1} = (I_B \pm \frac{I_{os}}{2})$

$= \frac{9.5}{\beta_N}\,\mu A$

$\hat{\beta}_N = \frac{9.5\,\mu A}{38\,nA} = \underline{250}$

$\check{\beta}_N = \frac{9.5\,\mu A}{42\,nA} = \underline{226}$

$\Rightarrow \Delta\beta_N = 24$

$\overline{\beta}_N = \frac{\hat{\beta}_N + \check{\beta}_N}{2} = 238$

34

$I_{C1} + I_{C2} = 19\,\mu A$

Mirror forces $I_{C2} = 0.9\,I_{C1}$

Thus $I_{C1} = \frac{19}{1.9}\,\mu A = 10\,\mu A$

and $I_{C2} = 9\,\mu A$

$V_{OS} = \Delta V_{BE}$

$= V_{BE1} - V_{BE2}$

$= V_T \ln\frac{10}{9} = \underline{\underline{2.63\,mV}}$

35 $\boxed{9}$

At $I_{C17} = 550\,\mu A$, $V_{BE17} = 618\,mV$

$I_{B17} = \frac{550}{200} = 2.75\,\mu A$

$\Rightarrow I_{C16} = 9.5\,\mu A - I_{B17} + \frac{I_{E17}R_8 + V_{BE17}}{R_9}$

or $R_9 = 99.7\,k\Omega$

36

Neglecting base currents

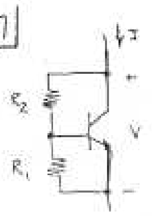
$I_{C18} = I_{C19} = \frac{180}{2} = 90\,\mu A$

$V_{BE18} = V_T \ln\frac{90 \times 10^{-6}}{10^{-14}} = 573\,mV$

Thus $R_{10} = \frac{V_{BE18}}{I_{C18}} = \underline{\underline{6.37\,k\Omega}}$

$I_{C14} = 3 \times 10^{-14}\, e^{573/26} = \underline{270\,\mu A} = I_{C20}$

37



$V_{R_1} = V_{BE}$

If we ignore base currents

$V_{R_1} = \frac{R_1}{R_1 + R_2}\,V$

and $V = V_{BE}\left(\frac{R_1 + R_2}{R_1}\right)$

$V = V_{BE}\left(1 + \frac{R_2}{R_1}\right)$

38

$I_{V_{CC}} = I_{C12} + I_{C13A} + I_{C13B} + I_{C14} + I_{C9} + I_{C8} + I_{C7} + I_{C16}$

$= (730 + 180 + 550 + 154 + 19 + 19 + 10.5 + 16.2)\,\mu A$

$= \underline{1.68\,mA}$

$P_{Diss} = P_D = I_{V_{CC}}(V_{CC} + V_{EE})$

$= 1.68(15 + 15)\,mW$

$= \underline{\underline{50.4\,mW}}$

39 $\boxed{9}$

Series connection of devices assures the same bias currents

$R_{id} = (\beta + 1)(4 r_e)$

$r_e = \frac{V_T}{9.5\,\mu A} = 2.63\,k\Omega$

$R_{id} = \underline{3.17\,M\Omega}$

$i_e = \frac{V_{id}}{4 r_e}$; $i_o = 2 i_e$

$\Rightarrow G_{m1} = \frac{i_o}{v_{id}} = \frac{2}{4 r_e} = \frac{1}{2 r_e}$

$= \underline{\underline{127\,\mu A/V}}$

$R_{o4} = r_o(1 + g_m(R_e \| r_\pi))$

$g_m = 1/r_e$

$R_e = 2 r_e = 5.26\,k\Omega$

$r_\pi = (\beta_P + 1)\, r_e = 134\,k\Omega$

Thus $R_{o4} = \underline{\underline{15.4\,M\Omega}}$

$R_{o6} = 18.2\,M\Omega$ (from text)

$R_{o1} = R_{o4} \| R_{o6} = \underline{\underline{8.34\,M\Omega}}$

$G_{m1} R_{o1} = 127 \times 8.34 = 1059\,V/V$

See gain decreases due to negative feedback.

40

$R_o = r_{o6}(1 + g_{m6}(R_2 \| r_{\pi 6}))$

– need to double the second factor

Since $r_{\pi 6} \gg R_2$

$R_{o6} \simeq r_{o6}(1 + g_{m6} R_2)$

Thus

$1 + g_{m6} R_2' = 2(1 + g_{m6} R_2)$

$g_{m6} = \frac{1}{2.63\,k\Omega}$, $R_2 = 1\,K$

$\Rightarrow R_2' = \underline{\underline{4.63\,k\Omega}}$

41 $I_{C5} = I_{C6} = I_{C7}$

$\Rightarrow r_{e5} = r_{e6} = r_{e7} = 2.63\,k\Omega$

(a) $v_{b6} = (r_{e6} + R_2)\,i_e = \underline{\underline{4.63\,k\Omega \times i_e}}$

(b) $R_B = 50k \,||\, r_{\pi5} \,||\, r_{\pi6}$

$= 45.1\,k\Omega$

$\Rightarrow i_{e7} = \dfrac{v_{b6}}{R_B} = \underline{\underline{0.103\,i_e}}$

(c) $i_{b7} = \dfrac{i_{e7}}{\beta+1} = \dfrac{0.103}{201}\,i_e = \underline{\underline{510\mu A \times i_e}}$

(d) $v_{b7} = v_{b6} + r_{e7}\,i_{e7}$

$= (4.63\,k\Omega + 2.63\,k\Omega \times 0.103)\,i_e$

$= \underline{\underline{4.9\,k\Omega \times i_e}}$

(e) $R_{in} = \dfrac{v_{b7}}{i_e} = \underline{\underline{49\,k\Omega}}$

---

42 From eqn 9.85

$$\frac{\Delta I}{I} = \frac{\Delta R}{R + \Delta R + r_e}$$

$$\Delta I = G_{m1} V_{os} = \frac{V_{os}}{2r_e}$$

Thus

$$\frac{V_{os}}{2r_e I} = \frac{\Delta R}{R + \Delta R + r_e} \quad ; \quad r_e I = V_T$$

$$\frac{V_{os}}{2V_T} = \frac{\Delta R}{R}\left[\frac{1}{1 + \frac{r_e}{R} + \frac{\Delta R}{R}}\right] \quad (*)$$

$$\frac{V_{os}}{2V_T}\left(1 + \frac{r_e}{R}\right) = \frac{\Delta R}{R}\left(1 - \frac{V_{os}}{2V_T}\right)$$

$$\underline{\underline{\frac{\Delta R}{R} = \frac{V_{os}}{2V_T}\;\frac{1 + r_e/R}{1 - \frac{V_{os}}{2V_T}}}}$$

(b) $V_{os} = 5mV$, $r_e = 2.63\,k\Omega$, $R = 1\,k\Omega$

$$\frac{\Delta R}{R} = \frac{5}{2(25)}\;\frac{1 + 2.63}{1 - \frac{5}{2(25)}} = \underline{\underline{0.40}}$$

(c) R_2 completely shorted

$\Rightarrow \dfrac{\Delta R}{R} = -1$

From (*) $\dfrac{V_{os}}{2V_T} = -1\;\dfrac{1}{r_e/R}$

$\Rightarrow V_{os} = \underline{\underline{-19\,mV}}$ (or 19mV)

---

43 Current in the collector of Q_3 remains unchanged at $9.5\mu A$

Thus $I_{E3} = I_{E4} = \frac{51}{50}\,9.5\mu A = 9.69\mu A$

$I_{C4} = \frac{25}{26}\,I_{E4} = 9.317\mu A$

$\Rightarrow \Delta I = 9.5 - 9.317 = 0.183\mu A$

$V_{os} = \dfrac{\Delta I}{G_{m1}} = 2r_e\,\Delta I = 2(2.63\,k\Omega)(0.183\mu A)$

$= \underline{\underline{0.96\,mV}}$

~~44~~ **19**

---

45 Working with common-mode half circuits

$$i_{c3} = \frac{\beta_P}{\beta_P + 1}\;\frac{v_{icm}}{r_{e1} + r_{e2} + \frac{2R_o}{\beta_P + 1}}$$

$$= \frac{\beta_P\,v_{icm}}{(\beta_P + 1)(r_{e1} + r_{e2}) + 2R_o}$$

Similarly

$$i_{c4} = \frac{K\beta_P\,v_{icm}}{(K\beta_P + 1)(r_{e1} + r_{e2}) + 2R_o}$$

---

46 **9** Recall that for a resistor degenerated mirror the current gain is given, approximately, by

$\dfrac{i_o}{i_1} = \dfrac{R_{E1}}{R_{E2}}$ where R_E is the total emitter resistance

(a) R_1 shorted

$\dfrac{i_o}{i_1} = \dfrac{r_{e1}}{r_{e2} + R_2} = \dfrac{2.63}{2.63 + 1} = 0.72$

Thus $i_o = (1.72)\,i_e$ (cf. $2i_e$)

Since the output impedance is unaffected, the gain is thus reduced by $\underline{\underline{14\%}}$ $\left(\frac{1.72 - 2}{2}\right)$

(b) R_2 shorted

$\dfrac{i_o}{i_1} = \dfrac{r_{e1} + R_1}{r_{e2}} = \dfrac{3.63}{2.63} = 1.38$

$\Rightarrow i_o = (1 + 1.38)\,i_e = 2.38\,i_e$

Note that the output of the mirror decreases because of the lack of degeneration from R_2. Neglecting this, since R_{o1} is largely determined by R_2,
$\Rightarrow$ gain increases by $\frac{2.38 - 2}{2} = \underline{\underline{19\%}}$

(c) Current gain remains at unity. Thus
$i_o = 2i_e$
and gain is unaffected.

---

47 **9** Since r_μ is typically very large we will ignore its effect

$r_{o1} = \dfrac{125}{9.5\mu A} = 13.16\,M\Omega$

$r_{o3} = \dfrac{50}{9.5\mu A} = 5.26\,M\Omega$

$R_i = (\beta_n + 1)\left[r_{e1} + \left(r_{e2} + \frac{2R_o}{\beta_P + 1}\right) || \, r_{o1} \,||\, r_{o3}\right]$

$= 201\left[2.63K + \left(2.63K + \frac{4.5M}{51}\right) || \, 13.16M \,||\, 5.26M\right]$

$= \underline{\underline{19.5\,M\Omega}}$

Since $R_i = 2R_{icm}$

$R_{icmf} = \dfrac{R_i}{2}(1 + \beta_P)$ See problem #8

$= \underline{\underline{497\,M\Omega}}$

48

$$R_{i2} = (\beta+1)\left[r_{e16} + R_{i17} \| R_9 \right]$$

$r_{e16} = 1.54\,k\Omega$

$r_{e17} = 45.5\,\Omega$

$R_{i17} = 201\,(45.5 + 50) = 19.2\,k\Omega$

$$\Rightarrow R_{i2} = 201\,(1.54 + 19.2 \| 50)\,k\Omega = \underline{\underline{3.1\,M\Omega}}$$

$$v_{b17} = \frac{R_{i17} \| R_9}{r_{e16} + R_{i17} \| R_9}\, v_{i2}$$

$$= 0.9\, v_{i2}$$

$$i_{c17} = \frac{\alpha}{r_{e17} + R_8}\, 0.9\, v_{i2}$$

$$\Rightarrow G_{m2} = \frac{\alpha(0.9)}{45.5 + 50} = \underline{\underline{9.38\,mA/V}}$$

49

$R_{o17} = 787\,k\Omega$

$I_{C13B} = 550\,\mu A$

$g_{m13B} = 22\,mA/V$; $r_{\pi 13B} = (\beta+1)/g_m = 2.32\,k\Omega$

$r_o = \dfrac{50}{550\,\mu A} = 90.9\,k\Omega$

$$R_{o13B} = r_o\,(1 + g_m(R_E \| r_\pi))$$

$$= 90.9\left[1 + 22(R_E \| 2.32)\right]$$

$$= 787$$

$\Rightarrow R_E \| 2.32 = 0.348$

and $\dfrac{1}{R_E} = 2.44$ or $R_E = 0.410\,k\Omega$

$\underline{\underline{R_E = 410\,\Omega}}$

Current $\dfrac{R_{E12}}{R_E} = \dfrac{550\,\mu A}{730\,\mu A} \Rightarrow \underline{\underline{R_{E12} = 309\,\Omega}}$

$$\frac{R_{E13A}}{R_E} = \frac{550}{180} = \underline{\underline{1.25\,k\Omega}}$$

50 9

$$\hat{v}_o = V_{CC} - V_{CEsat\,13A} - V_{BE14}$$

$$= \underline{\underline{4.2\,V}}$$

$$\check{v}_o \simeq -V_{EE} + V_{CEsat17} + V_{BE23} + V_{BE20}$$

$$= -5 + 0.2 + 0.6 + 0.6$$

$$= \underline{\underline{-3.6\,V}}$$

51

With Q_{23} removed, current in Q_{17} increases to $730\,\mu A$. This changes G_{m2}

$$r_{e17} = \frac{V_T}{730\,\mu A} = 34.2\,\Omega$$

$$\Rightarrow G_{m2} \simeq 0.923\,\frac{\alpha}{100 + 34.2} = 6.8\,mA/V$$

Because $r_{o17} \gg r_{o13B}$, R_{o2} remains virtually unchanged at $81\,k\Omega$

$R_3 = (\beta_P + 1) R_L \| r_{o13A} \simeq 74\,k\Omega$

$$\Rightarrow A_2 = -6.8(81)\,\frac{74}{74 + 81} = \underline{\underline{-263\,V/V}}$$

52

Ignore base currents of Q_{15}

$180\,\mu A = I_{C16} + \dfrac{I}{\beta + 1}$, where $I = I_{R6}$

$I_{C15} = I_S\, e^{V_{BE}/V_T}$ where $V_{BE} = I R_6$

Thus $$I = \frac{V_T}{27}\ln\left[\frac{180\,\mu A - \frac{I}{201}}{I_S}\right]$$

9

$= 191{,}422\,V/V$

$\equiv \underline{\underline{105.6\,dB}}$

output current is limited to $\pm 20\,mA$ (see problems 85 and 86)

$\Rightarrow |V_o| < 20\,mA\,(200)$

$\underline{\underline{|V_o| < 4\,V}}$

To obtain a seed solution, let $I = 0$ on RHS

$\Rightarrow I = \dfrac{V_T}{27}\ln\dfrac{180\,\mu A}{10^{-14}} = 21.9\,mA$

iterating $\underline{\underline{I = 21.0\,mA}}$

53

Maximum output current of the 1st stage $= 19\,\mu A$

$\Rightarrow I_{C22} = 19\,\mu A \Rightarrow V_{BE22} = V_{BE24} = 534\,mV$

$\Rightarrow I_{R11} = \dfrac{534}{50} = 10.7\,\mu A$

$\therefore I_{C21} = (19 + 10.7)\,\mu A = 29.7\,\mu A$

and $V_{BE21} = 545.3\,mV$

$V_{BE21} = I R_7 \Rightarrow I = \underline{\underline{20.2\,mA}}$

A simple doubling of R_7

54 $\frac{v_o}{v_i} = \frac{243{,}147}{0.97} = 250{,}667\ V/V = \underline{\underline{108\ dB}}$

$\frac{R_L}{R_o + R_L} = 0.97 \implies R_o = R_L\left(\frac{1}{0.97} - 1\right)$

or $R_o = \underline{\underline{61.9\Omega}}$

$\left.\frac{v_o}{v_i}\right|_{R_L = 200} = 250{,}667 \frac{200}{200 + 61.9}$

55 80° PM says that 2nd pole introduces 10° of phase shift at 1 MHz

i.e. $\tan^{-1} \frac{f_t}{f_{P2}} = 10°$

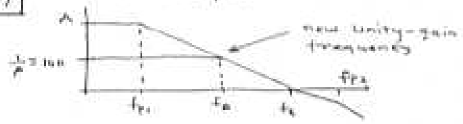
or $f_{P2} = \underline{\underline{5.67\ MHz}}$

56 Each pole adds 5° of phase shift

$\tan^{-1} \frac{10^6}{f_{2,3}} = 5°$

$\Rightarrow f_{2,3} = \underline{\underline{11.4\ MHz}}$

57 Consider Bode plot



85° of closed-loop phase margin

$\Rightarrow \tan^{-1} \frac{f_B}{f_{P2}} = 5°$

or $f_B = \underline{\underline{437\ kHz}}$

Recalling the "broadbanding" effect of negative feedback, we get

$f_B = f_{P1}(1 + A\beta) \simeq f_{P1} A\beta$

Loop gain $A\beta = 2.43 \times 10^5 \cdot \frac{1}{100} = 2.43 \times 10^3$

$\Rightarrow f_{P1} = \underline{\underline{180\ Hz}}$

$f_B = \frac{G_{m1}}{2\pi C_c} = 437\ kHz$

$\Rightarrow C_c = \frac{1}{5.26 \times 10^3 (2\pi) 437 \times 10^3}$

$= \underline{\underline{0.69\ pF}}$

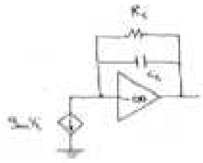
4.58 dominant pole $f_p = \frac{1}{2\pi R (A C_c)}$; $A = 1000$

with single pole response

$A_o f_p = f_t \implies f_p = \frac{5 \times 10^6}{10^6} = 5\ Hz$

$\Rightarrow R = \frac{1}{2\pi(5)\,1000\,(50\,pF)} = \underline{\underline{637\ k\Omega}}$

9.59



DC, gain is

$A_o = G_{m1} R_c$

$= 10 \times 10^{-3} \times 10^8$

$= \underline{\underline{10^6\ V/V}}$

$f_p = \frac{1}{2\pi R_c C_c} = \frac{1}{2\pi \times 10^8\ 50 \times 10^{-12}}$

$= 31.8\ Hz$

$f_t = A_o f_p$

$= 31.8\ MHz$

$SR = \frac{2I}{C_C}$

$G_{m1} = \frac{I}{2V_T} \implies 2I = 4 G_{m1} V_T$

$SR = \frac{4 G_{m1} V_T}{C_C} = \frac{4(10 \times 10^{-3})(25 \times 10^{-3})}{50 \times 10^{-12}}$

$= \underline{\underline{20\ V/\mu s}}$

9.60 $SR = 10\ V/\mu s$

$|V_{omax}| = 10\ V$

For sine wave of amplitude, V_{omax}, maximum rate of change

$\left.\frac{dv_o}{dt}\right|_{max} = \omega_m \hat{V}_o = 2\pi f_m \hat{V}_o$

Thus

$2\pi f_m \hat{V}_o = SR$

$\Rightarrow f_m = \frac{10 \times 10^6}{2\pi(10)} = \underline{\underline{159.2\ kHz}}$

$f_t = \frac{SR}{2\pi(4V_T)} = \frac{10 \times 10^6}{2\pi\,(4)\,25 \times 10^{-3}}$

$= \underline{\underline{15.9\ MHz}}$

9.61

$\underline{\underline{I_{E1} = I_{E2}}} = 50\mu A \simeq \underline{\underline{I_{E3} = I_{E4}}}$

$\underline{\underline{I_{E5} = 1mA}}$; $V_{BE5} = V_{BE6}$

$\therefore$ $\underline{\underline{I_{E6} = 1mA = I_{E7}}}$

$r_{e1} = r_{e2} = 500\Omega$

$r_{e5} = r_{e6} = r_{e7} = 25\Omega$

$G_{m1} = 2\left(\frac{1}{2r_{e1}}\right) = 2mA/V$

$R_{o1} = (\beta+1)(r_{e5} \| r_{e6})$

$= 1.25\ K\Omega$

and $A_1 = G_{m1}R_{o1} = 2.5\ V/V$

9.62 $\frac{1}{2}$ LSB must be less than 1%

i.e $\frac{1}{2}\frac{1}{2^N} \le \frac{1}{100} \Rightarrow N \ge 5.6$

$\therefore$ $\underline{\underline{N = 6 \text{ bits}}}$

Resolution $\frac{10V}{2^6} = \underline{\underline{0.156V}}$

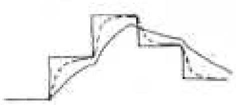
For same resolution need $\underline{\underline{7 \text{ bits}}}$

Still $\underline{\underline{7 \text{ bits}}}$

resolution $= \frac{15}{2^7} = \underline{0.117V}$

$Q = \frac{1}{2}\text{LSB} = \frac{1}{2}\frac{15}{2^7} = \underline{0.058V}$

9.63 9



--- $\tau = \frac{1}{5}T$

$\tau = T$

9.64 Require error in MSB $\le \frac{1}{2}$ LSB

$$\frac{V}{R} - \frac{V}{R\left(1+\frac{x}{100}\right)} \le \frac{1}{2}\frac{V}{2^{N-1}R}$$

$$\frac{1+\frac{x}{100}-1}{1+\frac{x}{100}} \le \frac{1}{2^N} \quad \text{or } \frac{x}{100}\left(2^N-1\right) \le 1$$

$$\Rightarrow x = \frac{1}{2^N-1} \times 100$$

$N = 2$ $\quad \underline{\underline{x = 33.3\%}}$

$N = 4$ $\quad \underline{\underline{x = 6.67\%}}$

$N = 8$ $\quad \underline{\underline{x = 0.39\%}}$

9.65 Since V_{BE}'s are equal, collector currents are scaled with respect to emitter areas

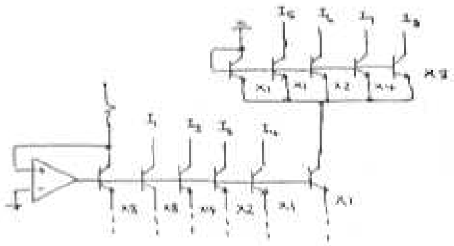
$I_1 + I_1 + I_2 + I_3 + I_4 = I$

$I_1(1+1+2+4+8) = I \Rightarrow I_1 = \underline{\underline{I/16}}$

$I_2 = \underline{\underline{I/8}}$

$I_3 = \underline{\underline{I/4}}$ $\qquad I_4 = \underline{\underline{\frac{I}{2}}}$

9.66 Circuit is sketched below 9



$A_{tot} = 8+8+4+2+1+1+1+1+2+4+8$

$= \underline{\underline{40}}$

For 8-bits binary weighted

$A_{tot} = (1 + 1 + 2 + 4 + \dots + 2^{N-1} + 2^{N-1})$

Q_0 $\qquad\qquad Q_{ref}$

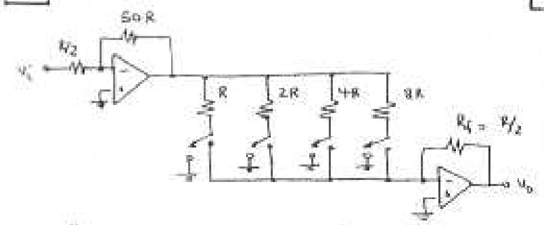
$= 2^{N-1} + 1 \quad \{1+2+4+\dots+2^{N-1}\}$

$= 2^{N-1} + 1 + 2^N - 1 = 2^{N-1} + 2^N$

$N = 8$

$A_{tot} = 2^7 + 2^8 = \underline{\underline{384}}$

9.67 9



$2^N - 1$ discrete outputs $= 2^4 - 1$

$= \underline{\underline{15}}$

Smallest sine wave $= \frac{10}{2^4} = 0.625V$

largest $= 10 \times \frac{G_{eq}}{G_f} = 10 \cdot \frac{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}{2}$

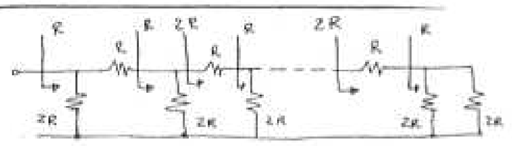
$= 5\ \frac{1}{8}(1+2+4+8) = \frac{5}{8}(2^4-1)$

$= \underline{\underline{9.375V}}$

10V pk-pk $\Rightarrow$ 5V pk or $\frac{1}{2}$ FS

$\therefore$ $D = \underline{\underline{1000}}$

9.68



$R_{in} = R$

9.69

$T_c = \frac{1}{f_{clk}} = 1\mu s$

$T_1 = 2^{12} T_c = \underline{\underline{4.096 ms}}$

$T = T_1 + T_2 = T_1 (1 + \frac{V_A}{V_{ref}})$

$= 2T_1 = \underline{\underline{8.19 ms}}$

$V_{peak} = 10 = \frac{V_A}{\tau} T_1$

$\Rightarrow \tau = \frac{V_A}{V_{peak}} T_1 = \underline{\underline{4.096 ms}}$

$\Delta\tau = -1\%$ and causes a -1% change in V_{peak}

$\Rightarrow V_{peak} = \underline{\underline{9.9V}}$

No : Final count does not depend on τ

9.70

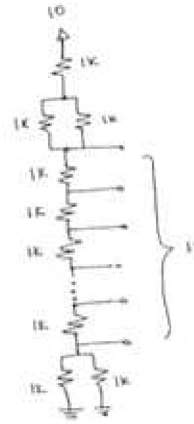
$2^N - 1$ comparators $= \underline{\underline{15}}$

Comparators are biased 1LSB apart starting from $\frac{1}{2}$ LSB

Thus, $V_{ref\,n} = (\frac{2n-1}{2})$ LSB

$1 LSB = \frac{10}{2^4} = 0.625V$

and references are 0.3125, 0.9375, 1.5625, 2.1875
2.8125 ...... 9.0625



# of resistor $= 2 + 14 + 2 + 1$
$= \underline{\underline{19}}$

$Rate = \frac{1}{(35+50) \times 10^{-9}}$

$= \underline{\underline{11.76 MHz}}$

(a) (0000......000) (15 in all)
and $(0000)_2$

(b) all comparators with references less than 5.1V will produces ones

Let's find 'em

$(\frac{2n-1}{2}) 0.625 \geq 5.1 \Rightarrow n \geq 8.6$

Thus the lowest 8 outputs will be ones
(0000000111111111) and $(0100)_2$

(c) Full scale input (1111...... 111)
and $(1111)_2$

$\boxed{10}$

Chapter 10 - Problems

$\boxed{1}$ Ideal 3V logic implies:

$V_{OH} = V_{DD} = \underline{\underline{3.0V}}$; $V_{OL} = \underline{\underline{0.0V}}$;

$V_{th} = V_{DD}/2 = 3.0/2 = \underline{\underline{1.5V}}$;

$V_{IL} = V_{DD}/2 = \underline{\underline{1.5V}}$; $V_{IH} = V_{DD}/2 = \underline{\underline{1.5V}}$

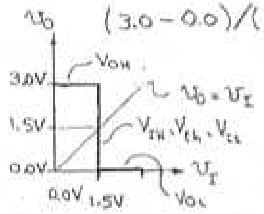
$NM_H = V_{OH} - V_{IH} = 3.0 - 1.5 = \underline{\underline{1.5V}}$

$NM_L = V_{IL} - V_{OL} = 1.5 - 0.0 = \underline{\underline{1.5V}}$

The gain in the transition region is:

$(V_{OH} - V_{OL})/(V_{IH} - V_{IL}) =$

$(3.0 - 0.0)/(1.5 - 1.5) = 3/0 = \underline{\underline{\infty V/V}}$



Inverting Transfer Characteristic

$\boxed{2}$ $\boxed{10}$ Nearly ideal 3.3V logic, assumed ideal:

$\rightarrow V_{OH} = \underline{\underline{3.3V}}$, $V_{OL} = \underline{\underline{0.0V}}$, $V_{th} = 0.4(3.3) = \underline{\underline{1.32V}}$

Now, at V_{th}, $v_O = v_I$, so to reach $v_O = 1.32V$

the required input is $1.32/(-50) = -26.4mV$

Thus, $V_{IL} = 1.32 - 26.4 \times 10^{-3} = \underline{\underline{1.294\ V}}$

Likewise, $V_{IH} = 1.32 + (3.3 - 1.32)/50 = \underline{\underline{1.360V}}$

Best possible noise margins are:

$NM_H = V_{OH} - V_{IH} = 3.30 - 1.360 = \underline{\underline{1.940\ V}}$

$NM_L = V_{IL} - V_{OL} = 1.294 - 0.0 = \underline{\underline{1.294V}}$

For noise margins only 7/10 of these, and V_{OH}, V_{OL} still ideal:

$V_{IH} = 3.3 - 0.7(1.940) = \underline{\underline{1.942\ V}}$, and
$V_{IL} = 0.0 + 0.7(1.294) = \underline{\underline{0.906V}}$

Correspondingly, the large-signal voltage gain is:
$G = (3.3 - 0.0)/(0.906 - 1.942) = \underline{\underline{-3.18\ V/V}}$

$\boxed{3}$ $\boxed{10}$ Here, $V_{OH} = 1.2V$, and $V_{OL} = 0.0V$.

Also, $V_{IH} - V_{IL} \leq 1.2/3 = 0.4V$ ---(1)

Now, the noise margins are "within 30% of one other". Thus $NM_H = (1 \pm 0.3)NM_L$ or

$NM_L = (1 \pm 0.3)NM_H$. Thus to remain "within" either $NM_H = 1.3NM_L$, or $NM_L = 1.3NM_H$,

in which case, either $NM_L = 0.769\ NM_H$, or

$NM_H = 0.769\ NM_L$

For the former case: $0.769(V_{OH} - V_{IH}) = (V_{IL} - V_{OL})$

or $0.769(1.2 - V_{IH}) = V_{IL} - 0$, whence

$V_{IL} = 0.923 - 0.769\ V_{IH}$

Now, from (1), $V_{IH} = V_{IL} + 0.4$.

Thus, $V_{IL} = 0.923 - 0.769(V_{IL} + 0.4) = 0.615 - 0.769V_{IL}$

and $V_{IL} = 0.615/1.769 = \underline{\underline{0.349V}}$,

whence $V_{IH} = 0.4 + 0.349 = \underline{\underline{0.749V}}$. $\boxed{10}$

Alternatively, $NM_H = 0.769\ NM_L$, and

$(V_{OH} - V_{IH}) = 0.769(V_{IL} - V_{OL})$, or

$1.2 - V_{IH} = 0.769\ V_{IL} - 0$, and $V_{IH} = 1.2 - 0.769V_{IL}$

With (1), $V_{IL} + 0.4 = 1.2 - 0.769V_{IL}$, and

$1.769V_{IL} = 0.8$, whence $V_{IL} = \underline{\underline{0.452V}}$

and $V_{IH} = 0.4 + 0.452 = \underline{\underline{0.852}}V$

Thus, overall, $V_{OH} = \underline{1.2V}$, $V_{OL} = \underline{\underline{0.0V}}$,

V_{IH} ranges from $\underline{0.749V}$ to $\underline{0.852V}$, and

V_{IL} ranges from $\underline{0.349V}$ to $\underline{0.451V}$, in which case, margins can be as low as:

$NM_L = V_{IL} - V_{OL} = \underline{\underline{0.349V}}$, and

$NM_H = V_{OH} - V_{IH} = 1.2 - 0.852 = \underline{\underline{0.348V}}$,

and as high as $\underline{\underline{0.452V}}$, and $\underline{\underline{0.451V}}$.

4 | 10

a) Generally, $t_P = (t_{PHL} + t_{PLH})/2$,

but due to current ratio, $t_{PHL} = 0.5\, t_{PLH}$.

Thus $1.5\, t_{PLH} = 2(1.2\text{ ns})$, whence

$t_{PLH} = 2.4/1.5 = \underline{\underline{1.6\text{ns}}}$, and $t_{PHL} = \underline{\underline{0.8\text{ns}}}$

Check: $t_P = (1.6 + 0.8)/2 = 1.2\text{ ns}$ ✓

b) Generally, $t_P = CV/I = kC$

Originally, $1.2 = kC$ --- (1)

Then, $1.7(1.2) = k(C+1)$ --- (2)

Dividing (2/1): $1.7 = (C+1)/C$

Thus, $1.7C = C+1$, $0.7C = 1$, $C = \underline{1.43\text{pF}}$

(the combined load and output capacitances)

c) With the load inverter removed:

$0.6(1.2) = k(1.43 - C_{in})$ --- (3)

Dividing (3/1): $0.6 = (1.43 - C_{in})/1.43$

Thus, $C_{in} = 1.43(1-0.6) = \underline{\underline{0.57}}\text{pF}$; $C_{out} = 1.43 - 0.57 = \underline{\underline{0.86}}\text{pF}$

5 | 10

Average static current (at 50% duty cycle) is $(0+40)/2 = 20\mu A$

When switching at 100MHz, current is $150\mu A$

Dynamic current is $(150 - 20) = 130\mu A$, and

dynamic power is $P_D = (3.3V)(130\mu A) = 429\mu W$.

But, $P_D = fCV_{DD}^2$, and $C = 429\times10^{-6}/(100\times10^6\times3.3^2)$

Thus $C_{eq} = C = \underline{0.394\text{pF}}$

6 | 10

Now, $P_D = fCV_{DD}^2$. Thus for V_{DD} reduced from 5V to 3.3V, power reduces by a factor $(3.3/5)^2 = \underline{0.436}$. For frequency reduced by a factor $(3.3/5)$, additional power saved is $((5-3.3)/5)\times10^{-3}\times0.436 = \underline{1.48\text{mW}}$.

Check: First reduction is to $10(0.436) = \underline{4.36\text{mW}}$, and the second to $4.36(3.3/5) = \underline{2.88\text{mW}}$, by $4.36 - 2.88 = 1.48\text{mW}$ ✓

7 | 10

a) For current proportional to V_{DD}, reduction in current (and frequency) is to $3.3/5 = \underline{\underline{0.66}}$ or 66% of previous (or by 34%)

b) For current proportional to V_{DD}^2, reduction is to $(3.3/5)^2 = \underline{\underline{0.436}}$ or 44% of previous (or by 56%).

For case a), delay increases by $1/0.66 = 1.515$ times, and power reduces (from Eq 13.4) to $0.66 \times (3.3/5)^2 = 0.287$ of previous, for a net change in DP to $1.515 \times 0.287 = \underline{\underline{0.435}}$ of previous, a net decrease by $\underline{56.5\%}$

For case b), delay increases by $1/0.436 = \underline{2.29}$ times, and power reduces to $0.436(3.3/5)^2 = 0.190$ of previous, for a net change in DP to $2.29\times0.190 = \underline{0.435}$ of previous (by 56.5%)

8 | 10

a) For current proportional to $(V_{DD} - V_t)$

i) For $V_t = 1V$: Change in current to $(3.3-1)/(5.0-1) = 0.575$, or $\underline{\underline{57.5\%}}$ of previous, a decrease of 42.5%

Change in delay to $1/0.575 = 1.74$ or an increase of 74%.

Change in frequency to 0.575 or 57.5% of previous.

Change in dynamic power due to change in both voltage and frequency is to $0.575 \times (3.3/5)^2 = 0.250$ or $\underline{25\%}$ of previous.

Change in DP is to $1.74 \times 0.25 = 0.435$, or $\underline{\underline{43.5\%}}$ of previous, a decrease of 56.5%

ii) For $V_t = 0.5V$:

Current: to $(3.3-0.5)/(5.0-0.5) = 0.622$, to $\underline{62.2\%}$

(next)

8 | 10

Delay: to $1/0.622 = 1.61$, to $\underline{161\%}$

Frequency: to $\underline{62.2\%}$

Dynamic Power: to $0.622\,(3.3/5.0)^2 = 0.271$, to $\underline{27.1\%}$

Delay-Power (DP): to $1.61 \times 0.271 = 0.436$, to $\underline{43.6\%}$

(iii) For $V_t = 0V$ (as in P10.7)

Current: to $3.3/5.0 = 0.66$, to $\underline{66\%}$

Delay: to $1/0.66 = 1.515$, to $\underline{151.5\%}$

Frequency: to $1/1.515 = 0.66$, to $\underline{66\%}$

Dynamic Power: to $0.66\,(3.3/5.0)^2 = 0.287$, to $\underline{28.7\%}$

Delay-Power (DP): to $1.515 \times 0.287 = 0.435$, to $\underline{43.5\%}$

b) For current proportional to $(V_{DD} - V_t)^2$

i) For $V_t = 1V$:

Current: to $[(3.3-1)/(5.0-1)]^2 = 0.331$, to $\underline{33.1\%}$

Delay: to $1/0.331 = 3.02$, to $\underline{302\%}$

(next)

---

18 | 10

Frequency: to $1/3.02 = 0.331$, to $\underline{33.1\%}$

Dynamic Power: to $0.331\,(3.3/5.0)^2 = 0.144$, to $\underline{14.4\%}$

Delay-Power (DP): to $3.02 \times 0.144 = 0.435$, to $\underline{43.5\%}$

ii) For $V_t = 0.5V$:

Current: to $[(3.3-0.5)/(5.0-0.5)]^2 = 0.387$, to $\underline{38.7\%}$

Delay: to $1/0.387 = 2.58$, to $\underline{258\%}$

Frequency: to $1/2.58 = 0.387$, to $\underline{38.7\%}$

Dynamic Power: to $0.387\,(3.3/5.0)^2 = 0.168$, to $\underline{16.9\%}$

Delay-Power (DP): to $2.58 \times 0.168 = 0.435$, to $\underline{43.5\%}$

(iii) For $V_t = 0V$ (as in P10.7):

Current: to $[3.3/5.0]^2 = 0.436$, to $\underline{43.6\%}$

Delay: to $1/0.436 = 2.30$, to $\underline{230\%}$

Frequency: to $1/2.30 = 0.436$, to $\underline{43.6\%}$

Dynamic Power: to $0.436\,(3.3/5.0)^2 = 0.190$, to $\underline{19.0\%}$

Delay-Power (DP): to $2.30 \times 0.190 = 0.437$, to $\underline{43.7\%}$

9

Chip area changes to $0.9 \times 0.9 = \underline{0.81}$ of pr

Current changes only due to oxide change, since

change cancels, to $1/0.9 = \underline{1.11}$ of previous

Effective Capacitance: half changes by 0.9^2 to c
half changes by $0.9^2/0.9$ t

Net Capacitance changes by $(0.81 + 0.90)/2 = \underline{0.}$

Propagation Delay changes by $(1/1.111)\,0.855 = \underline{0.7}$

Max. Operating Frequency changes by $1/0.77 = \underline{1.3}$

Dyn. Power Diss. (at max. freq.) changes by 1.30×0.855

Delay-Power Product changes by $0.770 \times 1.11 = \underline{0.8}$

Performance (f/A) changes by $1.30/0.81 = \underline{1.60}$

Now, if the supply voltage is reduced by

(but V_t is not, remaining at $0.2V_{DD}$),

$(V_{DD} - V_t)$ changes to $(0.9V_{DD} - 0.2V_{DD}) = 0.7$
or to $(0.7V_{DD}/0.8V_{DD}) = \underline{0.875}$ of previou

---

9 | 10

Thus, current changes by $(1/0.9)(0.875)^2 = \underline{0.851}$

Assume capacitances are voltage-independant.

Thus, propagation delay changes by $(1/0.851)\,0.855 = \underline{1.005}$

Maximum frequency changes by $1/1.005 = \underline{0.995}$

Dyn. Power changes by $(0.995)(0.855)(0.9)^2 = \underline{0.689}$

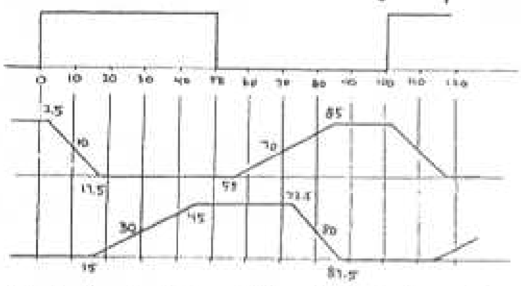
Delay-Power product changes by $1.005\,(0.689) = \underline{0.693}$

Performance (f/A) changes by $0.995/0.81 = \underline{1.23}$

10 | 10

The results depend on whether the gates are inverting or non-inverting.

For inverting gates, the timing diagram is:



Note: For simplicity, 0% to 100% (rather than 10% to 90%) both in the diagram above and calculation to follow:

For inverting gates (as shown, above):

a) For a rising input, time to 90% change of output of second gate is $10 + 20 + 30/2 = \underline{45\,ns}$

b) For a falling input, time to 90% change

10 | 10

of output of 2nd gate is $20 + 10 + 15/2 = \underline{37.5\,ns}$

For non-inverting gates:

a) Time to 90% rise is $10 + 10 + 15/2 = \underline{27.5\,ns}$

b) Time to 90% fall is $20 + 20 + 30/2 = \underline{55\,ns}$

The propagation delay for these gates is

$t_P = (t_{PHL} + t_{PLH})/2 = (10 + 20)/2 = \underline{15\,ns}$

11 | 10

Note that this question ignores the possibility of dynamic power dissipation:

Average propagation delay is $t_P = (50 + 70)/2 = 60\,ns$

Average power loss at 50% duty cycle $= (1 + 0.5)/2 = 0.75\,mW$

Delay-Power product is $DP = 60 \times 10^{-9} \times 0.75 \times 10^{-3}$

or $DP = 45 \times 10^{-12}\,J = \underline{45\,pJ}$

10.12

10.13

14 | 10

For $v_I = 1.5V$, the NMOS operates in triode mode, while the PMOS is cut off.

$r_{DSn} = [k_n(v_I - V_t)]^{-1} = [100 \times 10^{-6}(1.5 - 0.5)]^{-1} = \underline{10\,k\Omega}$

Thus $v_O = 100 \times 10^{-3} \times 10^4 / (10^4 + 10^5) = \underline{9.09\,mV}$

For $v_I = -1.5V$, the PMOS operates with

$r_{DSp} = [k_p(|v_I| - V_t)] = [(100 \times 10^{-6}/10)(1.5 - 0.5)]^{-1} = \underline{10^5\,\Omega}$

Thus $v_O = 100 \times 10^{-3} \times 10^5 / (10^5 + 10^5) = \underline{50\,mV}$

15 | 10

At V_{th}, both transistors operate in saturation with $V_{th} = v_I = v_O = v$, at which $i_D = \frac{k_n}{2}(v - V_{tn})^2 = \frac{k_p}{2}(V_{DD} - v - |V_{tp}|)^2$

Thus, $V_{DD} - v - |V_{tp}| = \sqrt{k_n/k_p}\,(v - V_{tn})$

$v(1 + (k_n/k_p)^{1/2}) = V_{DD} - |V_{tp}| + V_{tn}(k_n/k_p)^{1/2}$

Thus $v = V_{th} = \dfrac{V_{DD} - |V_{tp}| + V_{tn}(k_n/k_p)^{1/2}}{1 + (k_n/k_p)^{1/2}}$

as presented in Equation 10.8

10.16

10.17

10.18

19 | 10

NMOS width is $8\,\mu m$; PMOS width is $8(2.5) = 20\,\mu m$

Total output capacitance $= (20 + 8)2 + 50 = \underline{106\,fF}$.

Now, $t_P = 1.6C / [k'(W/L)V_{DD}]$

Thus $t_P = 1.6(106)10^{-15} / [75 \times 10^{-6}(8/0.8)3.3] = \underline{68.5\,ps}$

20 **10**

From Eq. 10.14, 10.15, 10.16:

$$i_{DnIAv} = \tfrac{1}{2}\left[\tfrac{1}{2} k_n'(W/L)_n (V_{DD}(1-\alpha))^2 + k_n'(W/L)_n \left(V_{DD}(1-\alpha)V_{DD}/2 - \tfrac{1}{2}(V_{DD}/2)^2\right)\right]$$
$$= \tfrac{1}{2} k_n'(W/L)_n \left[V_{DD}^2(1-\alpha)^2/2 + V_{DD}^2(1-\alpha)/2 - V_{DD}^2/8\right]$$
$$= \tfrac{1}{4} k_n'(W/L)_n V_{DD}^2 \left[(1-\alpha)^2 + (1-\alpha) - \tfrac{1}{4}\right]$$

Now, see the term [ ] becomes:

$$[1 - 2\alpha + \alpha^2 + 1 - \alpha - \tfrac{1}{4}] = \alpha^2 - 3\alpha + 1.75$$

Thus, $t_{PHL} = (CV_{DD}/2)/i_{DnIAv} = \dfrac{4C/2}{k_n'(W/L)_n V_{DD}(\alpha^2 - 3\alpha + 1.75)}$

or $t_{PHL} = \dfrac{2C}{k_n'(W/L)_n V_{DD}(\alpha^2 - 3\alpha + 1.75)}$

Check for $\alpha = 0.2$: $(\alpha^2 - 3\alpha + 1.75) = 0.2^2 - 3(0.2) + 1.75 = 1.19$

and the multiplier in the numerator is $\frac{2}{1.19} = 1.68 \simeq 1.7$ ✓

20 **10**

For $\alpha = 0.1$, $\alpha^2 - 3\alpha + 1.75 = 0.1^2 - 3(0.1) + 1.75 = 1.46$,

and the multiplier becomes $2/1.46 = \underline{1.37}$

For $\alpha = 0.5$, $\alpha^2 - 3\alpha + 1.75 = 0.5^2 - 3(0.5) + 1.75 = 0.50$

and the multiplier becomes $2/0.5 = \underline{4.00}$

10.21

22 **10**

Dynamic Power is $P_D = fCV_{DD}^2$; Static Power is P_S.

Now, $9.0 = P_S + 120 \times 10^6 C\, 5^2$

and $4.7 = P_S + 50 \times 10^6 C\, 5^2$

Subtracting, $4.3 = 70 \times 10^6 C(25)$

Whence $C = 4.3/(25 \times 70 \times 10^6) = \underline{2457\,pF}$

and $P_S = 9.0 - 120 \times 10^6(25)2457 \times 10^{-12} = 9.0 - 7.37 = \underline{1.63W}$

For 70% of the gates active, total gates $= 0.7 \times 10^6$

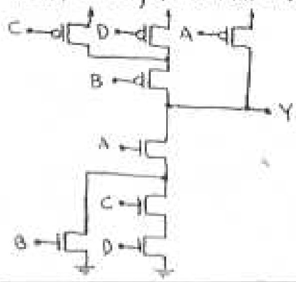
Capacitance per gate is $2457 \times 10^{-12}/(0.7 \times 10^6) = \underline{3.5\,fF}$

10.23

10.24

25 **10**

For $Y = \overline{A + B(C+D)}$, the PDN can be drawn directly, and then the PUN as direct dual:



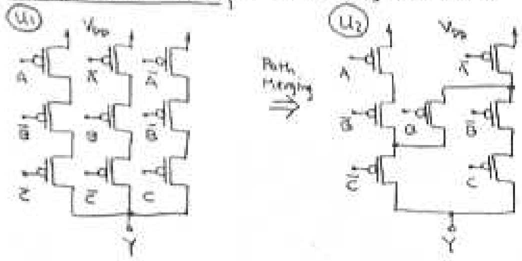
26 **10**

$Y = \bar{A}BC + A\bar{B}C + AB\bar{C}$

A very direct implimentation would need:

$2(3 \times 3) = 18$ MOS for the gate itself, plus $3 \times 2 = 6$ for the required inverters, for a total of $\underline{24}$ transistors.

For the PUN directly: (Inverting variables)



For the PDN, $Y = \bar{A}BC + A\bar{B}C + AB\bar{C}$.

Correspondingly: $\bar{Y} = \overline{\bar{A}BC + A\bar{B}C + AB\bar{C}}$

$= \overline{\bar{A}BC} \cdot \overline{A\bar{B}C} \cdot \overline{AB\bar{C}} = (A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$ ①

$= ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$, and replicating $\bar{A}\bar{B}\bar{C}$,

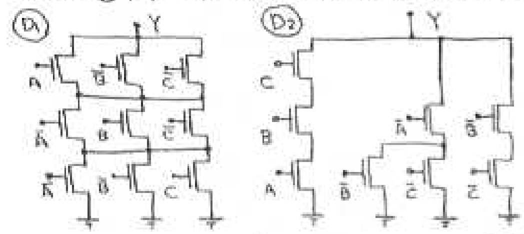
$\bar{Y} = ABC + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$ --- ②

(next)

26 [10]

For the PDN directly:

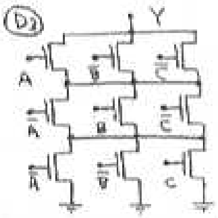
From ①, $\bar{Y} = (A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$



From ②, $\bar{Y} = ABC + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$, see (D2)
where path merging is included

For the PDN from the PUN (U1)

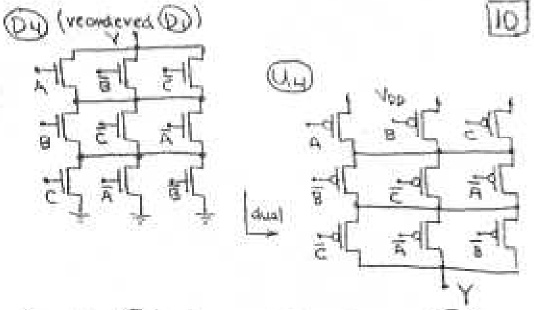
Simply, replacing series connections with parallel connections:



See that (D3) is identical to (D1) in detail!

(next)

(D4) (reordered (D1)) [10]



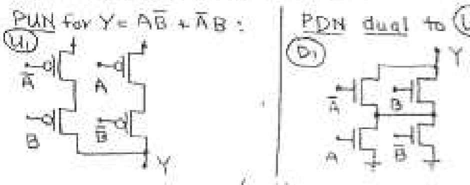
See that (U4), while not the same as (U1), is highly related, having some variable exchange in the middle and right columns.
Clearly, there are lots of variations of the completely-connected array.

27 [10]

$Y = A\bar{B} + \bar{A}B \rightarrow \bar{Y} = \overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B}} \cdot \overline{\bar{A}B}$

or $\bar{Y} = (\bar{A}+B)(A+\bar{B}) = AB + \bar{A}\bar{B}$

PUN for $Y = A\bar{B} + \bar{A}B$: | PDN dual to (U1):



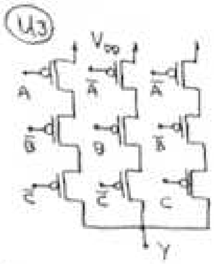
(next)

27 [10]

For the PUN from the PDN (D1)

See that both parallel and series connections can be identified for transformation, but that the series (vertical) ones (as drawn) contain variables and their complements, and are therefore always open.

Thus converting parallel paths to serial ones

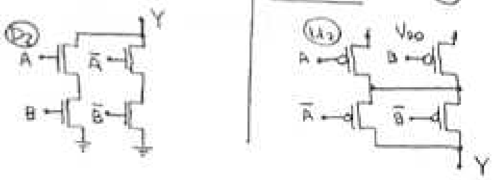


See this is the same as (U1)

*Note, however that (D1) can be redrawn as shown, then its colums (series links) converted to rows (parallel links of a PUN:

[10]

PDN for $\bar{Y} = AB + \bar{A}\bar{B}$: | PUN dual to (D2)



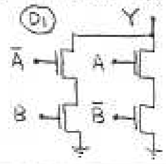
The two circuits required are (U1) with (D1) and (U2) with (D2)

28 [10]

$Y = AB + \bar{A}\bar{B}$. Directly, the PUN is as follows:

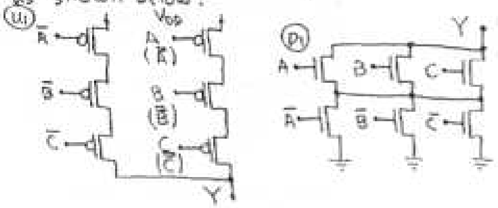
Now, $\bar{Y} = \overline{AB + \bar{A}\bar{B}} = \overline{AB} \cdot \overline{\bar{A}\bar{B}} = (\bar{A}+\bar{B})(A+B)$
or $\bar{Y} = \bar{A}B + A\bar{B}$.

Directly, the PDN is:



29 $\boxed{10}$

$Y = ABC + \bar{A}\bar{B}\bar{C}$. Directly, the PUN is as shown below:

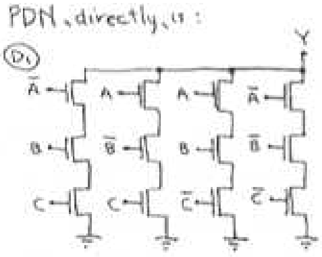


The corresponding dual PDN is shown above right.

30 $\boxed{10}$

Even-parity circuit: $\bar{Y} = \bar{A}BC + A\bar{B}C + AB\bar{C} + \bar{A}\bar{B}\bar{C}$

PDN, directly, is:



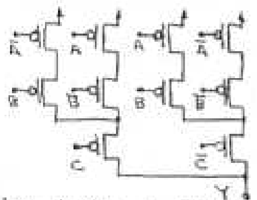
It uses 12 transistors

31 $\boxed{10}$

For output high with odd parity:

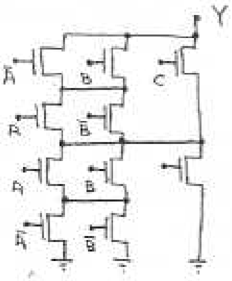
$Y = A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC = (A\bar{B} + \bar{A}B)\bar{C} + (\bar{A}\bar{B} + AB)C$

Directly the PUN is:



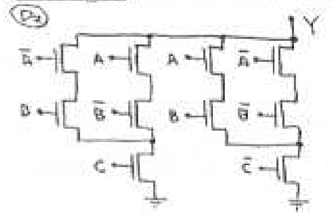
[Note that two transistors could be eliminated by combining the A and Ā inputs in a non-planar topology]

Using duality, the PDN becomes:

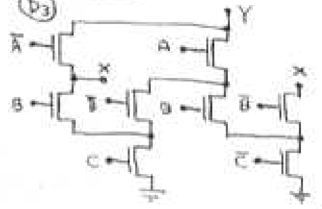


31 $\boxed{10}$

PDN reduced to 10 transistors:



PDN reduced to 8 transistors: (X and X are joined)

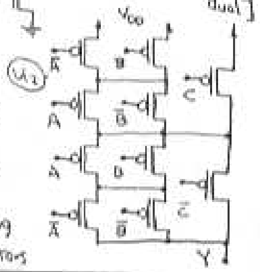


[This circuit is not "planar", but has one "cross-over" (X-X), it has no convenient dual]

PUN as the dual of D_2:

[Think of the structure of the dual of (D1) when constructing this.

The complete circuit, using (U2) and (D2) has 20 transistors



32 $\boxed{10}$

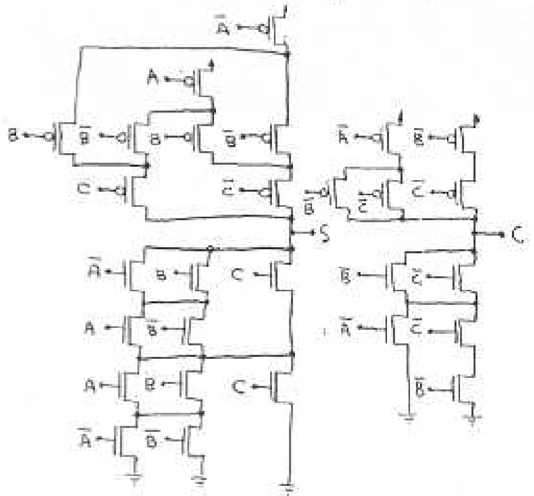
$\text{Sum}, S = A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

Carry $C_o = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$

$= AB + AC + BC = A(B+C) + BC$

Create the PUNs directly, simplifying that for S as in P10.30 above, as

$S = \bar{A}(B\bar{C} + \bar{B}C) + A(\bar{B}\bar{C} + BC)$



33 [10]

For matched-inverter equivalence of the circuit in Fig 13.14: $p_A = p$; $p_B = p_C = p_D = 2p$; and $n_A = n_B = 2n$; $n_C = n_D = 2(2n) = 4n$.

10.34

35 [10]

Ignore the capacitances of the transistors themselves: For the matched NAND, $t_{PLH} = t_{PHL} = t_P$. For the "uncompensated NAND, $t_{PLH} = t_P$, $t_{PHL} = t_P/4$. Thus t_{PLH} are the same, but t_{PHL} is 4 times greater with no matching.

36

For design a), there are $2(6) + 2 = \underline{14}$ transistors.

All 7 NMOS use $(W/L)_n = n$
1 PMOS uses $(W/L)_p = p$
6 PMOS use $(W/L)_p = 6p$

Total Area $= 7(1.2)0.8 + 1(3.6)0.8 + 6(6)(3.6)0.8 = \underline{113.3\,\mu m^2}$

For design b), there are $2(3)2 + 1(2)2 = \underline{16}$ transistors:
6 NMOS use $(W/L)_n = n$
6 PMOS use $(W/L)_p = 3p$
2 PMOS use $(W/L)_p = p$
2 NMOS use $(W/L)_n = 2n$

Total equivalent devices is $6n + 18p + 2p + 2n = 10n + 20p$
Total equivalent area is $[10 + 3(20)]n = 70n$, and
Total Area $= 70(1.2)0.8 = \underline{67.2\,\mu m^2}$, or $\underline{59\%}$ of a)

---

37 [10]

Corresponding to a matched inverter characterized by n and p where $k_p = k_n = k$, the two-input NOR uses transistors n and $2p$ where $k_p = 2k_n$

a) For A grounded, V_{thB} occurs near $V_{DD}/2$, with Q_{PB} and Q_{NB} in saturation and Q_{PA} in triode. Let $V_{th} = v$, and the voltage across Q_{PA} be x.

Thus $i_D = k_p[(5-1)x - x^2/2]$
and $i_D = \frac{1}{2}k_p(5 - x - v - 1)^2$
and $i_D = \frac{1}{2}k_n(v-1)^2$

For $k_p = 2k_n$, $i_D = 2k_n(4x - x^2/2) = k_n(8x - x^2)$ --- (1)

and $i_D = k_n(4 - x - v)$ --- (2)

and $i_D = \frac{1}{2}k_n(v-1)^2$ --- (3)

From 2) 3): $\pm(v-1)(0.707) = 4 - x - v$

Thus, $1.707v = 4.707 - x$
or $x = 4.707 - 1.707v$

(or) $0.293v = 3.293 - x$
or $x = 3.293 - 0.293v$

Now $x \cong 0$, in which case, $v = 4.707/1.707 = 2.78$ (ok)
or $v = 3.293/0.293 = 11.2$ (Clearly too large)

Thus $x = 4.707 - 1.707v$ --- (4)

[10]

Now, from 1), 3): $(v-1)^2 = 2(8x - x^2)$

With 4), $v^2 - 2v + 1 = 16(4.707 - 1.707v) - 2(4.707 - 1.707v)^2$

or $v^2 - 2v + 1 = 75.32 - 27.32v - 44.31 + 32.13v - 5.83v^2$

or $6.83v^2 + v(-2 + 27.32 - 32.13) + (1 - 15.32 + 44.31) = 0$

or $6.83v^2 - 6.81v - 30.01 = 0$

whence $v = (--6.81 \pm (6.81^2 - 4(6.83)30.01)^{1/2})/2(6.83)$

$= (6.81 \pm 29.43)/13.66 = 2.65V$

Check: [>2.5V probably OK since one PMOS is full on]

Thus $V_{th} = \underline{2.65V}$

b) For A and B joined, the PMOS can be approximated as a single device with twice the length, for which the width is twice that in a matched inverter. Thus, for the equivalent PMOS device, $(W/L)_{peq} = p$ and $k_p = k$. For each of the two NMOS, $(W/L)_n = n$ and $k_n = k$.

Thus at $V_{th} = v$ with all devices in saturation:

(next)

10

$i_D = 2k/2(v-1)^2 = k/2(5-v-1)^2$,

$2(v-1)^2 = (4-v)^2$, and $\pm\sqrt{2}(v-1) = (4-v)$

Thus, $1.414v - 1.414 = 4 - v$, $2.414v = 5.414$,

whence $V_{th} = v = \underline{2.24V}$

See this is reduced from the single-input value (of 2.65V)!

Note that this fact can be used to control the relative threshold of multiple gates connected to a single fanout node, in order to guarantee operation sequence for slowly changing signals.

38

For the resistor load, the output voltage rising is $v_O = V_{DD}(1 - e^{-t/R_DC})$

This reaches $V_{DD}/2$ when $(1 - e^{-t/R_DC}) = 0.5$,

or $e^{-t/R_DC} = 0.5$, and $t = -R_DC \ln 0.5 = 0.693 R_DC$

Thus $t_{PLH} = \underline{0.693 R_D C}$

For the current load, $I = V_{DD}/R_D$, the output reaches $V_{DD}/2$ when $t = (CV_{DD}/2)/I$,

or $t = (CV_{DD}/2)/(V_{DD}/R_D) = 0.5 R_D C$

(next)

10

Now, $k_n = k_n'(W/L)_n = 75 \times 10^{-6}(1.2/0.8) = 112.5\mu A/V^2$

and $k_p = 0.256 \times 112.5 = 28.8\mu A/V^2$

Now, $k_p'(W/L)_p = 28.8$; Thus $(W/L)_p = 28.8/(75/3) = \underline{1.152}$

For $L_p = 0.8\mu m$, $W_p = 1.152(0.8) = \underline{0.922\mu m}$

In summary, for this inverter: $k_n = \underline{112.5\mu A/V^2}$

$k_p = \underline{28.8\mu A/V^2}$

and $r = k_n/k_p = \underline{3.91}$

Now, $V_{OH} = \underline{+5.0V}$

For $V_{OL} = v \simeq 0$, $i_D = k_p/2(v_{GS} - V_t)^2$

or $i_D = (28.8/2)(5.0 - 0.8)^2 = 254\mu A$

For the NMOS, $254 = 112.5[(5-0.8)v - v^2/2]$

or $v^2 - 8.4v + 4.52 = 0$,

whence $v = (--8.4 \pm (8.4^2 - 4(1)4.52)^{1/2})/2$

$= (8.40 \pm 7.24)/2 = 0.58V$

Thus $V_{OL} = \underline{0.58V}$

10

Thus $t_{PLH} = \underline{0.5 R_D C}$, a reduction to

$(0.50/0.69) \times 100 = \underline{72.5\%}$ or by $\frac{0.69 - 0.50}{0.69} \times 100 = \underline{27.5\%}$

39

Here $V_{DD}/4 = 5/4 = 1.25V$

Now, for v_O rising, the NMOS is cutoff, and the PMOS is in triode mode with:

$i_{D_p} = k_p[(v_{SG} - V_t)v_{SD} - v_{SD}^2/2]$, and here

$i_D = k_p[(5-0.8)(5-1.25) - (5-1.25)^2/2]$

$= k_p(15.75 - 7.03) = \underline{8.72k_p}$

Now, for v_O falling, the net current extracted from the load is $i_{D_n} - i_{D_p}$ which should be i_{D_p}

Thus $i_{D_n} = 2i_{D_p} = 2(8.72)k_p$, for triode operation where $i_{D_n} = k_n[(5-0.8)1.25 - 1.25^2/2]$

Overall, $i_{D_n} = 2(8.72)k_p = k_n(5.25 - 0.78) = 4.47k_n$

Thus $k_p = (4.47/(2(8.72)))k_n = \underline{0.256}k_n$

Check using Eq 10.39, where $r = k_n/k_p = 3.91$:

$V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$

$= (5 - 0.8)[1 - (1 - 1/3.91)^{1/2}] = \underline{0.577V}$

From Eq 13.35, $V_{IL} = V_t + (V_{DD} - V_t)/(r(r+1))^{1/2}$

$= 0.8 + 4.2/(3.91 \times 4.91)^{1/2} = \underline{1.76V}$

From Eq 13.38, $V_{IH} = V_t + (2/\sqrt{3r})(V_{DD} - V_t)$

$= 0.8 + (2/\sqrt{3(3.91)})4.2 = \underline{3.25V}$

From Eq 13.36, $V_M = V_t + (V_{DD} - V_t)/(r+1)^{1/2}$

$= 0.8 + 4.2/(4.91)^{1/2} = \underline{2.70V}$

Now, $NM_H = V_{OH} - V_{IH} = 5.00 - 3.25 = \underline{1.75V}$

and $NM_L = V_{IL} - V_{OL} = 1.76 - 0.58 = \underline{1.18V}$

40 **10**

Note that a design with $r = 2$ sacrifices V_{OL} for improved propagation-delay symmetry.

From Eq. 10.39, $V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$
$= (5 - 0.8)[1 - (1 - 1/2)^{1/2}] = \underline{1.23V}$

Now, $k_n'(W/L)_n = 75(1.2/0.8) = r k_p = 2(75/3)(W/L)_p$
Thus, $(W/L)_p = (70/25) \times 1/2 \times (1.2/0.8) = \underline{(1.8/0.8)}$

Now $C_{gs_n} = 1.2(1.5) = 1.8fF$; $C_{gd_n} = 1.2(0.5) = \underline{0.6fF}$
Thus $C_{in} = 1.8 + 0.6 = \underline{2.4fF}$.

Now, the output capacitance includes C_{db} and C_{gd} of the load plus C_{db} and twice C_{gd} of the switch.

Thus, the total capacitance of the output and the loading gate is:

$C_{out} = 1.8(0.5 + 2.0) + 1.2(2.0 + 2(0.5)) + 2.4 = \underline{10.5fF}$

Now, from Eq. 10.43, $t_{PLH} = 1.7C/(k_p V_{DD})$,
and $t_{PLH} = 1.7 \times 10.5 \times 10^{-15} / [(75/3)10^{-6}(1.8/0.8)5] = \underline{63.5ps}$

Now, from Eq. 10.44, $t_{PHL} = 1.7C/[k_n(1 - 0.46/r)V_{DD}]$
and $t_{PHL} = 1.7 \times 10.5 \times 10^{-15} / [75 \times 10^{-6}(1.2/0.8)(1 - 0.46/2)5] = \underline{41.2ps}$

Thus $t_p = (t_{PLH} + t_{PHL})/2 = (63.5 + 41.2)/2 = \underline{52.4ps}$

Thus $(2r+1)^2(r-1) = (r+1)^3$
and $(4r^2 + 4r + 1)(r-1) = (r^2 + 2r + 1)(r+1)$
$4r^3 + 4r^2 + r - 4r^2 - 4r - 1 = r^3 + 2r^2 + r + r^2 + 2r + 1$
$4r^3 - 3r - 1 = r^3 + 3r^2 + 3r + 1$
$3r^3 - 3r^2 - 6r - 2 = 0$
$r^3 - r^2 - 2r - 2/3 = 0$

Test $r^3 - r^2 - 2r - 2/3$
$r = 2 \rightarrow 8 - 4 - 4 - 2/3 = -2/3$
$r = 3 \rightarrow 27 - 9 - 6 - 2/3 = 11\,1/3$
$r = 2.1 \rightarrow 2.1^3 - 2.1^2 - 4.2 - .67 = 9.26 - 4.4 - 4.2 - .67 = -0.01$

Thus $r = \underline{2.1}$, for which:
$NM_L = 0.8 - (5.0 - 0.8)[1 - (1 - 1/2.1)^{1/2} - (2.1(2.1+1))^{-1/2}]$
$= 0.8 - 4.2[1 - 0.724 - 0.392] = \underline{1.29V}$

41

From Eq. 10.41, $NM_L = V_t - (V_{DD} - V_t)[1 - (1 - \frac{1}{r})^{1/2} - (r(r+1))^{-1/2}]$

Now, $\partial NM_L / \partial r = -(V_{DD} - V_t)[-\frac{1}{2}(1 - \frac{1}{r})^{-1/2}(-\frac{-1}{r^2}) - (-\frac{1}{2}(r(r+1))^{-3/2}(2r+1))]$

Maximum occurs where:
$-1/2(1 - 1/r)^{-1/2}(1/r^2) = -1/2(r(r+1))^{-3/2}(2r+1)$

Square both sides: $(1 - 1/r)^{-1}/r^4 = r^{-3}(r+1)^{-3}(2r+1)^2$

or $\frac{1}{1 - 1/r} = \frac{r}{(r+1)^3}(2r+1)^2 = \frac{r}{r-1}$

10.42

43 **10**

From Eq. 10.41, $NM_H = (V_{DD} - V_t)(1 - 2/\sqrt{3r})$

This is zero, when $1 - 2/\sqrt{3r} = 0$,
or $\sqrt{3r} = 2$, or $3r = 4$, or $r = \underline{1.33}$

For $r = 1$, $NM_H = 4.2(1 - 2/\sqrt{3}) = \underline{-0.65V}$

For $r = 2$, $NM_H = 4.2(1 - 2/\sqrt{6}) = \underline{0.77V}$

For $r = 4$, $NM_H = 4.2(1 - 2/\sqrt{12}) = \underline{1.78V}$

For $r = 8$, $NM_H = 4.2(1 - 2/\sqrt{24}) = \underline{2.48V}$

For $r = 16$, $NM_H = 4.2(1 - 2/\sqrt{48}) = \underline{2.99V}$

But, what about NM_L? (For $r = 16$, it is $0.92V$)

44

From Eq. 10.41 and 10.42, noise margins are equal when
$V_t - (V_{DD} - V_t)[1 - (1 - \frac{1}{r})^{1/2} - 1/(r(r+1))^{1/2}] = (V_{DD} - V_t)[1 - 2/(3r)^{1/2}]$

or $V_t/(V_{DD} - V_t) = 2 - 2/(3r)^{1/2} - (1 - 1/r)^{1/2} - 1/(r(r+1))^{1/2}$ — (1)

Here $V_t/(V_{DD} - V_t) = 0.8/(5.0 - 0.8) = \underline{0.1904}$
Try various values of r to solve (1):

10

For $r = 2$, $f(r) = 2 - 2/6^{1/2} - (1 - 1/2)^{1/2} - 1/(2(3))^{1/2}$
$= 2 - 0.816 - 0.707 - 0.408 = \underline{0.069}$
For $r = 3$, $f(r) = 2 - 2/9^{1/2} - (1 - 1/3)^{1/2} - 1/(3(4))^{1/2}$
$= 2 - 0.667 - 0.816 - 0.289 = \underline{0.228}$
Try $r = 2.8$, $f(r) = 2 - 2/(3(2.8))^{1/2} - (1 - 1/2.8)^{1/2} - 1/(2.8(3.8))^{1/2}$
$= 2 - 0.690 - 0.802 - 0.307 = \underline{0.201}$
Try $r = 2.7$, $f(r) = 2 - 2/(3(2.7))^{1/2} - (1 - 1/2.7)^{1/2} - (1/(2.7(3.7)))^{1/2}$
$= 2 - 0.703 - 0.793 - 0.316 = \underline{0.188}$

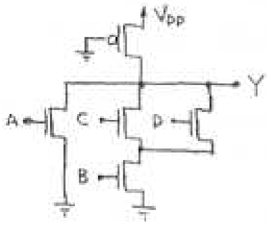
Conclude $r \simeq \underline{2.72}$, for which the margins are:
$NM = NM_H = NM_L = (V_{DD} - V_t)(1 - 2/(3r)^{1/2})$
$= 4.2(1 - 2/(3(2.72))^{1/2}) = \underline{1.26V}$

10.45

$\boxed{46}$

$Y = \overline{A + B(C+D)}$, whence $\overline{Y} = A + B(C+D)$

Thus the PDN can be formed directly as shown:



Now, $t_{PLH}/t_{PHL} = (k_n/k_p)(1 - 0.46/r)$
$= r(1 - 0.46/r) = 2.72 - 0.46 = \underline{\underline{2.26}}$

Now, $t_{PLH} = 1.7(1 \times 10^{-12})/(25 \times 10^{-6}(1.3/0.8)5) = \underline{\underline{8.24 ns}}$

and $t_{PHL} = 8.24/2.26 = \underline{\underline{3.65 ns}}$
and $t_P = (8.24 + 3.65)/2 = \underline{\underline{5.95 ns}}$

Now, dynamic power is approximately fCV_{DD}^2, since the output swing is not quite V_{DD}.

For equal static and dynamic power
$f \times 1 \times 10^{-12} \times 5^2 = 1.82 \times 10^{-3}$
whence $f = 1.82 \times 10^{-3}/(25 \times 10^{-12}) = \underline{\underline{72.8 MHz}}$,
for which the period is $1/(72.8 \times 10^6) = \underline{13.7 ns}$

Now, for transition times in the same proportion as propagation delays $t_{TLH}/t_{THL} = 8.24/3.65 = 2.26$

Now, for full output swing, there must be time for 2 full transitions in each cycle:
Thus $t_{THL} \le 13.7/(1 + 2.26) = \underline{4.19 ns}$, and
$t_{TLH} \le 4.19(2.26) = \underline{9.47 ns}$

Since these values are of the same order as the propagation delays, full swing operation is likely not possible at 72.8 MHz.

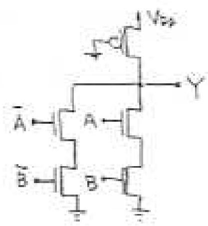
$\boxed{47}$ $\boxed{10}$

For an Exclusive OR, $Y = A\overline{B} + \overline{A}B$, and

$\overline{Y} = \overline{A\overline{B} + \overline{A}B} = \overline{A\overline{B}} \cdot \overline{\overline{A}B} = (\overline{A} + B)(A + \overline{B})$

or $\overline{Y} = \overline{A}\,\overline{B} + AB$

The PDN results directly:



$\boxed{48}$

For a pseudo-NMOS NOR gate, independent of the number of inputs, the worst-case value of V_{OL} occurs for one input high (and a single NMOS conducting

From Eq. 10.39, $V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$,
for which $0.2 = (5.0 - 0.8)[1 - (1 - 1/r)^{1/2}]$
and $(1 - 1/r)^{1/2} = 1 - 0.2/4.2 = 0.952$
Thus $1 - 1/r = 0.907$
$1/r = 0.093$, and $r = \underline{10.76}$
Thus $k_n/k_p = 10.76 = 75(1.8/1.2)/(25(W/L)_p)$,
Thus $(W/L)_p = (75/25)(1.8/1.2)/10.76 = \underline{0.418}$
Thus for $W_p = 1.8\mu m$, $L_p = 1.8/0.418 = \underline{4.31 \mu m}$
and $(W/L)_p = \underline{(1.8/4.31)}$

$\boxed{49}$ $\boxed{10}$

For a), see directly that $X = 1 \cdot \overline{A} = \overline{A}$

and $Y = X \cdot \overline{B} = \underline{\overline{A} \cdot \overline{B}}$

For b), see directly that $Y = \underline{\overline{A} \cdot \overline{B}}$

For each circuit, node Y nominally satisfies both conditions. However in a), with A high and B low, Y is not pulled down completely to ground, but remains at V_{tp}, due to the PMOS threshold. Circuit b) does not have this problem, but node X is floating for A, B both high. However, X is not an output node. The body effect makes this worse. Notice that b) is exactly a complementary CMOS NOR gate for which $Y = \overline{A} \cdot \overline{B} = \underline{\overline{A + B}}$

For V_{DD} replaced by an inverter driven by C, $Y = \overline{C}(\overline{A} \cdot \overline{B}) = \overline{A} \cdot \overline{B} \cdot \overline{C} = \underline{\overline{A + B + C}}$, a 3-input NOR (for both a) and b).

Practically speaking, however, there is a problem because, as noted above, the series PMOS do not operate well with a low input. In fact Y is pulled down only to one threshold drop below ground, when C is high.

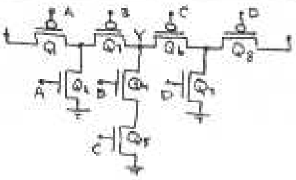
$\boxed{50}$ $\boxed{10}$

With these reversals, the output is high when B is low or when A is low.

Thus, $Y = \overline{A} + \overline{B}$, or $\underline{Y = \overline{A \cdot B}}$, a NAND function

Circuit b), being a fully complementary CMOS gate, functions ideally. However circuit b) provides a relatively low high output for A low with B high. In this case, the output becomes $V_{DD} - V_{tn}$, where V_{tn} is raised from V_{tn0} by the body effect.

51



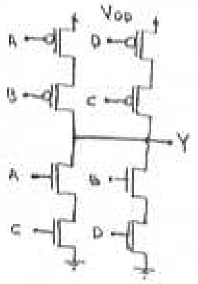
There are several deficiencies:

First, there are two possibilities for short-circuits when B and C are low and A and D are complementary, in which case current flows (for example) through Q_1, Q_3, Q_6, Q_7. This problem occurs for $\bar{A}\bar{B}\bar{C}D$ and $A\bar{B}\bar{C}\bar{D}$.

10

Second, node Y is pulled down to V_t (increased by the body effect) when B or C are low with A or D high, respectively. In this respect, this circuit is like that in Fig P13.49a.

With respect to Y as a function of A, B, C, D, the result will depend on device sizing. For Q_2, Q_7, relatively weaker than the PMOS, $Y = \bar{A}\bar{B} + \bar{C}\bar{D}$. A better circuit is shown below:



For Q_3, Q_6 relatively weaker, $\bar{Y} = A + D + BC$ and $Y = \overline{A+D} \cdot \overline{BC}$
$= \bar{A}\bar{D}(\bar{B} + \bar{C})$
$= \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}\bar{D}$

With input E driving an inverter to replace both V_{DD} connections, $Y = \bar{E}(\bar{A}\bar{E} + \bar{C}\bar{D})$ is possible, barring all the other problems this circuit brings.

52

10

For the switch gate and the input both at 3.3V, the switch output is

$V_{OH} = 3.3 - V_t$, where $V_t = V_{t0} + \gamma\left(\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f}\right)$

Thus $V_{OH} = 3.3 - 0.8 - 0.5(V_{OH} + 0.6)^{1/2} + 0.5(0.6)^{1/2}$
$= 2.89 - 0.5(V_{OH} + 0.6)^{1/2}$
or $2V_{OH} - 5.77 = (V_{OH} + 0.6)^{1/2}$

Squaring, $V_{OH} + 0.6 = 4V_{OH}^2 - 23.1V_{OH} + 33.29$
$4V_{OH}^2 - 24.1V_{OH} + 31.69 = 0$, and

$V_{OH} = (--24.1 \pm \sqrt{24.1^2 - 4(4)31.69})/8$
or $V_{OH} = (24.1 \pm 8.59)/8 = \underline{1.94V}$ (and a larger one)

Now, for input low and switch gate high, $V_{OL} = \underline{0V}$

For the inverter current: For $V_{OH} = 1.94V$, the PMOS is in saturation with

$i_D = \frac{1}{2}\mu_p C_{ox}(W/L)_p(3.3 - 1.94 - 0.8)^2 = \frac{1}{2}\left(25(10^{-6})3\left(\frac{1.2}{0.8}\right)(0.56)^2\right)$

Thus, the static inverter current is $\underline{17.64\mu A}$

For t_{PLH}: Here, $v_O = 0$, $V_t = V_{t0} = 0.8V$, $V_{GS} = V_{DD} = 3.3V$.
Thus, initially, $i_D(0) = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = \underline{351.6\mu A}$
At $v_O = 3.3/2 = 1.65V$,
$V_t = 0.8 + 0.5(1.65 + 0.6)^{1/2} - 0.5(0.6)^{1/2} = \underline{1.16V}$

and $i_D(t_P) = \frac{1}{2}(75)(1.2/0.8)(3.3 - 1.65 - 1.16)^2 = \underline{13.5\mu A}$

Thus $i_{D_{av}} = (351.6 + 13.5)/2 = \underline{183\mu A}$

and $t_{PLH} = (CV_{DD}/2)/i_{D_{av}} = 100(10^{-15})1.65/(183\times10^{-6}) = \underline{0.90ns}$

For t_{PHL}: Here, $V_t = V_{t0}$, and the initial current is

$i_D(0) = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = \underline{352\mu A}$

At half swing, operation is in triode mode, and

$i_D(t_P) = 75(1.2/0.8)[(3.3 - 0.8)1.65 - 1.65^2/2] = \underline{311\mu A}$

Thus, the average current is $(352 + 311)/2 = \underline{332\mu A}$

and $t_{PHL} = 100(10^{-15})1.65/311\times10^{-6} = \underline{0.53ns}$

53 $\boxed{10}$

a) For the inverter, with $v_{O2} = 3.3 - 0.8 = 2.5V$, Q_N is in saturation and the current is

$i_{DN} = \frac{1}{2}(75)(1.2/0.8)(v_{O1} - 0.8)^2 = 56.25(v_{O1} - 0.8)^2$,

and Q_P is in triode mode with current

$i_{DP} = (75/3)(3.6/0.8)[(3.3 - v_{O1} - 0.8)0.8 - 0.8^2/2]$

or $i_{DP} = 100[(2.5 - v_{O1})0.8 - 0.8^2/2] = 80(2.5 - v_{O1}) - 32$

Now $i_{DN} = i_{DP}$, or $56.25(v_{O1} - 0.8)^2 = 80(2.5 - v_{O1}) - 32$

Thus, $v_{O1}^2 - 1.6v_{O1} + 0.64 = 3.56 - 1.422v_{O1} - 0.569$

or $v_{O1}^2 - 0.178v_{O1} - 2.35 = 0$,

whence $v_{O1} = [- -0.178 \pm (0.178^2 - 4(-2.35))^{1/2}]/2$

$= [0.178 \pm 3.07]/2 = \underline{1.62V}$

For Q_1, $V_t = 0.8 + 0.5[(1.62 + 0.6)^{1/2} - 0.6^{1/2}] = \underline{1.158V}$

Capacitor charging current:

At v_{O1}: $i_D = \frac{1}{2}(75)(1.2/0.8)(3.3 - 1.62 - 1.158)^2 = \underline{15.3\mu A}$

At 0V: $i_D = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = \underline{351.6\mu A}$

Average: $i_D = (15.3 + 351.6)/2 = \underline{183\mu A}$

Now, $t_{PLH} = 20 \times 10^{-15} \times 1.62 / 183 \times 10^{-6} = \underline{177ps}$

b)

For the inverter, $V_{IH} = \frac{1}{8}(5(3.3) - 2(0.8)) = \underline{1.86V}$

For this value, $i_{D1} = 75(1.2/0.8)[(3.3 - 0.8)1.86 - 1.86^2/2] = \underline{328\mu A}$

Current in $Q_R = (75/3)(W/L)_R(3.3 - 0.8)^2 = 328/2$

for which $(W/L)_R = 1.05$, and for $L_R = 0.8\mu m$,

$(W/L)_R = (0.84/0.8)$

For t_{PHL}: For Q_1 with source grounded and gate at V_{DD}, while Q_R has a source-to-gate voltage V_{DD}:

Initially, at $v_{O1} = V_{DD}$, $i_{DR} = 0$, since $v_{SGR} = 0V$,

and $i_{D1} = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = \underline{352\mu A}$

At $v_{O1} = V_{IH}$, $i_{DR} = 25(0.84/0.80)[(3.3 - 0.8)(3.3 - 1.86) - (3.3 - 1.86)^2/2]$

$= \underline{67.3\mu A}$

and $i_{D1} = 75(1.2/0.8)[(3.3 - 0.8)1.86 - 1.86^2/2] = \underline{328\mu A}$

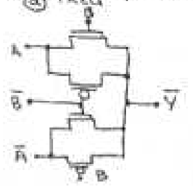
whence average current for capacitor charging is

$i_{DAV} = (352 + 328 - 67.3)/2 = \underline{306\mu A}$

Thus, $t_{PHL} = 20 \times 10^{-15}(3.3 - 1.86)/(306 \times 10^{-6}) = \underline{94ps}$

54

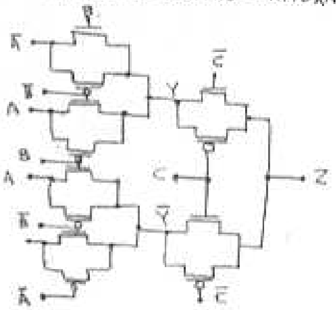
a) Need $\overline{Y} = AB + \overline{A}\overline{B}$. In direct analogy to Fig 10.31:



(next)

b) Need $Z = \overline{Y}C + Y\overline{C}$, where $Y = \overline{\overline{Y}} = (\overline{AB + \overline{A}\overline{B}})$,

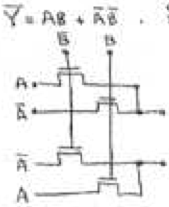
whence $Y = \overline{AB} \cdot \overline{\overline{A}\overline{B}} = (\overline{A} + \overline{B})(A + B) = A\overline{B} + \overline{A}B$.



55

Need a CPL circuit for $Y = A\overline{B} + \overline{A}B$ and

$\overline{Y} = AB + \overline{A}\overline{B}$. {See Exercise 13.9b}



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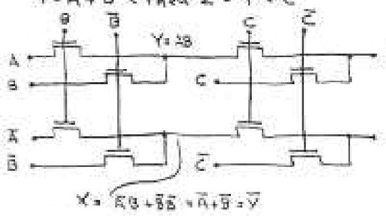
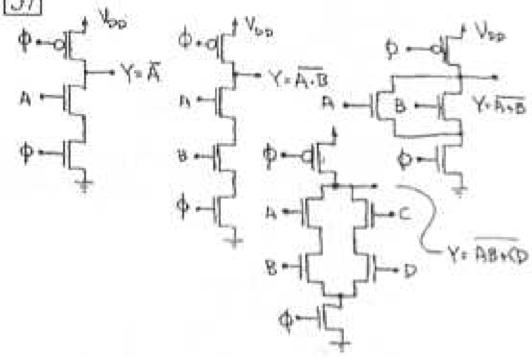
56 $\boxed{10}$

Require a CPL for $Z = ABC$ and $\overline{Z} = \overline{ABC} = \overline{A} + \overline{B} + \overline{C}$

Extend Fig 10.32 to 3 variables by dealing in pairs, creating $Y = AB$, then $Z = YC$ with $\overline{Y} = \overline{A} + \overline{B}$, then $\overline{Z} = \overline{Y} + \overline{C}$



57



58 | 10

At $v_Y = 0.3V$, $i_{DP} = \frac{1}{2}(75/3)(2.4/0.8)(3.0-0.8)^2 = \underline{181.5\mu A}$

At $v_Y = 2.7V$, $i_{DP} = (75/3)(2.4/0.8)[(3.0-0.8)0.3 - 0.3^2/2] = \underline{46.1\mu A}$

Thus $i_{Dav} = (181.5 + 46.1)/2 = \underline{114\mu A}$

and $t_{TLH} = t_r = 15\times10^{-15}(2.7-0.3)/(114\times10^{-6}) = \underline{316\,ps}$

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For 3 NMOS in series, the equivalent length is $3L_n = 3(0.8) = 2.4\mu m$

Thus for $v_Y = 3.0V$, $i_D = \frac{1}{2}(75)(1.2/2.4)(3.0-0.8)^2 = \underline{90.8\mu A}$

and for $v_Y = 1.5V$, $i_D = 75(1.2/2.4)[(3.0-0.8)1.5 - 1.5^2/2] = \underline{81.6\mu A}$

Thus $i_{Dav} = (90.8 + 81.6)/2 = \underline{86.2\mu A}$

and $t_{PHL} = 15\times10^{-15}(3.0/2)/(86.2\times10^{-6}) = \underline{261\,ps}$

60 | 10

a) $C_1 = 5fF$:

Now, for v_{C1} rising to $V_{DD} - V_t = 5 - 1 = 4V$, and assuming Q_1 continues to conduct, v_Y will fall by an amount $(C_1/C_L)(\Delta v_{C1}) = 5/30(4) = \underline{0.67V}$ to $5.0 - 0.67 = 4.33V$. Since this exceeds 4.0, the assumption that Q_1 continues to conduct is verified. Thus v_Y drops by $\underline{0.67V}$

Note that if the body effect is included, it will likely to be impossible to raise v_{C1} to 4V. Thus 0.67V is the largest possible change.

b) $C_1 = 10fF$:

In view of the previous analysis, assume that ultimately $v_Y = v_{C1} = v$. Now, the charge change in each capacitor is the same:

$Q = CV \rightarrow 10(v - 0) = 30(5 - v)$

and $10v = 150 - 30v$, $40v = 150$, and $v = \underline{3.75V}$

Thus v_Y drops by $5 - 3.75 = \underline{1.25V}$ to 3.75V

61 | 10

For a 0.5V change, $t = C\Delta V/I_L = 30(10^{-15})0.5/10^{-12} = \underline{15\,ms}$

Since the precharge interval is much shorter than the evaluate, the period of the minimum clocking frequency can be as great as 15ms, for which $f_{min} = 1/(15\times10^{-3}) = \underline{67\,Hz}$

62

In each cycle, the output must charge completely, then discharge to at least the threshold of the succeeding gate.

From Exercise 10.10, the precharge time is about $t_r = 0.4ns$. From Exercise 10.11, the discharge time is about $t_{PHL} = 0.5ns$.

Thus, the maximum allowed clocking frequency can be as high as $1/(0.4+0.5)\times10^{-9} = \underline{1.1\,GHz}$

Chapter 11 Problems

1 | **11**

For the inverters, using Eq 13.8, :

$$V_{th} = \left[V_{DD} - |V_{tp}| + (k_n/k_p)^{1/2} V_{tn}\right] / \left(1 + (k_n/k_p)^{1/2}\right)$$
$$= (5 - 1 + \sqrt{2.5} \times 1)/(1 + \sqrt{2.5}) = 5.58/2.58 = \underline{2.16V}$$

Current from the PMOS, for which $v_{SG} = 5V$ with $v_D = 2.16V$, is $i_D = (100/2.5)(2/1)[(5-1)(5-2.16) - (5-2.16)^2/2]$, or $i_D = \underline{586\mu A}$

To sustain this from the NMOS,
$586 = 100(W/L)_n[(5-1)2.16 - 2.16^2/2]$,
whence $(W/L)_{eq} = 5.86/6.31 = \underline{0.929}$

Thus, for Q_5, Q_6 in series, $(W/L)_{5,6} = 2(0.929) = \underline{1.86}$ and $W_5 = W_6 = \underline{1.86\mu m}$ for $L_{5,6} = 1\mu m$.

2 | **11**

Initially, the current from the PMOS (say Q_4) is essentially zero, since v_{SD} is 0.
Thus, for $v_{\bar{Q}} = 5V$, $i_{D_{7,8}} = \frac{1}{2}(100)(4/(1+1))(5-1)^2 = \underline{1600\mu A}$

Now, for $v_{\bar{Q}} = V_{DD}/2 = 2.5V$:
$i_{D_{7,8}} = 100(4/2)[(5-1)2.5 - 2.5^2/2] = \underline{1375\mu A}$
$i_{D_4} = (100/2.5)(2/1)[(5-1)2.5 - 2.5^2/2] = \underline{550\mu A}$

Thus, the average discharge current is $(1600 - 0 + 1375 - 550)/2 = \underline{1213\mu A}$
and $t_{PHL} = 30 \times 10^{-15} \times 2.5 / 1212 \times 10^{-6} = \underline{61.9ps}$

Now, $t_{PLH} = 1.7C/(k_p'(W/L)_p V_{DD}) = \dfrac{1.7 \times 30 \times 10^{-15}}{(100/2.5)(2/1)5} = \underline{127.5ps}$

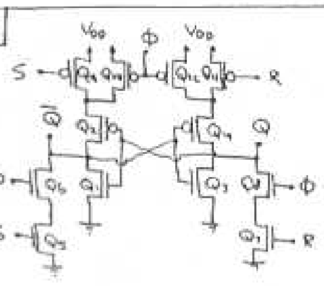
Thus the minimum set/reset pulse length is
$127.5 + 61.9 = \underline{189ps}$

3 | **11**

Assume that Q_2 is conducting, Q_1 is cut off, and Q_5 is conducting. Now to lower $\bar{Q}$ to $V_{DD}/2 = 2.5V$, Q_2 and Q_5 must be matched, since equivalent control voltages are applied to each.

Thus $k_2 = k_5$, and $25(W/L)_2 = 75/2$, whence $(W/L)_2 = (W/L)_p = \underline{6}$

4 | **11**

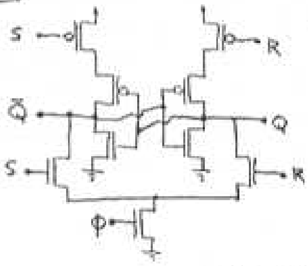


As noted, this fully-complementary circuit uses 12 transistors. However, a 10-T version exists in which Q_{10} and Q_{12} are omitted.

Note further that an effective 9-T version exists, in which Q_6 and Q_8 are moved below Q_5 and Q_7, then merged into a single grounded-source device. Note that all of the designs can employ the latter idea to reduce the transistor count by 1. See the sketch following:

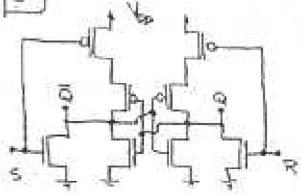
(next)

4 | **11**



This circuit suffers only from the fact that unclocked changes in S and R have a secondary impact on $Q\bar{Q}$ since raising S or R disconnects $Q\bar{Q}$ from V_{DD}. In some applications this may lead to system noise sensitivity in which case or the other or both of Q_{10}, Q_{12} (in the previous sketch) may be added.

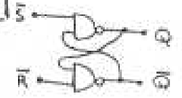
5 | **11**



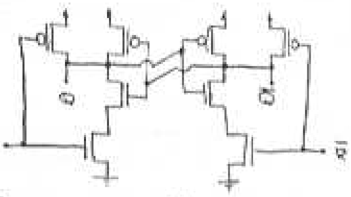
See that this is a complementary CMOS NOR-gate implementation.

6

11



R	S	$\bar{R}$	$\bar{S}$	Q_{n+1}
0	0	1	1	Q_n
0	1	1	0	1
1	0	0	1	0
1	1	0	0	not used



In this circuit, $\bar{R}$ and $\bar{S}$ rest high. Either going low forces the corresponding output to go high. [Thus, $\bar{S}=0 \rightarrow S=1 \rightarrow Q=1$]

If both $\bar{S}$ and $\bar{R}$ are low, the outputs Q and $\bar{Q}$ are no longer complementary, but are both high. Following this, the output retains the state defined by the last of $\bar{R}, \bar{S}$ to go high. If both $\bar{R}$ and $\bar{S}$ go high simultaneously, the outputs Q and $\bar{Q}$ oscillate, initially in phase, with a phase difference, initiated by noise, which

6

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grows increasingly rapidly until the outputs "snap" into a complementary state of 0/1 or 1/0. During the oscillation, the period is somewhat less than twice the propagation delay of the basic inverter, and the amplitude is a fraction of the full logic swing. This fraction depends on the ratio of propagation time to transition time of the inverters, being larger for faster logic transitions and longer pure propagation delays. Generally speaking, the outputs are usually nearly sinusoidal.

7

11

Note that the devices are matched, with

a. $k_n = k_p = 20(12/6) = 40\mu A/V^2$, and $|V_t| = 1V$.

For $v_I = 0V, 5V$: one device is on, one off ; $v_O = \underline{5V}, \underline{0V}$

For $v_I = 1V, 4V$: one on, one off ; $v_O = \underline{5V}, \underline{0V}$

For $v_I = 1.5V, 3.5V$: one in saturation, one in triode mode.

$i_D = \frac{1}{2}(40)(1.5-1)^2 = 40[(5-1.5)v_O - v_O^2/2]$

Thus $0.125 = 2.5v_O - v_O^2/2$

or $v_O^2 - 5v_O + 0.25 = 0$

and $v_O = (-\,-5 \pm \sqrt{5^2 - 4(0.25)})/2$

$= (5 \pm 4.8989)/2 = 0.05V$

Thus $v_O = \underline{0.05V}$ or $\underline{4.95V}$

For $v_I = 2.0V, 3.0V$:

$\frac{1}{2}(2-1)^2 = (5-2-1)v_O - v_O^2/2$

or $(2-1)^2 = 2 \cdot 2v_O - v_O^2$

and $v_O^2 - 4v_O + 1 = 0$

whence $v_O = (-\,-4 \pm \sqrt{4^2 - 4(1)})/2$

$= (4 \pm 3.464)/2 = 0.27V$

Thus $v_O = \underline{0.27V}$ or $\underline{4.73V}$

For $v_I = 2.25V$ or $2.75V$:

$(2.25-1)^2 = 2(5-2.25-1)v_O - v_O^2$

$1.5625 = 3.5v_O - v_O^2$

$v_O^2 - 3.5v_O + 1.5625 = 0$

7

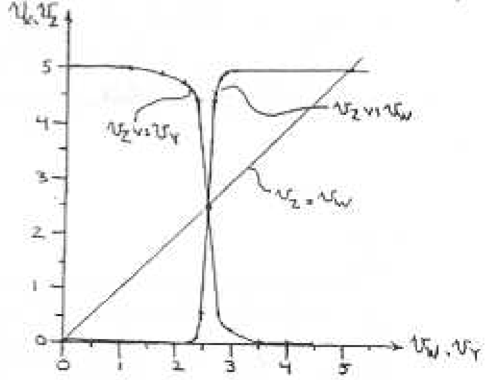
11

whence $v_O = (-\,-3.5 \pm \sqrt{3.5^2 - 4(1.5625)})/2$

$= (3.5 \pm 2.45)/2 = 0.525$

Thus $v_O = \underline{0.525V}$ or $\underline{4.475V}$

b. For $v_I = 2.5V$, $v_O = \underline{2.5V}$, by symmetry.



Now, having plotted v_Z versus v_Y (or v_X versus v_W), use the graph to find v_Z versus v_W :
Work backwards : first v_Z, then $v_Y = v_X$, then v_W.

For $v_Z = \underline{2.5V}$, $v_Y = v_X = v_W = \underline{2.5V}$

For $v_Z = \underline{4.4V}$, $v_Y = 2.25V$; for $v_X = 2.25V$, $v_W = \underline{2.55V}$

For $v_Z = \underline{4.5V}$, $v_Y = 1.50V$; for $v_X = 1.50V$, $v_W = \underline{2.65V}$

(next)

7 11

Plot these on the sketch above. See that the v_Z versus v_W characteristic crosses the

C. $v_Z = v_W$ line at:

point A: (0,0)
point B: (2.5,2.5)
point C: (5,5)

At point B, the current flow in each inverter is:

$I_D = \frac{1}{2}(40)(2.5-1)^2 = \underline{45\mu A}$

where for each transistor, $r_o = 100/(45\times10^{-6}) = 2.22 M\Omega$

and $g_m = 2(\frac{1}{2})40(2.5-1) = \underline{60\mu A/V}$

Thus for each inverter operating at (2.5, 2.5), the voltage gain is $-(g_m + g_m)(r_o \| r_o) =$

$-g_m r_o = -60\times10^{-6}\times2.22\times10^{6} = \underline{133\ V/V}$

Thus an estimate of the slope of the v_Z versus v_W curve at B is $(133)^2 = \underline{\underline{17.7\times10^3\ V/V}}$

Correspondingly a lower bound on the width of the transition region is $(5-0)/(17.7\times10^3)$, or $\underline{\underline{0.28\,mV}}$, that is $\pm 0.14\,mV$ around 2.5V.

Alternatively, a more detailed analysis can be done in a number of ways. For example, consider a sample-point analysis:

From earlier work above, for $v_Z = 4.95V$, $V_Y = 1.5V$

Now to produce $v_X = V_Y = 1.5V$, v_W must be around 2.5V with a value determined by

7 11

the PMOS in saturation and the NMOS in triode mode sharing the current i_D:

Thus $i_D = \frac{1}{2}(40)(5-v-1)^2 = 40[(v-1)1.5 - 1.5^2/2]$

or $(4-v)^2 = 2(v-1)1.5 - 1.5^2$

or $16 - 8v + v^2 = 3v - 3 - 2.25$

or $v^2 - 11v + 21.25 = 0$

whence $v = (--11 \pm (11^2 - 4(21.25))^{1/2})/2$

$= (11 \pm 6)/2 = 2.5V$

While this seems strange at first, it is simply an indicator that for $v_X = 1.5V$, the NMOS is actually just at the edge of saturation, such that the saturation-current expression still applies, for which the saturation currents of both devices are equal when $v_W = 2.5V$. Thus the output resistance and finite current gain must be considered (as we did earlier)

For the gain of $-133V/V$ found there, to produce a 1V change in v_X (from 2.5 to 1.5V) would require $1/133$ V or 7.5mV (or 15mV total) on v_W. Recall that this 15mV change on v_W moves v_Z from 0.05 to 4.95V!

We conclude that the transition region is more like 15mV than a fraction of a mV.

Note that this approach uses a combination of large- and small-signal modelling to better approximate the large-signal nonlinearities of logic.

8 11

The approximate transfer characteristic of each inverter passes through points: (0,5), (2.0,4.6), (2.42,0.4), (5,0).

For the linear centre segment between (2.0,2.46), (2.42,0.4) an equation is $v_O = a - bv_I$

Here: $4.60 = a - 2.00b$, and

$0.40 = a - 2.42b$

Subtract: $4.20 = 0.42b \rightarrow b = \underline{10}$

Now, $4.60 = a - 2(10) \rightarrow a = 4.6 + 20 = \underline{24.6}$

Check: $0.4 = 24.6 - 2.42(10)$ ✓

Thus the middle part of the characteristic is

$v_O = 24.6 - 10v_I$

For each device, $v_O = v_I = v$, when

$v = 24.6 - 10v$, or $11v = 24.6$, or $v = \underline{2.236V}$

where the gain is $\Delta v_O/\Delta v_I = -b = \underline{-10\ V/V}$

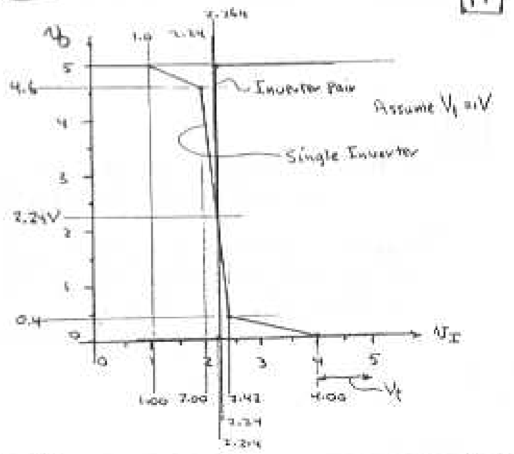
Thus point B on the open-loop characteristic is $v_W = v_Z = \underline{\underline{2.236V}}$, where the loop gain can be approximated to be at least $(-10)^2 = \underline{\underline{100\ V/V}}$

The open-loop characteristic reaches $v_O = 5V$, where $v_I = 2.236 + (5-2.236)/100 = \underline{2.264V}$

and it reaches 0V where

$v_I = 2.236 - 2.236/100 = \underline{2.214V}$

8 11



9 11

From Exercise , $T = 0.69\,RC$. From the equation just preceding the exercise, a 2% error introduced by R_{on} implies that

$R + R_{on} = 1.02R$

and $R = R_{on}/0.02 = 1k\Omega/0.02 = \underline{\underline{50k\Omega}}$,

for which $C = T/0.69R = 1\times10^{-3}/(0.69\times50\times10^{3}) = \underline{\underline{30nF}}$

10 | **11**

From page 1025, for the monostable:

$$T = C(R + R_{on})\ln\left[R/(R+R_{on})(V_{DD}/(V_{DD}-V_{th}))\right]$$

Neglecting R_{on}, $T = \underline{CR\ln(V_{DD}/(V_{DD}-V_{th}))}$

Now, $V_{th} = (0.5 \pm 0.1)V_{DD}$, and
$\ln(1/(1-0.5)) = 0.693$, $\ln(1/(1-0.4)) = 0.511$, and
$\ln(1/(1-0.6)) = 0.916$

Thus T can vary from $(0.511/0.693)100 = \underline{73.7\%}$
to $(0.916/0.693)100 = \underline{132.2\%}$ of nominal

or by $\underline{-26.3\%}$ to $\underline{+32.2\%}$ of nominal.

---

11 | **11**

$$T = C(R+R_{on})\ln\left[R/(R+R_{on}) \times V_{DD}/(V_{DD}-V_{th})\right]$$
$$= 0.001\times10^{-6}(10^4+200)\ln\left[10^4/(10^4+200) \times 10/(10-5)\right]$$
$$= 1.02\times10^{-5}\ln 1.96 = 1.04\times10^{-5}\,s = \underline{10.4\,\mu s}$$

Now, $\Delta V_1 = V_{DD}R/(R+R_{on}) = 10\times10^4/(1.02\times10^4) = \underline{9.80\,V}$

and $\Delta V_2 = V_{DD} + V_{D1} - V_{th} = 10 + 0.5 - 5 = \underline{5.70\,V}$

Change in v_{O1} is due to the variable current in R flowing in R_{on}.

Initially, the voltage across R_{on} is $V_{DD}R_{on}/(R+R_{on})$
Finally, it is $(V_{DD}-V_{DD}/2)R_{on}/(R+R_{on})$

---

11 | **11**

Thus, the total change is $(V_{DD}/2)R_{on}/(R+R_{on})$
or $(10/2)200/(10^4+200) = \underline{0.098\,V}$

The peak sinking current through G_1 occurs as v_{O1} rises and current flows through R_{on} and the diode. Its peak value is
$(V_{DD}-V_{th}-V_D)/R_{on} = (10-5-0.7)/200 = \underline{21.5\,mA}$

---

12 | **11**

$$T = C(R+R_{on})\ln\left[R/(R+R_{on}) \times V_{DD}/(V_{DD}-V_{th})\right]$$
Thus $1 = 10^{-6}(R+100)\ln\left[R/(R+100) \times 1/(1-0.4)\right]$

Now for R_{on} ignored:
$R = 10^{-6}/\ln[1/0.6] = 1.958\,M\Omega$.
Clearly, for this R, R_{on} can be neglected, and $R = \underline{1.96\,M\Omega}$

---

13 | **11**

a) From Ex 13.17, $T = CR\ln\left[V_{DD}/(V_{DD}-V_{th}) \times V_{DD}/V_{th}\right]$
Now, for $V_{th} = V_{DD}/2$, $T = CR\ln(2\times2) = \underline{1.39\,CR}$

b) For $f_0 = 100\,kHz$, $T = 1/10^5 = 1.39\,CR$
whence $CR = 0.721\times10^{-5}$

For a choice of $R = \underline{10\,k\Omega}$, $C = 0.721\times10^{-5}\times10^{-4} = \underline{721\,pF}$

---

14 | **11**

From Exercise 13.17, $T = CR\ln\left(\frac{V_{DD}}{V_{DD}-V_{th}} \times \frac{V_{DD}}{V_{th}}\right)$

For $V_{th} = 0.5V_{DD}$, $f_0 = 1/T = [CR\ln 4]^{-1} = 0.721/CR$

For $V_{th} = 0.6V_{DD}$, $f_0 = [CR\ln(1/0.4 \times 1/0.6)]^{-1} = 0.701/CR$

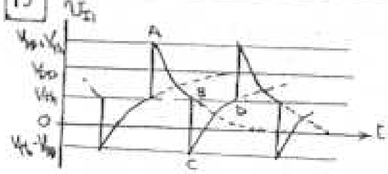
For $V_{th} = 0.4V_{DD}$, $f_0 = [CR\ln(1/0.6 \times 1/0.4)]^{-1} = 0.701/CR$

Thus, for V_{th}/V_{DD} over the range 0.4 to 0.6, f_0 varies from $(0.701/0.721)100 = \underline{97.2\%}$ of nominal, through nominal to 97.2% of nominal again.

Note that the frequency is relatively constant, independant of V_{th} variation. This is due to compensation of one half-cycle by the next, each varying quite strongly but in opposite directions.

---

15 | **11**



At Point A: $v_I = V_{DD} + V_{th}$, and thereafter
$v_I = (V_{DD} + V_{th} - 0)e^{-t/RC} + 0$

15 | **11**

At Point B: $v_I = V_{th}$ and $t = t_1$, where
$V_{th} = (V_{DD} - V_{th})e^{-t_1/RC}$
or $t_1 = -RC\ln[V_{th}/(V_{DD}+V_{th})]$
or $t_1 = RC\,[(V_{DD}+V_{th})/V_{th}]$

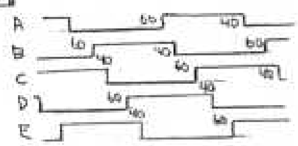
At Point C: $v_I = V_{th} - V_{DD}$, and thereafter
$v_I = (V_{th} - V_{DD} - V_{DD})e^{-t/RC} + V_{DD}$

At Point D: $v_I = V_{th}$ at $t = t_2$, where
$V_{th} = (V_{th} - 2V_{DD})e^{-t_2/RC} + V_{DD}$
or $t_2 = -RC\ln[(V_{DD}-V_{th})/(2V_{DD}-V_{th})]$
or $t_2 = RC\ln[(2V_{DD}-V_{th})/(V_D-V_{th})]$

Thus, the period $T = t_1 + t_2$
whence $T = RC\left[\ln\frac{V_{DD}+V_{th}}{V_{th}} + \ln\frac{2V_{DD}-V_{th}}{V_{DD}-V_{th}}\right]$

or $T = RC\ln\left[\frac{V_{DD}+V_{th}}{V_{th}} \times \frac{2V_{DD}-V_{th}}{V_{DD}-V_{th}}\right]$, as suggested.

16 | 11



16 | 11

Waveform A is low for 60+40+60+40+60 = <u>260 ns</u>
and high for 40+60+40+60+40 = <u>240 ns</u>

Period is (260 + 240) = <u>500 ns</u>
Frequency is 1/500 = <u>2 MHz</u>

Percentage of cycle for which output is high is <u>48%</u>
<u>Check</u>: $t_P = (60+40)/2 = 50$ ns ✓

17 | 11

For 11 inverters, there are 2(11) = 22 transitions whose average length is $t_P = (1/20\times10^6)/22 =$ <u>2.27 ns</u>

18 | 11

A 1 Mb array requires n address bits where $2^n = 10^6$, or $n\log_{10}2 = 6$, $n = 6/\log_{10}2 = 19.93$
Thus 20 bits are needed to address every cell.

For 16-bit words, $2^4 = 16$ and 4 bits are not needed.
Thus 20 − 4 = <u>16 bits</u> of address are sufficient.

<u>Check</u>: $m = \log_{10}(10^6/16)/\log_{10}(2) = 4.796/0.301 = 15.93$
Use 16 ✓
<u>Note</u>: A "1 Mb array" actually holds $2^{20} = 1024^2 = 1048576$ cells.

19 | 11

Since the array is square, there are about $(10^6)^{1/2}$ or 1000 word lines. More precisely there are $(2^{20})^{1/2} = 2^{10} =$ <u>1024</u> word lines.

For a 1-Mb square array, there are also 1024 bit l

Thus a straightforward design would use <u>1024</u> sense amplifiers/drivers.

Now for dynamic storage on C operating at f in voltage V, the dynamic power is $P_D = fCV^2$.
Thus $C = P_D/(fV^2) = 500\times10^{-3}/(1/200\times10^{-9}\times5^2)$
or C = <u>4000 pF</u>

If 90% of the dynamic loss is in the array, the array capacitance is 4000(0.9) = <u>3600 pF</u>
For 16 bit lines selected and active at a time, the capacitance per bit line could be 3600/16 = <u>225</u>
which is 225/1024 = <u>220 fF/bit</u>.
For voltage reduction to 3 V, power is reduced to a factor $3^2/5^2 =$ <u>0.36</u>. For the same power level the array could be larger by a factor 1/0.36 = <u>2.8 times</u>

20 | 11

Cell area = $\frac{1}{2}(21\times10^{-3}\times31\times10^{-3})/10^9 =$ <u>0.326 µm²</u>.
For 2 cells, the area is 0.65 µm², for which (if square) the side length is $\sqrt{0.65} =$ <u>0.807 µm</u>
and the cell dimension is about <u>0.4 µm by 0.8 µm</u>

21 | 11

The cell area is $10^9\times0.38\times10^{-6}\times0.76\times10^{-6} =$ <u>0.289×10^{-3} m²</u>

The chip area is $19\times10^{-3}\times38\times10^{-3} =$ <u>0.722×10^{-3} m²</u>

Thus the peripheral circuits and interconnect occupy $(0.722-0.289)10^{-3} = 0.433$ mm²,
or (433/722)×100 = <u>60%</u> of the chip area.

22 | 11

For 16 blocks, <u>4 bits</u> of block address are needed. The arrays are each of 256M/16 = 16 Mbit size. If each block array is square, it has 4096 rows and 4096 columns. It needs $\log_2 4096 =$ <u>12 bits</u> each of row and column address bits, for a total of 2(12) + 4 = <u>28 bits</u> in all.

<u>Check</u>: $2^{28} = 268\times10^6$, or 256M bits. ✓

$\boxed{23}$ $\boxed{11}$

a) For W high and $\bar{B}$ high, with $\bar{Q}$ originally low and the gate of Q_1 high:

At $v_{\bar{Q}} = 2.5V$, from Eq. 5.30, $V_t = V_{t0} + \gamma\left[\sqrt{V_{SB}+2\phi_f} - \sqrt{2\phi_f}\right]$
or $V_t = 0.8 + 0.5\left[\sqrt{2.5+0.6} - \sqrt{0.6}\right] = \underline{1.293V}$

Now, for $I_5 = I_1$, $\frac{1}{2}k_{n5}(v_{GS5} - V_t)^2 = \frac{1}{2}k_{n1}(v_{GS1} - V_t)^2$
or $(W/L)_5(2.5 - 1.293)^2 = n(5 - 0.8)^2$
or $(W/L)_5 = n(4.2)^2/1.207^2 = \underline{12.1n}$.

b) For W high and B low, with Q originally high and the gate of Q_6 high:

At $v_Q = 2.5V$, with Q_4 in saturation and Q_6 in triode:
$\frac{1}{2}k_{p4}(5-0.8)^2 = k_{n6}\left[(5-0.8)2.5 - 2.5^2/2\right]$
or $\frac{1}{2}n(4.2)^2 = (W/L)_6[7.375]$
or $(W/L)_6 = n(4.2^2)/(2(7.375)) = \underline{1.20n}$

c) Clearly, for a very conservative design, one in which both write mechanisms work, the larger of the choices in a) b) would be used, namely $(W/L)_5 = (W/L)_6 = 12.1\ n$, very large!

In practice, this design needs a lot of area, and, instead, a conservative version of b) would likely be used, with $(W/L)_5 = (W/L)_6$ larger than $1.2n$, say by a factor of 2, perhaps $\underline{2.4n}$

d. $\boxed{11}$

For $(W/L)_6 = 12.1n$, and matched inverters,
$k_4' = 50 \times 10^{-6} n = 50 \times 10^{-6} \times 2 = 100\mu A/V^2$,
and $k_6' = 1210\mu A/V^2$

For $v_Q = 5V$: $v_{SD4} = 0$ and $i_{D4} = 0$, but
$i_{D6} = \frac{1}{2}(1210 \times 10^{-6})(5-0.8)^2 = \underline{10.7mA}$

For $v_Q = 2.5V$:
$i_{D6} = 1210 \times 10^{-6}\left[(5-0.8)2.5 - 2.5^2/2\right] = \underline{8.92mA}$
and $i_{D4} = 100 \times 10^{-6}\left[(5-0.8)2.5 - 2.5^2/2\right] = \underline{0.74mA}$

Thus, the average discharge current is $(10.7 - 0 + 8.92 - 0.74)/2 = \underline{9.44\ mA}$

Thus, the discharge time is $t = 50 \times 10^{-15} \times 2.5/9.44 \times 10^{-3} = \underline{13.2ps}$

For a minimum-size design with $(W/L)_6 = 2.4n$, the average current would be:
$\left[(2.4/1.2)[10.7 - 0 + 8.92] - 0.74\right]/2 = \underline{1.58mA}$
and the time to $V_{DD}/2$ would be
$t = 50 \times 10^{-15} \times 2.5/1.58 \times 10^{-3} = \underline{79ps}$

$\boxed{24}$ $\boxed{11}$

a) Initially, $I_5 = \frac{1}{2}k_{n5}'(v_{GS} - V_t)^2 = \frac{1}{2}(50)10^{-6}(10/2)(5-0.8)^2$
or $I_5 = 2.205mA$, and $I_1 = 0$, since $v_{DS1} = 0$.
Thus, $I_{C\bar{Q}} = 2.205 - 0 = \underline{2.205mA}$

b) At the end of the interval, $v_{\bar{Q}} = 2.5V$, and
$V_{t5} = V_t + \gamma\left[\sqrt{V_{SB}+2\phi_f} - \sqrt{2\phi_f}\right]$
$= 1.0 + 0.5\left[\sqrt{2.5+0.6} - \sqrt{0.6}\right] = \underline{1.49V}$

Correspondingly, $I_5 = \frac{1}{2}(50)10^{-6}(10/2)(2.5-1.49)^2 = \underline{127.5\ \mu A}$
Now, Q_1 is in triode mode, with
$I_1 = 50 \times 10^{-6}(4/2)\left[(5-1)2.5 - 2.5^2/2\right] = \underline{687.5\mu A}$
Thus $I_{C\bar{Q}} = 127.5 - 687.5 = -\underline{560\mu A}$, negative!
Thus we see that Q_5 is incapable of raising $\bar{Q}$ to 2.5V. This is the case even if the body effect is ignored (check this for interest!).
(You may also be interested in calculating the highest voltage to which $\bar{Q}$ can be raised.)

c) The average value is not relevant.

d) Δt is ∞ as defined here!
Note that in Exercise 11.9, $\Delta t = 19.4ps$, during which time the voltage on the gate of Q_1 reduces, the current in Q_1 reduces and the current in Q_2 increases, allowing Q_5 to become more effective.

$\boxed{25}$ $\boxed{11}$

For $v_W = 2.5V$, $v_B = 2.5V$, $v_{\bar{Q}} = 0V$:
$i_5 = \frac{1}{2}k_n'(v_{GS} - V_t)^2 = \frac{1}{2}(50)10^{-6}(10/2)(2.5-1)^2 = \underline{281\mu A}$

However, the voltage at Q will rise due to this current where $281 = 50(4/2)\left[(5-1)v_{Q1} - v_{Q1}^2/2\right]$.
Thus $v_{Q1}^2 - 8v_{Q1} + 5.62 = 0$,
whence $v_{Q1} = (--8 \pm \sqrt{8^2 - 4(5.62)})/2$
$= (8 \pm 6.44)/2 = \underline{0.78V}$

This reduces i_5 to $i_5 = \frac{1}{2}(50)(10/2)(2.5-0.78-1)^2 = \underline{64.8\mu A}$
for which $v_{Q1}^2 - 8v_{Q1} + (0.648 \times 2) = 0$
whence $v_{Q1} = (--8 \pm \sqrt{8^2 - 4(0.648 \times 2)})/2$
$= \underline{0.165V}$

Try $v_{Q1} = (0.165 + 0.78)/2 = \underline{0.47V}$
for which $i_5 = 125(1.5 - 0.47)^2 = \underline{133\mu A}$
and $v_{Q1} = (8 \pm \sqrt{64 - 4(1.33 \times 2)})/2 = \underline{0.347V}$

Try $v_{Q1} = (0.347 + 0.47)/2 = 0.41 \simeq 0.40V$
for which $i_5 = 125(1.5 - 0.40)^2 = \underline{151\mu A}$
and $v_{Q1} = (8 \pm \sqrt{64 - 4(1.51 \times 2)})/2 = 0.397 \simeq \underline{0.40V}$
with $i_5 = \underline{150\mu A}$

Now, for the bit-line voltage to reduce by 0.2V, it will take

$t = (1 \times 10^{-12} \times 0.2)/150 \times 10^{-6} = \underline{1.33ns}$

26 | 11

The bit-line capacitance is $[n(2)+70] = 2n+70$ fF

For a 5V supply and precharge to $V_{DD}/2$, there is a 2.5V change on the 50fF cell capacitor to produce a 0.1V bit-line signal.

Thus $0.1 = 2.5(50/(2n+20+50))$

or $2n+70 = 2.5(50)/0.1 = 1250$

or $2n = 1180$ and $n = 590 > 512$

Thus we can use <u>512 rows</u> for which $\log_2 512 = 9$ or <u>9 bits</u> addressing is needed.

For a sense amplifier of 5× greater gain

$0.1/5 = 125/(2n+70)$

$2n+70 = 5(125)/0.1 = 6250$

$2n = 6180$, and $n = 3090 > 2048$

To address <u>2048</u> <u>rows</u> requires <u>11 bits</u>

---

27 | 11

If the memory array has n columns, it has 2n rows and $2n^2$ cells

Refresh time is $2n(20)10^{-9}$ s $= (1.00-0.98)8\times10^{-3}$ s

whence $n = 0.02 \times 8\times10^{-3}/40\times10^{-9} = 4000$

The corresponding memory capacity is $2n^2 = 2(4000)^2$ or <u>32 Mbits</u>

---

28 | 11

For a 1Mb square array there are 1024 rows and 1024 columns.

Thus the bit-line capacitance is $10^{-15}(1024\times1+12)$ or <u>1.036 pF</u>

When storing a '1', the voltage on C_S is $(V_{DD}-V_t)$ or $(5-1.5) = 3.5$V. With precharge to $V_{DD}/2 = 2.5$V, the change in voltage on $C_S = 3.5-2.5 = 1.0$V,

For $C_S = 25$fF, the bit-line voltage resulting is $25/(25+1036)\times1$ = <u>23.6 mV</u>

When storing a '0', the voltage on C_S is 0V and the change is $2.5-0 = 2.5$V with a resulting bit-line signal of $25/(25+1036)\times2.5$ = <u>58.9 mV</u>

---

29 | 11

For leakage current I, the voltage change on C in time T is $V = IT/C$

Correspondingly, $1 = I \times 10\times10^{-3}/20\times10^{-15}$, and the maximum leakage is $I = 20\times10^{-15}/10\times10^{-3}$ = <u>2pA</u>

---

30 | 11

Pattern the solution after the approach used in the solution of Example 11.3:

For the bit-line output to reach $0.9 V_{DD} = 2.7$V from $V_{DD}/2 = 1.5$V in 2ns for an initial bit-line signal of $0.1/2 = 0.05$V:

$2.7 = 1.5 + 0.05e^{2/\tau}$

whence $2/\tau = \ln[(2.7-1.5)/0.05] = 3.178$

and $\tau = 2/3.178 = 0.629$ ns

Thus $C/G_m = 0.629\times10^{-9}$ s, and $G_m = 1\times10^{-12}/(0.629\times10^{-9})$ = <u>1.589 mA/V</u>

For matched inverters, $g_{mn} = g_{mp} = G_m/2 = 1.589/2 = 0.795$ mA/V

Now, $g_m = k'(W/L)(V_{GS}-V_t)$

and $0.795\times10^{-3} = 100\times10^{-6}(W/L)_n[3.0/2-0.8]$

Thus $(W/L)_n = 0.795\times10^{-3}/(100\times10^{-6})/0.7$ = <u>11.36</u>

Now, for devices assumed to have length L = 1 μm (or, alternatively, for each micron of device length)

W_n = <u>11.36 μm</u> and $W_p = 3(11.36)$ = <u>34.1 μm</u>

Now, for a differential input signal of 0.2V (and 0.1V on each bit-line), the response time is t, where $2.7 = 1.5 + 0.1e^{t/0.629}$,

whence $t = 0.629 \ln(2.7-1.5)/0.1$ = <u>1.56 ns</u>

31 | 11

Note that for the inverters,

$k_n = k_n'(W/L)_n = 100(6/1.5) = 400\,\mu A/V^2$

$k_p = k_p'(W/L)_p = (100/2.5)(15/1.5) = 400\,\mu A/V^2$

Thus we see that the inverters are matched.

Generally, $i_D = \frac{1}{2}k_n(v_{GS}-V_t)^2$, and

$g_m = \partial i_D/\partial v_{GS} = k_n(v_{GS}-V_t)$

Now at $v_{GS} = v_B = V_{DD}/2 = 3.3/2 = 1.65V$,

$g_{m_n} = 400(1.65-0.8) = 340\,\mu A/V$

Thus $G_m = g_{m_n} + g_{m_p} = 2(340) = \underline{680\,\mu A/V}$

For a bit-line capacitance of 0.8pF, $\tau = C/G_m$

or $\tau = 0.8\times10^{-12}/680\times10^{-6} = \underline{1.176\,ns}$

Now, for $0.9V_{DD}$ reached in 2ns, for a signal Δv,

$0.9(3.3) = 1.65 + \Delta v\, e^{2/1.176}$

or $\Delta v = (2.97-1.65)/5.478 = 0.241V$

Thus the initial voltage between B lines must be $2(0.241) = \underline{0.482V}$

If an additional 1ns is allowed: $t = 2+1 = 3ns$,

and $\Delta v = (2.97-1.65)/e^{3/1.176} = 0.103V$

allowing a signal to be used of $2(0.103) = \underline{0.206V}$

Now, with the original bit-line signal of 0.241V and a delay of 3ns:

31 | 11

$2.97 = 1.65 + 0.241e^{3/\tau}$

and $e^{3/\tau} = (2.97-1.65)/0.241 = 5.477$

$3/\tau = \ln(5.477) = 1.7006$

whence $\tau = 3/1.7006 = 1.764\,ns$

Thus $C = G_m\tau = 680\times10^{-6}\times1.764\times10^{-9} = \underline{1.20pF}$

This is an increase (from 0.8pF) of $\left(\frac{1.2-0.8}{0.8}\right)100 = \underline{50\%}$

For the longer line, the initial delay to establish a suitable signal becomes 150% of 5ns = $\underline{7.5ns}$

32 | 11

a) For an initial difference between bit lines of ΔV, each bit-line signal is $\Delta V/2$.

For the rising line: $v_B = V_{DD}/2 + (\Delta V/2)e^{t/(C_B/G_m)}$

whence $e^{t/(C_B/G_m)} = (2/\Delta V)(0.9-0.5)V_{DD}$

$= 0.8V_{DD}/\Delta V$

Taking base-e logarithms and cross-multiplying,

$t = t_d = (C_B/G_m)\ln(0.8V_{DD}/\Delta V)$ as stated.

b) For a reduction of t_d to ½ the original value, G_m must be doubled. To do this, double the width of all transistors (or increase by a factor of $\underline{2.0\times}$)

(next)

32 | 11

c) Now, for $V_{DD} = 5V$, $\Delta V = 0.2V$, with the original design, $t_d = C_B/G_m \ln(0.8(5)/0.2)$

or $t_d = 3.00\, C_B/G_m$

For the modified design, $\Delta V = 0.2/4 = 0.05V$

and $t_d = C_B/G_m' \ln(0.8(5)/0.05) = 4.38\, C_B/G_m'$

For these to be equal, $3.00/G_m = 4.38/G_m'$

and $G_m' = 1.46\, G_m$.

Thus the transistors must be made 46% wider (or be increased by a factor of $\underline{1.46\times}$)

33 | 11

For the DRAM arrangement, the signal is applied to only one side; Thus in comparison to the SRAM treatment, the applied signal is only half as large.

Now, the specification must be met for either a '0' or a '1' stored. The worst case is a differential signal of 40mV (corresponding to a single-side signal of 20mV)

Thus $2.0 = 20\times10^{-3}e^{5/\tau}$, and $5 = \tau\ln(2/20\times10^{-3})$,

or $5 = \tau\ln(100) = 4.605\tau$, whence $\tau = \underline{1.086\,ns}$

(next)

33 | 11

For a 1pF bit-line capacitance, $G_m = C/\tau$

or $G_m = 1\times10^{-12}/1.086\times10^{-9} = 0.921\,mA/V$,

with $0.921 = 0.46\,mA/V$ from each transistor.

Now, for the n-channel device, $g_m = k_n'(W/L)_n(V_{GS}-V_t)$

or $0.46\times10^{-3} = 100\times10^{-6}(W/L)_n(2.5-1)$

Thus $(W/L)_n = (0.46/0.1)/1.5 = \underline{3.07}$

For matched inverters, $(W/L)_p = 2.5(3.07) = \underline{7.68}$

When a '1' is read, the response time will be

$t = \tau\ln(2/20\times10^{-3}) = 1.086\ln100 = \underline{5ns}$

(Note: this is as designed!)

When a '0' is read,

$t = 1.086\times10^{-9}\ln(2/(100/2)\times10^{-3}) = \underline{4.01ns}$

34 | 11

Here $2^n = 512$, $n\log_{10}2 = \log_{10}512$, $n = 2.709/0.301 = \underline{9.00}$

Thus the number of bits is $\underline{9}$

The decoder has $\underline{512}$ output lines, one of which is active (high). The NOR array requires true and complement input lines for each bit: $2\times9 = \underline{18}$

Each row uses 9 NMOS for a total of $9\times512 = \underline{4608}$ NMOS and $\underline{512}$ PMOS, for a total of $\underline{5120}$ transistors.

35 | 11

For a 256Kbit square array, there are $(256\times1024)^{1/2} = 512$ rows and columns.

Number of column-address bits is $\log_2 512 = \underline{9}$

Two multiplexors are needed, since both true and complement bit lines are required. For each multiplexor, there are 512 output lines.

For each (half) multiplexor, 512 NMOS are needed for a total of $\underline{1024}$ NMOS pass gates.
For the 512 output NOR decoder itself, $512\times9 = \underline{4608}$ NMOS and $\underline{512}$ PMOS are needed.
The address-bit inverters need $\underline{9}$ NMOS and $\underline{9}$ PMOS
Overall, the need is for $1024 + 4608 + 9 = \underline{5641}$ NMOS and $512 + 9 = \underline{521}$ PMOS, for a total of $\underline{6162}$ transistors.

36 | 11

From the solution to P11.35 above, a square 256K-bit array has 512 rows and 512 columns for which 9 row and 9 column address bits are needed.
Check: $2^{9+9} = 2^{18} = 262144$ ✓

For the tree of Fig 11.28, 9 levels of pass gates are needed.

36 | 11

The total number of pass gates is
$N = 2+4+8+16+32+64+128+256+512$
See that $N = 2 + 2(N-512)$, or $N = 2 + 2N - 1024$,
whence $N = 1022$
Thus a tree column decoder for 9 bits needs $\underline{1022}$ pass transistors

For true and complement bit lines, a total of $2(1022) = \underline{2044}$ pass transistors are needed. Compare this with the number required beyond the input inverters in P11.35 namely $6162 - 18 = \underline{6144}$

37 | 11

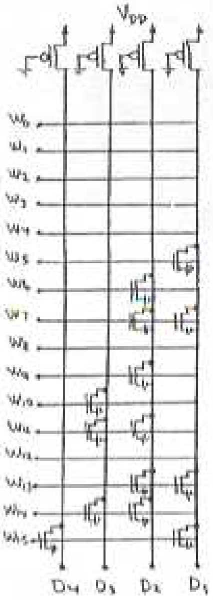
Note from Fig 11.29 that the output is high if no word is selected. Thus, logically, high must correspond to logic 0 (and no transistor, as noted in the text).

Correspondingly, the words stored in Fig 11.29 are 0100, 0000, 1000, 1001, 0101, 0001, 0110, and 0010.

38 | 11

Need $Z = X \times Y$

X	Y	Z
00	00	0000
00	01	0000
00	10	0000
00	11	0000
01	00	0000
01	01	0001
01	10	0010
01	11	0011
10	00	0000
10	01	0010
10	10	0100
10	11	0110
11	00	0000
11	01	0011
11	10	0110
11	11	1001



Note that a total of 14 NMOS and 4 PMOS are used.

39 | 11

a) For the PMOS, with $V_B = 2.5V$
$i_D = (90/3)10^{-6}(12/1.2)[(5-1)2.5 - 2.5^2/2]$
$= 30\times10^{-6}(10)[4(2.5) - 2.5^2/2] = 2.06$

Thus the average charging current is $\underline{2.06mA}$

Time for precharge, $t = CV/I$
whence $t = 1\times10^{-12}(5-0)/(2.06\times10^{-3}) = \underline{2.42}$

b) For the word-line rise, $\tau = RC = 5\times10^{3}\times2\times10^{-12} = 1$

Here, $v_W = 5(1-e^{-t/10})$
Thus the rise time (10% to 90%) is essentially the time t to 90%, where
$0.9(5) = 5(1-e^{-t/10})$
$e^{-t/10} = 0.1$
and $t = -10\ln(0.1) = \underline{23ns}$

At the end of one time constant, $t = \tau = 10ns$,
and $v_W = 5(1-e^{-10/10}) = \underline{3.16V}$

For discharge, $i_{D_{av}} = \frac{1}{2}k_n'(W/L)_n(v_{GS}-V_t)^2$
$= \frac{1}{2}(90)(3/1.2)(3.16-1)^2 = \underline{525}$
Thus, the bit-line voltage will lower by 1V in about $\Delta t = C\Delta V/i_{D_{av}} = 1\times10^{-12}\times1/(525\times10^{-6})$
$= \underline{1.90ns}$

40 (a) $V_{OH} = 0 - 0.75 = -0.75V$

$V_{OL} = 0 - 0.75 - IR = -(0.75 + IR)$ **11**

(b) $V_{th} = -(IR/2 + 0.75) = -(0.75 + IR/2)$

(c) For $i = 0.99I$, $V_{BE} = 750 + 25\ln 0.99 = 750$ mV

$i = 0.01I$, $V_{BE} = 750 + 25\ln 0.01 = 635$ mV

For $0.99I$ in Q_R, $v_I = -(0.75 + \frac{IR}{2}) - (0.750 - 0.635)$

$= -(0.875 + IR/2)$

(d) For $0.01I$ in Q_R, $v_I = -(0.75 + IR/2) + 0.115$

$= -(0.635 + IR/2)$

(e) $V_{IH} = -(0.635 + IR/2)$

$V_{IL} = -(0.875 + IR/2)$

(f) $NM_H = -0.75 - -(0.635 + IR/2)$

$= IR/2 - 0.115$

$NM_L = -(0.875 + IR/2) - -(0.75 + IR)$

$= IR/2 - 0.115$

(g) $V_{IH} - V_{IL} = -(0.635 + IR/2) - -(0.875 + IR/2)$

That is; $IR/2 - 0.115 = 0.230$

And $IR = 2(0.345) = 0.690V$

(h) $V_{OH} = \underline{-0.75V}$; $V_{OL} = -0.75 - 0.69 = \underline{-1.44V}$;

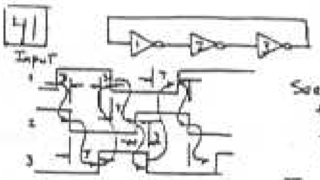
$V_{IL} = -(0.875 + 0.345) = \underline{-1.22V}$;

$V_{IH} = -(0.635 + 0.345) = \underline{-0.98V}$;

$V_R = -(0.750 + 0.345) = \underline{-1.095V}$.

---

41 **11**



See that once started, the process continues; that is we have an oscillation.

In each cycle, each gate output rises and falls.

Thus the period is $3(3+7) = 30$ns

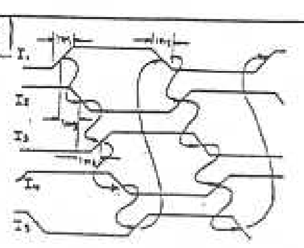
Frequency is $1/30 = \underline{33.3 \text{ MHz}}$.

Any output is high for $3+7+3 = \underline{13\text{ns}}$

and low for $7+3+7 = \underline{17\text{ns}}$

Check: $\underline{30\text{ ns}}$ ✓

42 **11**

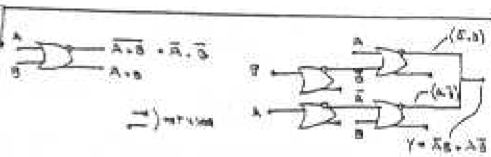


10 transitions per cycle, each of 1ns duration:

Period = $\underline{10\text{ns}}$

Frequency = $\underline{100 \text{ MHz}}$

43 **11**



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44 For $v_I = V_{IL} = -1.435V$, $I_E = 3.97$, $v_O = -1.77V$ **11**

For $v_I = V_R = -1.32V$, $I_E = 4.00$mA, $v_O = -1.31V$

For $v_I = V_{IH} = -1.205V$, $I_E = 4.12$mA, $v_O = -0.88V$

At V_{IL}: $I_A = 0.01(3.97) = 39.7\mu A \rightarrow r_e = \frac{25\times10^{-3}}{39.7\times10^{-6}} = 630\Omega$

$I_B = 0.99(3.97) = 3.93$mA $\rightarrow r_e = \frac{25}{3.93} = 6.4\Omega$

$I_{E2} = \frac{2-1.77}{50} = 4.6$mA $\rightarrow r_e = \frac{25}{4.6} = 5.4\Omega$

and $R_{in2} = 101(50+5.4) = 5.6k\Omega$

$\therefore$ gain $= \frac{50}{50+5.4} \times (5.6k\Omega \| .245) \times \frac{100}{101} \frac{1}{630+6.4} = \underline{0.339 \text{ V/V}}$

At V_R: $I_A = I_B = 2$mA $\rightarrow r_e = 25/2 = 12.5\Omega$

$I_{E2} = \frac{2-1.31}{50} = 13.8$mA $\rightarrow r_e = 1.81\Omega$

and $R_{in2} = 101(50+1.81) = 5.23k\Omega$

$\therefore$ gain $= \frac{50}{50+1.81} \times (5.23k\Omega \| .245) \times \frac{100}{101} \times \frac{1}{12.5+12.5} = \underline{8.94 \text{ V/V}}$

At V_{IH}: $I_A = 0.99(4.12) = 4.08$mA $\rightarrow r_e = \frac{25}{4.08} = 6.13\Omega$

$I_B = 0.01(4.12) = .041$mA $\rightarrow r_e = \frac{25}{.041} = 610\Omega$

$I_{E2} = \frac{2-0.88}{50} = 22.4$mA $\rightarrow r_e = 1.1\Omega$

and $R_{in2} = 101(50+1.1) = 5.1k\Omega$

$\therefore$ gain $= \frac{50}{50+1.1} \times (5.1 \| 0.245) \times \frac{100}{101} \times \frac{1}{610+6} = \underline{0.368 \text{ V/V}}$

---

45 Assume I_E is constant at 4mA. **11**

(a) Currents are: 3.6mA and 0.4mA

$\therefore$ Emitter-Base voltage difference $= V_T \ln \frac{3.6}{0.4}$

or $= 25 \ln 9 = 54.9$ mV.

Thus $V_{IL} = -1.32 - .055 = \underline{-1.375V}$

$V_{IH} = -1.32 + .055 = \underline{-1.265V}$

(b) Currents are: $4(0.999) = 3.996$mA and $.001 \times 4 = 4\mu A$

$\therefore$ Emitter-Base voltage difference $= V_T \ln \frac{3.996}{.004}$

or $= 25 \ln 999 = 173$mV.

Thus $V_{IL} = -1.32 - 0.173 = \underline{-1.493V}$

$V_{IH} = -1.32 + 0.173 = \underline{-1.147V}$

46 11

$I_{OL} = \frac{2-1.77}{50} = 4.6\,mA$

$P_D = 2(4.6) = 9.2\,mW$

$I_{OH} = \frac{2-0.88}{50} = 22.4\,mA$

$P_D = 2(22.4) = 44.8\,mW$

For the logic-gate core $P_D \simeq 4mA \times 5.2V = 20.8\,mW$.

Thus total $P_D = 9.2 + 44.8 + 20.8 = \underline{\underline{74.8\,mW}}$
(ignoring R_3)

47 11

$NM_H = 0.325V$, of which 50% is 162mV, for $\beta = 100$ and $V_{BE2} = 0.83V$, $I_{E2} = 22.4mA$.

Approximately:

$$-2 + \frac{50}{50 + \frac{245}{\beta+1}} \times (2 - 0.83) = -0.88 - 0.162$$

or

$$\frac{50(1.17)}{50 + \frac{245}{\beta+1}} = 0.958$$

$$50 + \frac{245}{\beta+1} = 61.06$$

whence $\beta = \frac{245}{11.06} - 1 = \underline{\underline{21.2}}$

Check: For $v_O = -.88 - .162 = -1.042V$

$I_{E2} = \frac{2-1.042}{50} = 19.2\,mA$

and $V_T \ln \frac{22.4}{19.2} = 3.85\,mV$ — OK, since small. Can ignore.

48 11

$v = -0.88 + (-.88 - 2)(1 - e^{-t/RC})$

or $v = -2 + 1.12\, e^{-t/50C}$

After 1ns, $v = -1.77V$

ie $-1.77 = -2 + 1.12\, e^{-1/50C}$

or $e^{-1/50C} = \frac{2 - 1.77}{1.12}$ and $-1/50C = -1.583$

Thus $C = \frac{10^{-9}}{50(1.583)} = 12.6 \times 10^{-12}F = \underline{\underline{12.6\,pF}}$

49 11

$v = 2/3 \times 30\,cm/ns = 20\,cm/ns$

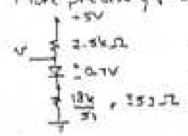
Ratio, $\frac{\text{Rise Time}}{\text{Return Time}} = 5/1 = \frac{3.5}{2L/20}$

$L = \frac{3.5\,ns \times 20\,cm/ns}{5 \times 2} = \underline{\underline{7\,cm}}$

50

$V_{OL} \simeq 0.7V$; $V_{OH} = +5.0V$

More precisely, for $V_{OL} = v$.



$v = \frac{0.353}{2.5 + .353} \times (5 - 0.7) + 0.7 = 1.$

ie $V_{OL} = \underline{\underline{1.23V}}$

Logically: A is high if one of A or B & one of C or D are high.

That is $\underline{A = (A+B) \cdot (C+D)}$

51 1

For $v_I = v_O = V_{DD}/2 = 5/2 = 2.5V$,

$i_{DN} = \frac{1}{2} k_n' (W/L)_n (v_{GS} - V_t)^2$

$= \frac{1}{2}(100)10^{-6}(2/1)(2.5 - 0.7 - 1)^2 = \underline{64\,\mu A}$

Now, the collector current of $Q_2 = \beta i_B = \beta i_{DN}$

$= 100(64 \times 10^{-6}) = \underline{6.4\,mA}$

Correspondingly, the totem-pole current is

$i_{EQ2} = (6400 + 64)10^{-6} = \underline{6.46\,mA}$

Now, for $i_{EQ1} = i_{EQ2}$, $i_{DP} = i_{DN} = 64\,\mu A$

Thus $64 = \frac{1}{2}(100/2.5)(W/L)_p (5 - 2.5 - 0.7 - 1)^2$

whence $(W/L)_p = 2.5(2/1) = (\underline{\underline{5\mu m/1\mu m}})$

Note that the latter could be seen directly!

52

At the threshold V_{th}, $v_O = v_I = V_{th} = v$, and the two MOS operate in saturation with equ currents. Thus $\frac{1}{2}(100/2.5)(2/1)(5 - v - 0.7 - 1)^2 = \frac{1}{2}(100)(2/1)(v - 0.7 - 1)^2$.

Thus, $(3.3 - v)^2 = 2.5(v - 1.7)^2$

and $(3.3 - v) = \pm\sqrt{2.5}(v - 1.7)$.

Usefully, $(3.3 - v) = (1.58v - 2.69)$,

whence $2.58v = 5.99$, and $v = V_{th} = \underline{\underline{2.32}}$

For this value, $i_{DN} = \frac{1}{2}(100)(2/1)(2.32 - 0.7 - 1)^2 = 38.4$

and the totem-pole current is $(\beta+1) i_{DN}$

or $101(38.4)10^{-6} = \underline{\underline{3.88\,mA}}$

53 (11)

The problem as stated is very general, and, correspondingly, its solution can be long and complex. Assume for simplicity that the specifications of Problem 11.51 apply, with matched MOS having $(W/L)_p = 2.5(W/L)_n$.

For R_2: With $v_{O1} = V_t/3 = 1/3 = 0.333V$

$i_{DN} = 100(10^{-6})(3/1)[(5-0.7-1)0.33 - 0.33^2/2] = 209\mu A$

Now, if 50% of this is lost in R_2,

$R_2 = 0.7/(0.50 \times 209) = \underline{6.70\,k\Omega}$

Now if 20% is lost in R_2,

$R_2 = 0.7/(0.20 \times 209) = \underline{16.7\,k\Omega}$

For R_1:

$i_{Dp} = (100/2.5)10^{-6}2.5(3/1)[(5-0-1)0.33 - 0.33^2/2] = \underline{256\mu A}$

Now, if 50% of this is lost in R_1,

$R_1 = (5-0.333)/(0.5 \times 256) = \underline{36.5\,k\Omega}$

Now, if 20% is lost in R_1

$R_1 = 2.5(36.5) = \underline{91.1\,k\Omega}$

In comparison:

For the 50% case, $R_1/R_2 = 36.5/6.70 = \underline{5.45}$

For the 20% case, $R_1/R_2 = 91.1/16.7 = \underline{5.45}$

(Why should their equality be obvious?)

Thus, in general $R_1/R_2 = \underline{5.45}$

---

54 (11)

For t_{PLH}:

At $v_O = 0V$, $i_{Dp} = 1/2(100/2.5)(3/1)(5-0-1)^2 = \underline{640\mu A}$

At $v_O = 2.5V$, $i_{Dp} = (100/2.5)(3/1)[(5-1)2.5 - 2.5^2/2] = \underline{550\mu A}$

Thus $i_{Dpav} = (640 + 550)/2 = \underline{595\mu A}$

and $i_{Oav} = (100+1)595 = \underline{60.1\,mA}$

Thus $t_{PLH} = CV/I = 2\times10^{-12} \times 2.5/(60.1\times10^{-3}) = \underline{83.2ps}$

For t_{PHL}:

At $v_O = 5.0V$, $i_{DN} = 1/2(100)(3/1)(5-0.7-1)^2 = \underline{1.09\,mA}$

At $v_O = 2.5V$, $i_{DN} = 100(3/1)[(5-0.7-1)(2.5-0.7) - (2.5-0.7)^2/2] = \underline{864\mu A}$

Thus $i_{Dnav} = (1089 + 864)/2 = 977\mu A$

and $i_{Oav} = 101(977\times10^{-6}) = \underline{98.6mA}$

Thus $t_{PHL} = CV/I = 2\times10^{-12}(2.5)/(98.6\times10^{-3}) = \underline{50.7ps}$

Thus $t_P = (83.2 + 50.7)/2 = \underline{67.0ps}$

Note that this solution embodies two assumptions:

1) Internal capacitances can be neglected.
2) Transitions are from ideal 0V and 5V output-signal levels.

If outputs of $(5-0.7) = 4.3V$ and $(0+0.7) = 0.7V$ apply, t_P becomes about

$67\times(2.5-0.7)/2.5 = \underline{48ps}$

55

For the circuit in Fig 11.45e, $R_1 = R_2 = 5k\Omega$ robs the base of some of its drive current, namely $0.7/5\times10^3 = \underline{140\mu A}$.

Using results from the solution of P11-52 above:

For t_{PLH}: $i_{Bav} = 595 - 140 = \underline{455\mu A}$

and $i_{Oav} = 101(455\times10^{-6}) = \underline{46.0mA}$

Thus $t_{PLH} = 2\times10^{-12}\times2.5/4.6\times10^{-3} = \underline{108.7ps}$

For t_{PHL}: $i_{Bav} = 977 - 140 = \underline{837\mu A}$

and $i_{Oav} = 101(837\times10^{-6}) = \underline{84.5mA}$

Thus $t_{PHL} = 2\times10^{-12}\times2.5/84.5\times10^{-3} = \underline{59.2ps}$

Thus $t_P = (59.2 + 108.7)/2 = \underline{84ps}$

---

56 (11)

For the BiCMOS NAND of Fig 11.47 to have a dynamic response somewhat like that of the inverter of Fig 11.45e):

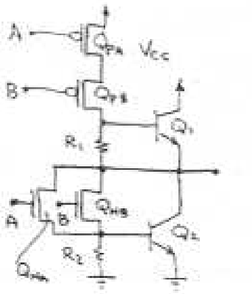
$$(W/L)_{PA} = (W/L)_{PB} = (W/L)_P$$

and $(W/L)_{NA} = (W/L)_{NB} = 2(W/L)_N$

---

57 (11)

A BiCMOS 2-input NOR is as shown:



In terms of the basic matched inverter:

$(W/L)_{PA} = (W/L)_{PB} = 2(W/L)_P$

$(W/L)_{NA} = (W/L)_{NB} = (W/L)_N$

where $(W/L)_P$ and $(W/L)_N$ characterize the inverter.

CHAPTER 12 - PROBLEMS

12.1

$$T(s) = \frac{\omega_0}{s+\omega_0} \qquad T(j\omega) = \frac{\omega_0}{j\omega+\omega_0}$$

$$|T(j\omega)| = \frac{\omega_0}{\sqrt{\omega_0^2+\omega^2}}$$

$$\phi(\omega) \triangleq \mathrm{Tan}^{-1}\left[\frac{\mathrm{Im}(T(j\omega))}{\mathrm{Re}(T(j\omega))}\right]$$

$$= -\tan^{-1} \omega/\omega_0$$

$$G = 20\log_{10}|T(j\omega)|$$

$$A = -20\log_{10}|T(j\omega)|$$

ω	$\lvert T(j\omega)\rvert$ [V/V]	G [dB]	A [dB]	ϕ °
0	1	0	0	0
$0.5\omega_0$	0.8944	-0.97	0.97	-26.57
ω_0	0.7071	-3.01	3.01	-45.0
$2\omega_0$	0.4472	-6.99	6.99	-63.43
$5\omega_0$	0.1961	-14.1	14.1	-78.69
$10\omega_0$	0.0995	-20.0	20.0	-84.29
$100\omega_0$	0.010	-40.0	40.0	-89.43

12.2

$$T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{1}{s^3+2s^2+2s+1}$$

$$T(j\omega) = \left[j(2\omega-\omega^3)+(1-2\omega^2)\right]^{-1}$$

$$|T(j\omega)| = \left[(2\omega-\omega^3)^2+(1-2\omega^2)^2\right.$$

$$= \left[4\omega^2-4\omega^4+\omega^6+1-4\omega^2+4\omega\right.$$

$$= \left[1+\omega^6\right]^{-1/2}$$

$$= \frac{1}{\sqrt{1+\omega^6}}$$

For Phase Angle:

$$\phi(\omega) = \tan^{-1}\left[\frac{\mathrm{Im}[T(j\omega)]}{\mathrm{Re}[T(j\omega)]}\right]$$

$$= -\tan^{-1}\left[\frac{2\omega-\omega^3}{1-2\omega^2}\right]$$

For $\omega = 0.1$:

$$|T(j\omega)| = (1+0.1^6)^{-1/2} \cong \underline{1}$$

$$\phi(\omega) = -11.5° = \underline{-0.20 \text{ rad}}$$

For $\omega = 1$ rad/s:

$$|T(j\omega)| = (1+1^6)^{-1/2} = 1/\sqrt{2} = \underline{0.7}$$

$$\phi = -\tan^{-1}(1/-1) = -135° = \underline{2.356}$$

Note: $G = -3\text{dB}$

Also: $\tan^{-1}(-1) = -45°$ or $-135°$
$\tan^{-1}(-1/1) = -45°$
$\tan^{-1}(1/-1) = -135°$

For $\omega = 10$ rad/s:

$$|T(j\omega)| = (1+10^6)^{-1/2} = \underline{0.001}$$

$$\phi = -\tan^{-1}\left[\frac{2(10)-10^3}{1-2(10^2)}\right]$$

CONT.

$$= -\tan^{-1}\left[\frac{-980}{-199}\right]$$

$$= -\left[180^\circ + \tan^{-1}\left(\frac{980}{199}\right)\right]$$

$$= -258.5^\circ$$

$$= \underline{\underline{4.512 \text{ rad}}}$$

Now consider an input of $A \sin \omega t$ to $T(s)$. The output is then given by:

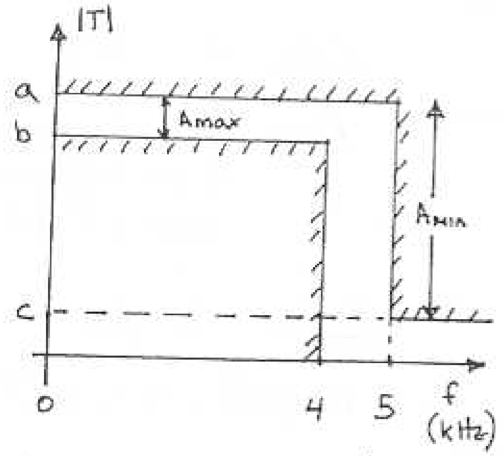
$$A\,|T(j\omega)| \sin(\omega t + \phi(\omega))$$

Using this result, the output to each of the following inputs will be:

INPUT	OUTPUT	
$2\sin(0.1t)$	$2\sin(0.1t - 0.2)$	ie $2\times 1 = 2$
$2\sin(1t)$	$\sqrt{2}\sin(t - 2.356)$	i.e. $2\times 1/\sqrt{2} = \sqrt{2}$
$2\sin(10t)$	$2\times 10^{-3}\sin(10t - 4.512)$	

12.3

12.4



Note |T| is shown in a linear scale but Amax and Amin are in dB

From the problem

$\frac{a}{b} = 1.1$, $c = 0.1\%\, a$ or $\frac{c}{a} = 0.001$

$$A_{max} = 20\log_{10} a - 20\log_{10} b$$

$$= 20\log_{10} a/b$$

$$= 20\log_{10}(1.1)$$

$$= \underline{\underline{0.83\text{dB}}}$$

$$A_{min} = 20\log_{10} a - 20\log_{10} c$$

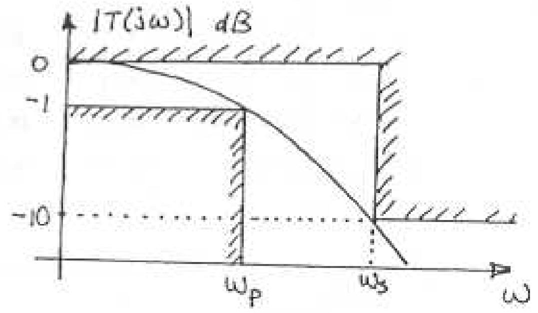
$$= 20\log_{10}(a/c)$$

$$= 20\log_{10}(0.001)$$

$$= \underline{60\text{dB}}$$

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{2\pi 5}{2\pi 4} = \underline{\underline{1.25}}$$

12.5



$$T(s) = \frac{k}{1+s\tau}$$

$$= \frac{1}{1+1s}$$

If $\tau = 1s$ & the DC gain = 1 then $\underline{\underline{k=1}}$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

At the passband edge:

CONT.

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\omega_p^2}} = 10^{-1/20}$$

$\therefore\ \omega_p = \underline{\underline{0.5088\ \text{rad/s}}}$

At the stopband edge:

$$|T(j\omega_s)| = \frac{1}{\sqrt{1+\omega_s^2}} = 10^{-10/20}$$

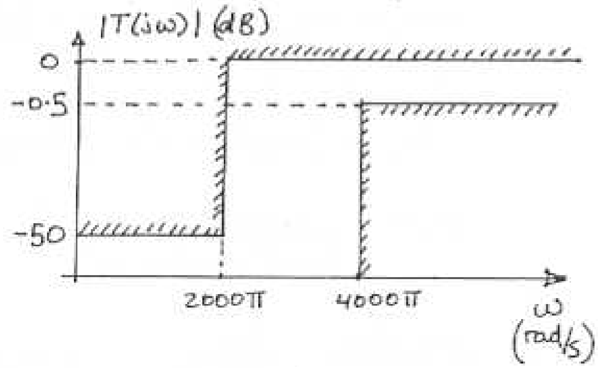
$\therefore\ \omega_s = \underline{\underline{3\ \text{rad/s}}}$

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{3}{0.5088} = \underline{5.9}$$

12.6

Passband is defined by: $f \geq 2\,kHz$
$\Rightarrow \omega_p = 2\pi(2000)$ rad/s

Stopband is defined by: $f \leq 1\,kHz$
$\Rightarrow \omega_s = 2\pi(1000)$ rad/s



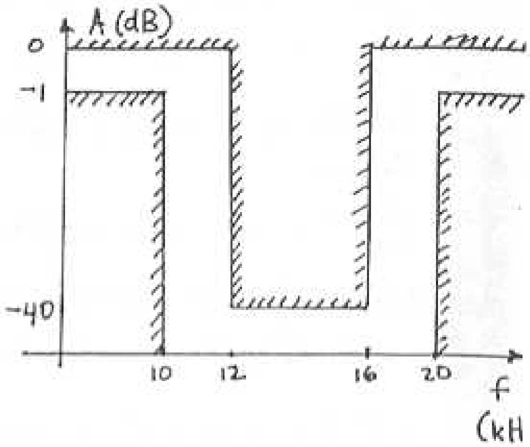
Note we assumed a maximum transmission of 0 dB.

12.7

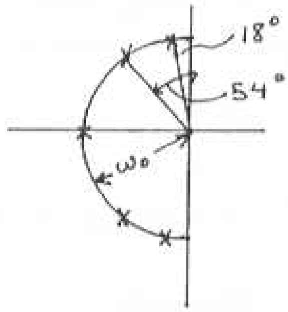
Passband: $f \in \{[0, 10kHz] \cup [20kHz, \infty]\}$

Stopband: $f \in [12kHz, 16kHz]$

$A_{max} = 1\,dB$, $A_{min} = 40\,dB$



12.8



$\omega_0 = 10^3$

Poles at 18°:

$$P_1 = \omega_0\left(-\cos(90^\circ - 18^\circ) \pm j\sin(90^\circ - 18^\circ)\right)$$
$$= \omega_0\left(-\cos 72^\circ \pm j\sin 72^\circ\right)$$
$$= \omega_0(-0.309 \pm j\,0.951)$$

Poles at 54°

$$P_2 = \omega_0\left(-\cos 36^\circ \pm j\sin 36^\circ\right)$$
$$= \omega_0(-0.809 \pm j\,0.588)$$

Poles on Real Axis

$P_3 = -\omega_0$

CONT.

Note: Given a pair of poles

$P_i = \omega_o(-\cos\alpha \pm j\sin\alpha)$,

introduces a second order term as follows:

$(s+\omega_o\cos\alpha - j\omega_o\sin\alpha)(s+\omega_o\cos\alpha + j\omega_o\sin\alpha)$

$= s^2 + s(\omega_o\cos\alpha - j\omega_o\sin\alpha + \omega_o\cos\alpha + j\omega_o\sin\alpha)$
$\quad + \omega_o^2[\cos^2\alpha + j\cos\alpha\sin\alpha - j\cos\alpha\sin\alpha + \sin^2\alpha]$

$= s^2 + s(2\omega_o\cos\alpha) + \omega_o^2$

So for P_1 we get a term:

$s^2 + s(2\omega_o 0.309) + \omega_o^2$
$\quad = s^2 + 0.618\omega_o s + \omega_o^2$

For P_2 we get:

$s^2 + 1.618\omega_o s + \omega_o^2$

For P_3 : $(s+\omega_o)$

∴ The denominator of $T(s)$ is given by

$$D(s) = (s+\omega_o)(s^2+0.618\omega_o s+\omega_o^2)\times(s^2+1.618\omega_o s+\omega_o^2)$$

Case (a) - If all the zeros are @ ∞, the numerator is a constant

$|T(0)| = \frac{k}{\omega_o^5} = 1$ for unity gain at DC

∴ $k = \omega_o^5$

$$T(s) = \frac{k}{D(s)} = \frac{\omega_o^5}{D(s)}$$

where $D(s)$ is given above.

Case (b) - For all zeros at 0, the numerator is given by ks^5

A $s = j\infty$ $|T(s\to j\infty)| = \frac{k}{1} = 1$

$$\therefore T(s) = \frac{s^5}{D(s)}$$

12.9

Poles at -1 and $-0.5 \pm j0.8$ gives a denominator:

$D(s) = (s+1)(s+0.5-j0.8)(s+0.5+j0.8)$

$\quad = (s+1)(s^2+2(0.5)s+0.5^2+0.8^2)$

$\quad = (s+1)(s^2+s+0.89)$

Zeros at ∞ and $\pm j2$ give a numerator:

$N(s) = k(s+j2)(s-j2) = k(s^2+4)$

Note there is one zero at ∞ because Degree$(D(s))$ - Degree$(N(s))$ = 1

$$T(s) = \frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$|T(j0)| = \frac{k(4)}{0.89} = 1$ ∵ DC gain = 1

$\Rightarrow k = 0.2225$

$$\therefore T(s) = \frac{0.2225(s^2+4)}{(s+1)(s^2+s+0.89)}$$

12.10

Numerator is given by

$$a_7 (s-0)(s^2+(10^3)^2)(s^2+(3\times10^3)^2) \times (s^2+(6\times10^3)^2)$$

$$= a_7 s (s^2+10^6)(s^2+9\times10^6)(s^2+36\times10^6)$$

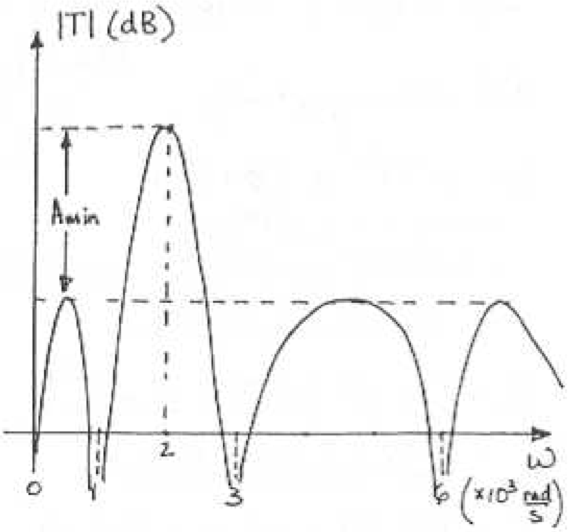
Degree of Numerator $\triangleq M = 7$

Degree of Denominator $\triangleq N$

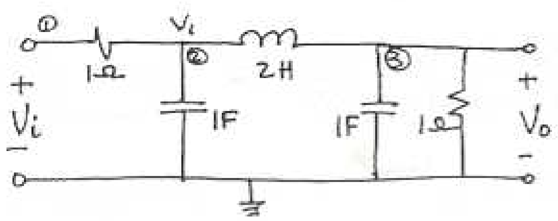
Given that there is one zero at ∞ :

$$N - M = 1 \implies \underline{\underline{N = 8}}$$

$$\therefore T(s) = \frac{a_7 s (s^2+10^6)(s^2+9\times10^6)(s^2+36\times10^6)}{s^8 + b_7 s^7 + b_6 s^6 + \cdots + b_0}$$



12.11



The easiest way to solve the circuit is to use nodal analysis at nodes ①, ②, ③

At node ③ $\Sigma I = 0$

$$\frac{V_o}{1} + \frac{V_o}{1/s} + \frac{V_o - V_1}{2s} = 0$$

$$\therefore V_1 = V_o (2s^2 + 2s + 1) \qquad \text{Eq. (a)}$$

At node ② $\Sigma I = 0$

$$\frac{V_1 - V_i}{1} + \frac{V_1}{1/s} + \frac{V_1 - V_o}{2s} = 0$$

$$\therefore V_1 (2s^2+2s+1) = V_o + 2s V_i \qquad \text{Eq. (b)}$$

(a) → (b)

$$V_o (2s^2+2s+1)^2 = V_o + 2s V_i$$

$$V_o (4s^4 + s^3(4+4) + s^2(2+4+2) + s(2+2)+1) = V_o + 2s V_i$$

$$\frac{V_o(s)}{V_i(s)} \triangleq T(s) = \frac{2s}{4s^4+8s^3+8s^2+4s}$$

$$\underline{\underline{T(s) = \frac{0.5}{s^3+2s^2+2s+1}}}$$

Poles are given by:

$$s^3 + 2s^2 + 2s + 1 = 0$$

$$(s+1)(s^2+s+1) = 0$$

$\therefore$ Poles are $s = \underline{-1}$ and $\underline{\underline{s = -\frac{1}{2} \pm j\sqrt{3}/2}}$

12.12

$A_{max} = 1\text{dB}$, $A_{min} = 20\text{dB}$, $\omega_s/\omega_p = 1.3$

Using:

$$A(\omega_s) = 10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] \quad \text{Eq (12.15)}$$

$$= A_{min}$$

CONT

$\epsilon = \left[10^{1/10} - 1\right]^{1/2} = 0.5088$

$A_{min} = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$

$10^{A_{MIN}/10} - 1 = \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}$

$\log\left(10^{A_{min}/10} - 1\right) = \log\left(\epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right)$

$N = \dfrac{\log\left[\left(10^{A_{MIN}/10} - 1\right)/\epsilon^2\right]}{2 \log(\omega_s/\omega_p)}$

$= 11.3 \Rightarrow$ choose $\underline{\underline{N = 12}}$

The actual value of stopband attenuation can be calculated using the integer value of N:

$A(\omega_s) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$; $N = 12$

$= \underline{\underline{27.35\,dB}}$ actual attenuation

If the stopband specs are to be met exactly we need to find Amax.

Eq. 12.15 can be rearranged to give

$\epsilon^2 = \dfrac{10^{A_{min}/10} - 1}{(\omega_s/\omega_p)^{2N}}$ $A_{min} = 20$, $N = 12$

$= 0.1824$

$\therefore A_{max} = 10 \log(1 + \epsilon^2)$

$= \underline{\underline{0.73\,dB}}$

12.13

$N = 7$, $A_{max} = 3\,dB$

We want attenuation at

$\omega = 1.6\,\omega_p$ or $\frac{\omega}{\omega_p} = 1.6$

$\epsilon = \sqrt{10^{A_{max}/10} - 1} = 0.998$

$A(\omega) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}\right]$

$= 10 \log\left[1 + 0.998^2 (1.6)^{14}\right]$

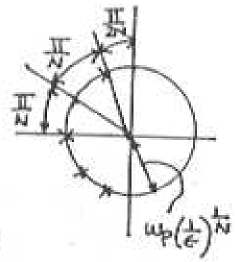
$= \underline{\underline{28.56\,dB}}$

12.14

$\omega_p = 10^3$ rad/s, $N = 5$

$A_{max} = 1dB \Rightarrow \epsilon = 0.5088$

Find solution graphically



$P_1 = \omega_p \left(\frac{1}{\epsilon}\right)^{1/N} \angle \left(\frac{\pi}{2} + \frac{\pi}{2N}\right)$

$= 873.59 \angle (6\pi/10)$

$= 873.59 \left[\cos(6\pi/10) \pm j \sin\left(\frac{6\pi}{10}\right)\right]$

$= \underline{-269.96 \pm j\,830.84}$

$P_2 = 873.59 \angle \left[\frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N}\right]$

$= \underline{-706.75 \pm j\,513.49}$

$P_3 = 873.59 \angle \pi = \underline{\underline{-873.59}}$

12.15

$f_p = 10\,kHz$ $\frac{\omega_s}{\omega_p} = 1.5$ $A_{MIN} = 15\,dB$

$f_s = 15\,kHz$ $A_{max} = 2dB$

$\epsilon^2 = 10^{A_{max}/10} - 1 \Rightarrow \epsilon = 0.76478$

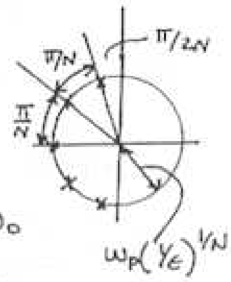
CONT.

Manipulation Eq (12.15) we get:

$$N = \frac{\log\left[\left(10^{A_{min}/10} - 1\right)/\epsilon^2\right]}{2\log(\omega_s/\omega_p)} = 4.88$$

$\therefore$ Use $\underline{\underline{N = 5}}$

Finding naturalmodes graphically :-



radius $= \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} \triangleq \omega_o$

$$\omega_o = 6.629\times10^4$$

$$P_1 = \omega_o \measuredangle (\pi/2 + \pi/2N) = \omega_o \measuredangle (6\pi/10)$$
$$= \omega_o\left(\cos\left(\tfrac{6\pi}{10}\right) \pm j\sin\left(\tfrac{6\pi}{10}\right)\right)$$
$$= \underline{\underline{\omega_o(-0.309 \pm j0.951)}}$$

$$P_2 = \omega_o\left(\cos\tfrac{8\pi}{10} \pm j\sin\tfrac{8\pi}{10}\right)$$
$$= \underline{\underline{\omega_o(-0.809 \pm j0.588)}}$$

$$P_3 = \omega_o(\cos\pi \pm j\sin\pi) = \underline{\underline{-\omega_o}}$$

Given a natural mode $-\alpha \pm j\beta$, the following term results

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$
$$= s^2 + 2\alpha s + \alpha^2 + \beta^2$$
$$= \underline{s^2 + 2\mathrm{Re}[P]s + |P|^2}$$

Also, note that for a Butterworth, all natural modes have a magnitude of ω_o.

P_1 yields: $s^2 + 0.618\omega_o s + \omega_o^2$
P_2 yields: $s^2 + 1.618\omega_o s + \omega_o^2$
P_3 yields: $s + \omega_o$

$$\therefore T(s) = \frac{k}{(s+\omega_o)(s^2 + .618\omega_o s + \omega_o^2)} \times \frac{1}{s^2 + 1.618\omega_o s + \omega_o^2}$$

For unity dc gain

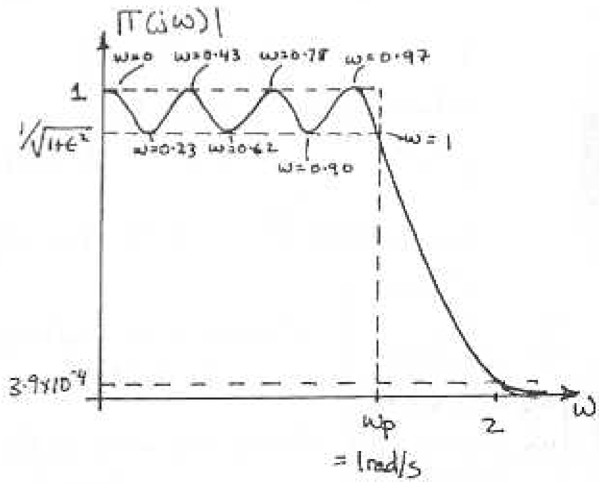
$$|T(j0)| = \frac{k}{\omega_o^5} = 1 \Rightarrow k = \omega_o^5$$

$$\therefore T(s) = \underline{\underline{\frac{\omega_o^5}{(s+\omega_o)(s^2 + .618\omega_o s + \omega_o^2)}}} \times \underline{\underline{\frac{1}{(s^2 + 1.618\omega_o s + \omega_o^2)}}}$$

For attenuation at 20kHz use Eq (12.15) with $\frac{\omega_s}{\omega_p} = \frac{20}{10} = 2$

$$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$
$$= \underline{27.8\,dB}$$

12.16



Given $A_{max} = 1dB \Rightarrow \epsilon = 0.5088$

CONT.

Using Eq 12.18

$$|T(j\omega)| = \left[1+\epsilon^2 \cos^2\left(N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-1/2}$$

for $\omega \le \omega_p$

If $|T(j\omega)| = 1$

$$1 = 1+\epsilon^2 \cos^2\left(N\cos^{-1}(\omega/\omega_p)\right) \qquad \omega_p = 1$$

$$N\cos^{-1}(\omega/1) = \cos^{-1}(0)$$

$$\cos^{-1}(\omega) = \frac{2i+1}{2N}\pi$$

ω's repeat after this value

$$\therefore \omega_i = \cos\left[\frac{2i+1}{2N}\pi\right] \qquad i = 0, 1, \ldots, \frac{N-1}{2}$$

$$\boxed{\begin{aligned} \omega_0 &= 0.9749 \\ \omega_1 &= 0.7818 \\ \omega_2 &= 0.4339 \\ \omega_3 &= 0 \end{aligned}}$$

ω values at which $|T| = 1$

note $\omega_4 = -0.4339 = -\omega_2$!

If $|T| = 1/\sqrt{1+\epsilon^2}$, then

$$1/\sqrt{1+\epsilon^2} = \left[1+\epsilon^2\cos^2\left(N\cos^{-1}(\omega/\omega_p)\right)\right]^{-\frac{1}{2}}$$

$$1 = \cos\left(N\cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right)$$

$$N\cos^{-1}(\omega) = \cos^{-1}(0) = i\pi \qquad i = 0, 1, 2 \ldots$$

$$\omega_i = \cos\left[\frac{i\pi}{N}\right] \qquad i = 0, 1, 2, \ldots \frac{N}{2}$$

$$\boxed{\begin{aligned} \omega_0 &= 1.0 \\ \omega_1 &= 0.9010 \\ \omega_2 &= 0.6235 \\ \omega_3 &= 0.2252 \end{aligned}}$$

ω values at which $|T| = (1+\epsilon^2)^{-1/2}$

Note $\omega_4 = -0.2252 = -\omega_3$!

To find $|T(j2)|$ use Eq(12.19) since $\omega > \omega_p$

$$|T(j\omega)| = \left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-\frac{1}{2}}$$

$$= \left[1+0.5088^2\cosh^2\left(7\cosh^{-1} 2\right)\right]^{-\frac{1}{2}}$$

$$= \underline{\underline{3.898 \times 10^{-4}\ \text{V/V}}}$$

$$|T|_{dB} = \underline{\underline{-68.2\ \text{dB}}}$$

For roll-off consider

$$T(s) = \frac{k}{s^7 + b_6 s^6 + \ldots + b_0}$$

for $\omega \gg \omega_p$ $\quad T(j\omega) \simeq \dfrac{k}{\omega^7}$

$\therefore$ Rolloff is $\frac{1}{2^7}$ or $20\log(1/2^7)$ per octave $= \underline{\underline{-42\ \text{dB/octave}}}$.

12.17

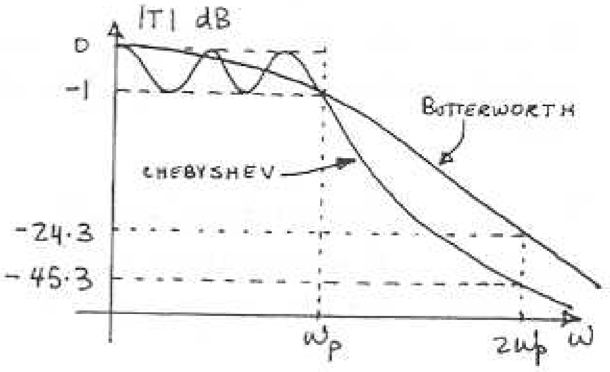
$\omega_s/\omega_p = 2 \qquad A_{max} = 1\text{dB} \Rightarrow \epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1} = 0.5088$

$$|T_B| = \left[1+\epsilon^2(\omega_s/\omega_p)^{2N}\right]^{-1/2}$$

$$|T_C| = \left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\left[\frac{\omega_s}{\omega_p}\right]\right)\right]^{-1/2}$$

$|T_B| = 6.13 \times 10^{-2} \Rightarrow \underline{-24.3\text{dB}}$

$|T_C| = 5.43 \times 10^{-3} \Rightarrow \underline{\underline{-45.3\text{dB}}}$



12.18

$f_p = 3.4\text{kHz}$ $\quad A_{max} = 1\text{dB} \Rightarrow \epsilon = 0.5088$

$f_s = 4\text{Khz}$ $\quad A_{min} = 35\text{dB}$

$\omega_s/\omega_p = 1.176$

Using Eq (12.22):

$$A(\omega_s) = 10\log\left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right)\right]$$

& trying different values for N

N	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	38.98 dB

$\therefore$ Use $\underline{\underline{N=10}}$

Excess attenuation $= 39 - 35 = \underline{\underline{4\text{dB}}}$

Poles are given by:

$$P_k = -\omega_p \sin\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right) + j\,\omega_p\cos\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

for $k = 1, 2, \ldots, N$.

Since $\epsilon = 0.5088$ and $N = 10$

$\sinh\left(1/N \sinh^{-1}(1/\epsilon)\right) = 0.1433$

$\cosh\left(1/N \sinh^{-1}(1/\epsilon)\right) = 1.010$

$$\therefore P_1 = \omega_p\left[-0.1433\sin\left(\tfrac{\pi}{20}\right) + j1.010\cos\left(\tfrac{\pi}{20}\right)\right] = \omega_p(-0.0224 + j0.9978)$$

$P_2 = \omega_p(-0.0650 + j0.900)$

$P_3 = \omega_p(-0.1013 + j0.7143)$

$P_4 = \omega_p(-0.1277 + j0.4586)$

$P_5 = \omega_p(-0.1415 + j0.1580)$

Now it should be realized that the remaining poles are complex conjugates of the above.

Pole-pair P_1 & P_1^* give a factor:

$$s^2 + 2(0.0224)\omega_p s + \omega_p^2(0.0224^2 + 0.9978^2) = \underline{s^2 + 0.0448\omega_p s + 1.023\omega_p^2}$$

i.e. this factor is from $(s-P_1)(s-P_1^*)$

P_2 yields: $\underline{s^2 + 0.130\omega_p s + 0.902\omega_p^2}$

P_3 yields: $\underline{s^2 + 0.203\omega_p s + 0.721\omega_p^2}$

P_4 yields: $\underline{s^2 + 0.265\omega_p s + 0.476\omega_p^2}$

P_5 yields: $\underline{s^2 + 0.283\omega_p s + 0.212\omega_p^2}$

Now T(s) is given by

$$T(s) = \frac{k\,\omega_p^{10}}{\epsilon\, 2^9 (s-P_1)(s-P_1^*)\cdots(s-P_5)(s-P_5^*)}$$

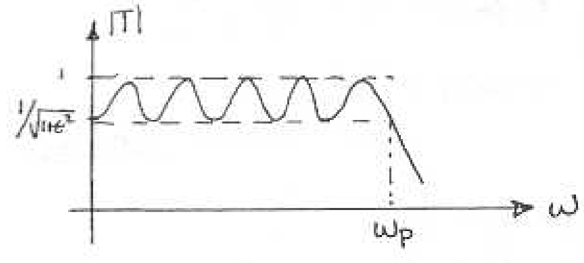
where the second order terms of the denominator are given above.

k is the dc gain

$\therefore$ we want the dc gain to be

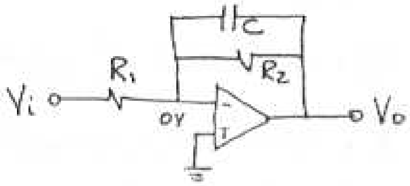
$k = \frac{1}{1+\epsilon^2} = \underline{\underline{0.8913}}$

$\omega_p = \underline{\underline{2\pi \times 3400}}$



12.19

$f_0 = 10kHz$ DC gain $= 10$ $R_{in} = 10k\Omega$



$R_{in} = R_1 = \underline{\underline{10k\Omega}}$

DC gain $= -R_2/R_1 = -10$

$R_2 = 10R_1 = \underline{\underline{100k\Omega}}$

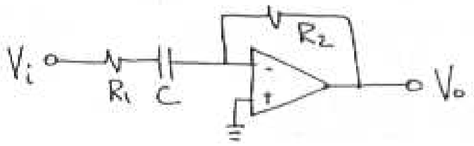
$R_2 C = 1/\omega_0$

$C = \dfrac{1}{\omega_0 R_2} = \dfrac{1}{2\pi\ 10^4 \times 100\times10^3}$

$= \underline{\underline{0.159nF}}$

12.20

$f_0 = 100kHz$ $R_i(\infty) = 100k\Omega$ $|T(\infty)| = 1$



$R_i(\infty) = R_1 = \underline{\underline{100k\Omega}}$

$|T(\infty)| = R_2/R_1 = 1$

$R_2 = R_1 = \underline{\underline{100k\Omega}}$

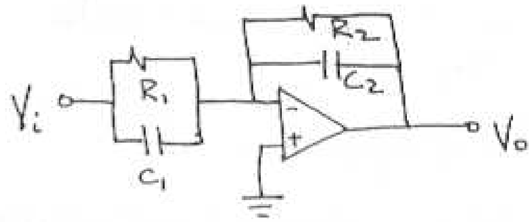
$CR_1 = 1/\omega_0$

$C = \dfrac{1}{\omega_0 R_1} = \dfrac{1}{2\pi\ 100\times10^3 \times 100\times10^3}$

$= \underline{\underline{15.9nF}}$

12.21

Use general first-order circuit:



- Zero at 1kHz ; Pole at 100kHz
- $|T(0)| = 1$; $R_i(0) = 1k\Omega$

Thus: $R_i(DC) = R_1 = \underline{\underline{1k\Omega}}$

$T(DC) = -R_2/R_1 = -1$

$R_2 = R_1 = \underline{\underline{1k\Omega}}$

For a pole at 100kHz

$C_2 R_2 = \dfrac{1}{\omega_0} \Rightarrow C_2 = \dfrac{1}{2\pi f_0 R_2}$

$= \underline{\underline{1.59nF}}$

For the circuit $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$

Thus the zero at $-a_0/a_1 = -2\pi\ 10^3$

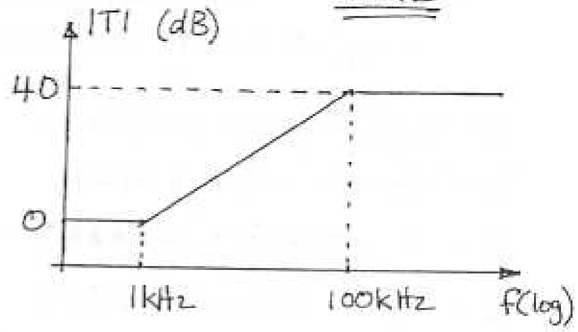
$C_1 R_1 = a_1/a_0$

$C_1 = \dfrac{1}{2\pi\ 10^3 R_1}$

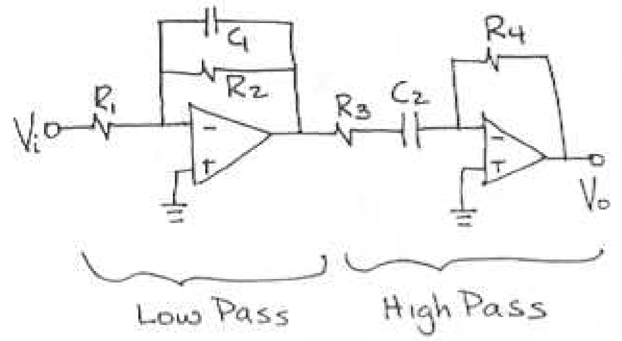
$= \underline{\underline{159nF}}$

High freq gain $= \dfrac{-C_1}{C_2} = \underline{\underline{-100}}$

$= \underline{\underline{40dB}}$



12.22



gain $= 10^{12/20} = 3.98 \approx \underline{4}$

want $R_i = R_1$ large

$\therefore R_1 = \underline{\underline{100k\Omega}}$

Total gain $= A_{LP} A_{HP} = 4$

$A_{LP} = -R_2/R_1 \implies R_2 = -A_{LP} R_1$ and $R_2 \leq 100k\Omega$

$\therefore$ make $A_{LP} = -1 \quad A_{HP} = -4$

$R_2 = \underline{\underline{100k\Omega}}$

$$R_2 C_1 = \frac{1}{\omega_{0,LP}}$$

$$C_1 = \frac{1}{2\pi f_{o,LP} R_2} = \frac{1}{2\pi(10\times10^3)\,100\times10^3}$$

$$= \underline{\underline{0.159nF}}$$

$$\left.\begin{array}{l} A_{HP} = -R_4/R_3 = -4 \\ R_4 = 4R_3 \end{array}\right\} \text{make } R_4 = \underline{\underline{100k\Omega}},\ R_3 = \underline{\underline{25k\Omega}}$$

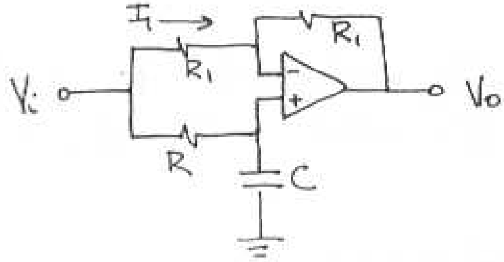
Now $R_3 C_2 = 1/\omega_{o,HP}$

$$C_2 = \frac{1}{2\pi f_{o,HP} R_3}$$

$$= \frac{1}{2\pi(100\times10^3)\,25\times10^3}$$

$$= \underline{\underline{63.7nF}}$$

12.23



At +ve terminal

$$V_+ = \frac{1/sC}{1/sC + R_1} V_i$$

$$= \frac{1}{1+s\tau} V_i \qquad \underline{\underline{\tau = RC}}$$

$V_- = V_+$ due to virtual short between terminals.

$$\therefore I_1 = \left(V_i - \frac{1}{1+s\tau}V_i\right)\frac{1}{R_1}$$

$$V_o = V_- - I_1 R_1$$

$$= \frac{V_i}{1+s\tau} - \left(V_i - \frac{V_i}{1+s\tau}\right)\frac{R_1}{R_1}$$

$$\frac{V_o}{V_i} = \frac{1-(1+s\tau)+1}{1+s\tau} = \frac{1-s\tau}{1+s\tau}$$

$$= \frac{\omega_o - s}{\omega_o + s} \qquad \omega_o = \frac{1}{\tau}$$

$$= \underline{\underline{-\frac{s-\omega_o}{s+\omega_o} = T(s)}}$$

$$T(j\omega) = -\frac{j\omega - \omega_o}{j\omega + \omega_o}$$

$$\phi(\omega) = 180° + \text{Tan}^{-1}\left(\frac{\omega}{-\omega_o}\right) - \text{Tan}^{-1}\left(\frac{\omega}{\omega_o}\right)$$

$$= 360° - 2\tan^{-1}\left(\frac{\omega}{\omega_o}\right) \qquad \because \tan^{-1}\left(\frac{\omega}{-\omega_o}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_o}\right)$$

$$= -2\tan^{-1}(\omega/\omega_o)$$

Now this equation can be rearranged:

$$\frac{\omega}{\omega_o} = \tan(-\phi/2) \Leftarrow \omega_o = \frac{1}{\tau} = \frac{1}{RC}$$

$$RC\omega = \tan(-\phi/2)$$

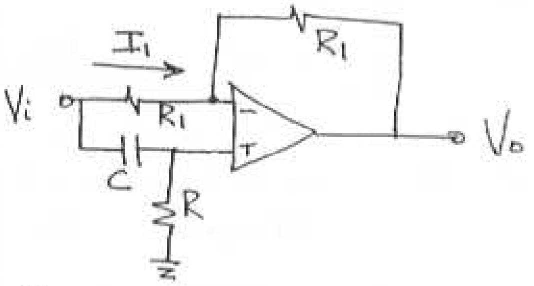
CONT.

$$\therefore R = \frac{\tan(-\phi/2)}{C\omega} = 10^4 \tan(-\phi/2)$$

$\phi = -30°, -60°, -90°, -120°, -150°$

$R = 2.68k\Omega, 5.77k\Omega, 10k\Omega, 17.32k\Omega, 37.32k\Omega$

12.24



$$V_+ = \frac{R}{R + 1/sC} V_i = \frac{s}{s+\omega_0} V_i$$

where $\omega_0 = \frac{1}{RC}$

$$I_1 = \frac{V_i - (s/s+\omega_0)V_i}{R_1}$$

$$V_o = \frac{s}{s+\omega_0} V_i - I_1 R_1$$

$$= \frac{s}{s+\omega_0} V_i - V_i\left(1 - \frac{s}{s+\omega_0}\right)$$

$$\frac{V_o}{V_i} = \frac{2s - s - \omega_0}{s+\omega_0} = \frac{s-\omega_0}{s+\omega_0}$$

Now:

NOTE $\tan^{-1}\frac{\omega}{-\omega_0} = 180° - \tan^{-1}\frac{\omega}{\omega_0}$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 180 - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Clearly $\phi(0) = 180°$ & $\phi(\omega \to \infty) = 0°$

12.25

Low Pass $\omega_0 = 10^3$ rad/s

$Q = 1$

DC gain = 1

$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(0) = a_0/\omega_0^2 = 1$$

$$a_0 = \omega_0^2 = 10^6$$

$$\therefore T(s) = \frac{10^6}{s^2 + 10^3 s + 10^6}$$

$$\omega_{max} = \omega_0\sqrt{1 - 1/2Q^2}$$

$$= \frac{\omega_0}{\sqrt{2}}$$

$$= 0.707 \text{ rad/s}$$

$$|T_{max}| = \frac{|a_0| Q}{\omega_0^2\sqrt{1 - \frac{1}{4Q^2}}} \Leftarrow a_0 = \omega_0^2$$

$$= \frac{|\omega_0^2|}{\omega_0^2\sqrt{3/4}}$$

$$= 2/\sqrt{3}$$

$$= 1.15 \text{ V/V}$$

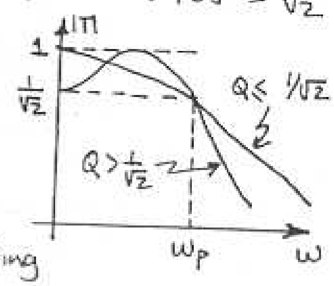
$$= 1.21 \text{ dB}$$

12.26

$\omega_p = 1$ rad/s

$A_{max} = 3$dB

$10^{-3/20} = 0.708 \cong \frac{1}{\sqrt{2}}$



There are many Q-values which may be used

$Q \leq 1/\sqrt{2}$ - no peaking

$Q > 1/\sqrt{2}$ - peaking

CONT.

Solution 1 $\quad Q \leq 1/\sqrt{2}$

For $Q = 1/\sqrt{2}$ the response is maximally flat. Because this is desirable, use: $\underline{\underline{Q = \frac{1}{\sqrt{2}}}}$

$$T(s) = \frac{a_0}{s^2 + s\omega_0\sqrt{2} + \omega_0^2}$$

$$|T(0)| = \frac{a_0}{\omega_0^2} = 1$$

$$\underline{\underline{a_0 = \omega_0^2}}$$

$$|T(j1)|^2 = \frac{\omega_0^2}{(\omega_0^2-1) + 2\omega_0^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\omega_0 = 1 \text{ rad/s}$$

$$\therefore \underline{\underline{T_1(s) = \frac{1}{s^2 + \sqrt{2}s + 1}}}$$

Solution 2 $\quad Q > 1/\sqrt{2}$

From the figure: $|T(0)| = 1/\sqrt{2} = \frac{a_0}{\omega_0^2}$

$$\therefore \underline{\underline{a_0 = \omega_0^2/\sqrt{2}}}$$

$$\text{Now } |T|_{max} = \frac{|a_0|Q}{\omega_0^2\sqrt{1 - 1/4Q^2}} = 1$$

$$\frac{Q}{\sqrt{2}\sqrt{1 - 1/4Q^2}} = 1$$

$$Q = \sqrt{2}\sqrt{1 - 1/4Q^2}$$

$$\therefore Q^2 = 2\left(1 - \frac{1}{4Q^2}\right)$$

$$= 2 - \frac{1}{2Q^2}$$

$$Q^4 - 2Q^2 + \tfrac{1}{2} = 0$$

Solving for Q^2 gives:-

$$Q^2 = 1 \pm \sqrt{2}$$

ASIDE:

$\because Q > 1/\sqrt{2}$

$Q^2 > 1/2$

$4Q^2 > 2$

$\frac{1}{4Q^2} < \frac{1}{2}$

$\therefore 1 - \frac{1}{4Q^2} > 1/2$

$\therefore \left|1 - \frac{1}{4Q^2}\right| = 1 - \frac{1}{4Q^2}$

$\Rightarrow Q = 0.5412$ or 1.3066

$\therefore Q > \frac{1}{\sqrt{2}}$ $\quad$ use $\underline{\underline{Q = 1.3066}}$

Now at the passband edge

$$|T(j1)| = 1/\sqrt{2}$$

$$|T(j1)|^2 = \frac{(\omega_0^2/\sqrt{2})^2}{(\omega_0^2-1)^2 + \frac{\omega_0^2}{Q^2}} = \frac{1}{2}$$

$$\frac{\cancel{\omega_0^4}}{\cancel{2}} = \frac{1}{\cancel{2}}\left[\cancel{\omega_0^4} - 2\omega_0^2 + 1 + \frac{\omega_0^2}{Q^2}\right]$$

$$\omega_0^2(2 - 1/Q^2) = 1$$

$$\underline{\omega_0 = 0.841}$$

$$\therefore T_2(s) = \frac{\omega_0^2/\sqrt{2}}{s^2 + \omega_0/Q\, s + \omega_0^2}$$

$$= \underline{\underline{\frac{0.5}{s^2 + 0.644s + 0.707}}}$$

If $\omega_s = 2$

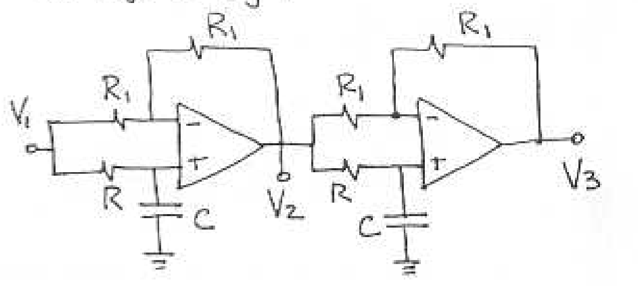
$|T_1(j2)| = 0.242 \quad |T_2(j2)| = 0.1414$

$\therefore \underline{\underline{A_{min,1} = -12.3\text{dB}}} \quad \underline{\underline{A_{MIN,2} = -17\text{dB}}}$

12.27

V_2 lags V_1 by $120°$

V_3 lags V_2 by $120°$



$\omega = 2\pi 60$ & $C = 1\,\mu F$

$$T(s) = \frac{s - \omega_0}{s + \omega_0} \qquad \omega_0 = \frac{1}{RC}$$

CONT.

$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Sub: $\tan\left(\frac{\omega}{-\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

$$\Rightarrow \phi(\omega) = -2\tan(\omega/\omega_0)$$

Now $\phi = -120^\circ$ at $\omega = 2\pi 60$

$$-120 = -2\tan(\omega RC)$$

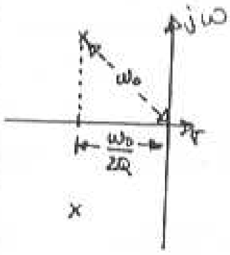
$$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$$

$$\underline{\underline{R = 4.59\ k\Omega}}$$

R_1 can be arbitrarily chosen

use $\underline{\underline{R_1 = 10k\Omega}}$

12.28



Natural Modes:

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\omega_0 = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2}$$

$$= \underline{1.0}$$

$$\frac{\omega_0}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_0}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore \quad \underline{\underline{T(s) = \frac{s^2}{s^2 + s + 1}}}$$

12.29

For a 2nd-order bandpass

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_0^2 - \omega^2) + \frac{j\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{a_1\omega}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{a_1\omega_1}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_1\omega_0}{Q}\right)^2}} = \frac{a_1\omega_2}{\sqrt{(\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_2\omega_0}{Q}\right)^2}}$$

$$\omega_1^2\left[(\omega_0^2 - \omega_2^2)^2 + \cancel{\left(\frac{\omega_2\omega_0}{Q}\right)^2}\right] = \omega_2^2\left[(\omega_0^2 - \omega_1^2)^2 + \cancel{\left(\frac{\omega_1\omega_0}{Q}\right)^2}\right]$$

$$\omega_1^2(\omega_0^4 - \cancel{2\omega_0^2\omega_2^2} + \omega_2^4) =$$

$$\omega_2^2(\omega_0^4 - \cancel{2\omega_0^2\omega_1^2} + \omega_1^4)$$

$$\omega_1^2\omega_0^4 + \omega_1^2\omega_2^4 = \omega_2^2\omega_0^4 + \omega_2^2\omega_1^4$$

$$\omega_0^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^4 - \omega_1^2\omega_2^4$$

$$\omega_0^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^2(\omega_1^2 - \omega_2^2)$$

$$\omega_0^4 = \omega_1^2\omega_2^2$$

$$\underline{\underline{\omega_0^2 = \omega_1\omega_2}} \quad \text{Q.E.D.}$$

CONT.

For Fig. 12.4 : $\omega_{p1} = 8100$ rad/s
$\omega_{p2} = 10\,000$ rad/s
$A_{max} = 1$ dB

$$\omega_0^2 = (8100)(10\,000)$$

$$\omega_0 = \underline{9000 \text{ rad/s}}$$

$$|T(j\omega_{p1})| = |T(j\omega_{p2})| = 10^{-1/20} = \underline{0.8913}$$

$$|T(j\omega_0)| = \frac{\omega_0 a_1}{\sqrt{(\omega_0^2-\omega_0^2)^2 + \left(\frac{\omega_0^2}{Q}\right)^2}} = 1$$

$$\Rightarrow \frac{\omega_0 a_1}{\omega_0^2/Q} = 1$$

$$\therefore \frac{Q a_1}{\omega_0} = 1 \Rightarrow a_1 = \frac{\omega_0}{Q}$$

$$|T(j\omega_{p1})|^2 = |T(j0.9\omega_0)|^2 = 0.8913^2$$

$$\frac{(\omega_0/Q)^2(0.9\omega_0)^2}{(\omega_0^2 - (0.9\omega_0)^2)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_0}{Q}(.9\omega_0)\right)^2 = 0.8913^2\left[\left(\omega_0^2 - (0.9\omega_0)^2\right)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2\right]$$

$$\frac{0.81\omega_0^4}{Q^2} = 0.8913^2\left[\omega_0^4(1-0.81)^2 + \frac{0.81\omega_0^4}{Q^2}\right]$$

$$\frac{0.81\omega_0^4}{Q^2}\left(1-0.8913^2\right) = 0.8913^2\omega_0^4 \times (1-0.81)^2$$

SUB $\omega_0 = 9000$ gives
$\underline{Q = 2.41}$

Now $a_1 = \frac{\omega_0}{Q} = 0.415\omega_0$

$$\therefore T(s) = \frac{0.415\omega_0 s}{s^2 + 0.415\omega_0 s + \omega_0^2}$$

If $\omega_{s1} = 3000$ rad/s

$$|T(j3000)| = \frac{0.415\omega_0(3000)}{\sqrt{(\omega_0^2 - 3000^2)^2 + (\omega_0 3000 \times .415)^2}}$$

$$= 0.1537$$

$$\therefore A_{MIN} = -20\log(0.1537) = \underline{16.3\text{dB}}$$

Now ω_{s1} and ω_{s2} are geometrically symmetrical about ω_0:

$$\omega_{s1}\omega_{s2} = \omega_0^2$$

$$\omega_{s2} = \frac{9000^2}{3000} = \underline{27\,000 \text{ rad/s}}$$

$\boxed{12.30}$

From exercise 12.15

$$Q = \frac{\omega_0}{BW\sqrt{10^{A/10}-1}} \quad \begin{cases}\omega_0 = 2\pi(60)\\ BW = 2\pi 6\\ A = 20\text{dB}\end{cases}$$

$$= \underline{1.005}$$

$$T(s) = a_2\frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$|T(0)| = \frac{a_2\omega_0^2}{\omega_0^2} = 1 \leftarrow \text{DC Gain}$$

CONT.

$\underline{a_2 = 1}$

$$T(s) = \frac{s^2 + (2\pi 60)^2}{s^2 + s\,\frac{2\pi 60}{1.005} + (2\pi 60)^2}$$

$$T(s) = \frac{s + 1.421\times10^5}{s^2 + 375.1s + 1.421\times10^5}$$

12.31

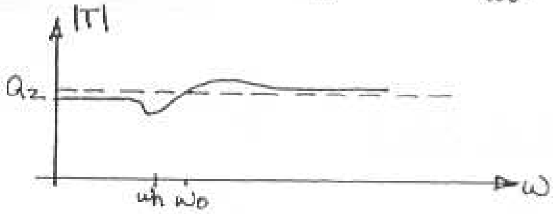
FOR ALL PASS:

$$T(s) = a_2\,\frac{s^2 - s\,\omega_0/Q + \omega_0^2}{s^2 + s\,\omega_0/Q + \omega_0^2}$$

If zero frequency < pole frequency

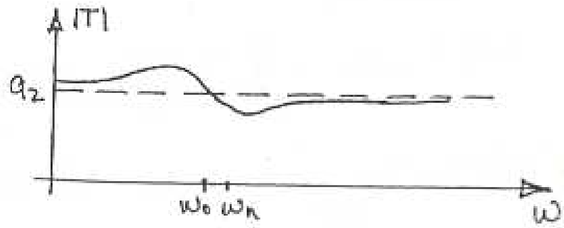
$$T(s) = a_2\,\frac{s^2 - s\,\omega_n/Q + \omega_n^2}{s^2 + s\,\omega_0/Q + \omega_0^2} \quad \omega_n < \omega_0$$

At DC: $|T| = a_2\,\frac{\omega_n^2}{\omega_0^2}$ where $\frac{\omega_n^2}{\omega_0^2} < 1$



If zero frequency > pole frequency then $\omega_n > \omega_0$

At DC: $|T| = a_2\,\omega_n^2/\omega_0^2$ where $\frac{\omega_n^2}{\omega_0^2} > 1$



12.32

$$T(s) = \frac{s^2 - s\,\omega_0/Q' + \omega_0^2}{s^2 + s\,\omega_0/Q_0 + \omega_0^2}\,a_2$$

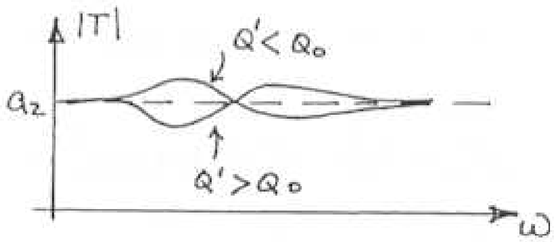
Zero Q < Pole Q $\Rightarrow Q' < Q_0$

At $\omega = \omega_0$:

$$|T| = \frac{a_2\,\omega_0^2/Q'}{\omega_0^2/Q_0} = \frac{a_2 Q_0}{Q'} > a_2$$

If $Q' > Q_0$

$$|T(j\omega_0)| = \frac{a_2 Q_0}{Q'} < a_2$$



12.33

$\omega_0 = 10^4$ rad/s, $Q = 2$, $R = 10k\Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC} \Leftarrow L = \frac{R^2 C}{Q^2}$$

$$= \frac{Q^2}{R^2C^2}$$

$$C = \frac{Q}{R\omega_0} = \underline{20nF}$$

$$L = \frac{1}{C\omega_0^2} = \underline{500mH}$$

12.34

$\omega_0 = 1/\sqrt{LC}$

If $L' = 1.01L$

$\omega_0' = (1.01LC)^{-1/2}$

$= 0.9950 \frac{1}{\sqrt{LC}}$

$= 0.9950\,\omega_0$

$\therefore \underline{\underline{\Delta\omega_0 = -0.5\%}}$

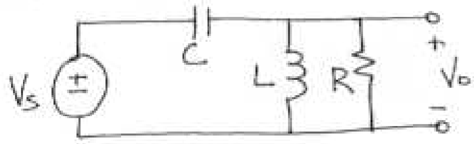
If $C' = 1.01C$

$\omega_0' = 0.9950\,\omega_0$

$\underline{\underline{\Delta\omega_0' = -0.5\%}}$

Changing R has no effect on ω_0

12.35



Use voltage divider rule:

$$V_o = \frac{Z_{R\parallel L}}{Z_{R\parallel L} + Z_c} V_s$$

$$\frac{V_o}{V_s} = \frac{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1} + \frac{1}{sC}}$$

$$= \frac{sC}{\left(\frac{1}{sL} + \frac{1}{R}\right) + sC}$$

$$\therefore T(s) = \frac{V_o(s)}{V_s(s)} = \frac{s^2}{s^2 + s/RC + \frac{1}{LC}}$$

12.36

Low Pass: $\omega_0 = 10^5$, $C = 0.1\mu F$

$Q = 1/\sqrt{2}$

$Q = \omega_0 C R$

$R = \frac{Q}{\omega_0 C}$

$= \frac{1}{\sqrt{2} \times 10^5 \times 0.1 \times 10^{-6}}$

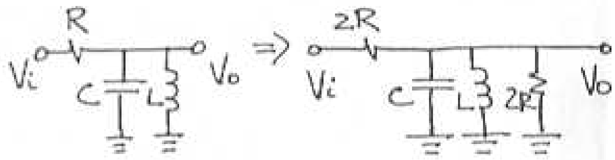
$= \underline{\underline{70.7\,\Omega}}$

$\omega_0 = 1/\sqrt{LC}$

$L = \frac{1}{\omega_0^2 C}$

$= \underline{\underline{1mH}}$

12.37



$\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \omega_0 C R$

$A_{MID} = 1$

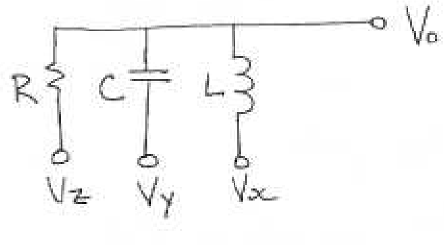
$\omega_0 = 1/\sqrt{LC}$

$Q = \omega_0 C (2R \parallel 2R)$

$= \omega_0 C R$

$A_{MID} = \frac{2R}{2R + 2R} = 1/2$

12.38



$V_o/V_z \big|_{V_y = V_x = 0} = T_{BP}(s)$

$V_o/V_y \big|_{V_x = V_z = 0} = T_{HP}(s)$

CONT.

$$\left.\frac{V_o}{V_x}\right|_{V_y=V_z=0} = T_{LP}(s)$$

Using superposition

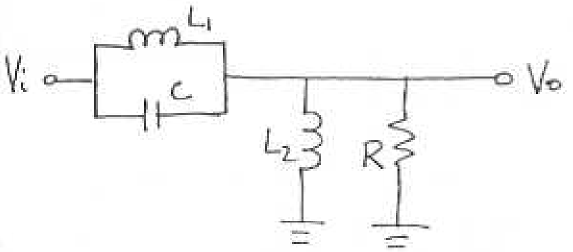
$$V_o = \frac{V_o}{V_x}V_x + \frac{V_o}{V_y}V_y + \frac{V_o}{V_z}V_z$$

$$= T_{LP}V_x + T_{HP}V_y + T_{BP}V_z$$

$$= \frac{\frac{1}{LC}V_x + s^2V_y + \frac{s}{RC}V_z}{s^2 + s/RC + 1/LC}$$

$$\therefore V_o = V_x\frac{1/LC}{s^2+s/RC+1/LC} + V_y\frac{s^2}{s^2+s/RC+1/LC} + V_z\frac{s/RC}{s^2+s/RC+1/LC}$$

12.39



From Eq 12.46

$$T(s) = \frac{s^2 + 1/L_1C}{s^2 + s\,(1/CR) + \frac{1}{(L_1\|L_2)C}}$$

Required notch $\omega_n^2 = \frac{1}{L_1C} = (0.9\omega_o)^2$

but:

$$\omega_o^2 = \frac{1}{(L_1\|L_2)C} \quad \text{where } L_1\|L_2 = \frac{1}{L_1^{-1}+L_2^{-1}} = \frac{L_1L_2}{L_1+L_2}$$

$$= \frac{L_1+L_2}{L_1L_2C}$$

$$= \frac{L_1+L_2}{L_2}(.9\omega_o)^2$$

$$1 = \left(\frac{L_1}{L_2}+1\right)0.9^2$$

$$\therefore L_1/L_2 = \frac{1}{0.9^2} - 1 = \underline{\underline{0.2346}}$$

For $\omega \ll \omega_o$:-

$$|T| \cong \frac{1/L_1C}{1/(L_1\|L_2)C} = \frac{L_2}{L_1+L_2}$$

i.e. inductors dominate!

For $\omega \gg \omega_o$ L_1 & L_2 are "open" C is shorted

$$\underline{\underline{|T| \cong 1}}$$

12.40

$L = C_4 R_1 R_3 R_5 / R_2$

Choose $\underline{\underline{R_1 = R_2 = R_3 = R_5 = 10k\Omega}}$

$\therefore L = C_4 \times 10^8$

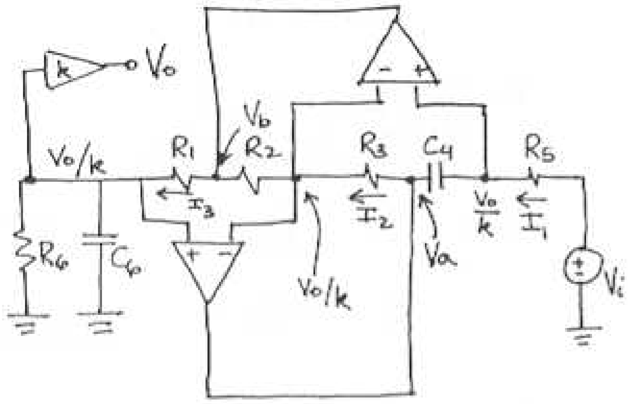
For:

$L = 10H = C_4 \times 10^8 \Rightarrow \underline{\underline{C_4 = 100nF}}$

$L = 1H \Rightarrow \underline{\underline{C_4 = 10nF}}$

$L = 0.1H \Rightarrow \underline{\underline{C_4 = 1nF}}$

12.41



Because of the virtual short at opamp input terminals, the voltages are : V_o/k

$$I_1 = \frac{V_i - V_o/k}{R_5}$$

$$V_a = V_o/k - I_1/sC_4$$

$$= V_o/k - \frac{V_i - V_o/k}{sC_4 R_5}$$

$$= \frac{V_o(sC_4R_5+1) - kV_i}{sC_4R_5k}$$

$$I_2 = \frac{V_a - V_o/k}{R_3}$$

$$= -\frac{V_i}{sC_4R_3R_5} + \frac{V_o/k}{sC_4R_3R_5}$$

$$V_b = \frac{V_o}{k} - I_2R_2$$

$$= \frac{V_o}{k} - \frac{V_o/k - V_i}{sC_4R_3R_5/R_2}$$

$$I_3 = \frac{V_b - V_o/k}{R_1} = \frac{V_i - V_o/k}{sC_4R_1R_3R_5/R_2}$$

Now, I_3 flows only into R_6 & C_6 since for ideal opamps, $R_{IN} = \infty$!

$$\therefore \frac{V_o}{k} = I_3\left(R_6 \parallel \frac{1}{sC_6}\right)$$

Let $L \triangleq \frac{R_1R_3R_5C_4}{R_2}$

So: $I_3 = \frac{V_i - V_o/k}{sL}$ &

$$V_o/k = I_3\left(R_6 \parallel \frac{1}{sC_6}\right)$$

$$\frac{V_o}{k} = \frac{V_i - V_o/k}{sL} \cdot \frac{1}{1/R_6 + sC_6}$$

$$= \frac{V_i - V_o/k}{sL/R_6 + s^2LC_6}$$

$$V_o/k\left(1 + sL/R_6 + s^2LC_6\right) = V_i$$

$$\frac{V_o}{V_i} = \frac{k/LC_6}{s^2 + \frac{s}{R_6C_6} + \frac{1}{LC_6}}$$

Recall $L = R_1R_3R_5C_4/R_2$

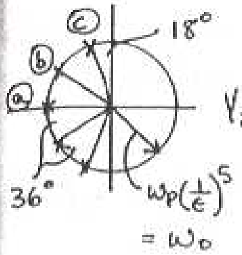
$$\therefore \frac{V_o}{V_i} = \frac{\frac{kR_2}{C_4R_1R_3R_5}}{s^2 + \frac{s}{R_6C_6} + \frac{R_2}{C_6C_4R_1R_3R_5}}$$

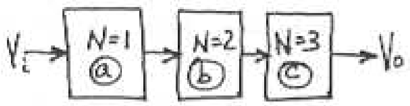
12.42

$$A_{max} = 10\log(1+\epsilon^2) = 3\,dB$$

$$\therefore \epsilon = 0.998 \simeq 1$$

$$\omega_0 = \omega_p = 10^4$$





For circuit ⓐ use fig 12.13(a)

DC Gain $= 1 = R_2/R_1 \Rightarrow \underline{R_1 = R_2 = 10\,k\Omega}$

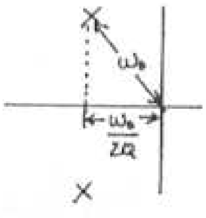
$CR_2 = 1/\omega_0 \Rightarrow C = 1/R_2\omega_0 = \frac{1}{10^4 \cdot 10^4}$
$= \underline{10\,nF}$

For circuit ⓑ use Fig. 12.22(a)

$\omega_0 = 10^4$ rad/s

$\frac{\omega_0}{2Q} = \omega_0 \cos 36^\circ$

$Q = \frac{1}{2\cos 36} = \underline{\underline{0.618}}$



From Table 12.1

$$T(s) = \frac{\frac{kR_2}{C_4 C_6 R_1 R_3 R_5}}{s^2 + s/C_6R_6 + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$$

$\omega_0^2 = \frac{R_2}{C_4 C_6 R_1 R_3 R_5}$ Let $R_1 = R_3 = R_5 = R_2 = R$
$C_4 = C_6 = C$

$\omega_0^2 = \frac{1}{R^2 C^2}$ USE $\underline{\underline{C_4 = C_6 = 100\,nF}}$

$\therefore R = \frac{1}{\omega_0 C} \Rightarrow \underline{\underline{R_1 = R_2 = R_3 = R_5 = 1\,k\Omega}}$

Now using:

$\frac{\omega_0}{Q} = \frac{1}{C_6 R_6}$ & $Q = 0.618$

$R_6 = \frac{Q}{C_6\omega_0} = \underline{\underline{618\,\Omega}}$

For circuit ⓒ use Fig 12.22 (a)

$\omega_0 = 10^4$ which is the same as for circuit ⓑ.

$\therefore \underline{C_4 = C_6 = 100\,nF}$
$\underline{R_1 = R_2 = R_3 = R_5 = 1\,k\Omega}$

Now: $Q = \frac{1}{2\cos 72^\circ} = 1.618$

$R_6 = Q/\omega_0 C_6 = \underline{\underline{1.618\,k\Omega}}$

12.43

$f_0 = 4\,kHz$ $f_N = 5\,kHz$ $Q = 10$

now $C_4 = 10\,nF$ and $k = 1 \equiv$ dc gain

$\omega_0 = \left[C_4(C_{61} + C_{62})R_1R_3R_5/R_2\right]^{-1/2}$

$C_{61} + C_{62} = C_6$

Choose $\underline{\underline{C_6 = C_4 = 10\,nF}}$ &
$R_1 = R_3 = R_5 = R_2 = R$

$\therefore \omega_0 = (C_4 C_6 R^2)^{-1/2}$

$R = \frac{1}{\omega_0 C_4}$

$\Rightarrow \underline{\underline{R_1 = R_3 = R_5 = R_2 = 3.979\,k\Omega}}$

$\omega_n = (C_4 C_{61} R^2)^{-1/2}$

$C_{61} = \frac{1}{\omega_n^2 R^2 C_4} \Rightarrow \underline{C_{61} = 6.4\,nF}$

& $\underline{C_{62} = 3.6\,nF}$

$Q = R_6\sqrt{\frac{C_{61} + C_{62}}{C_4} \cdot \frac{R_2}{R_1R_3R_5}}$

$= R_6\sqrt{\frac{1}{R_1^2}} = R_6/R_1 \Rightarrow \underline{R_6 = 39.79\,k\Omega}$

12.44

From Fig. 12.16 (g) $\phi = 180°$ at f_0 !

$\therefore$ Use $f_0 = 1\,kHz$ $\quad Q = 1$

$$\omega_0^2 = \frac{R_2}{C_4 C_6 R_1 R_3 R_5} \qquad \underline{\underline{\overset{\text{LET}}{C = C_4 = C_6 = 1nF}}} \quad R_1 = R_3 R_5 = R_2 = R$$

$$= 1/C^2 R^2$$

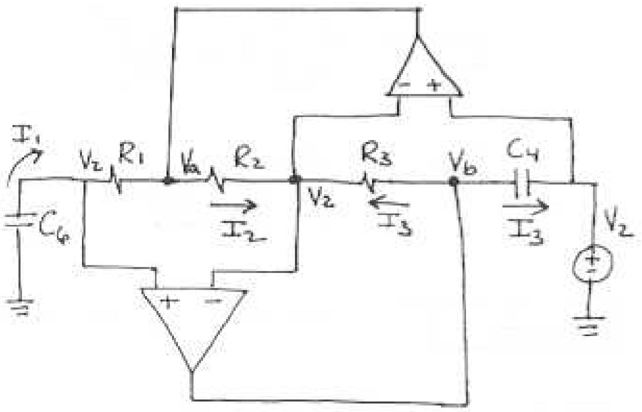
$$R = 1/\omega_0 C = \underline{\underline{159.16\,k\Omega = R_1 = R_3 = R_5 = R_2}}$$

$$\frac{\omega_0}{Q} = \frac{1}{R_6 C_6} \Rightarrow R_6 = \frac{Q}{C_6 \omega_0}$$

$$= \frac{1}{10^{-9}\, 2\pi 10^3}$$

$$\therefore \underline{\underline{R_6 = 159.16\,k\Omega}}$$

12.45



Because of virtual short at opamp input terminals all nodes are at V_2 !

$$I_1 = -sC_6 V_2$$

Since no current goes into the opamp input terminals we have:

$$V_a = V_2 - I_1 R_1$$
$$= V_2 (1 + sC_6 R_1)$$

$$I_2 = (V_a - V_2)\frac{1}{R_2}$$
$$= \frac{sC_6 R_1}{R_2}$$

$$V_b = V_2 - I_2 R_3$$
$$= V_2 - \frac{sC_6 R_1 R_3}{R_2}$$

$$I_3 = (V_b - V_2)\, sC_4$$
$$= -\frac{s^2 C_4 C_6 R_1 R_3}{R_2} V_2$$

Now the voltage source sees an input impedance given by:

$$Z_{in} = -V_2/I_3 = \frac{R_2}{s^2 C_4 C_6 R_1 R_3}$$

As required.

for $s = j\omega \Rightarrow s^2 = -\omega^2$

$$Z_{in}(j\omega) = \frac{-R_2}{C_4 C_6 R_1 R_3} \cdot \frac{1}{\omega^2}$$

$$= -R(\omega)$$ i.e. A PURE NEGATIVE RESISTANCE!

12.46

For LPN: See Table 12.1 & Fig. 12.22(e)

$$T(s) = k\frac{C_{61}}{C_{61}+C_{62}} \cdot \frac{s^2 + \frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}{s^2 + s/C_6 R_6 + \frac{R_2}{C_4 (C_{61}+C_{62}) R_1 R_3 R_5}}$$

At DC $\rightarrow s = 0$

$$T(0) = k\frac{C_{61}}{C_{61}+C_{62}} \quad \frac{R_2/C_4 C_6 R_1 R_3 R_5}{R_2/C_4(C_{61}+C_{62}) R_1 R_3 R_5}$$

CONT.

$\Rightarrow T(0) = k \triangleq$ DC Gain!

Note that $C_{61} + C_{62}$ is the total capacitance across R_6

$\therefore C_6 = C_{61} + C_{62}$

$$\frac{\omega_n^2}{\omega_o^2} = \frac{R_2/C_4 C_{61} R_1 R_3 R_5}{R_2/C_4 (C_{61}+C_{62}) R_1 R_3 R_5}$$

$$= \frac{C_{61}}{C_{61}+C_{62}}$$

$$\frac{\omega_n^2}{\omega_o^2} = \frac{C_{61}}{C_6}$$

$$\therefore C_{61} = C_6 \left(\frac{\omega_n}{\omega_o}\right)^2 = C\left(\frac{\omega_n}{\omega_o}\right)^2$$

Clearly from $T(s)$ above:

$$\omega_n^2 = R_2/C_4 C_{61} R_1 R_3 R_5$$

$$\Rightarrow \omega_n = \sqrt{\frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}$$

$$\omega_o^2 = R_2/C_4 C_6 R_1 R_3 R_5$$

$$\Rightarrow \omega_o = \sqrt{\frac{R_2}{C_4 (C_{61}+C_{62}) R_1 R_3 R_5}}$$

12.47

For HPN: See Table 12.1 & Fig 11.22(f)

$$T(s) = k \frac{s^2 + (R_2/C_4 C_6 R_1 R_3 R_{51})}{s^2 + s/C_6 R_6 + \left(\frac{R_2}{C_4 C_6 R_1 R_3}\right)\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$$

Clearly: $\omega_n = \sqrt{\frac{R_2}{C_4 C_6 R_1 R_3 R_{51}}}$

$$\omega_o = \sqrt{\frac{R_2}{C_4 C_6 R_1 R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$$

At high frequencies $s \to \infty$

$T(\infty) = k \triangleq$ high freq gain.

Observe that the equivalent resistance at the +ve terminal of A_1 is:

$$\frac{1}{R_5} = \frac{1}{R_{51}} + \frac{1}{R_{52}} \quad \text{AND}$$

for the resonator (table 12.1)

$$R_5 = 1/\omega_o C \Rightarrow \frac{1}{R_5} = \omega_o C$$

$$\frac{\omega_o^2}{\omega_n^2} = \frac{R_2/C_4 C_6 R_1 R_3 R_5}{R_2/C_4 C_6 R_1 R_3 R_{51}} \Rightarrow R_{51} = R_5 \frac{\omega_o^2}{\omega_n^2}$$

Now $\frac{1}{R_5} = \frac{1}{R_5 \frac{\omega_o^2}{\omega_n^2}} + \frac{1}{R_{52}}$

$$\frac{1}{R_{52}} = \frac{1}{R_5}\left[1 - \frac{\omega_n^2}{\omega_o^2}\right]$$

$$R_{52} = \frac{R_5}{1 - \omega_n^2/\omega_o^2}$$

12.48

$$T(s) = \frac{0.4508\,(s^2 + 1.6996)}{(s+0.7294)(s^2 + 0.27865 + 1.0504)}$$

PART (a) Replace s with s/ω_p

$$T(s) = \frac{0.4508\left(s^2/\omega_p^2 + 1.6996\right)}{\left(\frac{s}{\omega_p} + 0.7294\right)\left(\frac{s^2}{\omega_p^2} + \frac{0.27865}{\omega_p} + 1.0504\right)}$$

CONT.

$$T(S) = \frac{0.4508\omega_p\,(s^2 + 1.6996\,\omega_p^2)}{(s + 0.7294\omega_p)(s^2 + 0.2786\omega_p s + 1.0504\,\omega_p^2)}$$

SUB $\omega_p = 10^4$ rad/s

$$T(S) = \frac{4508\,(s^2 + 1.6996 \times 10^8)}{(s + 7294)(s^2 + 2786 s + 1.0504 \times 10^8)}$$

Part (b)

First decompose T(s) into 1st-and 2nd-order sections with unity DC gain!

$$T_1(s) = \frac{k_1}{s + 7294} \qquad T_1(0) = \frac{k_1}{7294} = 1$$

$$\Rightarrow \underline{\underline{k_1 = 7294}}$$

Now $k_1 k_2 = 4508 \Rightarrow \underline{k_2 = 0.6180}$

$$\therefore T_2(s) = \frac{0.6180\,(s^2 + 1.6996 \times 10^8)}{s^2 + 2786 s + 1.0504 \times 10^8}$$

As a check:

$$T_2(0) = \frac{0.6180\,(1.6996 \times 10^8)}{1.0504 \times 10^8} = 1.000$$

As EXPECTED!

$\therefore\ T(S) = T_1(s) \cdot T_2(s)$

For first-order section use Fig 12.13(a)

$\omega_0 = 7294$ rad/s DC Gain = 1

Let $\underline{\underline{C = 10nF}}$

$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow \underline{\underline{R_1 = R_2 = 13.71 k\Omega}}$

For second-order section

$\omega_n^2 = 1.6996 \times 10^8 \Rightarrow \omega_n = 13.037 \times 10^3$

$\omega_0^2 = 1.0504 \times 10^8 \Rightarrow \omega_0 = 10.249 \times 10^3$

$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$

For LPN use table 12.1 and Fig 12.22(e)

Make $R_1 = R_2 = R_3 = R_5 = R$ and $\underline{\underline{C = C_4 = C_6 = 10nF}}$

$R = \frac{1}{\omega_0 C} = \underline{\underline{9.757 k\Omega = R_1 = R_2 = R_3 = R_5}}$

$\omega_n^2 = \frac{1}{C R^2 C_{61}} \Rightarrow \underline{\underline{C_{61} = 6.18nF}}$

$C_{62} = C - C_{61}$

$= \underline{\underline{3.82 nF}}$

$$Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}} = \frac{R_6}{\sqrt{R^2}}$$

$$= \frac{R_6}{R}$$

$\therefore\ R_6 = RQ = (9.757 \times 10^3)(3.6787)$

$= \underline{\underline{35.89 k\Omega}}$

$\boxed{12.49}$

$f_0 = 1kHz$

The 3dB bandwidth for a 2nd order filter is given by:

$B = \omega_0 / Q \Rightarrow Q = \frac{2\pi 10^3}{2\pi 50} = \underline{\underline{20}}$

Choose $C = 10nF$

$R = \frac{1}{\omega_0 C} = \underline{\underline{15.92 k\Omega}}$

Use $\underline{\underline{R_1 = R_f = 10k\Omega}}$

CONT.

$$\frac{R_3}{R_2} = 2Q - 1 = 39$$

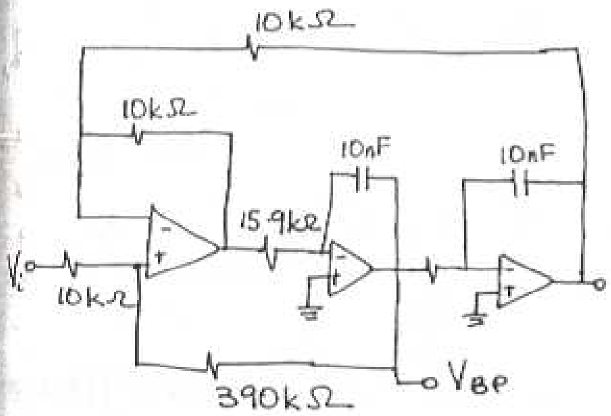
choose $\underline{R_2 = 10k\Omega}$ $\underline{R_3 = 390k\Omega}$

Now $T(s) = \dfrac{-k\omega_0 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$\Rightarrow |T(j\omega_0)| = \dfrac{k\omega_0^2}{\omega_0^2/Q} = kQ$

but $k = 2 - 1/Q = 1.95$

$\therefore$ Centre-freq gain $= kQ = \underline{39}$



12.50

$R_L = R_H = R_B/Q \Rightarrow R_B = QR_H$
$R_L = R_H$

Using Eq 12.66:

$$\frac{V_o}{V_i} = -k\,\frac{\frac{R_F}{R_H}s^2 - s\left(\frac{R_F}{R_B}\right)\omega_0 + \left(\frac{R_F}{R_L}\right)\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$= -k\frac{R_F}{R_H}\,\frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Flat Gain $= \underline{-k\,R_F/R_H}$

Part (b) – $\omega_0 = 10^4$ rad/s $Q = 2$ Flat Gain $= 10$

choose $\underline{C = 10nF} \Rightarrow R = \frac{1}{\omega_0 C} = \underline{10k\Omega}$

choose $\underline{R_F = R_1 = 10k\Omega}$

$\frac{R_3}{R_2} = 2Q - 1 = 3 \Rightarrow \underline{R_2 = 10k\Omega}$
$\underline{R_3 = 30k\Omega}$

Now $k = 2 - 1/Q = 1.5$

$\therefore$ Flat Gain $= 10 = (1.5)\frac{R_F}{R_H}$

$\therefore \frac{R_H}{R_F} = 0.15$

choose $\underline{R_F = 100k\Omega}$

$\underline{R_H = R_L = 15k\Omega}$

$R_B = QR_H = \underline{30k\Omega}$

12.51

Note ω_n does not depend on R or C
From eq. 12.67:

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

$\therefore \omega_n = \omega_0\sqrt{\frac{R_H}{R_L}}$ Nominally $R_H = R_L \pm 1\%$

Thus:

$\omega_n' = \omega_0\sqrt{\frac{1.01}{0.99}} = 1.01\omega_0$ $\omega_n'' = \omega_0\sqrt{\frac{0.99}{1.01}} = 0.99\omega_0$

$\therefore \omega_n$ can deviate from ω_0 by $\underline{\pm 1\%}$

12.52

Use Tow Thomas to realize a LPN (Fig 12.26)

$\omega_0 = 10^4 \quad \omega_n = 1.2\omega_0 \quad Q = 10 \quad$ DC Gain $= 1$

$C = 10nF \quad r = 20k\Omega$

$$R = \frac{1}{\omega_0 C} = \underline{\underline{10k\Omega}}$$

from 12.16 (e):

$$\text{DC Gain} = a_2 \frac{\omega_n^2}{\omega_0^2} = 1$$

$$a_2 \frac{1.2^2 \omega_0^2}{\omega_0^2} = 1$$

$$a_2 = \frac{1}{1.2^2} = \text{HF. Gain}$$

$$C_1 = C a_2 = \frac{10 \times 10^{-9}}{1.2^2} = \underline{\underline{6.94nF}}$$

$$R_2 = \frac{R(\omega_0/\omega_n)^2}{\text{HF Gain}} = R\left(\frac{1}{1.2}\right)^2 \times (1.2)^2$$

$$= R = \underline{\underline{10k\Omega}}$$

$$\underline{\underline{R_1 = R_3 = \infty}}$$

12.53

For all pass:

$$T(s) = \frac{-s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_z^2 = \frac{1}{C^2RR_2} \cdot \frac{C}{C_1} \Rightarrow \underline{\underline{\omega_z = \frac{1}{C\sqrt{RR_2}} \cdot \sqrt{\frac{C}{C_1}}}}$$

$$Q_z = \frac{\omega_z}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\frac{C}{C_1}}$$

$$Q_z = \frac{\sqrt{\frac{1}{C^2RR_2}\frac{C}{C_1}}}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\left(\frac{C}{C_1}\right)}$$

$$= \underline{\underline{\frac{1}{\sqrt{RR_2}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\sqrt{\frac{C}{C_1}}}}}$$

For All Pass $R_1 \to \infty$

To adjust Q_z, trim r or R_2 (independent of ω_z!)

Now $\omega_0 = \frac{1}{CR}$ so do not trim R or C!

Note if we trim R_2 or C_1 to adjust ω_z, this will also affect Q_z. So the options are:

For ω_z: (a) trim R_2 AND (r or R_3) to maintain the value of Q_z

OR

(b) trim C_1, and r or R_3

Prefer not to trim a capacitor so use (a)!

12.54

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + 0.27865 + 1.0504)}$$

Part (a) Replace s with s/ω_p

$\omega_p = 10^4$ rad/s.

$$T(s) = \frac{0.4508\left(s^2/\omega_p^2 + 1.6996\right)}{\left(\frac{s}{\omega_p} + 0.7294\right)\left(\frac{s^2}{\omega_p^2} + \frac{0.27865}{\omega_p} + 1.0504\right)}$$

CONT.

$$T(s) = \frac{0.4508\,\omega_p(s^2 + 1.6996\omega_p^2)}{(s+0.7294\omega_p)(s^2+0.2786\omega_p s + 1.0504\omega_p^2)}$$

$$= \frac{4508(s^2 + 1.6996\times10^8)}{(s+7294)(s^2+2786s+1.0504\times10^8)}$$

For First Order Section use Fig 12.13(a)

$\omega_0 = 7294$ Dc gain $= 1$

choose $\underline{C = 10nF}$

$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow \underline{R_1 = R_2 = 13.71k\Omega}$

For Second Order Section - use Fig 12.26

$\omega_n^2 = 1.6996\times10^8 \Rightarrow \omega_n = 13.037\times10^3$

$\omega_0^2 = 1.0504\times10^8 \Rightarrow \omega_0 = 10.249\times10^3$

$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$

Dc gain $= 1$

For Tow Thomas LPN use table 12.2

choose $\underline{C = 10nF}$

$R = \frac{1}{\omega_0 C} = \frac{1}{10.249\times10^3\ 10\times10^{-9}}$

$= \underline{9.757k\Omega}$

choose $\underline{r = 20k\Omega}$

Now from Fig 12.16 (e):

$T(0) = a_2\frac{\omega_n^2}{\omega_0^2} = 1 \Rightarrow a_2 = \frac{\omega_0^2}{\omega_n^2} = 0.618$

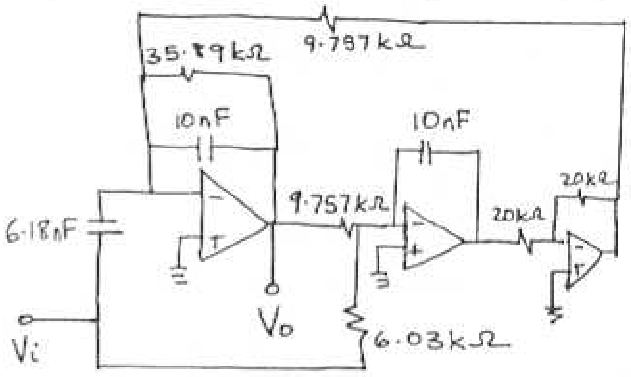
$\therefore$ HF gain $= a_2 = 0.618$

$C_1 = C\times HF gain \Rightarrow \underline{C_1 = 6.18nF}$

$R_2 = R(\omega_0/\omega_n)^2$

$R_2 = 0.618R \Rightarrow \underline{R_2 = 6.03k\Omega}$

$\underline{R_1 = R_3 = \infty}$ $QR = \underline{35.89k\Omega}$



$\boxed{12.55}$

Make $C_1 = C_2 = 1nF = C$

$\omega_0 = \frac{1}{\sqrt{C_1C_2R_3R_4}}$ $Q = \left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]^{-1}$

Let $R_3 = R$ $m = 4Q^2$

$R_4 = \frac{R}{m}$ $= 4/2 = 2$

$\therefore \omega_0 = \frac{1}{\sqrt{C^2R^2/2}} = \frac{\sqrt{2}}{RC} = 10^4$

$\Rightarrow R = \frac{\sqrt{2}}{10^4 10^{-9}} = \underline{141.42k\Omega}$

$\underline{R_3 = 141.4k\Omega}$

$R_4 = \frac{R_3}{2} \Rightarrow \underline{R_4 = 70.7k\Omega}$

12.56

For Fig 12.28 (a)

$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_1R_4} + \frac{1}{C_2R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

But $C_1 = C_2 = C$ & $R_3 = R_4 = R$, $RC = \tau$

$$\therefore t(s) = \frac{s^2 + s\,{}^2/RC + {}^1/R^2C^2}{s^2 + s\,{}^3/RC + \frac{1}{R^2C^2}}$$

$$= \frac{s^2 + s\,{}^2/\tau + {}^1/\tau^2}{s^2 + s\frac{3}{\tau} + {}^1/\tau^2}$$

Zeros defined by $\omega_z = 1/\tau$

$Q_z = \frac{1}{2}$

$\Rightarrow$ Double Root at $s = -1/\tau$

Poles of $t(s)$ are given by the quadratic formula:

$$s = \frac{-3}{2\tau} \pm \frac{\sqrt{5}}{2\tau} = \frac{-3 \pm \sqrt{5}}{2\tau}$$

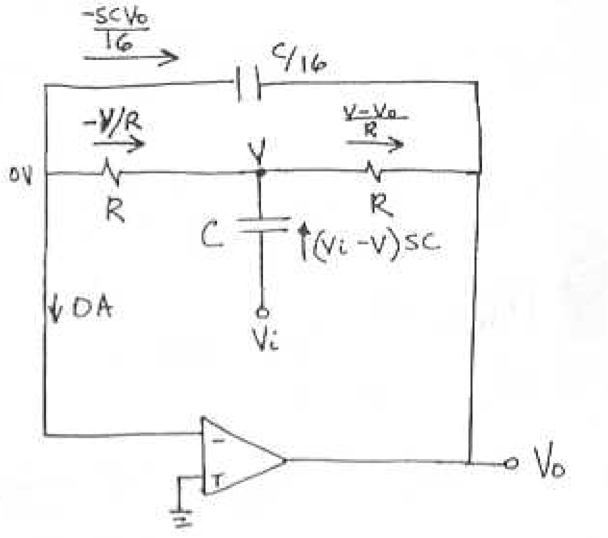
i.e. two roots on the negative real axis

If the network is placed in the negative feedback path of an ideal amplifier ($A = \infty$) then the poles are given by the zeros of $t(s)$:

Closed loop poles:

$$s = -1/\tau \quad \text{(multiplicity = 2)}$$

12.57



Note first $-\frac{sCV_o}{16} = -\frac{-V}{R}$

$$V = -\frac{sCRV_o}{16}$$

ΣI at V

$$-\frac{V}{R} + sC(V_i - V) - \frac{V - V_o}{R} = 0$$

$$\frac{sCV_o\cancel{R}}{\cancel{R}\,16} + sCV_i + s^2\frac{C^2RV_o}{16} + \frac{sCV_o}{16} + \frac{V_o}{R} = 0$$

mult by: $16R$ and let $RC = \tau$

$$s\tau V_o + 16\tau V_i s + s^2\tau^2 V_o + s\tau V_o + 16V_o = 0$$

$$V_o\left[s^2\tau^2 + s \times 2\tau + 16\right] = -16 s\tau V_i$$

$$\therefore \frac{V_o}{V_i} = -\frac{16s\tau}{s^2\tau^2 + 2\tau s + 16}$$

$$\therefore T(s) = \frac{s\,{}^{16}/RC}{s^2 + s\,{}^2/RC + {}^{16}/R^2C^2}$$

CONT.

Let $\omega_0^2 = \frac{16}{(RC)^2} \Rightarrow \omega_0 = \frac{4}{RC}$

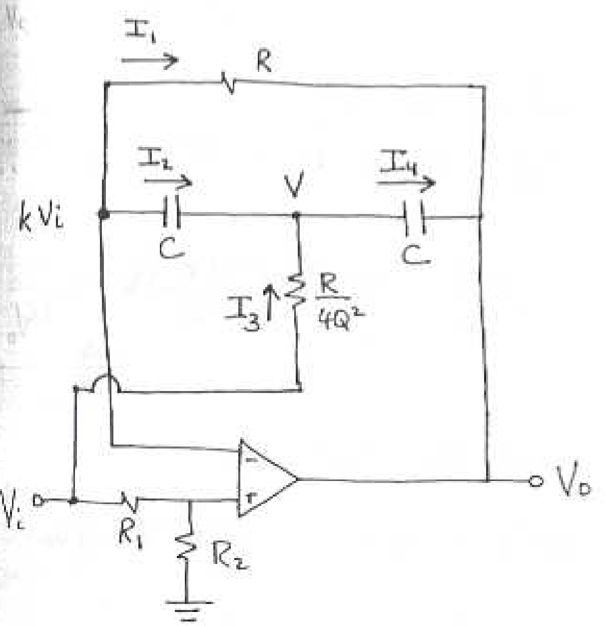
$\frac{\omega_0}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_0}{2} = \underline{\underline{2}}$

$\therefore \frac{V_o}{V_i} = \frac{-4\omega_0 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$\left.\begin{array}{l} |T|_{s=0} = 0 \\ |T|_{s=\infty} = 0 \end{array}\right\}$ Bandpass

$|T(j\omega_0)| = 4/\tfrac{1}{2} = \underline{\underline{8\tfrac{V}{V}}}$ = CENTRE FREQ GAIN

$\boxed{12.58}$



$RC = 2Q/\omega_0$

$k = \frac{R_2}{R_1+R_2}$

$V_{+ve} = V_{-ve} = kV_i$ due to virtual short

$$I_1 = -I_2$$

$$\frac{kV_i - V_o}{R} = \frac{V - kV_i}{1}(sC)$$

$$V = \frac{1}{sCR}\left(kV_i - V_o + sCRkV_i\right)$$

ΣI at $V = 0$

$$I_2 + I_3 - I_4 = 0$$

$$sC(kV_i - V) + \frac{4Q^2}{R}(V_i - V) - sC(V - V_o) = 0$$

$$sC\left(kV_i - \frac{kV_i}{sCR} + \frac{V_o}{sCR} - kV_i\right)$$
$$+ \frac{4Q^2}{R}\left(V_i - \frac{kV_i}{sCR} + \frac{V_o}{sCR} - kV_i\right)$$
$$- sC\left(\frac{kV_i}{sCR} - \frac{V_o}{sCR} + kV_i - V_o\right) = 0$$

$\Rightarrow$

$$-\frac{kV_i}{\cancel{R}} + \frac{V_o}{\cancel{R}}$$
$$+ \frac{4Q^2}{\cancel{R}}\left(V_i - \frac{kV_i}{sCR} + \frac{V_o}{sCR} - kV_i\right)$$
$$- \frac{sC}{\cancel{R}}\left(\frac{kV_i}{sC} - \frac{V_o}{sC} + kRV_i - V_oR\right) = 0$$

$\Rightarrow$ SUB $CR = \frac{2Q}{\omega_0}$ & $R = \frac{2Q}{C\omega_0}$

$$-kV_i + V_o$$
$$+ 4Q^2V_i - \frac{{}^{2}\cancel{4}Q^{\cancel{2}}kV_i\,\omega_0}{s\,\cancel{2}\cancel{Q}} + \frac{V_o\,\omega_0\,\cancel{4}Q^{\cancel{2}}}{s\,\cancel{2}\cancel{Q}} - 4Q^2kV_i$$
$$-kV_i + V_o - sk\,2Q/\omega_0\,V_i + sV_o\frac{2Q}{\omega_0} = 0$$

$$V_o\left[1+\frac{2Q\omega_o}{s}+1+\frac{2Qs}{\omega_o}\right]$$

$$=V_i\left[k-4Q^2+\frac{2kQ\omega_o}{s}+4Q^2k+k+\frac{2kQs}{\omega_o}\right]$$

$$\Rightarrow V_o\left[s^2\frac{2Q}{\omega_o}+2s+2Q\omega_o\right]=V_i\left[s^2\frac{2kQ}{\omega_o}+s(4Q^2k-4Q^2+2k)+2kQ\omega_o\right]$$

$$\Rightarrow \frac{V_o}{V_i}=\frac{s^2\frac{2kQ}{\omega_o}+s(4Q^2k-4Q^2+2k)+2kQ\omega_o}{s^2\frac{2Q}{\omega_o}+2s+2Q\omega_o}$$

$$=k\,\frac{s^2+s\frac{\omega_o}{Q}\left(2Q^2-\frac{2Q^2}{k}+1\right)+\omega_o^2}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$$

Recall $k=\frac{R_2}{R_1+R_2}$ and $\frac{1}{k}=1+\frac{R_1}{R_2}$

$$\Rightarrow \frac{V_o}{V_i}=\left(\frac{R_2}{R_1+R_2}\right)\frac{s^2+s\frac{\omega_o}{Q}\left(1-\frac{R_1}{R_2}\cdot 2Q^2\right)+\omega_o^2}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$$

$$\therefore T(s)=\frac{R_2}{R_1+R_2}\,\frac{s^2+s\frac{\omega_o}{Q}\left(1-\frac{2Q^2R_1}{R_2}\right)+\omega_o^2}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$$

For All Pass

we want $T(s)\propto\frac{s^2+\frac{\omega_o}{Q}(-1)s+\omega_o^2}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$

$$\Rightarrow 1-\frac{2Q^2R_1}{R_2}=-1$$

$$2Q^2\frac{R_1}{R_2}=2$$

$$\frac{R_1}{R_2}=\frac{1}{Q^2}$$

$$\therefore \underline{\underline{\frac{R_2}{R_1}=Q^2}} \quad \& \quad \frac{R_2}{R_1+R_2}=\frac{R_2/R_1}{1+R_2/R_1}=\underline{\underline{\frac{Q^2}{1+Q^2}}}$$

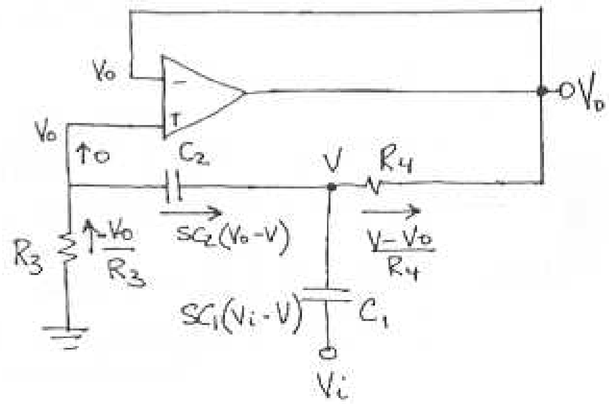
For Notch:

$$1-2Q^2\frac{R_1}{R_2}=0$$

$$\frac{R_1}{R_2}=\frac{1}{2Q^2}$$

$$\underline{\underline{\frac{R_2}{R_1}=2Q^2}} \quad \& \quad \underline{\underline{\frac{R_2}{R_1+R_2}=\frac{2Q^2}{1+2Q^2}}}$$

12.59



$\because$ No current can't flow into the terminal

$$-\frac{V_o}{R_3}=sC_2(V_o-V)$$

$$V=V_o\left(1+\frac{1}{sC_2R_3}\right)$$

CONT.

ΣI @ $V = 0$

$$-\frac{V_o}{R_3} + \frac{V_i - V}{1} sC_1 = \frac{V - V_o}{R_4}$$

$$V_o\left[-\frac{1}{R_3} + \frac{1}{R_4}\right] + V\left[-sC_1 - \frac{1}{R_4}\right] = -sC_1 V_i$$

$$V_o[R_4 - R_3] + V[sC_1R_3R_4 + R_3] = V_i sC_1R_3R_4$$

$$V_o(R_4 - R_3) + V_o\left(1 + \frac{1}{sC_2R_3}\right)(sC_1R_3R_4 + R_3)$$
$$= sC_1R_3R_4 V_i$$

$$V_o\left(R_4 - \cancel{R_3} + sC_1R_3R_4 + \cancel{R_3} + \frac{C_1}{C_2}R_4 + \frac{1}{sC_2}\right)$$
$$= sC_1R_3R_4V_i$$

$$V_o\left(s^2C_1C_2R_3R_4 + sC_1R_4 + sC_2R_4 + 1\right)$$
$$= s^2C_1R_3R_4C_2V_i$$

$$\therefore \frac{V_o}{V_i} = \frac{s^2C_1C_2R_3R_4}{s^2C_1C_2R_3R_4 + sC_1R_4 + sC_2R_4 + 1}$$

$$= \frac{s^2}{s^2 + s\left(\frac{1}{C_2R_3} + \frac{1}{C_1R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

Note $\left.\begin{array}{l}|T(0)| = 0 \\ |T(\infty)| = 1\end{array}\right\}$ $\therefore$ HighPass
High Freq Gain $= 1 \frac{V}{V}$

3dB freq $= 10^3$ rad/s, $Q = \frac{1}{\sqrt{2}}$ for max flat.

$\therefore \omega_o = 10^3 \frac{rad}{s}$ $C_1 = C_2 = 10nF$

clearly $\omega_o^2 = \frac{1}{C_1C_2R_3R_4}$ and

$$\frac{\omega_o}{Q} = \frac{1}{C_2R_3} + \frac{1}{C_1R_3} = \frac{C_1 + C_2}{C_1C_2R_3}$$

$$= \frac{2C}{C^2R_3} = \frac{2}{CR_3} = \sqrt{2} \times 10^3$$

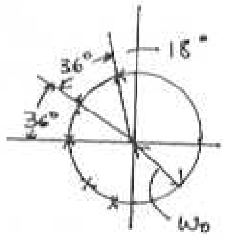
$$R_3 = \frac{2}{10 \times 10^{-9} \times 10^3 \times \sqrt{2}}$$

$R_3 = 141.4\ k\Omega$

$R_4 = \frac{1}{\omega_o^2 C_1C_2R_3} \Rightarrow R_4 = 70.7k\Omega$

12.60

$A_{max} = 3dB$



$\epsilon = \left(10^{3/10} - 1\right)^{-\frac{1}{2}} \cong 1$

$\omega_o = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} = \omega_p = 2\pi 5000 = 10^4\pi$

$Q_1 = \frac{1}{2\cos 36} = 0.618$

$Q_2 = \frac{1}{2\cos 72} =$

For first order section:

$\omega_o = 10^4\pi$ dc gain $= 1$

From 12.13 (a)

$R_1 = R_2 = 10k\Omega$

$C = \frac{1}{\omega_o R_2} = \frac{1}{10^4\pi 10^4} = 3.18\ nF$

Second-Order Section $Q = 0.618$:

from 12.34 (c): $m = 4Q^2 = 1.528$

CONT.

$RC = \dfrac{2Q}{\omega_0}$ let $\underline{\underline{R_1 = R_2 = 10k\Omega}}$

$C = \dfrac{2Q}{\omega_0 R} \Rightarrow \underline{\underline{C_4 = C = 3.93nF}}$

$\underline{\underline{C_3 = \dfrac{C}{m} = 2.57nF}}$

Second Order Section $Q = 1.618$:

$C = \dfrac{2Q}{\omega_0 R}$ $m = 4Q^2 = 10.472$

$= 10.3nF \Rightarrow \underline{\underline{R_1 = R_2 = 10k\Omega}}$

$\underline{\underline{C_4 = C = 10.3nF}}$

$\underline{\underline{C_3 = \dfrac{C}{m} = 0.984nF}}$

12.61

For a bandpass filter

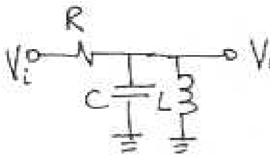
$$t(s) = \frac{\omega_0/Q\, s}{s^2 + s\omega_0/Q + \omega_0^2}$$

centre freq. gain = 1

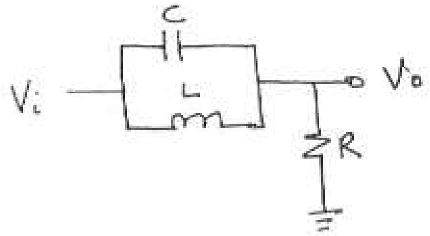
complementary transfer function:

$$t' = 1 - t = \frac{s^2 + \omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2} \equiv \text{NOTCH!}$$

From FIG 12.18(d)



$\Rightarrow$ INTERCHANGE Vi & gnd to get:



this is the same as Fig 12.18(e)

12.62

for Fig 12.18(d):

$$T(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = R\sqrt{\frac{C}{L}}$$

FOR ω_0

$$\frac{\partial \omega_0}{\partial L} = \frac{\partial (LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{-\omega_0}{2L}$$

$$\frac{\partial \omega_0}{\partial C} = -\frac{\omega_0}{2C}$$

$$\frac{\partial \omega_0}{\partial R} = 0$$

$$\therefore S_L^{\omega_0} = \frac{\partial \omega_0}{\partial L}\frac{L}{\omega_0} = \underline{\underline{-1/2}}$$

$$S_C^{\omega_0} = \frac{\partial \omega_0}{\partial C} \times \frac{C}{\omega_0} = \underline{\underline{\frac{-1}{2}}}$$

$$S_R^{\omega_0} = \frac{\partial \omega_0}{\partial R}\frac{R}{\omega_0} = \underline{\underline{0}}$$

FOR Q

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right)$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

CONT.

$$S_L^Q = \frac{-Q}{2L} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = \frac{Q}{R} \cdot \frac{R}{Q} = 1$$

12.63

$y = uv$

$$S_x^y = \frac{\partial(uv)}{\partial x} \frac{x}{uv}$$

$$= v\frac{\partial u}{\partial x} \frac{x}{uv} + u\frac{\partial v}{\partial x} \frac{x}{uv}$$

$$= \frac{\partial u}{\partial x} \frac{x}{u} + \frac{\partial v}{\partial x} \frac{x}{v}$$

$$= S_x^u + S_x^v$$

Part (b) $\quad y = u/v$

$$S_x^y = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(u/v)}{\partial x} \frac{xv}{u}$$

$$= \frac{1}{v} \frac{\partial u}{\partial x} \frac{vx}{u} + \frac{-u}{v^2} \frac{\partial v}{\partial x} \cdot \frac{xv}{u}$$

$$= \frac{\partial u}{\partial x} \frac{x}{u} - \frac{\partial v}{\partial x} \frac{x}{v}$$

$$= S_x^u - S_x^v$$

Part (c) $\quad y = ku$

$$S_x^y = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(ku)}{\partial x} \frac{x}{ku}$$

$$= k\frac{\partial u}{\partial x} \frac{x}{ku}$$

$$= \frac{\partial u}{\partial x} \frac{x}{u}$$

$$= S_x^u$$

Part (d) $\quad y = u^n$

$$S_x^y = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(u^n)}{\partial x} \frac{x}{u^n}$$

$$= nu^{n-1} \frac{\partial u}{\partial x} \cdot \frac{x}{u^n}$$

$$= n\frac{\partial u}{\partial x} \frac{x}{u} = nS_x^u$$

Part (e) $\quad y = f_1(u) \quad u = f_2(x)$

$$S_x^y = \frac{\partial y}{\partial x} \frac{x}{y} = \frac{\partial f_1(u)}{\partial x} \frac{x}{f_1(u)}$$

$$= \frac{\partial f_1(u)}{\partial x} \frac{\partial u}{\partial x} \cdot \frac{x}{f_1(u)}$$

$$= \frac{\partial f_1}{\partial u} \cdot \frac{\partial f_2}{\partial x} \cdot \frac{x}{f_1} \cdot \frac{u}{u}$$

But $u = f_2$

$$\therefore S_x^y = \frac{\partial f_1}{\partial u} \cdot \frac{\partial f_2}{\partial x} \cdot \frac{x}{f_1} \cdot \frac{u}{f_2}$$

$$= \frac{\partial f_1}{\partial u} \cdot \frac{u}{f_1} \cdot \frac{\partial f_2}{\partial x} \frac{x}{f_2}$$

$$= S_u^{f_1} \cdot S_x^{f_2}$$

$$= S_u^y \, S_x^u$$

12.64

Since the characteristic equation of Fig. 12.33 (b) is the same as for Fig. 12.29 the poles are given by Eq 12.86:

$$s^2 + s\frac{\omega_0}{Q}\left[1 + \frac{2Q^2}{A+1}\right] + \omega_0^2 = 0$$

This is because 12.33 (b) is based on the complementary transform of 12.29 and hence the pole

locations are preserved!
Now the actual ω_o and Q are given by:

$$\omega_{o,a} = \omega_o \quad \text{and} \quad Q_a = \frac{Q}{1+\frac{2Q^2}{(A+1)}}$$

Thus from Ex. 12.3

$$S_A^{\omega_{o,a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1} \quad \frac{2Q^2/(A+1)}{1+2Q^2/(A+1)}$$

$$\therefore S_A^{Q_a} \cong \frac{2Q^2}{A}$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^{Q} = +1/2$$

$$\frac{\partial Q}{\partial R_1} = \frac{1/\sqrt{R_1} - \sqrt{R_1}/R_2}{R_1\left(\frac{1}{\sqrt{R_1}} + \sqrt{R_1}/R_2\right)} \cdot \frac{Q}{2}$$

$$= \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{R_1\left(\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}}\right)} \cdot \frac{Q}{2}$$

$$\therefore S_{R_1}^{Q} = \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{\sqrt{R_2/R_1} + \sqrt{R_1/R_2}}$$

if $R_1 = R_2 \Rightarrow S_{R_1}^{Q} = 0$ &

$$S_{R_2}^{Q} = 0$$

12.65

If $R_1 = R_2$, then from (12.77) & (12.78)

$$\omega_o = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}}$$

$$Q = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$\frac{\partial \omega_o}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3 C_4 R_1 R_2}}$$

$$\frac{\partial \omega_o}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3 C_4 R_1 R_2}} = -\frac{\omega_o}{2C_3}$$

$$S_{C_3}^{\omega_o} = \frac{\partial \omega_o}{\partial C_3}\frac{C_3}{\omega_o} = -\frac{1}{2}$$

clearly $S_{C_3}^{\omega_o} = S_{C_4}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = -1/2$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3 C_4 R_1 R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^{Q} = -1/2$$

12.66

From table 12.1

$$\omega_o = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5/R_2}}$$

$$Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1 R_3 R_5}}$$

$$\frac{\partial \omega_o}{\partial C_4} = \frac{-\omega_o}{2C_4}$$

$$\therefore S_{C_4}^{\omega_o} = \frac{-\omega_o}{2C_4} \times \frac{C_4}{\omega_o} = -1/2$$

Similarly $S_{C_6}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_3}^{\omega_o} = S_{R_5}^{\omega_o} = -\frac{1}{2}$

$$\frac{\partial \omega}{\partial R_2} = \frac{\omega_o}{2R_2} \Rightarrow S_{R_2}^{\omega_o} = \frac{1}{2}$$

Now for Q:

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^{Q} = \frac{\partial Q}{\partial R_6}\frac{R_6}{Q} = +1$$

CONT.

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^{Q} = S_{R_2}^{Q} = +\underline{\underline{\frac{1}{2}}}$$

$$\frac{\partial Q}{\partial C_4} = \frac{-Q}{2C_4} \Rightarrow S_{C_4}^{Q} = S_{R_1, R_3, R_5}^{Q} = -\underline{\underline{\frac{1}{2}}}$$

12.67

$$R_{eq} = \frac{T_c}{C_1} = \frac{1/100\times10^3}{C_1}$$

for $1pF \rightarrow R_{eq} = 10^{-5}/10^{-12} = \underline{\underline{10\,M\Omega}}$

for $10pF \rightarrow R_{eq} = 10^{-5}/10\times10^{-12} = \underline{\underline{1\,M\Omega}}$

12.68

charge transferred $\Rightarrow Q = CV$
$= 10^{-12}(1)$
$= \underline{\underline{1pC}}$

For $f_0 = 100kHz$, average current is given by:

$$I_{AVE} = \frac{Q}{T} = 1pC \times \frac{1}{100\times10^3}$$
$$= \underline{\underline{0.1\,\mu A}}$$

For each clock cycle, the output will change by the same amount as the change in voltage across C_2!

$$\therefore \Delta V = Q/C_2 = \frac{1pC}{10pF} = \underline{\underline{0.1V}}$$

For $\Delta V = 0.1V$ for each clock cycle, the amplifier will saturate in

$$\#\text{cycles} = \frac{10V}{0.1V} = \underline{\underline{100 \text{ cycles}}}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10V}{(100 \text{ cycles})(1/100\times10^3)}$$
$$= \underline{\underline{10^4 \frac{V}{s}}}$$

12.69

$f_c = 400kHz$ $f_0 = 10kHz$ $Q = 20$

$C_1 = C_2 = 20pF = C$

$$C_3 = C_4 = \omega_0 T_c C = 2\pi(10^4)\frac{1}{400\times10^3}20\times10^{-12}$$
$$= \underline{\underline{3.14pF}}$$

$$C_5 = \frac{\omega_0 T_c C}{Q}$$
$$= \frac{C_3}{Q} = \underline{\underline{0.157pF}}$$

$$C_6 = \frac{\omega_0 T_c C}{Q} \times \text{centre frequency gain}$$
$$= \underline{\underline{0.157\,pF}}$$

Note that the clock frequency has doubled. Hence the period, T_c, is halved. Therefore, for the same integrating capacitors, the resistors (switched capacitors) will change by the factor of 2. So compensate for this by changing the switched caps by a factor of 1/2.

12.70

Ex 12-31 for $Q = 40$ $f_c = 200kHz$
$f_0 = 10kHz$

$C_1 = C_2 = 20pF = C$

$$C_3 = C_4 = \omega_0 T_c C$$
$$= 2\pi(10^4)\left(\frac{1}{200\times10^3}\right)20\times10^{-12}$$
$$= \underline{\underline{6.28\,pF}}$$

CONT.

$$C_5 = \frac{\omega_0 T_c C}{Q} = \frac{C_3}{Q} = \underline{\underline{0.157\,pF}}$$

$$C_6 = \frac{\omega_0 T_c C}{Q} = C_5 = \underline{\underline{0.157\,pF}}$$

12.71

$\omega_0 = 10^4$, $Q = 1/\sqrt{2}$, $f_c = 100\,kHz$

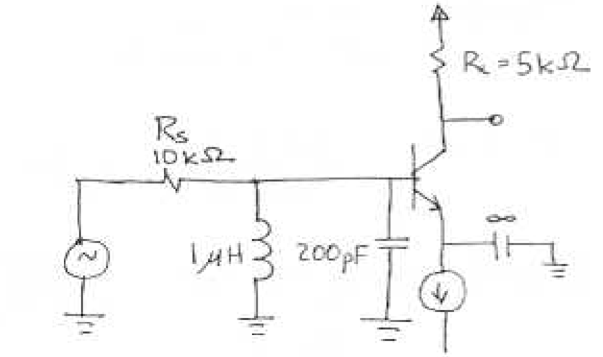
DC gain $\Rightarrow \frac{R_4}{R_6} \Rightarrow \frac{C_6}{C_4} = 1$

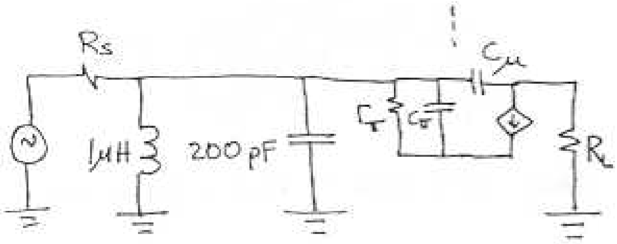
$C_1 = C_2 = 10\,pF$

$$C_3 = C_4 = C_6 = \omega_0 T_c C = 10^4\left(\frac{1}{100\times10^3}\right) 10\times10^{-12} = \underline{1\,pF}$$

$$C_5 = C_4/Q = \underline{\underline{1.41\,pF}}$$

12.72





$r_e = 25\,\Omega$, $C_\mu = 1\,pF$, $C_\pi = 10\,pF$, $\beta = 200$

$r_\pi = (\beta+1) r_e = 5.025\,k\Omega$

From base to collector

$$\frac{V_c}{V_b} = -\frac{\beta}{\beta+1} \cdot \frac{R_L}{r_e} = -199 = k$$

Total capacitance at base

$$C_T = C_\pi + 200p + C_\mu(1-k)$$ Miller Effect

$$= 10 + 200 + 1(1+199)$$
$$= 410\,pF$$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}\times 410\times10^{-12}}} = \underline{\underline{49.4\times10^6\ rad/s}}$$

Centre frequency gain =

$$\frac{r_\pi}{R_s + r_\pi} \cdot k = \frac{5.025}{10+5.025} \times -199 = \underline{\underline{-66.6\ V/V}}$$

$$BW = \frac{1}{RC} = \frac{1}{(R_s \| r_\pi)\, 410\,pF} = \underline{\underline{729\times10^3\ \frac{rad}{s}}}$$

$$Q = \frac{\omega_0}{BW} = 49.4/0.7293 = \underline{\underline{67.7}}$$

12.73

$$Q_0 = \frac{R_p}{\omega_0 L} \Rightarrow R_p = Q_0 \omega_0 L$$
$$= 200(2\pi 10^6)(10\times10^{-6})$$
$$= \underline{\underline{12.57\,k\Omega}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L}$$
$$= \frac{1}{(2\pi 10^6)^2\, 10\times10^{-6}}$$
$$= \underline{\underline{2.533\,nF}}$$

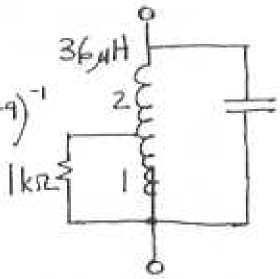
$$B = \frac{1}{RC} \qquad R_T = \frac{1}{(2\pi\times10\times10^3)(2.533\times10^{-9})}$$
$$= 6.283\,k\Omega$$

$$\therefore \frac{1}{R_1} + \frac{1}{R_P} = \frac{1}{R_T}$$

$$\Rightarrow R_1 = \underline{\underline{12.57\,k\Omega}} \quad \text{i.e. } R_1 // R_p = R_T$$

12.74

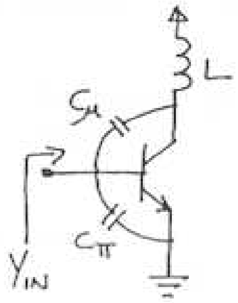
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$
$$= \left(2\pi(36\times10^{-6})\,10^{-9}\right)^{-1}$$
$$= \underline{\underline{838.8\,kHz}}$$



$$R_p = n^2 R$$
$$= 9\,(1k\Omega)$$
$$= 9k\Omega$$

$$Q = R_P/\omega_0 L$$
$$= \frac{9\times10^3}{2\pi\, 838.8\times10^3 \times 36\times10^{-6}}$$
$$= \underline{47.4}$$

12.75



for $\omega C_\mu \ll \frac{1}{\omega L}$

$$\therefore \omega^2 \ll \frac{1}{L C_\mu}$$

ie well below resonance

$$\therefore \text{gain} = -g_m(j\omega L)$$

$$\therefore Y_{in} = \frac{1}{r_\pi} + j\omega C_\pi + j\omega C_\mu(1 + g_m j\omega L)$$
$$= \left(\frac{1}{r_\pi} - \omega^2 g_m C_\mu L\right) + j\omega\,(C_\pi + C_\mu)$$

AS REQUIRED!

12.76

From Fig 12.16 (c):

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j a_1 \omega}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}}$$

$$T(j\omega_0) = \frac{j a_1 \omega_0}{j\omega_0^2/Q} = \frac{a_1 Q}{\omega_0}$$

$$|T(j\omega)| = a_1 \omega \left[(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{a_1 \omega\; Q/\omega\omega_0}{\sqrt{1 + Q^2\left(\frac{\omega_0^2 - \omega^2}{\omega_0 \omega}\right)^2}}$$

Now $\omega = \omega_0 + \delta\omega$, $\frac{\delta\omega}{\omega_0} \ll 1$

and $\omega^2 \simeq \omega_0^2\left(1 + \frac{2\delta\omega}{\omega_0}\right)$

CONT.

so $\omega_0^2-\omega^2 = -2\delta\omega\omega_0$

$\therefore \quad |T(j\omega)| \cong \dfrac{a_1 Q/\omega_0}{\sqrt{1+Q^2\left(\frac{2\delta\omega}{\omega}\right)^2}}$

for $Q>>1$: $Q^2\left(\frac{2\delta\omega}{\omega}\right)^2 \cong Q^2\left(\frac{2\delta\omega}{\omega_0}\right)$

$\therefore \omega \approx \omega_0$!

$\Rightarrow |T(j\omega)| \cong \dfrac{|T(j\omega_0)|}{\sqrt{1+Q^2\left(\frac{2\delta\omega}{\omega_0}\right)^2}}$

$= \dfrac{|T(j\omega_0)|}{\sqrt{1+4\,Q^2\frac{\delta\omega}{\omega_0}}}$

For N bandpass sections, synchronously tuned in cascade, half power is given by:

$$\left(\frac{1}{\sqrt{1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2}}\right)^N = \frac{1}{\sqrt{2}}$$

$$\left(1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2\right)^N = 2$$

$$4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2 = 2^{1/N}-1$$

$$\delta\omega = \frac{\omega_0}{2Q}\sqrt{2^{1/N}-1}$$

$\therefore$ Bandwidth:

$$\underline{\underline{B = 2\delta\omega = \frac{\omega_0}{Q}\sqrt{2^{1/N}-1}}}$$

12.77

For first order low pass:

$T(s) = \dfrac{\omega_0'}{s+\omega_0'} \qquad |T(j\omega)| = \dfrac{\omega_0'}{\sqrt{\omega^2+\omega_0'^2}}$

for a bandpass response around ω_0 with $\omega_0' = \dfrac{\omega_0}{2Q}$:

$$|T(j\omega)| \cong \frac{\omega_0/2Q}{(\delta\omega)^2+\left(\frac{\omega_0}{2Q}\right)^2}$$

$$= \frac{\omega_0/2Q}{\frac{\omega_0}{2Q}\sqrt{\left(\frac{2Q}{\omega_0}\right)^2(\delta\omega)^2+1}}$$

$$= \frac{1}{\sqrt{1+4Q^2\left(\delta\omega/\omega_0\right)^2}}$$

Now at $\omega=\omega_0$ or $\delta\omega=0$ $|T(j\omega_0)|=1$, then

$$T(j\omega) \simeq \frac{|T(j\omega_0)|}{\sqrt{1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2}}$$

Part (b)

For N synchronously tuned sections in cascade; 3dB bandwidth is given by:

$$\left(|T|/|T_0|\right)^N = \frac{1}{\sqrt{2}}$$

$$\left(|T|/|T_0|\right)^2 = \frac{1}{2^{1/N}} \quad \text{OR}$$

$$1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2 = 2^{1/N} \quad \text{OR}$$

$$2\delta\omega = \frac{\omega_0}{Q}\sqrt{2^{1/N}-1} \qquad (12.110)$$

CONT.

Thus : $|T(j\omega)|_{overall} = |T(j\omega)|^N$

$$= \frac{|T(j\omega_0)|_{overall}}{\left[1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2\right]^{N/2}}$$

NOTE $Q = \frac{\omega_0}{B}\sqrt{2^{1/N}-1}$

$$= \frac{|T(j\omega_0)|_{overall}}{\left(1+4\frac{\omega_0^2}{B^2}\left(2^{1/N}-1\right)\left(\frac{\delta\omega}{\omega_0}\right)^2\right)^{\frac{N}{2}}}$$

$$= \frac{|T(j\omega_0)|_{overall}}{\left[1+4\left(2^{1/N}-1\right)\left(\frac{\delta\omega}{B}\right)^2\right]^{\frac{N}{2}}}$$

Part (c) (i)

for bandwidth = 2B, ie $\delta\omega = \pm B$

$$Att = -20\log\left(1+4\left(2^{1/N}-1\right)(1)\right)^{-N/2}$$

$$= -10N\log\left(1+2^{2+1/N}-4\right)$$

$$= 10N\log\left(2^{2+1/N}-3\right)$$

N =	1	2	3	4	5
Att (dB)	6.70	8.49	9.28	9.79	10.13

Part (c) (ii)

3 dB bandwidth $\delta\omega = \pm B/2$

30dB bandwidth $\frac{\delta\omega}{B} = x$

$$-30 = -20\frac{N}{2}\log\left(1+4\left(2^{1/N}-1\right)x^2\right)$$

$$3 = N\log\left(1+4\left(2^{1/N}-1\right)x^2\right)$$

$$x = \left[\frac{10^{3/N}-1}{4\left(2^{1/N}-1\right)}\right]^{1/2}$$

Ratio of 30 dB to 3dB

$$BW = \frac{2Bx}{B} = 2x$$

N =	1	2	3	4	5
Ratio =	31.6	8.6	5.7		4.5

12.78

See fig 12.48 and Eq 12.115 and 12.116

(a) For the narrowband approximation variation of Ω around 0 is equivalent to $\delta\omega$ around ω_0. Thus, a low-pass maximally flat filter of bandwidth B/2 and order N for which $|T| = \left[\left(1+ \Omega/_{B/2}\right)^{2N}\right]^{-\frac{1}{2}}$

is transformed to a band-pass maximally flat filter of bandwidth B/2 and order 2N, and centre frequency ω_0, for which:

$$|T| = \left(1+\left(\frac{\delta\omega}{B/2}\right)^{2N}\right)^{-1/2}$$

(b) For bandwidth 2B, $\delta\omega = B$ &

$$|T| = \left(1+\left(\frac{B}{B/2}\right)^{2N}\right)^{-1/2}$$

$$= \left(1+2^{2N}\right)^{-1/2}$$ thus :

N	1	2	3	4	5
$\lvert T\rvert$	0.447	0.242	0.124	0.062	0.031
$\lvert T\rvert_{dB}$	-6.99	-12.3	-18.1	-24.1	-30.1

For 30 dB bandwidth,

$$-30 = 20\log x \Rightarrow x = 10^{-3/2} = \frac{1}{31.6}$$

$$\therefore \quad 1 + \left(\frac{\delta' \omega}{B/2}\right)^{2N} = (31.6)^2$$

$$\left(\frac{\delta' \omega}{B/2}\right)^{2N} = 999 - 1 = 998$$

Now the ratio of 30dB to 3dB bandwidths is

$$\text{ratio} = \frac{2\delta' \omega}{B} = \frac{\delta' \omega}{B/2} = 998^{\frac{1}{2N}}$$

N	1	2	3	4	5
ratio	31.6	5.62	3.16	2.37	1.99

12.79

$$A_{max} = 3\text{dB} \Rightarrow \epsilon = \sqrt{10^{A_{max}/10} - 1} \approx 1$$

Poles of low pass prototype are given by FIG 12.10(c)

Poles: $-\omega_p$, $\omega_p\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)$

Make $\omega_p = B/2$

$$\Rightarrow \text{poles: } \left\{-\frac{B}{2},\ +\frac{B}{2}\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)\right\}$$

Using the low-pass to bandpass transformation :

Poles of the bandpass filter :

$-\frac{B}{2} \pm j\omega_0$,

$-\frac{B}{4} \pm j\left(\frac{\sqrt{3}}{4}B + \omega_0\right)$ and

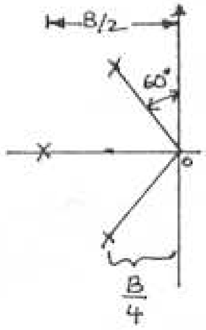
$-\frac{B}{4} \pm j\left(\frac{\sqrt{3}}{4}B - \omega_0\right)$

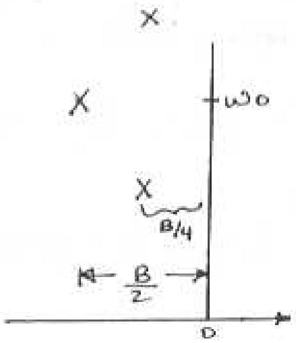
For the three circuits :

① $\omega_{01} = \omega_0$ $\quad B_1 = B$ $\quad Q_1 = \omega_0/B$

② $\omega_{02} \simeq \frac{\sqrt{3}}{4}B + \omega_0$ $\quad B_2 = \frac{B}{2}$ $\quad Q_2 \simeq \frac{2\omega_0}{B}$

③ $\omega_{03} \simeq \frac{\sqrt{3}}{4}B - \omega_0$ $\quad B_3 = \frac{B}{2}$ $\quad Q_3 \simeq \frac{2\omega_0}{B}$

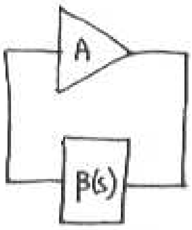




CHAPTER 13 - PROBLEMS

13.1

$$A = A_0 > 0$$



$$\beta(s) = \frac{k\frac{\omega_0}{Q}s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

(a) for oscillations $1 - A\beta(s) = 0$

$$\therefore A_0 K \frac{\omega_0}{Q} s = s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$

$$\omega_0^2 - \omega^2 = j\omega\left(\frac{\omega_0}{Q}\right)(A_0K - 1)$$

at the freq of oscillation, both Real & Imaginary parts are 0.

$$\therefore \underline{\underline{\omega = \omega_0}} \quad \& \quad \underline{\underline{A_0K = 1}}$$

(b)

$$L(j\omega) \triangleq A\beta(j\omega) = \frac{AK\frac{\omega_0}{Q}j\omega}{(\omega_0^2 - \omega^2) + j\omega\left(\frac{\omega_0}{Q}\right)}$$

$$\therefore \phi(\omega) = 90° - \tan^{-1}\left(\frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2}\right)$$

Now $\frac{\partial}{\partial x}\tan^{-1} v = \frac{1}{1+v^2}\cdot\frac{\partial v}{\partial x}$

$$\therefore \frac{\partial\phi}{\partial\omega} = \frac{-1}{1 + \left(\frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2}\right)^2}\cdot\frac{\partial}{\partial\omega}\left(\frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2}\right)$$

$$= \frac{-(\omega_0^2 - \omega^2)^2}{(\omega_0^2 - \omega^2) + \left(\frac{\omega\omega_0}{Q}\right)^2}\cdot\left[\frac{\frac{\omega_0}{Q}(\omega_0^2 - \omega^2) - 2\omega\frac{\omega\omega_0}{Q}}{(\omega_0^2 - \omega^2)^2}\right]$$

$$\left.\frac{d\phi}{\partial\omega}\right|_{\omega=\omega_0} = \frac{-1}{\omega_0^4/Q^2}\cdot\frac{2\omega_0^3}{Q}$$

$$= \underline{\underline{-\frac{2Q}{\omega_0}}}$$

(c) $$\Delta\omega_0 = \frac{\Delta\phi}{\partial\phi/\partial\omega} = \frac{\Delta\phi}{-2Q/\omega_0}$$

$$= \frac{-\Delta\phi\,\omega_0}{2Q}$$

$\therefore$ Per unit change in ω_0 is given by

$$\underline{\underline{\frac{\Delta\omega_0}{\omega_0} = -\frac{\Delta\phi}{2Q}}}$$

13.2

For the circuit of problem 13.1, the poles, which are the zeros of the characteristic equation, are given by:

$$1 - L(s) = 0$$

$$L(s) = 1$$

$$\frac{AK\left(\frac{\omega_0}{Q}\right)s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = 1$$

$$s^2 + s\frac{\omega_0}{Q}(1 - AK) + \omega_0^2 = 0$$

$\therefore$ Poles are at:

$$s = \frac{-\frac{\omega_0}{Q}(1-AK) \pm \sqrt{\left(\frac{\omega_0}{Q}\right)^2(1-AK)^2 - 4\omega_0^2}}{2}$$

$$= -\omega_0\left[\frac{1-AK}{2Q} \pm \sqrt{\left(\frac{1-AK}{2Q}\right)^2 - 1}\right]$$

$$= -\omega_0\left(\frac{1-AK}{2Q}\right)\left[1 \pm j\sqrt{\left(\frac{2Q}{1-AK}\right)^2 - 1}\right]$$

CONT.

Radial distance of w_0 =>

$$|s^2| = w_0^2 \left(\frac{1-AK}{2Q}\right)^2 \left[1 + \left(\frac{2Q}{1-AK}\right)^2 - 1\right]$$

$$= w_0^2$$

$\therefore$ $|s| = w_0$ ~ independent of A or K!

(a) For poles on $j\omega$- axis => real part = 0

$\therefore$ $-(1-AK) = 0$ => $AK = 1$

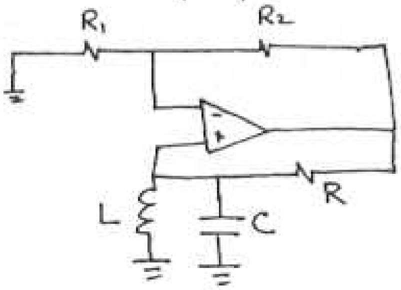
(b) For poles in RHS => Real Part > 0

$-(1-AK) > 0$

$AK > 1$

13.3

Assume ideal opamp.



At resonance $w_0 = \frac{1}{\sqrt{LC}}$

$K = 1$

Thus A must also be 1 (or slightly higher).

For $R_1 = 10k\Omega$, $R_2 = 100\Omega$

$$A = 1 + \frac{R_2}{R_1} = 1.01$$

(a) If $L' = 1.01L$

$$w_0' = \frac{1}{\sqrt{1.01}} w_0 = 0.995 w_0$$

or $-\frac{1}{2}\%$

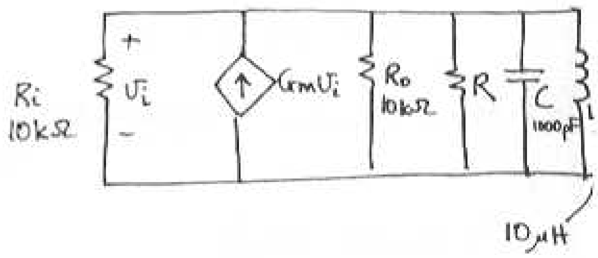
(b) If C changes by +1%

=> w_0 changes by $-\frac{1}{2}\%$

(c) If R changes by +1%

=> w_0 ~ unchanged

13.4



For resonator: $w_0 = \frac{1}{\sqrt{LC}} = 10^7 \frac{rad}{s}$

$$\frac{w_0}{Q} = \frac{1}{RC}$$

$$R = \frac{Q}{w_0 C} = \frac{100}{10^7 \times 1000 \times 10^{-12}} = 10k\Omega$$

Oscillations will occur at $w_0 = 10^7 \frac{rad}{s}$ when $G_m (R_i \| R_o \| R) = 1$ i.e. gain = 1

$$\therefore G_m = \frac{1}{10k \| 10k \| 10k} = \frac{3}{10^4} = 300 \frac{\mu A}{V}$$

13.5

At w_0 $A\beta = 1$

If $\beta(w_0)$ is $-20\,dB$ with a phase

CONT.

shift of 180° then clearly A should have a gain of 20dB (i.e. $A(\omega_0) = 10$) with a phase shift of ± 180°

i.e. $\underline{\underline{A = -10}}$

13.6

From Eq(13.8)

$$L_- = -V\frac{R_3}{R_2} - V_D\left(1 + \frac{R_3}{R_2}\right)$$

$$6 = 10\frac{R_3}{R_2} + 0.7\left(1 + \frac{R_3}{R_2}\right)$$

$$= 10.7\frac{R_3}{R_2} + 0.7$$

$\frac{R_3}{R_2} = 0.495$ By symmetry $\frac{R_4}{R_5} = 0.495$

Use $\underline{R_2 = R_5 = 10k\Omega}$

$\therefore \underline{\underline{R_3 = R_4 \cong 5k\Omega}}$

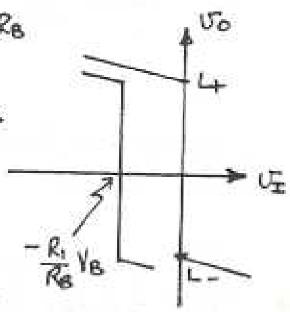
Slope of limiting characteristic

$$= \frac{R_4}{R_5} = 0.1$$

$\therefore R_1 = \frac{1}{0.1}R_4 = \underline{\underline{50k\Omega}}$

13.7

For V_B connected via R_B to the virtual ground, a current $= \frac{V_B}{R_B}$ flows into the node. To compensate, v_I must be moved by Δv_I, in a direction opposite to V_B to produce a current $\Rightarrow$



$$\frac{\Delta v_I}{R_1} = \frac{-V_B}{R_B}$$

$$\therefore \underline{\underline{\Delta v_I = -\frac{R_1}{R_B}V_B}}$$

$V_D = 0$ ~ assumed

$$L_- = -5 = -15\, R_3/R_2$$

$$\frac{R_3}{R_2} = \frac{1}{3} = R_4/R_5$$

Given $R_{in} = 100k\Omega \Rightarrow R_1 = \underline{\underline{100k\Omega}}$

Slope $= R_4/R_1 \leq 0.05$

$R_4 \leq R_1 \times 0.05$

$R_4 \leq 5k\Omega \Rightarrow$ let $\underline{\underline{R_4 = 4.3k\Omega}}$

$\therefore R_3 = R_4 \Rightarrow$ $\underline{\underline{R_3 = 4.3k\Omega}}$

$R_2 = R_5 = 3R_4 = 12.9k\Omega$

For standard resistance values:

$R_2 = R_5 = \underline{\underline{13k\Omega}}$

$$\therefore L_- = -15\frac{R_3}{R_2} = -15 \times \frac{4.3}{12.9}$$

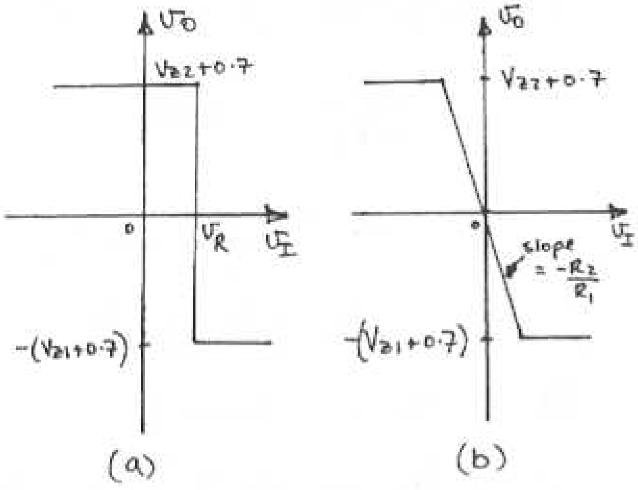
$$= -4.96V \cong -5V$$

Offset is $+5V \Rightarrow$ Use $V_B = -15V$

and $5 = R_1/R_B\ 15$

$\therefore R_B = 3R_1 = \underline{\underline{300k\Omega}}$

13.8



13.9

$$\frac{V_a}{V_o} = \frac{1/sC \,||\, R}{1/sC\,||\,R + 1/sC + R}$$

$$= \frac{\left(R/sC\right)/\left(\frac{1}{sC}+R\right)}{\frac{(R/sC)}{\left(\frac{1}{sC}+R\right)} + \frac{1}{sC} + R}$$

$$= \frac{\frac{R}{sC}}{\frac{R}{sC} + \left(\frac{1}{sC}+R\right)^2} \times \frac{s^2C^2}{s^2C^2}$$

$$= \frac{sCR}{sCR + (1+sCR)^2}$$

$$= \frac{sCR}{sCR + 1 + 2sCR + s^2C^2R^2}$$

$$= \frac{\frac{1}{RC}s}{s^2 + s\frac{3}{RC} + \frac{1}{R^2C^2}}$$

Note $\frac{V_a}{V_o}$ has zeros at 0 and ∞

i.e. A Bandpass!

$$\omega_0^2 = \frac{1}{R^2C^2} \Rightarrow \omega_0 = \frac{1}{RC}$$

$$\frac{\omega_0}{Q} = \frac{3}{RC} \Rightarrow Q = \frac{1}{3}$$

For centre frequency gain:

$s = j\omega_0 = j/RC$

$$\therefore \left.\frac{V_a}{V_o}\right|_{s=j/RC} = \frac{\frac{1}{RC}\,j/RC}{-\frac{1}{R^2C^2} + \frac{3}{RC}\left(\frac{j}{RC}\right) + \frac{1}{R^2C^2}}$$

$= \frac{1}{3}$ = centre freq. gain

13.10

$$L(j\omega) = \frac{1 + R_2/R_1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)} \qquad \text{Eq}(13.11)$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega CR - \frac{1}{\omega CR}}{3}\right)$$

using $\frac{\partial \tan^{-1} v}{\partial x} = \frac{1}{1+v^2}\frac{\partial v}{\partial x}$

$$\frac{\partial\phi}{\partial\omega} = \frac{-1}{1 + \left(\frac{\omega CR - \frac{1}{\omega CR}}{3}\right)^2} \cdot \frac{1}{3}\left(CR + \frac{1}{\omega^2 CR}\right)$$

$$\left.\frac{\partial\phi}{\partial\omega}\right|_{\omega=\frac{1}{RC}} = -\frac{1}{3}(CR + CR) = -\frac{2}{3}CR$$

for $\Delta\phi = -0.1$ rad

$$\Delta\omega_0 = \frac{\Delta\phi}{\partial\phi/\partial\omega} = \frac{-0.1}{-2/3\,\frac{1}{\omega_0}}$$

$$= 0.15\,\omega_0$$

$\therefore$ New frequency of oscillation

$$= 1.15\,\omega_0 = \frac{1.15}{RC}$$

13.11

Using Eq (13.10)

$$L(s) = \frac{1 + R_2/R_1}{3 + sCR + 1/sCR}$$

Poles of closed loop given by : $L(s) = 1$

$$1 + R_2/R_1 = 3 + sCR + \frac{1}{sCR}$$

$$0 = s^2 + \frac{s}{RC}\left(2 - \frac{R_2}{R_1}\right) + \frac{1}{R^2C^2}$$

$$Q = \frac{1}{(2 - R_2/R_1)}$$

for $Q = \infty$ ~ poles on $j\omega$ axis

~ $R_2/R_1 = 2$

For poles in R.H.P. $R_2/R_1 > 2$

13.12

From Fig(13.5) assuming resistance of limiting network is very low

At positive peak

$$v_O = \left(1 + \frac{20.3k}{10k}\right) v_I = 3.03\, v_I \quad (1)$$

$$v_O - \left[\frac{R_5}{R_5 + R_6} \cdot (v_O - (-15))\right] - 0.7 = v_I \quad (2)$$

Now for $10 V_{p\text{-}p}$ out

$\hat{v}_O = 5V$

$\hat{v}_I = \frac{5}{3.03} = 1.65\,V$

Using (2) $R_5 = 1k\Omega$

$$5 - \left(\frac{1}{1 + R_6} \cdot (V_O + 15)\right) - 0.7 = 1.65$$

$$\frac{20}{1 + R_6} = 2.65$$

$$R_6 = \frac{20}{2.65} - 1$$

$$R_6 = 6.5\,k\Omega = R_3$$

If $R_3 = R_6 = \infty$ from (2)

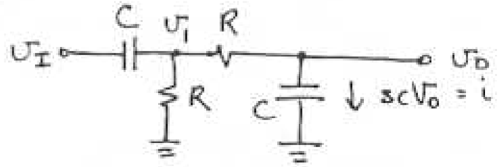
$$v_O - \left(\frac{1}{1+\infty}(V_O + 15)\right) - 0.7 = \frac{v_O}{3.03}$$

$$v_O - 0.7 = \frac{v_O}{3.3}$$

$v_O = 1.04V$

$\therefore$ opp output is $2v_O = 2.08\,V_{p\text{-}p}$.

13.13



$$\frac{v_1 - v_O}{R} = sCV_O \Rightarrow v_1 = v_O(1 + sCR)$$

ΣI at v_1

$$\frac{v_1}{R} + sC(v_1 - v_I) + sC\,v_O = 0$$

$$v_O(1 + sCR) + sCR(v_O + v_O sCR) - sCRv_I + sCRv_O = 0$$

$$v_O(1 + sCR + sCR + s^2C^2R^2 + sCR) = sCR\,v_I$$

CONT.

$$\beta(s) \triangleq \frac{v_o}{v_I} = \frac{sCR}{s^2C^2R^2 + 3sCR + 1}$$

$$= \frac{1}{3 + sCR + 1/sCR}$$

From Fig P(13.B) $\quad A = 1 + R_2/R_1$

$$\beta(jw) = \frac{1}{3 + j\left(wCR - \frac{1}{wCR}\right)}$$

Zero phase when $\quad wCR = \frac{1}{wCR}$

$$\underline{\underline{w = \frac{1}{CR}}}$$

$$\left|\beta(w = 1/RC)\right| = \frac{1}{3}$$

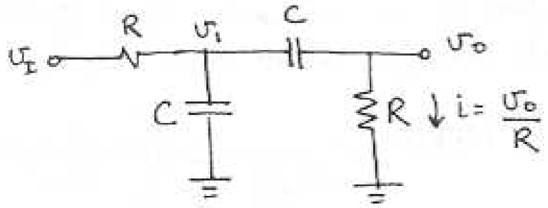
for oscillations $\quad 1 + R_2/R_1 \geq 3$

$$\Rightarrow \underline{\underline{\frac{R_2}{R_1} \geq 2}}$$

$$L(s) = A\beta = \underline{\underline{\frac{1 + R_2/R_1}{3 + sCR + \frac{1}{sCR}}}}$$

$$L(jw) = \underline{\underline{\frac{1 + R_2/R_1}{3 + j\left(wCR - \frac{1}{wCR}\right)}}}$$

$\boxed{13.14}$



$$sC(v_1 - v_o) = \frac{v_o}{R}$$

$$v_1 = v_o\left(1 + \frac{1}{sRC}\right)$$

ΣI at v_1:

$$\frac{-v_I + v_1}{R} + sCv_1 + \frac{v_o}{R} = 0 \quad \leftarrow \text{SUB FOR } v_1 \text{ and mult by } \frac{1}{sC}$$

$$\frac{1}{sCR}\left[-v + v_o\left(1 + sCR\right)\right] + v_o\left(1 + \frac{1}{sCR}\right) + \frac{v_o}{sCR} = 0$$

$$v_o\left[\frac{1}{sCR} + 1 + \frac{1}{sCR} + \frac{1}{s^2C^2R^2} + \frac{1}{sCR}\right] = \frac{v_I}{sCR}$$

$$\frac{v_o}{v_I} \triangleq \beta(s) = \frac{1}{3 + sCR + \frac{1}{sCR}}$$

$$\beta(jw) = \frac{1}{3 + j\left(wCR - \frac{1}{wCR}\right)}$$

phase is zero when $w_0CR = \frac{1}{w_0CR}$

$$\underline{\underline{w_0 = \frac{1}{RC}}}$$

$$\beta(jw_0) = \frac{1}{3}$$

$\therefore$ For oscillations

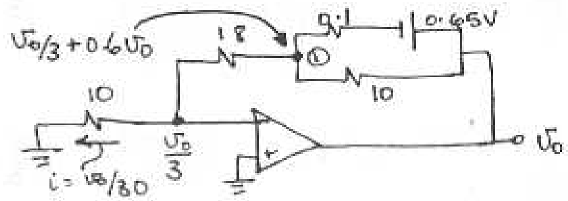
$$A = 1 + \frac{R_2}{R_1} \geq 3$$

$$\underline{\underline{\frac{R_2}{R_1} \geq 2}}$$

$$L(s) = \underline{\underline{\frac{1 + R_2/R_1}{3 + sCR + 1/sCR}}}$$

$$L(jw) = \underline{\underline{\frac{1 + R_2/R_1}{3 + j\left(wCR - \frac{1}{wCR}\right)}}}$$

$\boxed{13.15}$



CONT.

ΣI at node ①

$$\frac{v_o}{30} = \frac{v_o - \frac{v_o}{3} - 0.6v_o}{10} + \frac{v_o - 0.65 - \frac{v_o}{3} - 0.6v_o}{0.1}$$

$$= 0.00666\,v_o + 0.666\,v_o - 0.65$$

$$v_o = 10.156\ \text{V}$$

$\therefore$ Max. output $= \underline{\underline{20.3\text{V}_{p\text{-}p}}}$

13.16

$\omega_0 = \frac{1}{RC} = 2\pi 10^4 \qquad R = 10\text{k}\Omega$

$C = \frac{1}{10^4 \times 2\pi \times 10^4} \Rightarrow \underline{\underline{C \cong 1.6\text{nF}}}$

Now from Eq (13.11)

$$\beta(j\omega) = \left[3 + j\left(\omega CR - \frac{1}{\omega CR}\right)\right]^{-1}$$

$$\therefore \phi(\omega) = -\tan^{-1}\left(\frac{\omega CR - \frac{1}{\omega CR}}{3}\right)$$

Using $\frac{\partial \tan^{-1} v}{\partial x} = \frac{1}{1+v^2}\frac{\partial v}{\partial x}$ we get

$$\frac{\partial \phi(\omega)}{\partial \omega} = \frac{-1}{1+\left(\frac{\omega CR - \frac{1}{\omega CR}}{3}\right)^2}\left[\frac{RC + 1/\omega^2 RC}{3}\right]$$

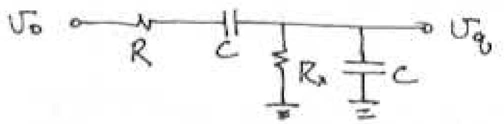
At $\omega = \omega_0 = \frac{1}{RC}$ $\quad \frac{\partial \phi(\omega)}{\partial \omega} = -\frac{2}{3}RC$

Now $5.7^\circ \approx 0.1\text{rad}$ (lag $= -0.1$rad)

$$\therefore \Delta\omega_0 = \frac{-0.1}{-2/3\,RC} \approx 0.15\,\omega_0 = 1.5\text{kHz}$$

$\therefore$ New frequency of oscillation $= 8.5$kHz

To restore operation:



$$\beta(s) = \frac{R_x \| \frac{1}{sC}}{R_x \| \frac{1}{sC} + R + \frac{1}{sC}}$$

$$= \frac{\frac{R_x/sC}{R_x + 1/sC}}{\frac{R_x/sC}{R_x + \frac{1}{sC}} + R + \frac{1}{sC}}$$

$$= \frac{R_x/sC}{R_x/sC + RR_x + \frac{R}{sC} + \frac{R_x}{sC} + \frac{1}{s^2C^2}}$$

$$\therefore \beta(s) = \frac{1}{2 + \frac{R}{R_x} + sCR + \frac{1}{sCR_x}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega CR - \frac{1}{\omega R_x C}}{2 + R/R_x}\right)$$

Now it is required that $\phi = 5.7^\circ$ at $\omega = \omega_0$! where $\omega_0 = 1/RC$

$$\therefore \omega_0 RC - \frac{1}{\omega_0 R_x C} = \left(2 + \frac{R}{R_x}\right)\tan(5.7)$$

$$1 - \frac{1}{\omega_0 R_x C} = \left(2 + R/R_x\right)(-0.1)$$

$$1 + 0.2 = \frac{1}{\omega_0 R_x C} - 0.1\frac{R}{R_x}$$

CONT.

$$R_x = \frac{1/\omega_0 C - 0.1R}{1.2}$$

given:
$\omega_0 = 2\pi 10^4$
$C = 1.6\times10^{-9}$
$R = 10^4$

$$R_x = \underline{\underline{7.5\,k\Omega}}$$

Now:

$$\beta(j\omega_0) = \frac{1}{2 + 10/7.5 + j(1 - 1/\omega_0 C R_x)}$$

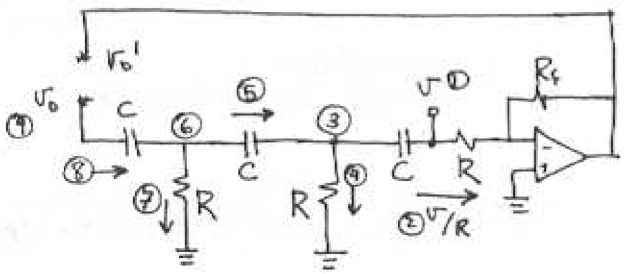
$$= (3.333 - j0.326)^{-1}$$

$$|\beta(j\omega_0)| = \frac{1}{3.35}$$

$\therefore$ $1 + R_2/R_1 = 3.35$ for oscillations

$\frac{R_2}{R_1} = \underline{\underline{2.35}}$ (not 2 as before)

13.17



③ $v + \frac{1}{sC}\frac{v}{R} = v\left(1 + \frac{1}{sCR}\right)$

④ $\frac{v\left(1+\frac{1}{sCR}\right)}{R} = \frac{v}{R} + \frac{v}{sCR^2}$

⑤ $i_s = \frac{2v}{R} + \frac{v}{sCR^2}$

⑥ ③ + ⑤$/sC$

$$= v + \frac{v}{sCR} + \frac{1}{sC}\left(2\frac{v}{R} + \frac{v}{sCR^2}\right)$$

$$= v + \frac{3v}{sCR} + \frac{v}{s^2C^2R^2}$$

⑦ $\frac{v}{R} + \frac{3v}{sCR^2} + \frac{v}{s^2C^2R^3}$

⑧ = ⑤ + ⑦

$$= \frac{3v}{R} + \frac{4v}{sCR^2} + \frac{v}{s^2C^2R^3}$$

⑨ $v_0 =$ ⑥ $+ \frac{1}{sC}$ ⑧

$$= v + \frac{3v}{sCR} + \frac{v}{s^2C^2R^2} + \frac{3v}{sCR} + \frac{4v}{s^2C^2R^2} + \frac{v}{s^3C^3R^3}$$

$$= v + \frac{6v}{sCR} + \frac{5v}{s^2C^2R^2} + \frac{v}{s^3C^3R^3}$$

Now loop gain $\equiv$

$$L(s) = -\frac{v_0'}{v_0}$$

$$\therefore L(s) = \frac{R_f/R\; v}{v\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)}$$

$$= \frac{s^3 R_f/R}{s^3 + \frac{6s^2}{RC} + \frac{5s}{C^2R^2} + \frac{1}{C^3R^3}}$$

$$L(j\omega) = \frac{-j\omega^3 R_f/R}{\frac{1}{C^3R^3} - \frac{6\omega^2}{RC} + j\left(\frac{5\omega}{C^2R^2} - \omega^3\right)}$$

CONT.

$L(j\omega)$ is real if

$$\frac{6\omega_0^2}{RC} = \frac{1}{R^3C^3}$$

$$\omega_0 = \frac{1}{\sqrt{6}\,RC}$$

$$L(j\omega_0) = \frac{\omega_0^2\, R_f/R}{-\omega_0^2 + 5/R^2C^2}$$

$$= \frac{R_f/R\ \omega_0^2}{-\omega_0^2 + 30\omega_0^2}$$

$$= \frac{R_f/R}{29}$$

Now Loop Gain = 1 if $R_f = 29R$

∴ Minimum value for $R_f = 29R$

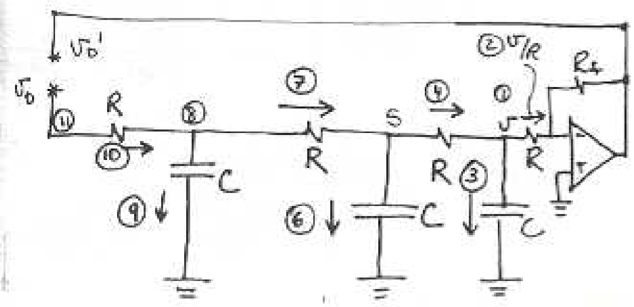
Given $C = 16nF$, $R = 10k\Omega$

$$f_0 = \frac{1}{2\pi\sqrt{6}\ 16\times10^{-9}\ 10\times10^{3}}$$

$$= 406.1Hz$$

$$\text{or } f_0 = \frac{0.065}{RC}$$

13-18



③ $i = scv$ ④ $scv + v/R$

⑤ $v + \left(scv + \frac{v}{R}\right)R = 2v + sCRv$

⑥ $2scv + s^2c^2Rv$

⑦ = ⑥ + ④ $= 3scv + s^2c^2Rv + v/R$

⑧ $2v + scRv + v + 3scRv + s^2c^2R^2v$
$= 3v + 4scRv + s^2c^2R^2v$

⑨ $3scv + 4s^2c^2Rv + s^3c^3R^2v$

⑩ = ⑦ + ⑨
$= 6scv + 5s^2c^2Rv + \frac{v}{R} + s^3c^3R^2v$

⑪ = ⑧ + ⑩ × R
$v_0 = 4v + 10scRv + 6s^2c^2R^2v + s^3c^3R^3v$

Now:

$$L(s) = -\frac{v_0'}{v_0} = \frac{v\ R_f/R}{v\left(s^3c^3R^3 + 6s^2c^2R^2 + 10scR + 4\right)}$$

$$= \frac{R_f/R}{s^3c^3R^3 + 6s^2c^2R^2 + 10scR + 4}$$

$$L(j\omega) = \frac{R_f/R}{(4 - 6\omega^2c^2R^2) + j(10\omega cR + \omega^3R^3c^3)}$$

$L(j\omega)$ is purely real if

$$10\omega_0 CR = \omega_0^3R^3C^3$$

$$\omega_0 = \frac{1}{\sqrt{10}}\ \frac{1}{RC}$$

Given $R = 10k\Omega$, $f_0 = 10kHz$

$$C = \frac{1}{\sqrt{10}\times10^4\times2\pi10^4}$$

$$= 0.503nF$$

CONT.

Now,

$$|L(j\omega_0)| = \frac{R_f/R}{4 - 6\omega_0^2R^2C^2} \quad \text{Sub for } \omega_0$$

$$= \frac{R_f/R}{4 - 6\frac{1}{10R^2C^2}R^2C^2}$$

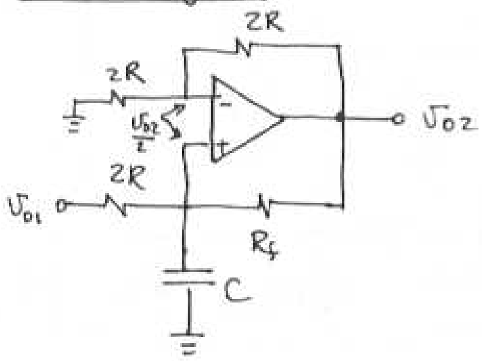
$$= \frac{R_f/R}{4 - 6/10} \geq 1$$

$$\therefore \quad R_f/R \geq 3.4$$

$$\underline{\underline{R_f \geq 34k\Omega}}$$

13.19

for 2^{nd} integrator



From the voltage divider around the upper branch: $V_+ = V_- = \frac{1}{2}V_{o2}$

$\Sigma I = 0$ at +ve input

$$\frac{\frac{1}{2}V_{o2} - V_{o1}}{2R} + sC\frac{V_{o2}}{2} + \frac{\frac{V_{o2}}{2} - V_{o2}}{R_f} = 0$$

$$\frac{V_{o2} - 2V_{o1}}{2R} + sCV_{o2} - \frac{V_{o2}}{R_f} = 0 \quad R_f = \frac{2R}{1+\Delta}$$

$$V_{o2}\left(\frac{1}{2R} + sC - \frac{1+\Delta}{2R}\right) = \frac{V_{o1}}{R}$$

$$V_{o2}\left(sCR - \frac{\Delta}{2}\right) = V_{o1}$$

$$\therefore \frac{V_{o2}}{V_{o1}} = \frac{1}{sCR - \Delta/2}$$

Now: $\frac{V_{o1}}{V_x} = \frac{-1}{sCR}$ $\quad \therefore L(s) = \frac{-1/sCR}{sCR - \Delta/2}$

Characteristic equation $L(s) = 1$

$$\therefore s^2C^2R^2 - \frac{sCR\Delta}{2} + 1 = 0$$

$\therefore$ Poles are

$$S_p = \frac{\frac{RC\Delta}{2} \pm \sqrt{\frac{R^2C^2\Delta^2}{4} - 4C^2R^2}}{2R^2C^2}$$

$$= \frac{\Delta/2 \pm 2j\sqrt{1 - (\Delta/4)^2}}{2RC}$$

for $\Delta \ll 1$ $\quad \left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{1/2} \approx \left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)$

$$\therefore S_p \cong \left[\Delta/2 \pm j2\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)\right]\frac{1}{2RC}$$

$$= \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC}$$

Now:

$Re[S_p] > 0 \Rightarrow$ Poles in R.H.P.!

for $\Delta \ll 1$

$$S_p \cong \frac{\Delta/2 \pm j2}{2RC} = \underline{\underline{\frac{1}{RC}\left(\frac{\Delta}{4} \pm j\right)}}$$

Q.E.D.

13.20

The transmission of the filter normalized to the centre frequency, w_0 is:

$$|T(j\omega)| = \frac{\omega\omega_0/Q}{(\omega_0^2-\omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}$$

$$= \frac{1/Q\left(\frac{\omega_0}{\omega}\right)}{\left(\left(\frac{\omega_0}{\omega}\right)^2-1\right)^2 + \frac{1}{Q^2}\left(\frac{\omega_0}{\omega}\right)^2}$$

Relative to the amplitude of the fondamental

(a) The second harmonic $=0$

(b) The third harmonic

$$= \frac{1}{3} \quad \frac{1/20 \times 1/3}{\left(\frac{1}{9}-1\right)^2 + \left(\frac{1}{20}\right)^2\left(\frac{1}{9}\right)} = 6.25\times10^{-3}$$

(c) The fifth harmonic

$$= \frac{1}{5} \quad \frac{\frac{1}{20}\times\frac{1}{5}}{\left(\frac{1}{25}-1\right)^2 + \left(\frac{1}{20}\right)^2\left(\frac{1}{25}\right)} = 2.08\times10^{-3}$$

(d) The 4^{th} harmonic $= 6^{th} = 10^{th} = 0$

7^{th} harmonic $= 1.04 \times 10^{-3}$

9^{th} " $= 0.625 \times 10^{-3}$

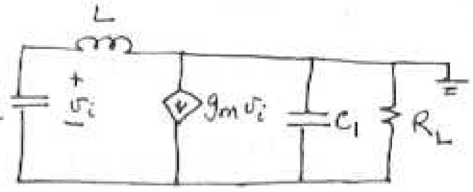
$\therefore$ $\dfrac{\text{RMS of } 2^{nd} \text{ to } 10^{th} \text{ harmoic}}{\text{RMS of fundamental}}$ is

$$\left[6.25^2 + 2.08^2 + 1.04^2 + 0.625^2\right]^{1/2} \times 10^{-3}$$

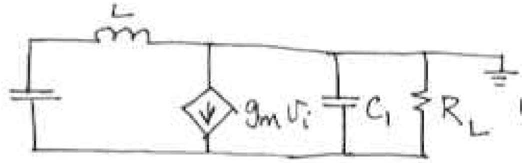
$$= \underline{\underline{6.7\times10^{-3}}} \text{ OR } \underline{\underline{0.7\%}}$$

13.21

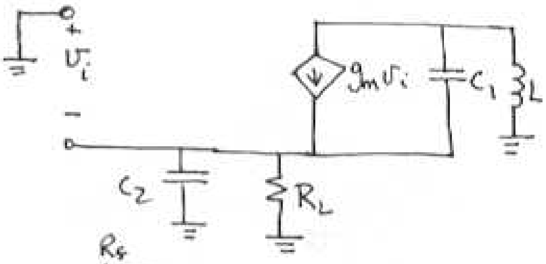
Consider the small signal models for each circuit. Assume r_π very large:



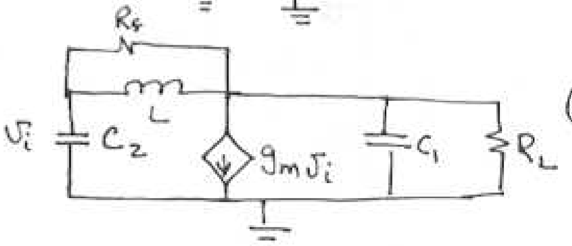
(a)



(b)



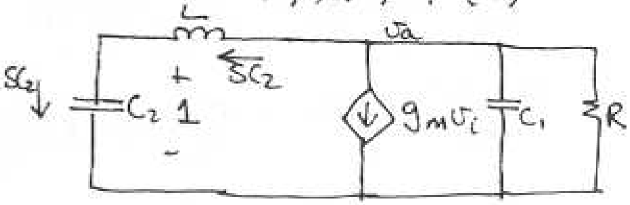
(c)



(d)

Given $R_f >> \omega_0 L$, circuits (a), (b) and (d) are the same except for the reference (ground) node.

For circuits (a), (b) & (d)



- Break the loop at v_i and assume unit return.

CONT.

$$v_a = 1 + sC_2 sL$$
$$= 1 + s^2 C_2 L$$

$\Sigma I = 0$ at v_a

$$g_m + sC_2 + sC_1\left(1 + s^2 C_2 L\right) + \frac{\left(1 + s^2 C_2 L\right)}{R} = 0$$

$$\therefore\ g_m + \frac{1}{R} + s(C_1 + C_2) + \frac{s^2 C_2 L}{R} + s^3 C_1 C_2 L = 0$$

This is the characteristic equation.
For $s = j\omega$:

$$g_m + \frac{1}{R} - \frac{\omega^2 C_2 L}{R} + j\left((C_1 + C_2)\omega - \omega^3 C_1 C_2 L\right) = 0$$

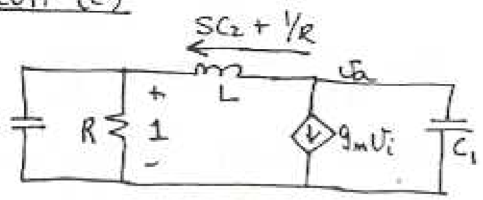
IMAGINARY PART = 0:

$$C_1 + C_2 = \omega^2 C_1 C_2 L$$
$$\omega = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} \equiv \text{Frequency of Oscillation}$$

REAL PART = 0

$$g_m + \frac{1}{R} = \frac{\omega^2 C_2 L}{R}$$
$$g_m R = \left(\frac{C_1 + C_2}{C_1 C_2 L}\right) C_2 L - 1$$
$$g_m R = \frac{C_2}{C_1} \equiv \text{LIMIT ON GAIN}$$

FOR CIRCUIT (c)



$$v_a = \left(sC_2 + \frac{1}{R}\right) sL + 1$$

$\Sigma I = 0$ at v_a, $v_i = 1$

$$g_m + sC_2 + \frac{1}{R} + sC_1\left[sL\left(sC_2 + \frac{1}{R}\right) + 1\right] = 0$$

$$g_m + \frac{1}{R} + sC_2 + s^3 C_1 C_2 L + \frac{s^2 C_1 L}{R} + sC_1 = 0$$

THE CHARACTERISTIC EQUATION ≡

$$g_m + \frac{1}{R} + s(C_1 + C_2) + \frac{s^2 C_1 L}{R} + s^3 C_1 C_2 L = 0$$

Note this is the same as above, with $C_1 \leftrightarrow C_2$

$$\therefore\ \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} \quad \text{and} \quad g_m R = \frac{C_1}{C_2}$$

13.22

(a) frequency of oscillations $\omega_0 = \frac{1}{\sqrt{LC}}$

gain ≥ 1 gain $= \frac{R_C}{2r_e} = \frac{R_C}{2V_T/I/2}$
$$= \frac{IR_C}{4V_T}$$

for $V_T = 0.025\text{V}$ then

$$IR_C \geq 4V_T$$

$$R_C \geq 0.1/I$$ for oscillations to start.

(b) for $R_C = \frac{1}{I}$ (kΩ) we have

$$\text{gain} = \frac{1/I}{2\left(\frac{2V_T}{I}\right)} = \frac{1}{4 \times 0.025} = 10$$

CONT.

Oscillations will start ($10>1$) and grow until Q_1, Q_2, go into cutoff. Output will go from V_{cc} to $V_{cc} - IR_c = V_{cc} - 1$. Therefore, output will be $1V_{p-p}$. Fundamental has a p-p amplitude of $\frac{4}{\pi} = 1.27\ V_{p-p}$.

13.23

From Exercise 13.10,

$L = 0.52H$

$C_s = 0.012\ pF$

$C_p = 4pF$

From (13.27)

$$C_{eq} = \frac{C_s\left(C_p + \frac{C_1C_2}{C_1+C_2}\right)}{C_s + C_p + \frac{C_1C_2}{C_1+C_2}}$$

$C_2 = 10pF \qquad C_1 = 1 \text{ to } 10pF$

$$C_L = \frac{0.012\left(4+\frac{10\times1}{10+1}\right)}{\left(0.012+4+\frac{10}{11}\right)} = 0.01197pF$$

$$C_H = \frac{0.012\left(4+\frac{10\times10}{10+10}\right)}{\left(0.012+4+\frac{100}{20}\right)} = 0.01198pF$$

$$\therefore f_{0H} = \frac{1}{2\pi\left[0.52\times0.01197\times10^{-12}\right]^{1/2}}$$

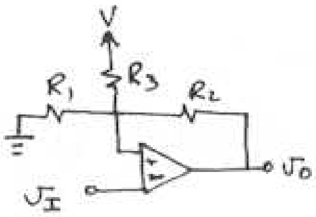
$$= \underline{\underline{2.0172\ MHz}}$$

$$f_{0L} = \left[2\pi\left(0.52\times0.01198\times10^{-12}\right)^{1/2}\right]^{-1}$$

$= 2.01612\ MHz$

Difference $= \underline{1kHz}$!

13.24



ΣI at +ve node:

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_+ - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = \frac{V}{R_3} + \frac{L_+}{R_2}$$

$$V_{TH} = \left(V/R_3 + L_+/R_2\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}$$

$$= \underline{\underline{\left(\frac{V}{R_3}+\frac{L_+}{R_2}\right) R_1 \| R_2 \| R_3}}$$

Similarly

$$\underline{\underline{V_{TL} = \left(\frac{V}{R_3}+\frac{L_-}{R_2}\right)\left(R_1 \| R_2 \| R_3\right)}}$$

(b) Now

$$V_{TH} = 5.1 = \left(\frac{15}{R_3}+\frac{13}{R_2}\right)\left(\frac{1}{10}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}$$

$$\frac{5.1}{10} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{15}{R_3} + \frac{13}{R_L}$$

$$0.51 = \frac{7.9}{R_2} + \frac{9.9}{R_3} \qquad (1)$$

AND

$$V_{TL} = 4.9 = \left(\frac{15}{R_3} - \frac{13}{R_2}\right)\left(\frac{1}{10}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}$$

$$0.49 = -\frac{17.9}{R_2} + \frac{10.1}{R_3} \qquad (2)$$

$$\left.\begin{aligned}&(1)\times\frac{10.1}{9.9} \Rightarrow 0.52 = \frac{8.06}{R_2} + \frac{10.1}{R_3}\\ &(2) \Rightarrow 0.49 = -\frac{17.9}{R_2} + \frac{10.1}{R_3}\end{aligned}\right\}\text{SUBTRACT TO GET} \Downarrow$$

CONT.

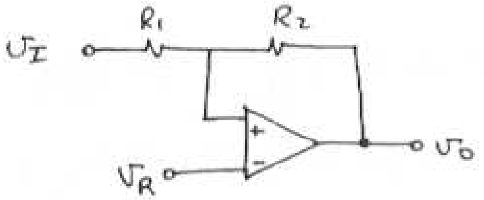
$$0.52-0.49 = \frac{8.06+17.9}{R_2}$$

$$R_2 = \frac{25.96}{0.0303} = \underline{\underline{856.8\,k\Omega}}$$

$$\frac{10.1}{R_3} = 0.49 + \frac{17.9}{856.8}$$

$$R_3 \cong \underline{\underline{19.8\,k\Omega}}$$

13.25



for $v_I = v_{TL}$ and $v_O = L_+$ initially

$$\frac{L_+ - v_R}{R_2} = \frac{v_R - v_{TL}}{R_1}$$

$$v_{TL} = v_R - \frac{R_1}{R_2} v_R - \frac{R_1}{R_2} L_+$$

$$\therefore\ v_{TL} = v_R\left(1 - \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} L_+$$

Similarly

$$\frac{L_- - v_R}{R_2} = \frac{v_R - v_{TH}}{R_1}$$

$$v_{TH} = v_R\left(1 + R_2/R_1\right) - \frac{R_1}{R_2} L_-$$

(b) Given $L_+ = -L_- = V$

$R_1 = 10\,k\Omega$

$V_{TL} = 0$

$V_{TH} = V/10$

Substituting these values we get:

$$0 = V_R(1 + 10/R_2) - 10/R_2\,V \quad (1)$$

$$\frac{V}{10} = V_R(1 + 10/R_2) + 10/R_2\,V \quad (2)$$

$$(1)-(2) \quad -\frac{V}{10} = -\frac{20}{R_2} V$$

$$R_2 = \underline{\underline{200\,k\Omega}}$$

$$0 = V_R(1 + 10/200) - \frac{10}{200} V$$

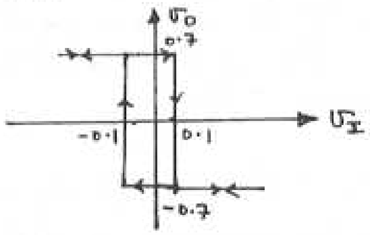
$$V_R = \frac{10/200\,V}{1 + 10/200} = \underline{\underline{47.62\,mV}}$$

13.26

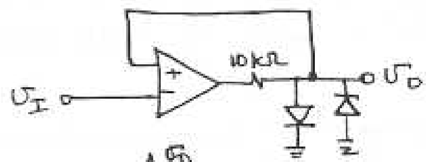
Output levels $= \pm 0.7V$

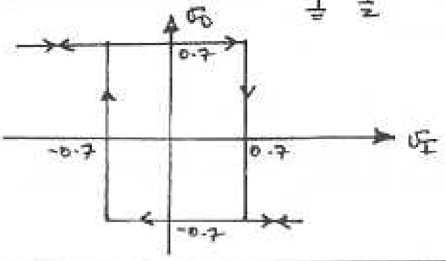
Threshold levels $= \pm \frac{10}{10+60} \times 0.7 = 0.1V$

$$i_{D,max} = \frac{12-0.7}{10} - \frac{0.7}{10+60} = \underline{\underline{1.12\,mA}}$$



13.27





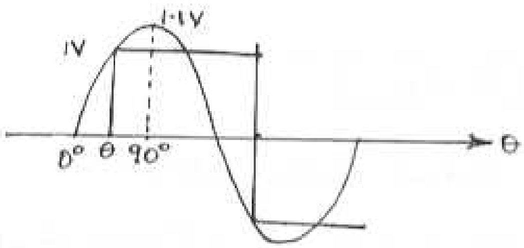
13.28

(a) A 0.5V peak sine wave, is not large enough to change the state of the circuit. Hence, the output will be either +12V or -12V at DC.

(b) The 1.1 V peak will change the state when

$$1.1\sin\theta = 1$$

$$\theta = 65.4^\circ$$



∴ The output is a symmetric square wave at frequency f, and lags the sine wave by an angle of 65.4°. The square wave has a swing of $\underline{\pm 12V}$.

Since $V_{TH} = -V_{TL} = 1V$, if the average shifts by an amount so either the +ve or -ve swing is <1V, then no change of state will occur. Clearly, if the shift is 0.1V, the output will be a DC voltage.

13.29

For $L_+ = -L_- = 7.5V$

$\underline{V_Z = 6.8V}$ with $V_D = 0.7V$.

For $V_{TH} = -V_{TL} = 7.5V \Rightarrow R_1 = R_2$

for $v_I = 0$ $I_{R_2} = 0.1mA = \frac{7.5}{R_1+R_2}$

$\Rightarrow R_1 = R_2 = \underline{\underline{37.5\ k\Omega}}$

$$I_D = 1mA = \frac{12-7.5}{R} - \frac{7.5}{2R_1}$$

$$1 = \frac{4.5}{R} + 0.1$$

$$R = \underline{\underline{4.1k\Omega}}$$

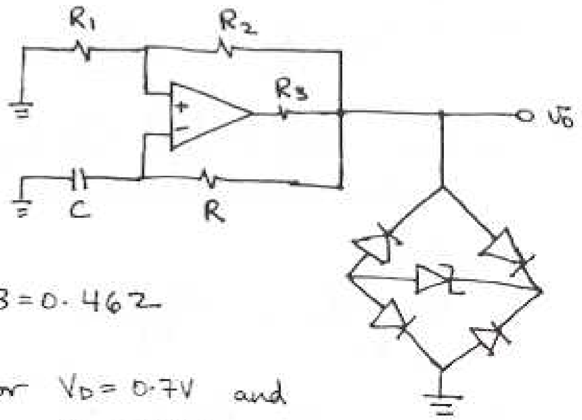
13.30

$T = 2\tau \ln \frac{1+\beta}{1-\beta}$ $\quad \beta = \frac{R_1}{R_1+R_2} = \frac{10}{26}$

$$T = 2(10\times10^{-9})(62\times10^3)\ln\left(\frac{1+10/26}{1-10/26}\right)$$

$T = 1.006ms \Rightarrow f = \underline{\underline{994.5Hz}}$

13.31



$\beta = 0.462$

for $V_D = 0.7V$ and

$v_O = \pm 5V$

$V_Z = 5 - 2V_D$

$\underline{\underline{V_Z = 3.6V}}$

CONT.

$$T = 2\tau \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$10^{-3} = 2\tau \ln\left(\frac{1.462}{1-0.462}\right) \Rightarrow \tau = 0.5\text{ msec}$$

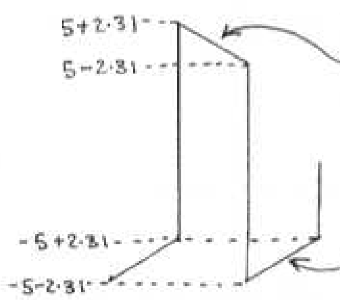
$$\tau = RC \qquad \Rightarrow R = \tau/C = \underline{\underline{50k\Omega}}$$

Thresholds $= \pm 0.462 \times 5 = \pm 2.31V$

Average current in R in ½ cycle:

$$I \cong \frac{1}{R}\left(\frac{5-2.31+2.31+5}{2}\right)$$

$$= \frac{5}{R} = \frac{5}{50k} = \underline{0.1mA}$$



$$R_1 + R_2 = \frac{5V}{0.1mA} = 50k\Omega$$

$$\frac{R_1}{R_1+R_2} = 0.462 \rightarrow R_1 = 50(0.462)$$
$$= 23.1k\Omega$$

$$\therefore R_2 = 26.9k\Omega$$

$$1 = \frac{13-5}{R_3} - 0.1 - 0.1$$

$$= I_{DIODE}$$

use $\underline{R_1 = 23k\Omega}$

$\underline{\underline{R_2 = 27k\Omega}}$

$$R_3 = \frac{8}{1.2}$$
$$= \underline{\underline{6.67k\Omega}}$$

13.32

From Fig 13.23(b), for ±5V outputs
$V_Z = 5 - 2V_{DIODE} = 5 - 1.4 = \underline{\underline{3.6V}}$

For ±5V out:

$R_1 = R_2 \qquad L_+ = -L_- = 5V$

$V_{TH} = -V_{TL} = 5V$

Max current in feedback network $= 0.2mA$

$$\therefore 0.2 = \frac{5}{R_1+R_2} \Rightarrow \underline{\underline{R_1 = R_2 = 25k\Omega}}$$

Max diode current $= 1mA$

$$\therefore \frac{13-5}{R_Z} = (0.2+1)\,mA$$

$$R_Z = \frac{8}{1.2} = \underline{\underline{6.67k\Omega}}$$

Now from Fig 13.25(c)

$$\text{slope} = \frac{-L_-}{RC} = \frac{V_{TH}-V_{TL}}{T/2}$$

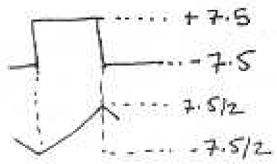
for $f = 1kHz$
$T = 10^{-3}s$
$\underline{\underline{C = 0.01\mu F}}$

$$\frac{5}{RC} = \frac{10}{10^{-3}/2} \Rightarrow \underline{\underline{R = 25k\Omega}}$$

13.33

For 15 Vpp output $v_Z = 15/2 - 0.7$
$= \underline{\underline{6.8V}}$

For the integrator:



i.e. V_C should ramp between V_{TH} & V_{TL}!

CONT.

$$v_c(t_1) = \frac{1}{RC}\int_{t_0}^{t_1} v\,dt + v_c(t_0)$$ – v is a square wave

$$\frac{7.5}{2} = \frac{1}{RC}(t_1-t_0)(7.5-(-7.5)) - \frac{7.5}{2}$$

$(t_1-t_0)=\frac{T}{2}$

$$7.5 = \frac{1}{RC}\frac{T}{2}(15)$$

$$1 = \frac{T}{RC} \Rightarrow R = \frac{T}{C} = \frac{1}{fC}$$

$$= \frac{1}{10^4(0.5\times10^{-9})}$$

$$\therefore \underline{\underline{R = R_{1-6} = 200k\Omega}}$$

Minimum zener current = 1mA

$$\frac{13-7.5}{R_7} = 1 + \frac{7.5}{R_1+R_2} + \frac{7.5-V_c}{R_5}$$

Maximum current into the integrator when $V_c = -\frac{7.5}{2}$

$$\therefore \frac{5.5}{R_7} = 1 + \frac{7.5}{400} + \frac{11.25}{200}$$

$$\therefore R_7 = 5.12k\Omega \overset{use}{\Longrightarrow} \underline{\underline{R_7 = 5.1k\Omega}}$$

Integrator output is triangular, with period = 100μs and ±7.5V peaks. (ie. 2× voltage at capacitor)

13.34

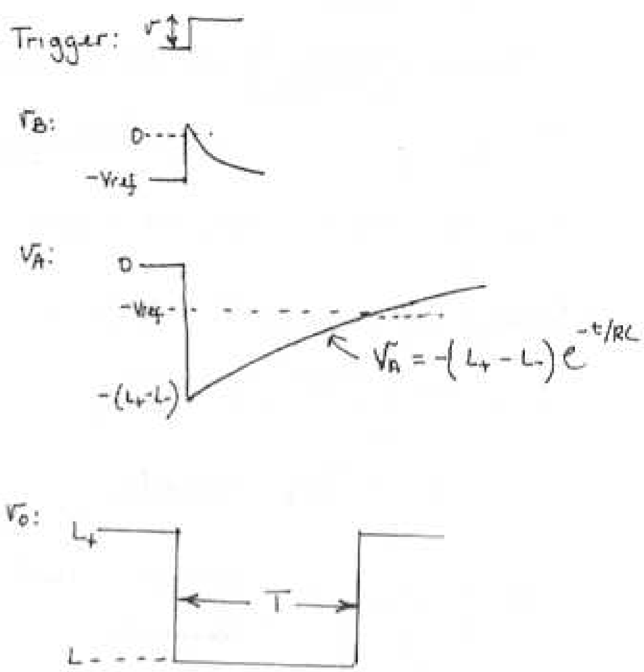
See sketches that follow:

$$V_A(t=T) = -V_{ref} = -(L_+-L_-)e^{-T/RC}$$

$$\frac{V_{ref}}{L_+-L_-} = e^{-T/RC}$$

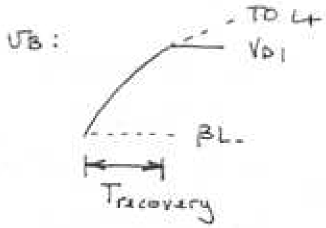
$$T = -RC\ln\left(\frac{V_{ref}}{L_+-L_-}\right) = RC\ln\left(\frac{L_+-L_-}{V_{ref}}\right)$$

<u>Q.E.D.</u>



13.35

For recovery, v_B goes from βL_- to L_+ until D_1 conducts at $V_{D1}=0.7V$



For recovery

$$v_B = -0.1(12) + (12+1.2)(1-e^{-t/\tau})$$

$$= 12 - 13.2\,e^{-t/\tau}$$

At Trecovery:

$$V_{D1} = 12 - 13.2\,e^{-T_r/\tau} \qquad \tau = R_3C_1$$

$$T_r = -R_3C_1\ln\left(\frac{V_{D1}-12}{13.2}\right)$$

$$= -(6171)(0.1\times10^{-6})\ln\left(\frac{11.3}{13.2}\right)$$

$$= \underline{96\mu s}$$

13.36

choose $\underline{C_1 = 1nF}$ $\underline{C_2 = 0.1nF}$

$R_1 = R_2 = 100k\Omega \Rightarrow \beta \doteq \frac{1}{2}$

$T \cong C_1 R_3 \ln\left(\frac{0.7+13}{-13(0.5-1)}\right)$

$10^{-4} = 10^{-9} R_3 \ln\left(\frac{13.7}{13(0.5)}\right)$

$\underline{R_3 = 134.1k\Omega}$

Need $R_4 >> R_1 \Rightarrow$ choose $\underline{R_4 = 470k\Omega}$

Min trigger voltage $= \beta L_+ - V_{D2} + V_{D1}$
$= \underline{6.5V}$

For recovery

$v_B = 13 - (13 - \beta L_-) e^{-t/\tau}$

$= 13 - 19.5 e^{-t/\tau} = 0.7$

$\therefore t_{recovery} = -\tau \ln\left(\frac{12.3}{19.5}\right)$

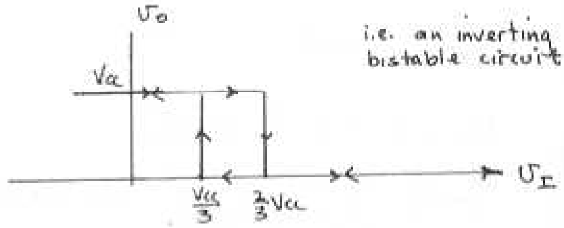
$= (134.1 \times 10^3)(10^{-9})(-0.4608)$

$= \underline{61.8 \mu s}$

13.37

For $v_I > 2/3 V_{cc}$ comp-1 = "1" and comp-2 = "0" and flip flop is reset. I.E. $v_O = 0V$. Now v_O will not change until $v_I = 1/3 V_{cc}$, when comp-2 = "1" and comp-1 = "0" and FF is set : I.E. $V_O = V_{cc}$

For $\frac{1}{3}V_{cc} < v_I < \frac{2}{3}V_{cc}$, comp-1 = comp-2 = "0" and no change of state will occur.



i.e. an inverting bistable circuit

13.38

(a) $C = 1nF$

$v_C = V_{cc}(1 - e^{-t/\tau})$ where $\tau = RC$

Pulse width of $10\mu s$ when $v_C = V_{TH} = \frac{2}{3}V_{cc}$

$\therefore \frac{2}{3} = 1 - e^{-t/RC}$ $t = T = 10\mu s$

$-\frac{T}{RC} = \ln\left(\frac{1}{3}\right) \Rightarrow R = \frac{-T}{C\ln(1/3)}$

$= \underline{9.1k\Omega}$

(b) for $T = 20\mu s$ $R = 9.1k\Omega$, $C = 1nF$

$\therefore V_{TH} = 15(1 - e^{-T/RC})$

$= 15\left(1 - e^{-\frac{20\times10^{-6}}{9.1\times10^3\times10^{-9}}}\right)$

$= \underline{13.3V}$

13.39

$C = 680pF$ $f = 50kHz$

$T = 20\mu s = T_H + T_L$

For 75% Duty $T_H = 15\mu s$
$T_L = 5\mu s$

From Eq (13.43) we have:

CONT.

$T_L = C R_B \ln 2$

$$\therefore R_B = \frac{5\times10^{-6}}{680\times10^{-12}\ln(2)} = \underline{\underline{10.6\ k\Omega}}$$

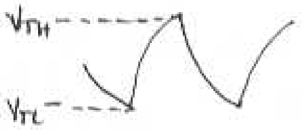
From Eq (13.41)

$$T_H = C(R_A+R_B)\ln(2)$$

$$R_A = \frac{15\times10^{-6}}{680\times10^{-12}\ln(2)} - 10.6\times10^3$$

$$= \underline{\underline{21.2\ k\Omega}}$$

13.40



For the rise:

$$V_C = V_{CC} - (V_{CC}-V_{TL})\,e^{-t/C(R_A+R_B)}$$

$$V_{TH} = V_{CC} - (V_{CC}-V_{TL})\,e^{-T_H/C(R_A+R_B)}$$

$$\frac{V_{CC}-V_{TH}}{V_{CC}-V_{TL}} = e^{-T_H/C(R_A+R_B)}$$

$$\underline{\underline{T_H = C(R_A+R_B)\ln\left(\frac{V_{CC}-V_{TL}}{V_{CC}-V_{TH}}\right)}}$$

For exponential fall:

$$v_C = V_{TH}\,e^{-t/CR_B}$$

$$\therefore V_{TL} = V_{TH}\,e^{-T_L/CR_B}$$

$$T_L = CR_B\ln\left(\frac{V_{TH}}{V_{TL}}\right)$$

for $V_{TH} = 2V_{TL} \Rightarrow T_L = CR_B\ln(2)$

(b) $C = 1\text{nF}$, $R_A = 7.2\text{k}\Omega$, $R_B = 3.6\text{k}\Omega$
$V_{CC} = 6\text{V}$ $\quad V_{TH,\,ext} = 0$

$$\therefore T_H + T_L = T = \ln 2\,(R_A + 2R_B)C$$

$T = 9.98\,\mu s \rightarrow \underline{\underline{f = 100\text{kHz}}}$

$$\text{Duty cycle} = \frac{T_H}{T_H+T_L} = \frac{R_A+R_B}{R_A+2R_B} = 0.75$$

$\Rightarrow \underline{\underline{75\%}}$

(c) $V_{CC} = 5\text{V}$, $\quad V_{TH} = \frac{2}{3}\times5 = \frac{10}{3} = 3.33\text{V}$

for 1V input $\quad V_{TH}' = 4.33\text{V}$
$\qquad V_{TL}' = \frac{1}{2}V_{TH}' = 2.17\text{V}$

$$T_H' = 10^{-9}(3.6+7.2)\times10^3\ln\left(\frac{5-2.17}{5-4.33}\right)$$

$$= \underline{\underline{15.6\,\mu s}}$$

$$T_L' = 10^{-9}\times3.6\times10^3\ln 2 = \underline{\underline{2.5\,\mu s}}$$

$$\therefore f = \frac{1}{(15.6+2.5)10^{-6}} = \underline{\underline{55.2\text{kHz}}}$$

$$\text{duty cycle} = \frac{15.6}{2.5+15.6} = \underline{\underline{86.2\%}}$$

for 1V input $\quad V_{TH}'' = 2.33$
$\qquad V_{TL}'' = 1.17$

$$\therefore T_H'' = 10^{-9}(3.6\ \ 7.2)10^3\ln\left(\frac{5-1.17}{5-2.33}\right)$$

$$= 3.92\,\mu s$$

$$T_L'' + T_L' = 2.5\,\mu s$$

$$\therefore f = \frac{10^6}{(3.92+2.5)} = \underline{\underline{156\text{kHz}}}$$

$$\text{duty cycle} = \frac{3.92}{2.5+3.92} = \underline{\underline{61\%}}$$

13.41

for sine wave: $v_o = 0.7 \sin \omega t$

slope at zero crossing

$0.7\omega \cos \omega t \,|_{t = 1/f}$

$= 0.7\omega = 0.7(2\pi f)$

Slope of triangular wave with peak of V volts and period 1/f is:

$$\frac{V - -V}{T/2} = \frac{2V}{\frac{1}{2f}} = 4Vf$$

Equating the slopes:

$$4Vf = 0.7(2\pi f)$$

$$V = 0.7 \times \frac{2\pi}{4} = \underline{\underline{1.0996}}$$

$$\text{Now } R = \frac{1.0996 - 0.7}{1\text{mA}} = 399.6\,\Omega$$

$$\simeq \underline{\underline{400\,\Omega}}$$

v_D changes by 0.1V per decade change in current.

$$\therefore v_o = 0.7 + 0.1 \log\left(\frac{1\text{mA}}{i_x}\right)$$

$$\Rightarrow i_x = 10^{\frac{v_o - 0.7}{0.1}} \text{ mA}$$

For output of 0.7V $i_x = 1\text{mA}$ and

$\theta = 90°$

Error = 0%

output = 0.65 $i_x = 0.316\text{mA}$

$\therefore v_I = 0.65 + 0.316 \times 0.4$

$= 0.7765\text{V}$

$$\theta = \frac{0.7765}{1.0996} \times 90° = 63.6°$$

output = 0.6V $i_x = 0.1\text{mA}$

$v_I = 0.6 + 0.1 \times 0.4 = 0.640\text{V}$

$$\theta = \frac{0.640}{1.0996} \times 90 = 52.4°$$

out = 0.55V $i_x = 0.0316\text{mA}$

$v_I = 0.55 + 0.0316 \times 0.4$

$= 0.563\text{V}$

$$\theta = \frac{0.563}{1.0996} \times 90 = 46.1°$$

OUT = 0.5V $i_x = 0.01\text{mA}$

$v_I = 0.5 + 0.01 \times 0.4 = 0.504\text{V}$

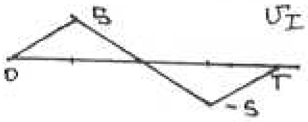
$$\theta = \frac{0.504}{1.0996}\, 90° = 41.3°$$

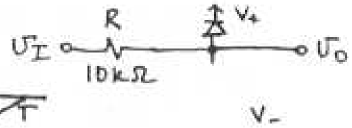
OUT (V)	
0.4	32.8°
0.3	24.6°
0.2	16.4°
0.1	8.2°
0	0°

Summarizing in Table Form:

v_O(V)	θ	$0.7\sin\theta$	% Error
0.7	90	0.7	0
0.65	63.6	0.627	3.7
0.6	52.4	0.554	8.2
0.55	46.1	0.504	9.1
0.50	41.3	0.462	8.2
0.40	32.8	0.379	5.5
0.30	24.6	0.291	3.0
0.20	16.4	0.198	1.2
0.10	8.2	0.0998	0.1
0.00	0	0.0	0.0

13.42





$v_O = A \sin \frac{2\pi}{T} t$

Slope of v_O at $t=0$,

$$\frac{\partial v_o}{\partial t} = A\frac{2\pi}{T}\cos\left(\frac{2\pi}{T}\right)t\Big|_{t=0}$$

$$= \frac{A\,2\pi}{T} \equiv \text{SLOPE AT ZERO CROSSING}$$

Slope of Δ-wave $= \frac{5}{T/4} = \frac{20}{T}$

$$\therefore \frac{20}{T} = \frac{A\,2\pi}{T}$$

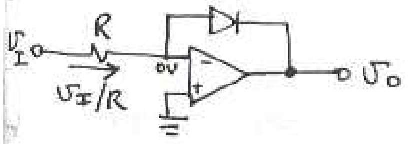
$$\underline{A = 3.18\text{V}}$$

$\therefore$ Clamp voltage:

$$V_+ = -V_- = 3.18 - 0.7$$

$$= 2.48 = \underline{\underline{2.5\text{V}}}$$

13.43



$\rightarrow v_I > 0$
$\rightarrow$ voltage across diode is $-v_O$

$$i_D = \frac{v_I}{R} = I_s e^{-v_O/nV_T}$$

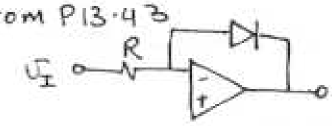
$$-\frac{v_O}{nV_T} = \ln\left(\frac{v_I}{RI_s}\right)$$

$$v_O = -nV_T \ln\left(\frac{V_I}{RI_s}\right), \quad v_I > 0$$

Q.E.D.

13.44

from P13.43



$$v_O = -nV_T \ln\left(\frac{V_I}{RI_s}\right)$$

Now.

$$V_A = -nV_T \ln \frac{V_1}{RI_s} \qquad R = 1k\Omega$$

$$V_B = -nV_T \ln \frac{V_2}{RI_s} \qquad V_1, V_2 > 0$$

$$V_C = +nV_T \ln \frac{1}{RI_s}$$

$$V_D = -(V_A + V_B + V_C)$$

$$= nV_T\left(\ln\left[\frac{V_1}{RI_s}\times\frac{V_2}{RI_s}\times\frac{RI_s}{1}\right]\right)$$

$$= nV_T \ln\left(\frac{V_1V_2}{RI_s}\right)$$

$$i_{D4} = I_s e^{v_D/nV_T}$$

$$= I_s \times \frac{V_1V_2}{RI_s} = \frac{V_1V_2}{R}$$

$$v_O = -i_{D4}R = -\frac{V_1V_2}{R}\times R$$

$$\therefore \underline{\underline{v_O = -V_1V_2}} \quad \text{ANALOG MULTIPLIER}$$

To check $\underline{v_1 = 0.5,\ v_2 = 2}$

$$I_{D1} = 0.5\text{mA} \rightarrow V_A = -0.7 + nV_T\ln\left(\frac{0.5}{1}\right)$$

$$= 0.7 + 2(0.025)\ln\left(\tfrac{1}{2}\right)$$

$$= -0.6653\text{V}$$

$$I_{D2} = 2\text{mA} \rightarrow V_B = (0.7 + 0.05\ln(2))(-1)$$

$$= -0.7347\text{V}$$

$$I_{D3} = 1\text{mA} \rightarrow V_C = 0.700\text{V}$$

$$V_D = -(0.6653 - 0.7347 + 0.7) = 0.7\text{V}$$

CONT.

$V_D = V_{D4} = 0.7V \Rightarrow I_{D4} = 1mA$

$\therefore v_O = -1V$ i.e. $2 \times 0.5 = 1$

For $v_1 = 3$, $v_2 = 2$:

$I_{D1} = 3mA \rightarrow V_A = -(0.7 + 0.05\ln 3) = -0.7549V$
$I_{D2} = 2mA \rightarrow V_A = -(0.7 + 0.05\ln 2) = -0.7347V$
$I_{D3} = 1mA \rightarrow V_C = 0.7V$

$\therefore V_D = V_{D4} = -(V_A + V_B + V_C) = +0.7896V$

$$\therefore \frac{I_{D4}}{1mA} = \frac{I_S e^{V_D/0.05}}{I_S e^{0.7/0.05}}$$

$$I_{D4} = e^{\frac{0.7986-0.7}{0.05}} = 6mA$$

$\therefore v_O = -6V$ i.e. $2 \times 3 = 6$.

For squarer: $v_1 = 2$ through $\frac{1}{2}k\Omega$ resistor

$I_{D1} = 4mA \rightarrow V_A = -(0.7 + 0.05\ln 4) = -0.7693$

$V_D = -(-0.7693) = 0.7693V$

$$I_{D4} = e^{\frac{0.7693-0.7}{0.05}} = 3.999mA$$

$\therefore V_O = -3.999V$ i.e. $2^2 = 4$.

13.45

Say $V_{BE} = \tilde{V}_D$ @ I $n = 1$

for $V_D = 0.25\,V_T$:

$$I_R = \frac{0.25V_T}{R} = \frac{0.25\,V_T}{\frac{2.5V_T}{I}} = \frac{I}{10}$$

$$V_{BE1} = \tilde{V}_D + nV_T \ln\left(\frac{I + I/10}{I}\right) \cong \tilde{V}_D + V_T\ln(1.1)$$

$$V_{BE2} = \tilde{V}_D + nV_T \ln\left(\frac{I - I/10}{I}\right) \cong \tilde{V}_D + V_T\ln(0.9)$$

$$V_I = -V_{BE2} + V_O + V_{BE1}$$
$$= V_T\left[\ln(1.1) + 0.25 - \ln(0.9)\right]$$
$$= \underline{\underline{0.451V_T}}$$

For $V_O = 0.5V_T$

$$I_R = \frac{0.5\,I}{2.5} = 0.2I$$

$$V_I = V_T\left[\ln(1.2) + 0.5 - \ln(0.8)\right]$$
$$= \underline{\underline{0.905V_T}}$$

$V_O = V_T$ $I_R = 0.4I$

$$V_I = V_T\left[\ln 1.4 + 1 - \ln 0.6\right]$$
$$= \underline{\underline{1.847V_T}}$$

$V_O = 1.5V_T$ $I_R = 0.6I$

$$V_I = V_T\left(\ln 1.6 + 1.5 - \ln 0.4\right)$$
$$= \underline{\underline{2.886V_T}}$$

$V_O = 2V_T$ $I_R = 0.8I$

$$V_I = V_T\left(\ln 1.8 + 2 - \ln 0.2\right) = \underline{4.197V_T}$$

$V_O = 2.4V_T$ $I_R = 0.96I$

$$V_I = V_T\left(\ln 1.96 + 2.4 - \ln 0.04\right) = \underline{\underline{6.292V_T}}$$

$V_O = 2.42V_T$ $I_R = 0.968I$

$$V_I = V_T\left(\ln 1.968 + 2.42 - \ln 0.032\right)$$
$$= \underline{\underline{6.519V_T}}$$

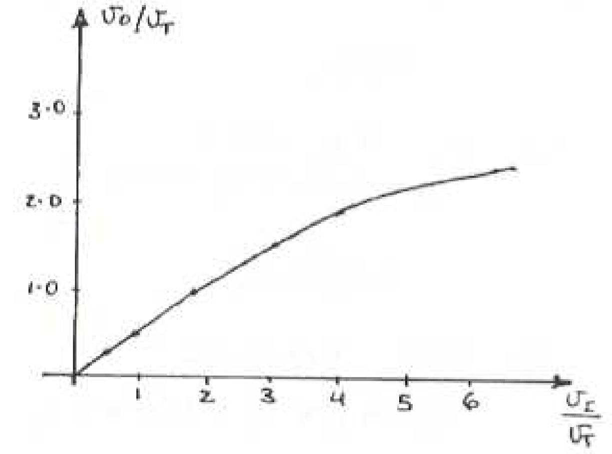
CONT.

Ideal curve given by

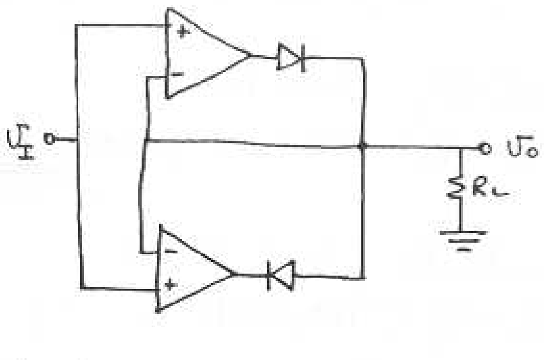
$$v_O = 2.42\,V_T \sin\left(\frac{v_I}{6.6\,V_T} \times 90^\circ\right)$$

$$\frac{v_I}{V_T} = \frac{6.6}{90} \sin^{-1}\left(\frac{v_O}{2.42\,V_T}\right)$$

v_O/V_T	0.25	0.50	1.00	1.50	2.00	2.40	2.42
v_I/V_T	0.451	0.905	1.85	2.89	4.20	6.29	6.52
v_I/V_T (ideal)	0.435	0.874	1.79	2.81	4.09	6.06	6.60



13.46



for high opamp gain $v_O = v_I$

$\therefore$ v_O is also a 10Vpp sine wave.

for $v_I > 0$

$$v_O = v_I - \frac{0.7}{A} \cong v_I$$

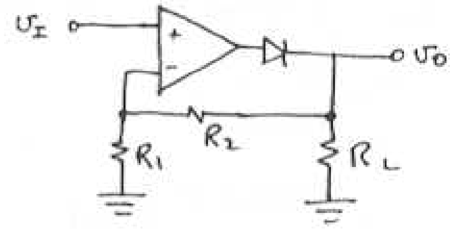
with D_1 "on"
D_2 "off"

for $v_I < 0$

$$v_O = v_I + \frac{0.7}{A} \cong v_I$$

with D_1 "off"
D_2 "on"

13.47

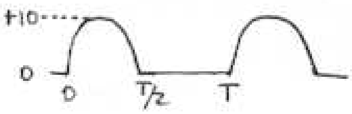


for $v_I \geq 0$

$$v_O = \left(1 + \frac{R_2}{R_1}\right) v_I$$

for a gain of 2 $\underline{\underline{R_1 = R_2 = 10k\Omega}}$

for v_I = 10Vpp Sine wave $v_O \Rightarrow$



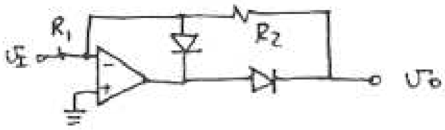
$$\text{Avg} = \frac{1}{T}\int_0^{T/2} 10 \sin\frac{2\pi}{T}t\,dt$$

$$= \frac{1}{T}\frac{T}{2\pi}\cos\frac{2\pi}{T}t \times (-10)\Big|_{t=0}^{T/2}$$

$$= \frac{-10}{2\pi}\left(\cos\pi - \cos 0\right)$$

$$= 10/\pi = \underline{3.18\text{ V}}$$

13.48



CONT.

for $v_I < 0 \Rightarrow v_O = -R_2/R_1$

$R_{in} = R_1 = \underline{\underline{100k\Omega}}$ $\therefore \underline{\underline{R_2 = 200k\Omega}}$

13.49

for high R_{in}, use $\underline{\underline{R_1 = 1M\Omega}}$

AC gain is given by R_2/R_1

$$\Rightarrow \underline{\underline{R_2 = 1M\Omega}}$$

Now for 1Vrms sine, peak is 1.414V.
The value of V_1 is then $\frac{1.414}{\pi} = 0.450V$

For 10V out at second stage

$$\text{gain (dc)} = \frac{10}{0.450} = 22.2$$

$\therefore R_4/R_3 = 22.2$

& choose $\frac{1}{2\pi R_4 C} = 10Hz$ (i.e. corner frequency)

To make C small, make $R_4 = 1M\Omega$

$$\therefore C = \underline{\underline{15.9nF}}$$

$$R_3 = \frac{1M\Omega}{22.2} = \underline{\underline{45k\Omega}}$$

13.50

At the +ve terminal $V_+ = -5V$

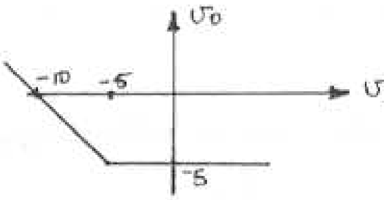
for $v_I > -5$ D_1 is "ON" and faces virtual short. $\therefore V_- = -5$, and no current will flow in feedback R.

$$\therefore \underline{v_O = -5V}$$

for $v_I < -5$ - D_1 "off" and

$$\frac{v_O}{v_I} = \frac{-5 - v_I}{R} = \frac{v_O + 5}{R}$$

$$\Rightarrow \underline{v_O = -v_I -}$$



13.51

for $v_I < 0$

D_2 "off"

$\frac{v_{O1}}{v_I} = -1$

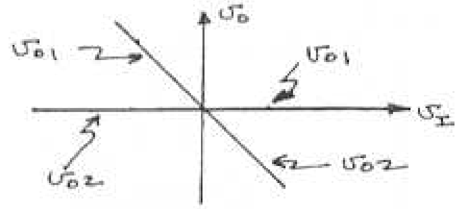
$\frac{v_{O2}}{v_I} = 0$

for $v_I > 0$

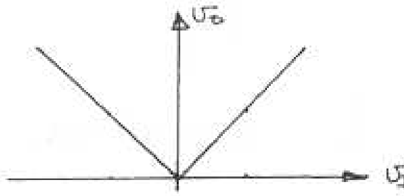
D_1 - "off"

$\frac{v_{O1}}{v_I} = 0$

$\frac{v_{O2}}{v_I} = -1$



13.52



For $v_I < 0$ ~ Diode is on, and cathode is forced to $\approx 0V$.

$\therefore v_O/v_I = -1$

For $v_I > 0$ ~ Diode is off, and the cathode now follows v_I since no current flows in Resistor. So v_O must follow

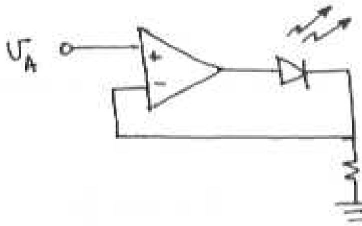
CONT.

v_I so that no current flows in feedback resistor.

$\therefore \frac{v_O}{v_I} = +1$

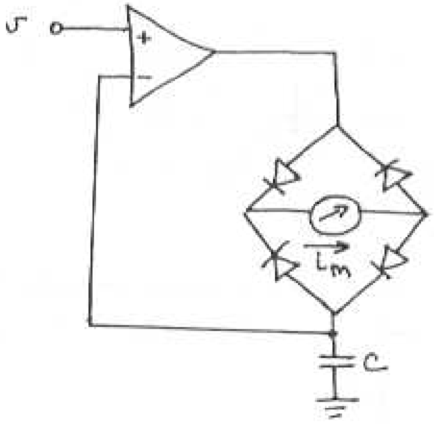
13.53

Simply place the LED in the feedback path.



$i_D = \frac{V_A}{R_1}$

13.54



$i_m = |i_C| = C\frac{dv}{dt}$ using $R = 1k\Omega$

$i_m = |i_R| = \frac{|v|}{R} = \frac{|v|}{1k\Omega} \Rightarrow i_m = |v|$ mA

Now $v = V \sin 2\pi 60t$

$\Rightarrow i_m = C \times 2\pi 60 |\cos(2\pi 60t)|$

for equivalence:

$\frac{V}{10^3}|\sin 2\pi 60t| = 2\pi 60 VC\, |\cos 2\pi 60t|$

$\therefore C = \frac{1}{2\pi 60 10^3} = \underline{\underline{2.65\mu F}}$

At 120Hz: $i_m = 2\pi\, 120 VC\, |\cos 2\pi 60t|$

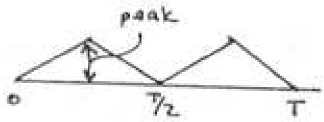
$i_{m_{120}} = 2\, i_{m_{60}}$

At 180Hz: $i_{m_{180}} = 3\, i_{m_{60}}$

For Δ-wave

with R_1

$i_m = 1mA$, $R = 1k\Omega$



$\therefore$ Full wave rectified wave has average voltage = 1V. $\therefore V_{peak} = 2V$

with C: slope $= \frac{V_{peak}}{T/4} = 4V_{peak} f$

$= 4 \times 2 \times 60 = 480$

Now, current through the capacitor will be a square wave (50% duty cycle)

Peak current $= 2.65 \times 10^{-6} \times 480$

$= 1.27$ mA.

$\therefore i_m = i_{avg} = \underline{1.27\, mA}$

13.55

10V pulses of 10μs, and large C_{LOAD}, will cause the op amp to current limit.

Charge transfered in one pulse:

$Q = (10mA)(10\mu s)$

$= 10^{-7}$ C

CONT

Voltage change per pulse:

$$\Delta V = Q/C = \frac{10^{-7}}{10\times10^{-6}} = 10\,mV$$

After:	1 pulse	$V_c = 10\,mV$
	2 pulses	20mV
	10 pulses	100mV

to reach	0.5V	require	50 pulses
	1.0V		100 "
	2.0V		200 "

13.56

For 1Vpp, peak detector output =
$V_o = 0.5\,V$.

Ripple Voltage = $(1\%)\,0.5 = 5\,mV$

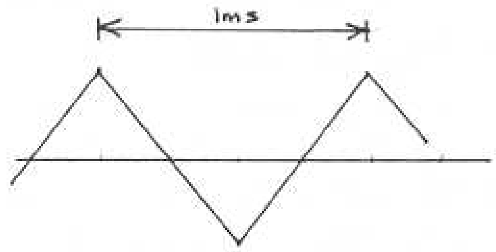
Total leakage = $10 + 1 = 11\,nA$

∴ total charge lost:

$$\Delta Q = 11\,nA \times 1\,ms = 11\,pC$$

∴ Required capacitance:

$$C = \frac{Q}{\Delta V} = \frac{11\times10^{-12}}{5\times10^{-3}} = \underline{\underline{2.2\,nF}}$$



Chapter 14 _ Problems

14

14.1 Refer to Fig. 14.2 (page 1232). The upper limit of the output voltage is determined by the saturation of Q_1 as

$$v_{O\max} = V_{CC} - V_{CEsat}$$
$$= 5 - 0.3 = \underline{4.7\ V}$$

The corresponding input is

$$v_I = 4.7 + 0.7 = \underline{5.4\ V}$$

The bias current I is

$$I = \frac{0 - (-V_{CC} + V_{BE2})}{R}$$
$$= \frac{5 - 0.7}{1} = 4.3\ mA$$

The lower limit of v_O is determined by either Q_1 cutting off,

$$\frac{-v_O}{R_L} = I \Rightarrow v_O = -4.3\ V$$

or by Q_2 saturating,

$$v_O = -V_{CC} + V_{CEsat} = -4.7\ V$$

Obviously, $v_{O\min} = \underline{\underline{-4.3\ V}}$

and the corresponding input is

$$v_I = -4.3 + 0.7 = \underline{-3.6\ V}$$

If the emitter-base junction area of Q_3 is made twice as large as that of Q_2, I becomes one half its previous value,

14

$$I = \frac{4.3}{2} = 2.15\ mA$$

and thus the lower limit of v_O changes to

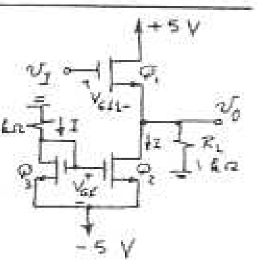
$$v_{O\min} = -IR_L = \underline{-2.15\ V}$$

The corresponding value of v_I is

$$v_I = -2.15 + 0.7 = \underline{-1.45\ V}$$

The upper limit does not change.

14.2 First we determine the bias current I as follows:



$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$$

But $V_{GS} = 5 - IR$
$= 5 - I$

Thus

$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(5 - I - V_t)^2$$
$$I = 10\,(5 - I - 1)^2$$
$$\Rightarrow I^2 - 8.1I + 16 = 0$$

$I = 3.416\ mA$ and $V_{GS} = 1.584\ V$

14

The upper limit on v_O is determined by Q_1 leaving the saturation region (and entering the triode region). This occurs when v_I exceeds V_{D1} by V_t volts,

$$v_{I\max} = 5 + 1 = \underline{+6\ V}$$

To obtain the corresponding value of v_O we must find the corresponding value of V_{GS1}, as follows:

$$v_O = v_I - V_{GS1}$$
$$i_L = \frac{v_O}{R_L} = \frac{v_I - V_{GS1}}{1}$$
$$= v_I - V_{GS1} = 6 - V_{GS1}$$
$$i_1 = I + i_L$$
$$= 3.416 + 6 - V_{GS1}$$
$$= 9.416 - V_{GS1}$$

But $i_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_t)^2$

Thus, $9.416 - V_{GS1} = 10\,(V_{GS1} - 1)^2$

$$\Rightarrow V_{GS1}^2 - 1.9\,V_{GS1} + 0.0584 = 0$$
$$V_{GS1} = 1.869\ V$$
$$v_{O\max} = 6 - 1.869$$
$$= \underline{\underline{+4.131\ V}}$$

14

The lower limit of v_O is determined either by Q_1 cutting off,

$$v_O = -IR_L = -3.416 \times 1 = -3.416\ V$$

or by Q_2 leaving saturation,

$$v_O = V_{G2} - V_t$$
$$= -5 + 1.584 - 1 = -4.416\ V$$

Thus, $v_{O\min} = \underline{\underline{-3.416\ V}}$

The corresponding value of v_I is determined by noting that since Q_1 is on the verge of cut-off, $V_{GS1} = V_t = 1\ V$ and

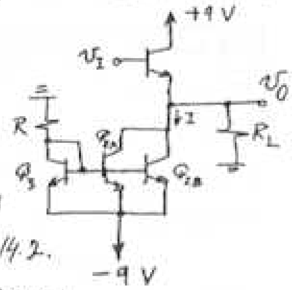
$$v_I = -3.416 + 1 = \underline{\underline{-2.416\ V}}$$

14.3 Refer to Fig. 14.2. With $V_{CC} = +9\ V$, the upper limit on v_O is $+8.7\ V$, which is greater than the required value of $+7\ V$. To obtain a lower limit of $-7\ V$ we select I so that

$I R_L = 7$

$\Rightarrow \quad I = 7\text{ mA}$ $\boxed{14}$

Since we are provided with four devices, we can minimize the total supply current by paralleling two devices to form Q_2 as shown below. The resulting supply current will be $3 \times \frac{I}{2}$ rather than $2I$ which is the value obtained in the circuit of Fig. 14.2. Thus the supply current is 10.5 mA. The value of R is found from



$$R = \frac{8.3\text{ V}}{3.5\text{ mA}} = \underline{2.37\text{ k}\Omega}$$

In a practical design we would select a standard value for R that results in I somewhat larger than 7 mA. Say, $R = \underline{2.2\text{ k}\Omega}$

$\boxed{14.4}$ Refer to Fig. 14.2. For a load resistance of $100\ \Omega$ and v_O ranging between −5 V and +5 V, the maximum current through Q_1 is $\boxed{14}$

$$I + \frac{5}{0.1} = I + 50\text{, mA}$$

and the minimum current is

$$I - \frac{5}{0.1} = I - 50\text{, mA}.$$

For a current ratio of 10,

$$\frac{I+50}{I-50} = 10$$

$$\Rightarrow \quad I = 61.1\text{ mA}$$

$$R = \frac{9.3\text{ V}}{61.1\text{ mA}} = \underline{152\ \Omega}$$

The incremental voltage gain is

$$A_v = \frac{R_L}{R_L + r_{e1}}$$

For $R_L = 100\ \Omega$;

At $v_O = +5\text{ V}$, $I_{E1} = 61.1 + 50 = 111.1\text{ mA}$

$$r_{e1} = \frac{25}{111.1} = 0.225\ \Omega$$

$$A_v = \frac{100}{100 + 0.225} = \underline{0.998\text{ V/V}}$$

At $v_O = 0\text{ V}$, $I_{E1} = 61.1\text{ mA}$ $\boxed{14}$

$$r_{e1} = \frac{25}{61.1} = 0.409\ \Omega$$

$$A_v = \frac{100}{100.409} = \underline{0.996\text{ V/V}}$$

At $v_O = -5\text{ V}$, $I_{E1} = 61.1 - 50 = 11.1\text{ mA}$

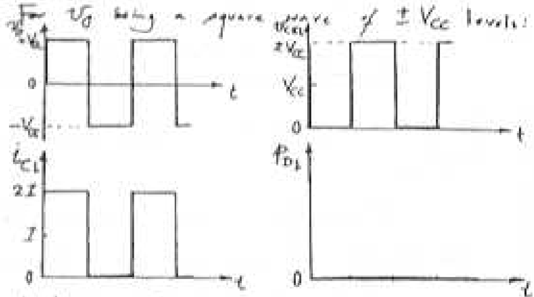
$$r_{e1} = \frac{25}{11.1} = 2.25\ \Omega$$

$$A_v = \frac{100}{102.25} = \underline{0.978\text{ V/V}}$$

Thus the incremental gain changes by $0.998 - 0.978 = 0.02$ or about $\underline{2\%}$ over the range of v_O.

$\boxed{14.5}$ Refer to Fig. 14.2 and 14.4

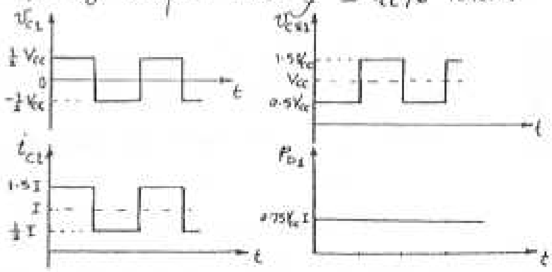
For v_O being a square wave of $\pm V_{CC}$ levels:



$P_{D1}|_{average} = \underline{0}$

For the corresponding sine-wave case [Fig. 14.4], $P_{D1}|_{av} = \underline{\frac{1}{2} V_{CC} I}$

For v_O a square wave of $\pm V_{CC}/2$ levels:



$P_{D1}|_{average} = \underline{0.75\, V_{CC}\, I}$

For a sine-wave output of $V_{CC}/2$ peak amplitude,

$$v_{C1} = \tfrac{1}{2} V_{CC} \sin\theta$$

$$i_{C1} = I + \frac{\frac{1}{2}V_{CC}}{R_L} \sin\theta = I + \tfrac{1}{2} I \sin\theta$$

$$v_{CE1} = V_{CC} - \tfrac{1}{2} V_{CC} \sin\theta$$

$$P_{D1} = \left(V_{CC} - \tfrac{1}{2} V_{CC} \sin\theta\right)\left(I + \tfrac{1}{2} I \sin\theta\right)$$

$$= V_{CC} I - \tfrac{1}{4} V_{CC} I \sin^2\theta$$

$$= V_{CC} I - \tfrac{1}{4} V_{CC} I \times \tfrac{1}{2}(1 - \cos 2\theta)$$

$$= \tfrac{7}{8} V_{CC} I + \tfrac{1}{8} V_{CC} I \cos 2\theta$$

$$P_{D1}|_{average} = \underline{\tfrac{7}{8} V_{CC} I}$$

14.6 In all cases, the average voltage across Q_2 is equal to V_{CC}. Thus, since Q_2 conducts a constant current I, its average power dissipation is $\underline{V_{CC}\, I}$. **14**

14.7 V_{CC} = 16, 12, 10 and 8 V

$I = 100$ mA $\quad R_L = 100\ \Omega$

$\hat{V}_o = 8$ V

$$\eta = \frac{1}{4}\left(\frac{\hat{V}_o}{I R_L}\right)\left(\frac{\hat{V}_o}{V_{CC}}\right)$$

$$= \frac{1}{4}\left(\frac{8}{10}\right)\left(\frac{8}{V_{CC}}\right) = \frac{1.6}{V_{CC}}$$

V_{CC}	16	12	10	8
η	10%	13.3%	16%	20%

14.8 Refer to Fig. P14.8. The bias current I is

$$I = \tfrac{1}{2}\mu_n C_{ox}\tfrac{W}{L}(0-(-2))^2 = 10 \times 4 = 40 \text{ mA}$$

For $R_L = \infty$, the maximum v_O is limited by the saturation of Q_1 to $V_{CC} - V_{CEsat} = \underline{+4.7\text{ V}}$, and the minimum v_O is limited by Q_2 leaving saturation to $-V_{CC} + |V_t| = \underline{-3\text{ V}}$.

For $R_L = 100\ \Omega$, the maximum v_O is still $\underline{+4.7\text{V}}$, and the minimum v_O is still $\underline{-3\text{V}}$ because at this level the current in R_L is -30 mA and thus Q_1 is still conducting 10 mA. (That is, the cut off limit of $-40\text{ mA} \times 100\ \Omega = -4$ V is not applicable since Q_2 would have entered the triode region.) **14**

To obtain a 1-V peak sine-wave output, R_L must be limited to ($\leq$)

$$R_L = \frac{1\text{ V}}{40\text{ mA}} = \underline{\underline{25\ \Omega}}$$

For this case,

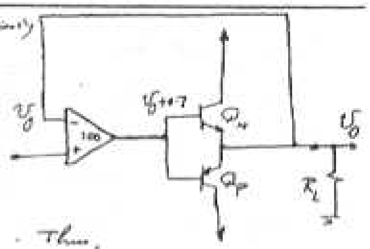
$$\eta = \frac{1}{4}\left(\frac{\hat{V}_o}{I R_L}\right)\left(\frac{\hat{V}_o}{V_{CC}}\right)$$

$$= \frac{1}{4}\left(\frac{1}{0.04 \times 25}\right)\left(\frac{1}{5}\right)$$

$$= \underline{\underline{5\ \%}}$$

14.9 Refer to Figs. 14.6 and 14.7. A 10% loss in peak amplitude is obtained when the amplitude of the input signal is $\underline{5\text{ V}}$. **14**

14.10 With v_I sufficiently positive so that Q_N is conducting, the situation shown obtains. Thus,



$$(v_I - v_O) \times 100 = v_O + 0.7$$

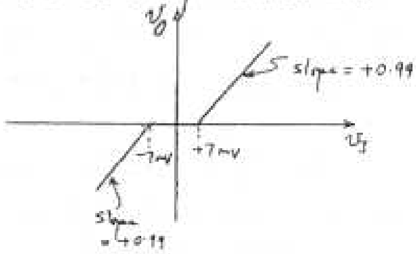
$$\Rightarrow v_O = \frac{1}{1.01}(v_I - 0.007)$$

This relationship applies for $v_I \geq 0.007$. Similarly, for v_I sufficiently negative so that Q_P conducts, the voltage at the output of the amplifier becomes $v_O - 0.7$, thus

$$(v_I - v_O) \times 100 = v_O - 0.7$$

$$\Rightarrow v_O = \frac{1}{1.01}(v_I + 0.007)$$

This relationship applies for $v_I \leq -0.007$ **14**
The result is the transfer characteristic

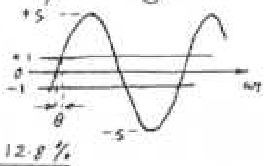


Without the feedback arrangement, the deadband becomes ± 700 mV and the slope changes a little (to nearly +1 V/V).

14.11 With $R_L = \infty$ and $v_I = +5$ V, v_O will be $v_I - V_{GS1} = v_I - V_t = 4$ V (since the current is virtually zero and thus $V_{GS} \simeq V_t$). Thus the resulting peak output voltage will be $\underline{4\text{ V}}$.

$\sin\theta = \frac{1}{5}$

$\Rightarrow \theta = 11.54°$

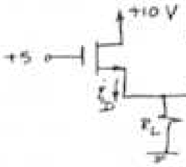


Cross-over interval $= 4\theta$

Fraction of cycle $= \frac{4\theta}{360°} = \underline{\underline{12.8\%}}$

For $v_I = +5$ V and $v_O = +2.5$ V, $V_{GS} = 2.5$ V, thus

14



$$i_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$$

$$= 0.1\,(2.5-1)^2 = 0.225 \text{ mA}$$

Thus, $R_L = \frac{2.5}{0.225} = \underline{\underline{11.1\ k\Omega}}$

14.12 For $V_{CC} = 10$ V and $R_L = 100\ \Omega$, the maximum sine-wave output power occurs when $\hat{V}_o = V_{CC}$ and is

$$P_{L\,max} = \frac{1}{2}\frac{V_{CC}^2}{R_L}$$

$$= \frac{1}{2} \times \frac{100}{100} = \underline{\underline{0.5\text{ W}}}$$

Correspondingly,

$$P_{S+} = P_{S-} = \frac{1}{\pi}\frac{\hat{V}_o}{R_L}V_{CC}$$

$$= \frac{1}{\pi} \times \frac{10}{100} \times 10 = 0.318\text{ W}$$

for a total supply power of

$$P_S = 2 \times 0.318 = \underline{\underline{0.637\text{ W}}}$$

The power conversion efficiency η is

$$\eta = \frac{P_L}{P_S} \times 100 = \frac{0.5}{0.637} \times 100 = \underline{\underline{78.5\%}}$$

For $\hat{V}_o = 5$ V, 14

$$P_L = \frac{1}{2}\frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{25}{100} = \underline{\underline{\frac{1}{8}\text{ W}}}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi}\frac{\hat{V}_o}{R_L}V_{CC}$$

$$= \frac{1}{\pi} \times \frac{5}{100} \times 10 = \frac{1}{2\pi}$$

$$P_S = \frac{1}{\pi}\text{ W} = \underline{\underline{0.318\text{ W}}}$$

$$\eta = \frac{1/8}{1/\pi} \times 100 = \frac{\pi}{8} \times 100 = \underline{\underline{39.3\%}}$$

14.13 $V_{CC} = 5$ V

For maximum η,

$$\hat{V}_o = V_{CC} = \underline{\underline{5\text{ V}}}$$

The output voltage that results in maximum device dissipation is given by Eq. (9.20),

$$\hat{V}_o = \frac{2}{\pi}V_{CC}$$

$$= \frac{2}{\pi} \times 5 = \underline{\underline{3.18\text{ V}}}$$

If operation is always at full output voltage, $\eta = 78.5\%$ and thus

$$P_{dissipation} = (1-\eta)P_S$$

$$= (1-\eta)\frac{P_L}{\eta} = \frac{1-0.785}{0.785}P_L = 0.274\,P_L$$

$$P_{dissipation}/\text{device} = \frac{1}{2} \times 0.274\,P_L = 0.137\,P_L$$

For a rated device dissipation of 1 W, and using a factor of 2 safety margin, 14

$$P_{dissipation}/\text{device} = 0.5\text{ W}$$

$$= 0.137\,P_L$$

$$\Rightarrow P_L = 3.65\text{ W}$$

$$3.65 = \frac{1}{2} \times \frac{25}{R_L}$$

$$\Rightarrow R_L = \underline{\underline{3.425\ \Omega}} \quad (\text{i.e. } R_L \geq 3.425\ \Omega)$$

The corresponding output power (i.e. greatest possible output power) is $\underline{\underline{3.65\text{ W}}}$.

If operation is allowed at $\hat{V}_o = \frac{1}{2}V_{CC} = 2.5$ V,

$$\eta = \frac{\pi}{4}\frac{\hat{V}_o}{V_{CC}} \quad (\text{Eq. } 9.15)$$

$$= \frac{\pi}{4} \times \frac{1}{2} = 0.393$$

$$P_{dissipation}/\text{device} = \frac{1}{2}\frac{1-\eta}{\eta}P_L = 0.772\,P_L$$

$$0.5 = 0.772\,P_L$$

$$\Rightarrow P_L = \underline{\underline{0.647\text{ W}}}$$

$$= \frac{1}{2}\frac{2.5^2}{R_L}$$

$$\Rightarrow R_L = \underline{\underline{4.83\ \Omega}} \quad (\text{i.e. } \geq 4.83\ \Omega)$$

14.14 $P_L = \frac{1}{2}\frac{\hat{V}_o^2}{R_L}$ 14

$100 = \frac{1}{2}\frac{\hat{V}_o^2}{16}$

$\hat{V}_o = 56.6$ V

$V_{CC} = 56.6 + 6 = 60.6 \longrightarrow \underline{61\text{ V}}$

Peak current from each supply $= \frac{\hat{V}_o}{R_L} = \frac{56.6}{16}$
$= \underline{3.54\text{ A}}$

$P_{S+} = P_{S-} = \frac{1}{\pi} \times 3.54 \times 61$

Thus, $P_S = \frac{2}{\pi} \times 3.54 \times 61$

$= \underline{137.4\text{ W}}$

$\eta = \frac{P_L}{P_S} = \frac{100}{137.4} = \underline{73\%}$

Using Eq. (9.22),

$P_{DNmax} = P_{DPmax} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{61^2}{\pi^2 \times 16}$

$= \underline{23.6\text{ W}}$

14.15 $P_L = \frac{\hat{V}_o^2}{R_L}$

$P_{S+} = P_{S-} = \frac{1}{2}\left(\frac{\hat{V}_o}{R_L}\right) V_{SS}$

$P_S = \frac{\hat{V}_o}{R_L} V_{SS}$

$\eta = \frac{P_L}{P_S} = \frac{\hat{V}_o^2/R_L}{\hat{V}_o V_{SS}/R_L} = \frac{\hat{V}_o}{V_{SS}}$

$\eta_{max} = 1$ (100%), obtained for $\hat{V}_o = V_{SS}$

$P_{Lmax} = \frac{V_{SS}^2}{R_L}$ 14

$P_{dissipation} = P_S - P_L$

$= \frac{\hat{V}_o}{R_L} V_{SS} - \frac{\hat{V}_o^2}{R_L}$

$\frac{\partial P_{dissipation}}{\partial \hat{V}_o} = \frac{V_{SS}}{R_L} - \frac{2\hat{V}_o}{R_L}$

$= 0$ for $\underline{\hat{V}_o = \frac{V_{SS}}{2}}$

Correspondingly, $\eta = \frac{V_{SS}/2}{V_{SS}} = \frac{1}{2}$ or $\underline{50\%}$

14.16 $A_v = \frac{R_L}{R_L + R_{out}}$

where, $R_{out} = \frac{r_e}{2} = \frac{V_T}{2 I_Q}$

For $A_v \geq 0.99$ with $R_L \geq 100\ \Omega$,

$0.99 = \frac{100}{100 + R_{out}} \Rightarrow R_{out} = 1\ \Omega$

$\frac{V_T}{2 I_Q} = 1 \Rightarrow I_Q = \underline{12.5\text{ mA}}$

$V_{BB} = 2 V_{BE}$

$= 2\left[0.7 + V_T \ln \frac{12.5}{100}\right]$

$= \underline{1.296\text{ V}}$

14.17 $R_{out} = \frac{1}{g_m} // \frac{1}{g_m} = \frac{1}{2g_m} = 10\ \Omega$ 14

$\Rightarrow g_m = \frac{1}{20} = 50$ mA/V

But, $g_m = \mu C_{ox}\frac{W}{L}(V_{GS} - V_t)$

$50 = 2 \times 100\,(V_{GS} - 1)$

$\Rightarrow V_{GS} = 1.25$ V

$V_{GG} = \underline{2.5\text{ V}}$

14.18 $I_Q = \underline{1\text{ mA}}$

For output of -1 V, $i_L = -\frac{1}{100} = -10$ mA

Using Eq. (14.27),

$i_N^2 - i_L i_N - I_Q^2 = 0$

$i_N^2 + 10\, i_N - 1 = 0$

$i_N = 0.1$ mA

$i_P = \underline{10.1\text{ mA}}$

Thus v_{EBp} increases by $V_T \ln \frac{10.1}{1} = 0.06$ V and the input step must be $\underline{-1.06\text{ V}}$

Largest possible positive output from 6 to 10, i.e. $\underline{4\text{V}}$.

Largest negative output from 6 to 0, i.e. $\underline{6\text{V}}$

14.19 $R_{out} = r_e/2 = 10\ \Omega$ 14

$\Rightarrow r_e = 20\ \Omega$

$I_Q = \frac{V_T}{r_e} = \frac{25}{20} = 1.25$ mA

Thus, $n = \frac{1.25}{0.1} = \underline{12.5}$

14.20 $I_Q \simeq I_{bias} = 0.5\ mA$, neglecting the base current of Q_N. More precisely,

$$I_Q = I_{bias} - \frac{I_Q}{\beta+1}$$

$$\Rightarrow I_Q = \frac{I_{bias}}{1+\frac{1}{\beta+1}} \simeq 0.98 \times 0.5 = \underline{\underline{0.49\ mA}}$$

The largest positive output is obtained when all of I_{bias} flows into the base of Q_N, resulting in

$$v_O = (\beta_N+1) I_{bias} R_L$$

$$= 51 \times 0.5 \times 100\Omega = \underline{\underline{2.55\ V}}$$

The largest possible negative output voltage is limited by the saturation of Q_P to $-10 + V_{EC\,sat} = \underline{\underline{-10\ V}}$.

To achieve a maximum positive output of 10 V without changing I_{bias}, β_N must be

$$10 = (\beta_N+1) \times 0.5 \times 100\Omega$$

14

$$\Rightarrow \beta_N = \underline{\underline{199}}$$

Alternatively, if β_N is held at 50, I_{bias} must be increased so that

$$10 = 51 \times I_{bias} \times 100\Omega$$

$$\Rightarrow I_{bias} = \underline{\underline{1.96\ mA}}$$

for which,

$$I_Q = \frac{I_{bias}}{1+\frac{1}{\beta+1}} = \underline{\underline{1.92\ mA}}$$

14.21 At 20°C, $I_Q = 1\ mA = I_S\, e^{(0.6/0.025)}$

$$\Rightarrow I_S\,(\text{at } 20°C) = 3.78 \times 10^{-11}\ mA$$

At 70°C, $I_S = 3.78 \times 10^{-11} \times (1.14)^{50}$

$$= 2.64 \times 10^{-8}\ mA$$

At 70°C, $V_T = 25\ \frac{273+70}{273+20} = 29.3\ mV$

Thus, $I_Q\,(\text{at } 70°C) = 2.64 \times 10^{-8}\ e^{0.6/0.0293}$

$$= \underline{\underline{20.7\ mA}}$$

Additional current $= 20.7 - 1 = 19.7\ mA$

Additional power $= 2 \times 20 \times 19.7 = \underline{\underline{788\ mW}}$

Additional temperature rise $= 10 \times 0.788 = \underline{\underline{7.9°C}}$

At 77.9°: $V_T = \frac{25}{293}(273+77.9) = 29.9\ mV$

$$I_Q = 3.78 \times 10^{-11} \times (1.14)^{57.9}\ e^{(0.6/0.0299)}$$

14

$$= \underline{\underline{37.6\ mA}}$$

etc., etc.

14.22 Refer to Fig. P14.22.

$$0.100 = 1\,(V_{GS3} - 1)^2$$

$$V_{GS3} = 1.316\ V = |V_{GS4}|$$

$$A_v = \frac{R_L}{R_L + R_{out}} = 0.99$$

$$\frac{1}{1+R_{out}} = 0.99$$

$$\Rightarrow R_{out} = 0.01\ k\Omega = 10\Omega$$

But $R_{out} = (1/g_{m1}) \,//\, (1/g_{m2})$

$$= \frac{1}{2g_m} \Rightarrow g_m = 50\ mA/V$$

$$50 = k_1\,(V_{GS1} - V_t)$$

$$= k_1\,(1.316 - 1)$$

$$\Rightarrow k_1 = \frac{50}{0.316} = 158.2\ \ mA/V^2$$

$$n = \frac{158.2}{2} = \underline{\underline{79.1}}$$

14.23 Since the peak positive output current is 200 mA, the base current of Q_N can be as high as $\frac{200}{\beta_N+1} = \frac{200}{51} \simeq 4\ mA$. We select $I_{bias} = 5\ mA$, thus providing the multiplier with a minimum current of 1 mA. 14

Under quiescent conditions ($v_O = 0$ and $i_L = 0$) the base current of Q_N can be neglected. Selecting $I_R = 0.5\ mA$ leaves $I_{C1} = 4.5\ mA$.

To obtain a quiescent current of 2 mA in the output transistors, V_{BB} should be

$$V_{BB} = 2V_T \ln\frac{2\times10^{-3}}{10^{-13}} = 1.19\ V$$

Thus

$$R_1 + R_2 = \frac{V_{BB}}{I_R} = \frac{1.19}{0.5} = 2.38\ k\Omega$$

At a collector current of 4.5 mA, Q_1 has

$$V_{BE1} = V_T \ln\frac{4.5\times10^{-3}}{10^{-14}} = 0.671\ V$$

The value of R_1 can now be determined as

$$R_1 = \frac{0.671}{0.5} = \underline{\underline{1.34\ k\Omega}}$$

and

$$R_2 = 2.38 - 1.34 = \underline{\underline{1.04\ k\Omega}}$$

14.24 (a) $V_{BE} = 0.7\text{ V}$ at 1 mA

14

At 0.5 mA, $V_{BE} = 0.7 + 0.025 \ln \frac{0.5}{1} = 0.683\text{ V}$

Thus $R_1 = \frac{0.683}{0.5} = \underline{1.365\text{ k}\Omega}$

and $R_2 = \underline{1.365\text{ k}\Omega}$

(b) For $I_{bias} = 2\text{ mA}$, I_C increases to nearly 1.5 mA for which

$V_{BE} = 0.7 + 0.025 \ln \frac{1.5}{1} = 0.710\text{ V}$

Note that $I_R = \frac{0.710}{1.365} = 0.52\text{ mA}$ is very nearly equal to the assumed value of 0.50 mA, thus no further iterations are required.

$V_{BB} = 2V_{BE} = \underline{1.420\text{ V}}$

(c) For $I_{bias} = 10\text{ mA}$, assume that I_R remains constant at 0.5 mA, then $I_{C1} = 9.5\text{ mA}$

and $V_{BE} = 0.7 + 0.025 \ln \frac{9.5}{1} = 0.756\text{ V}$

at which

$I_R = \frac{0.756}{1.365} = 0.554\text{ mA}$

Thus, $I_{C1} = 10 - 0.554 = 9.45\text{ mA}$

and $V_{BE} = 0.7 + 0.025 \ln \frac{9.45}{1} = 0.756\text{ V}$

Thus $V_{BB} = 2 \times 0.756 = \underline{1.512\text{ V}}$

(d) Now for $\beta = 100$,

14

$I_{R1} = \frac{0.756}{1.365} = 0.554\text{ mA}$

$I_{R2} = 0.554 + \frac{9.45}{101} = 0.648\text{ mA}$

$I_C = 10 - 0.648 = 9.352\text{ mA}$

Thus, $V_{BE} = 0.7 + 0.025 \ln \frac{9.352}{1} = 0.756\text{ V}$

$V_{BB} = 0.756 + I_{R2} R_2$

$= 0.756 + 0.648 \times 1.365$

$= \underline{1.641\text{ V}}$

14.25 Power rating $= \frac{130 - 30}{2} = \underline{50\text{ W}}$

$I_{Cav} \le \frac{50}{20} = \underline{2.5\text{ A}}$

14.26 $\theta_{JA} = \frac{150 - 25}{0.2} = 625\,^\circ\text{C/W} = \underline{0.625\,^\circ\text{C/mW}}$

At 70°C, Power rating $= \frac{150 - 70}{0.625} = \underline{128\text{ mW}}$

$T_J = 50 + 0.625 \times 100 = \underline{112.5\,^\circ\text{C}}$

14.27 $T_J \le 50 + 3 \times 30 = \underline{140^\circ\text{C}}$

$V_{BE} = 800 - 2 \times (140 - 25) = 570\text{ mV}$

$= \underline{0.57\text{ V}}$

14.28 (a) $\theta_{JA} = \frac{T_{Jmax} - T_{A0}}{P_{D0}}$

14

$= \frac{100 - 25}{2} = \underline{37.5\,^\circ\text{C/W}}$

(b) At $T_A = 50^\circ\text{C}$,

$P_{Dmax} = \frac{T_{Jmax} - T_A}{\theta_{JA}}$

$= \frac{100 - 50}{37.5} = \underline{1.33\text{ W}}$

(c) $T_J = 25^\circ + 37.5 \times 1 = \underline{62.5\,^\circ\text{C}}$

14.29 $T_C - T_A = \theta_{CA} P_D$

$= (\theta_{CS} + \theta_{SA}) P_D$

$\Rightarrow P_D = \frac{T_C - T_A}{\theta_{CS} + \theta_{SA}} = \frac{90 - 30}{0.5 + 0.1} = \underline{100\text{ W}}$

$T_J - T_C = \theta_{JC} P_D$

$130 - 90 = \theta_{JC} \times 100$

$\Rightarrow \theta_{JC} = \underline{0.4\,^\circ\text{C/W}}$

14.30 $\theta_{JC} = \frac{T_J - T_C}{P_D} = \frac{180^\circ - 50^\circ}{50} = \underline{2.6\,^\circ\text{C/W}}$

$T_J - T_S = \theta_{JS} P_D$

$180^\circ - T_S = (\theta_{JC} + \theta_{CS}) P_D$

$\Rightarrow T_S = 180 - (2.6 + 0.6) \times 30 = \underline{84^\circ}$

14

$T_S - T_A = \theta_{SA} P_D$

$84 - 39 = \theta_{SA} \times 30$

$\Rightarrow \theta_{SA} = \underline{1.5\,^\circ\text{C/W}}$

Required heat-sink length $= \frac{4.5\,^\circ\text{C/W/cm}}{1.5\,^\circ\text{C/W}}$

$= \underline{3\text{ cm}}$

14.31 $r_\pi = \frac{nV_T}{I_B} = \frac{2 \times 25 \times 10^{-3}}{0.5} = 0.1\,\Omega$

$r_x \simeq r_i - r_\pi = 0.98 - 0.1 = \underline{0.85\,\Omega}$

14.32 $V_{B'E} = V_{BE} - I_B r_x$

$= 1.05 - 0.19 \times 0.8$

$= 0.898\text{ V}$ (at $I_C = 5\text{ A}$)

For $I_C = 2\text{ A}$,

$V_{B'E} = 0.898 + mV_T \ln \frac{2}{5}$

$= 0.898 + 2 \times 0.025 \ln\left(\frac{2}{5}\right)$

$= 0.852\text{ V}$

$V_{BE} = 0.852 + I_B r_x$

$= 0.852 + \left(0.19 \times \frac{2}{5}\right) \times 0.8$

$= \underline{0.913\text{ V}}$

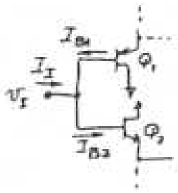
14.33 (a) For $R_L = \infty$:

14

At $v_I = 0$ V,

$I_{B1} = I_{B2} = \frac{2.87}{200}$

$I_I = I_{B2} - I_{B1} = \underline{0}$



At $v_I = +10$ V,

$I_{B1} = \frac{0.88}{200}$ mA $= 4.4\ \mu A$

$I_{B2} = \frac{4.87}{200}$ mA $= 24.4\ \mu A$

$I_I = I_{B2} - I_{B1} = \underline{20\ \mu A}$

At $v_I = -10$ V,

$I_{B1} = \frac{4.87}{200}$ mA $= 24.4\ \mu A$

$I_{B2} = \frac{0.88}{200}$ mA $= 4.4\ \mu A$

$I_I = I_{B2} - I_{B1} = \underline{-20\ \mu A}$

(b) For $R_L = 100\ \Omega$:

At $v_I = 0$, $I_I = \underline{0}$

At $v_I = +10$ V,

$I_{B1} = \frac{0.38}{200} = 1.9\ \mu A$

$I_{B2} = \frac{4.87}{200} = 24.4\ \mu A$

$I_I = I_{B2} - I_{B1} = \underline{22.5\ \mu A}$

At $v_I = -10$ V, $I_I = \underline{-22.5\ \mu A}$

14.34 For the design process:

14

At $v_I = +5$ V, the voltage across R_1, v_{R1}, is

$$v_{R1} = 10 - 5 - \overset{V_{EB1}}{0.7} = 4.3 \text{ V}$$

and, $i_{R1} = 2 \times 10$ mA (to allow for i_{B3} of as much as 10 mA and only a 2 to 1 variation in i_{E1})

Thus, $R_1 = \frac{4.3}{20} = \underline{215\ \Omega}$

Correspondingly,

$R_2 = \underline{215\ \Omega}$

Now, for $v_I = 0$ and $V_{EB1} \simeq 0.7$ V

$i_{R1} = \frac{10 - 0.7 - 0}{215} = 43.3$ mA

for which $v_{EB1} = 700 + 25 \ln \frac{43.3}{10} = 736.6$ mV

Since $I_Q = 40$ mA and $I_{S3} = 3 I_{S1}$,

$v_{BE3} = 700 + 25 \ln \frac{40}{3 \times 10} = 707.2$ mV

Thus, $R_3 = \frac{736.6 - 707.2}{40 \text{ mA}} = \underline{0.74\ \Omega}$

Correspondingly,

$R_4 = \underline{0.74\ \Omega}$

Output Resistance at $v_I = 0$:

14

As a result of 2 paths to output,

$$R_{out} = \frac{1}{2}\left(R_3 + r_{e3} + \frac{r_{e1}}{\beta_3 + 1}\right)$$

where $r_{e1} \simeq r_{e3} = \frac{25 \text{ mV}}{40 \text{ mA}} = 0.625\ \Omega$

Thus, $R_{out} = \frac{1}{2}\left(0.74 + 0.625 + \frac{0.625}{51}\right)$

$= \underline{0.69\ \Omega}$

Output Voltage for $v_I = +1$ V and $R_L = 2\ \Omega$

To start the solution, let $v_O \simeq 1$ V,

then $i_L \simeq \frac{1}{2} = 0.5$ A $= 500$ mA

$i_{B3} = \frac{500}{50} = 10$ mA

$i_{E1} \simeq \frac{10 - 1 - 0.7}{0.215} - 10 = 28.6$ mA

for which,

$v_{EB1} = 0.7 + 0.025 \ln \frac{28.6}{10} = 0.726$ V

$v_{B3} = 1 + 0.726 = 1.726$ V

Assuming $i_{E4} \simeq 0$,

$v_{BE3} \simeq 700 + 25 \ln \frac{500}{3 \times 10} = 770$ mV

Thus, $i_L = \frac{1.726 - 0.770}{0.74 + 2} = 0.349$ A

for which

14

$v_O = 2 \times 0.349 = 0.698$ V

Thus, the voltage drop across the series combination of R_4 and the EB junction of Q_4 can be found as follows,

$V_{B4} = V_{E4} = 1 - 0.74 = 0.26$ V

leaving a drop across the series combination of R_4 and EBJ4 of $0.698 - 0.26 = 0.438$. It follows that $i_{E4} \simeq 0$, as assumed.

Iterate again:

$i_L \simeq 0.35$ A

$i_{B3} \simeq \frac{0.35}{51} \simeq 7$ mA

$i_{E1} \simeq \frac{10 - 1 - 0.73}{0.215} - 7 = 31.5$ mA

$v_{EB3} = 0.7 + 0.025 \ln \frac{31.5}{10} = 0.729$ V

$v_{B3} = 1 + 0.729 = 1.729$ V

$v_{BE3} = 0.7 + 25 \ln \frac{350}{3 \times 10} = 0.761$ V

$i_L = \frac{1.729 - 0.761}{0.74 + 2} = 0.353$ A

14

$v_O = 2 \times 0.353$

$= \underline{0.706\ V}$

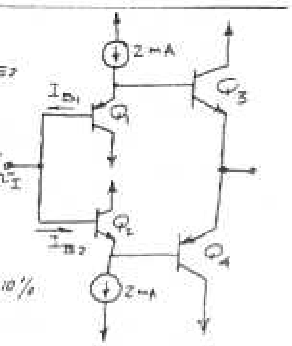
14.35 $I_{E1} = I_{E2} \simeq 2\ mA$

$I_Q = I_{E3} = I_{E4} = I_{E1} = I_{E2}$

Thus, $I_Q \simeq \underline{2\ mA}$

More precisely, $I_Q = \underline{\underline{1.96\ mA}}$

$I_{B1} = I_{B2} = \frac{2}{51} \simeq \underline{\underline{0.04\ mA}}$



The net input bias current is ideally $\underline{\text{zero}}$

For a β mismatch of 10%

$I_I = 0.10 \times 0.04\ mA$

$\simeq \underline{\underline{4\ \mu A}}$

For $R_L = 100\ \Omega$, the equivalent half circuit becomes

Thus, $2R_i = (\beta_1+1)[r_{e1} + (\beta_3+1)(r_{e3} + 2R_L)]$

where $r_{e1} = r_{e3} = \frac{25}{2}$

$= 12.5\ \Omega$

$R_i = \frac{51}{2}[0.0125 + 51(0.0125 + 0.200)]$

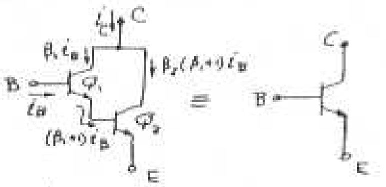
$= \underline{\underline{276.7\ k\Omega}}$

The gain can be determined using the equivalent half-circuit, as follows: **14**

$$A_v = \frac{v_o}{v_i} = \frac{2R_L}{2R_L + r_{e3} + \frac{r_{e1}}{\beta_3+1}}$$

$$= \frac{2 \times 100}{2\times 100 + 12.5 + \frac{12.5}{51}} = \underline{\underline{0.94\ V/V}}$$

14.36



$i_C = \beta_1 i_B + \beta_2(\beta_1+1) i_B$

$\beta_{eq} \equiv \frac{i_C}{i_B} = \underline{\underline{\beta_1 + \beta_2(\beta_1+1)}}$

$\simeq \underline{\underline{\beta_1\beta_2}}$ (for $\beta_1 \gg 1$ and $\beta_2 \gg 1$)

If the composite transistor is operated at a collector current I_C, then

$$I_B = \frac{I_C}{\beta_1 + \beta_2(\beta_1+1)}$$

$$I_{C1} = \frac{\beta_1}{\beta_1 + \beta_2(\beta_1+1)} I_C \simeq I_C/\beta_2$$

$$I_{C2} = \frac{\beta_2(\beta_1+1) I_C}{\beta_1 + \beta_2(\beta_1+1)} \simeq I_C$$ **14**

Thus,

$V_{BE2} = V_{BE}(1\ mA) + V_T \ln\left(\frac{I_C}{1\ mA}\right)$

$V_{BE1} = V_{BE}(1\ mA) + V_T \ln\left(\frac{I_C}{\beta_2 \times 1\ mA}\right)$

$V_{BEeq} = V_{BE1} + V_{BE2}$

$= \underline{\underline{2V_{BE}(1\ mA) + V_T \ln\left(\frac{I_C^2}{\beta_2}\right)}}$

$r_{e2} = \frac{V_T}{I_{E2}} \simeq \frac{V_T}{I_{C2}} \simeq \frac{V_T}{I_C}$

$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{V_T}{I_{C1}} \simeq \frac{V_T}{I_C/\beta_2} = \beta_2 r_{e2}$

$r_{\pi eq} = (\beta_1+1)[r_{e1} + (\beta_2+1) r_{e2}]$

$\simeq \beta_1[\beta_2 r_{e2} + \beta_2 r_{e2}]$

$= \underline{\underline{2\beta_1\beta_2 r_{e2}}} \simeq \underline{\underline{2\beta_1\beta_2\left(\frac{V_T}{I_C}\right)}}$

To determine g_{meq}, apply a signal v_{be} and find the corresponding current i_c,

$i_c = i_{c1} + i_{c2} = g_{m1} v_{be1} + g_{m2} v_{be2}$

$= g_{m1} v_{be} \frac{r_{e1}}{r_{e1} + (\beta_2+1) r_{e2}} + g_{m2} \frac{(\beta_2+1) r_{e2}}{r_{e1} + (\beta_2+1) r_{e2}}$

$i_c \simeq v_{be} \frac{1}{2\beta_2 r_{e2}} + \frac{\beta_2}{2\beta_2 r_{e2}} v_{be}$ **14**

$g_{meq} \equiv \frac{i_c}{v_{be}} \simeq \frac{1}{2 r_{e2}} = \underline{\underline{\frac{1}{2}\frac{I_C}{V_T}}}$

For operation at $I_C = 10\ mA$:

$\beta_{eq} \simeq \beta_1\beta_2 = 50^2 = \underline{\underline{2500}}$

$V_{BEeq} = 1.4 + 0.025 \ln \frac{10^2}{50}$

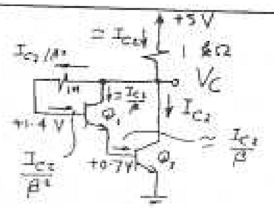
$= \underline{\underline{1.42\ V}}$

$r_{\pi eq} \simeq 2 \times 50 \times 50 \times \frac{25}{10} = \underline{\underline{12.5\ k\Omega}}$

$g_{meq} \simeq \frac{1}{2} \times \frac{10}{25} = \underline{\underline{200\ mA/V}}$

14.37 DC analysis:

$5 = I_{C2} \times 1\ k\Omega + \frac{I_{C2}}{\beta^2} \times 1\ M\Omega + 1.4$



$$I_{C2} = \frac{3.6}{1 + \frac{1000}{100\times 100}}$$

$= 3.3\ mA$

$I_{C1} \simeq \frac{3.3}{100} = 0.033\ mA$

$V_C \simeq \underline{1.7\ V}$

14.37 CONT'D

14

$$i_c = g_{m2}\, v_{be2}$$

$$= g_{m2} \frac{(\beta_2+1) r_{e2}}{r_{e1} + (\beta_2+1) r_{e2}}\, v_i$$

But $r_{e1} = \frac{V_T}{I_{E1}} = \frac{V_T}{I_{E2}/(\beta_2+1)} = r_{e2}(\beta_2+1)$

Thus, $i_c \simeq \frac{v_i}{2 r_{e2}}$

$$g_{meq} \simeq \frac{i_c}{v_i} = \frac{1}{2 r_{e2}}$$

$$= \frac{1}{2 \times (25/3.3)} = \underline{\underline{66\ \mathrm{mA/V}}}$$

$$v_o \simeq -i_c \times 1\,k\Omega$$

$$= -g_{meq} v_i \times 1\,k\Omega$$

$$\frac{v_o}{v_i} = -66 \times 1 = \underline{\underline{-66\ \mathrm{V/V}}}$$

To find R_{in},

$$i_i = i_{b1} + i_{1M\Omega\ \text{resistor}}$$

$$\simeq \frac{i_c}{\beta^2} + \frac{v_i - v_o}{1\,M\Omega}$$

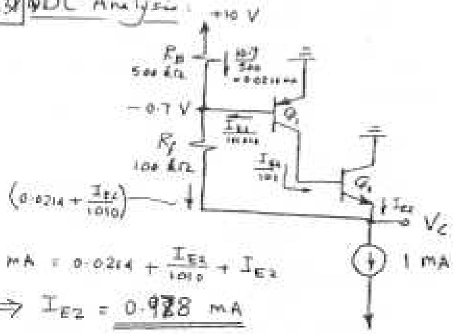
$$= \frac{v_i}{2\beta^2 r_{e2}} + \frac{67\, v_i}{1\,M\Omega}$$

$$= v_i \left[\frac{1}{2 \times 100^2 \times \frac{25}{3.3}} + \frac{67}{1\,M\Omega}\right] = 73.6\, v_i\ (nA)$$

$$R_{in} = \frac{v_i}{i_i} = \frac{1}{73.6}\ M\Omega = \underline{13.6\ k\Omega}$$

14.38 (a) DC Analysis:

14



$$1\ \mathrm{mA} = 0.0214 + \frac{I_{E2}}{1010} + I_{E2}$$

$$\Rightarrow I_{E2} = \underline{\underline{0.978\ \mathrm{mA}}}$$

$$I_{C2} = 0.99 \times 0.978 = \underline{\underline{0.97\ \mathrm{mA}}}$$

$$I_{C1} = \frac{0.978}{101} = \underline{\underline{9.7\ \mu A}}$$

$$V_C = -0.7 - 100\left(0.0214 + \frac{0.978}{1010}\right)$$

$$= \underline{\underline{-2.94\ V}}$$

(b) Small-signal parameters:

$$g_{m1} = \frac{9.7 \times 10^{-6}}{25 \times 10^{-3}} = 0.388\ \mathrm{mA/V}$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = 25.77\ k\Omega$$

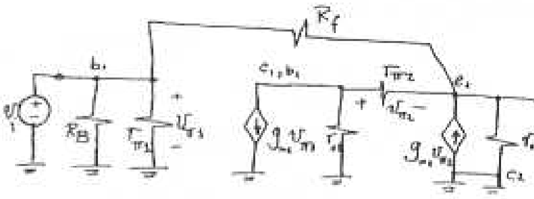
$$r_{o1} = \frac{|V_A|}{I_{C1}} = \frac{100}{9.7\,\mu A} = 10.31\ M\Omega$$

$$g_{m2} = \frac{0.97 \times 10^{-3}}{25 \times 10^{-3}} = 38.8\ \mathrm{mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = 2.58\ k\Omega$$

$$r_{o2} = |V_A|/I_{C2} = 103.1\ k\Omega$$

14



Node equation at b_2:

$$g_{m1} v_{\pi 1} + \frac{v_{b2}}{r_{o1}} + \frac{v_{\pi 2}}{r_{\pi 2}} = 0$$

But $v_{b2} = v_o + v_{\pi 2}$, then

$$g_{m1} v_{\pi 1} + \frac{v_o + v_{\pi 2}}{r_{o1}} + \frac{v_{\pi 2}}{r_{\pi 2}} = 0$$

$$\Rightarrow v_{\pi 2}\left(\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}\right) = -\left(\frac{v_o}{r_{o1}} + g_{m1} v_{\pi 1}\right)$$

or,
$$v_{\pi 2} = -\frac{\frac{v_o}{r_{o1}} + g_{m1} v_{\pi 1}}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}}$$

Node equation at output:

$$\frac{v_o}{r_{o2}} + \frac{v_o - v_{\pi 1}}{R_f} = g_{m2} v_{\pi 2} + \frac{1}{r_{\pi 2}} v_{\pi 2}$$

$$= \left(g_{m2} + \frac{1}{r_{\pi 2}}\right) v_{\pi 2}$$

$$= -\frac{\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)\left[\frac{v_o}{r_{o1}} + g_{m1} v_{\pi 1}\right]}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}}$$

Substituting $v_{\pi 1} = v_i$ and collecting terms, results in

14

$$v_o \left[\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)}{r_{o1}\left(\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}\right)}\right]$$

$$= -v_i \left[\frac{g_{m1}\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}} - \frac{1}{R_f}\right]$$

$$\frac{v_o}{v_i} = -\frac{\frac{g_{m1}\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)}{\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}} - \frac{1}{R_f}}{\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{g_{m2} + \frac{1}{r_{\pi 2}}}{r_{o1}\left(\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}}\right)}}$$

Since $r_{\pi 2} \ll r_{o1}$,

$$\frac{v_o}{v_i} \simeq -\frac{g_{m1}(g_{m2} r_{\pi 2} + 1) - \frac{1}{R_f}}{\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{1}{r_{o1}}(g_{m2} r_{\pi 2} + 1)}$$

$$= -\frac{g_{m1}(\beta_2 + 1) - \frac{1}{R_f}}{\frac{1}{r_{o2}} + \frac{1}{R_f} + \frac{1}{r_{o1}}(\beta_2 + 1)}$$

Since $\frac{1}{R_f} \ll g_{m1}(\beta_2 + 1)$

$$\frac{v_o}{v_i} \simeq -\frac{g_{m1}(\beta_2 + 1)}{\left(\frac{1}{r_{o2}} + \frac{1}{R_f}\right) + \frac{1}{r_{o1}}(\beta_2 + 1)}$$

Substituting $\beta_2 = \beta_N$ and noting that $\beta_N \gg 1$,

$$\frac{v_o}{v_i} \simeq -g_{m1} \frac{1}{\frac{1}{\beta_N}\left(\frac{1}{r_{o2}} + \frac{1}{R_f}\right) + \frac{1}{r_{o1}}}$$

$$= -g_{m1}\left[r_{o1} \,//\, \beta_N (r_{o2} \,//\, R_f)\right]$$

Q.E.D.

14.38 CONT'D

(c) $\frac{v_o}{v_i} = -0.388\,[10.31\times10^3 // 100\,(103.1 // 100)]$ $\boxed{14}$

$= \underline{-1320\ V/V}$

$R_{in} = R_B // r_{\pi 1} // \left[\frac{v_i}{\frac{v_i - v_o}{R_f}}\right]$

$= 500 // 25.77 // \left[\frac{R_f}{1 - \frac{v_o}{v_i}}\right]$

$= 500 // 25.77 // \frac{100}{1+1320}$

$= 500 // 25.77 // 0.0757$

$= \underline{75.5\ \Omega}$

14.39 Refer to Fig. 14.27 (p.1258).

The quiescent current through Q_2 and Q_4 is to be 2 mA. Then

$V_{BE2} = V_{BE4} = 0.7 + 0.025\ln\left(\frac{2}{10}\right) = 0.660\ V$

For Q_1 and Q_3, $I_C \simeq \frac{2}{100} = 0.02$ mA, thus

$V_{BE1} = V_{EB3} = 0.7 + 0.025\ln\frac{0.02}{1} = 0.602\ V$

$I_{B1} = \frac{20\ \mu A}{100} = 0.2\ \mu A$

$I_{bias} = 100\times0.2 = 20\ \mu A$

$I_{R_1,R_2} = \frac{1}{10}\times20\ \mu A = 2\ \mu A$

$I_{C5} = 20 - 2 = 18\ \mu A$

$V_{BE5} = 0.7 + 0.025\ln\frac{0.018}{1} = 0.600\ V$

$V_{BB} = V_{BE1} + V_{BE2} + V_{EB3} = \underline{1.864\ V}$

$R_1 + R_2 = \frac{1.864}{2\ \mu A} = 932\ k\Omega$ $\boxed{14}$

$R_1 = \frac{0.600}{2\ \mu A} = \underline{300\ k\Omega}$

$R_2 = 932 - 300 = \underline{632\ k\Omega}$

For $v_I = -10$ V and $R_L = 1\ k\Omega$:

$i_L = \frac{-10}{1} = -10$ mA

Assume that the current through Q_2 becomes almost zero, thus

$I_{C4} = 10$ mA

i.e. the current through Q_4 increases by a factor of 5. It follows that the current through Q_3 must increase by the same factor, thus V_{EB3} becomes

$V_{EB3} = 0.602 + 0.025\ln 5$

$= 0.642\ V$ (an increase of 0.04 V)

Let us check the current through Q_2. Since we assumed Q_1 and Q_2 to be almost cut off, all of I_{bias} now flows through the V_{BE} multiplier, an increase of 0.2 μA. Assuming that most of this increase occurs in I_{C5}, V_{BE5} becomes

$V_{BE5} = 0.7 + 0.025\ln\frac{0.0182}{1} = 0.600\ V$

Thus the voltage across the V_{BE}-multiplier remains approximately constant and the voltage $(V_{BE1} + V_{BE2})$ decreases by the same value that V_{EB3} increases by. That is $\boxed{14}$

$V_{BE1} + V_{BE2} = 0.660 + 0.602 - 0.04$

Since the current through each of Q_1 and Q_2 decreases by the same factor (call it m),

$0.025\ln m + 0.025\ln m = -0.04\ V$

$\Rightarrow m = 0.45$

Thus $I_{C2} = 0.45\times2 = 0.9$ mA

New iteration: $I_{C4} = 10.9$ mA (an increase by a factor $\simeq 5.5$).

$V_{EB3} = 0.602 + 0.025\ln 5.5$

$= 0.645\ V$

$v_I \simeq \underline{-10.645\ V}$

For $v_I = +10$ V and $R_L = 1\ k\Omega$:

Assume that Q_4 is now conducting a negligible current. Thus $I_{C2} \simeq I_L = 10$ mA. i.e. the current through each of Q_1 and Q_2 increases by a factor of 5. Thus $\boxed{14}$

$V_{BE2} = 0.66 + 0.025\ln 5$

$= 0.700\ V$

$V_{BE1} = 0.602 + 0.025\ln 5 = 0.642\ V$

$I_{B1} = 5\times0.2 = 1\ \mu A$

Thus the current through the multiplier becomes 19 μA, and assuming that most of the decrease occurs in I_{C5},

$V_{BE5} = 0.7 + 0.025\ln\frac{0.017}{1}$

$= 0.598\ V$

Thus the voltage across the multiplier becomes

$V_{BB} = 0.598\times\frac{932}{300} = 1.858\ V$

It follows that V_{EB3} becomes

$V_{EB3} = 1.858 - 0.700 - 0.642 = 0.516\ V$

i.e. V_{EB3} decreases by $0.600 - 0.516 = 0.084\ V$ and correspondingly I_{C3} decreases by a factor of $e^{\frac{-0.084}{0.025}} = 0.035$. ~~leaving~~ Thus I_{C4} becomes $0.035\times2 = 0.07$ mA, close to the zero value assumed. Thus no further iteration are required and

$v_I \simeq 10 + 0.7 + 0.642 - 1.858$

$= \underline{+9.484\ V}$

14.40 Refer to Exercise 14.13 and Fig. 14.28.

Now Q_5 has $I_S = 10^{-13}$ A. Then,

$$2 \times 10^{-3} = 10^{-13}\, e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln \frac{2\times 10^{-3}}{10^{-13}} = 0.593 \text{ V}$$

$$R_{E1} = \frac{0.593}{150 \text{ mA}} \simeq \underline{4\ \Omega}$$

For a normal peak current of 100 mA, the voltage drop across R_{E1} is $\underline{400 \text{ mV}}$ and its collector current is $10^{-13}\, e^{400/25} = \underline{0.89\ \mu A}$

14.41 Refer to Exercise 14.13 and Fig. 14.28.

$$2 \times 10^{-3} = 10^{-14}\, e^{V_{BE}/V_T}$$

$$\Rightarrow V_{BE} = 0.650 \text{ V}$$

$$R_{E1} = \frac{0.650 \text{ V}}{50 \text{ mA}} = \underline{13\ \Omega}$$

For a peak output current of 33.3 mA,

$$V_{BE} = 13 \times 33.3 = \underline{433 \text{ mV}}$$

$$I_{C5} = 10^{-14}\, e^{433/25} = \underline{0.33\ \mu A}$$

14.42 Refer to Fig. P.14.42. **14**

$$2 \times 10^{-3} = 10^{-14}\, e^{V_{EB5}/V_T}$$

$$V_{EB5} = 0.025 \ln (2 \times 10^{11}) = 0.650 \text{ V}$$

$$R = \frac{0.650 \text{ V}}{150 \text{ mA}} = \underline{4.3\ \Omega}$$

For a peak output current of 100 mA,

$$V_{EB5} = \underline{430 \text{ mV}}$$

$$I_{C5} = 10^{-14}\, e^{430/25} = \underline{0.3\ \mu A}$$

14.43 Refer to Fig. 14.29 (p. 1260)

At 125°C, $V_Z = 6.8 + (125-25) \times 2 = 7.0$ V

$$V_{E1} = 7.0 - (0.7 - 100 \times 0.002) = 6.5 \text{ V}$$

$$V_{BE2} = 0.5 \text{ V}$$

$$R_2 = \frac{0.5 \text{ V}}{100\ \mu A} = \underline{5 \text{ k}\Omega}$$

$$R_1 = \frac{6.5 - 0.5}{100\ \mu A} = \underline{60 \text{ k}\Omega}$$

At 25°C, $V_Z = 6.8$ V, $V_{E1} = 6.8 - 0.7 = 6.1$ V

$$V_{B2} = 6.1 \times \frac{5}{60+5} = 0.469 \text{ V}$$

$$I_{C2} = 100\, e^{(469-700)/25} = \underline{0.01\ \mu A}$$

14.44 Refer to the circuit of Fig. 14.30 (p. 1262). **14**

Resistors R_2 and R_3 control the gain,

$$A_v = -\frac{2R_2}{R_3} \quad \text{(See analysis on pages 681-682)}$$

Resistor R_3 controls the gain alone. Resistor R_2 affects both the gain and the dc output level. To see the latter point, Equate I_3 and I_4 from equations (9.93) and (9.44) to obtain

$$\frac{V_S - 3V_{EB}}{R_1} = \frac{V_O - 2V_{EB}}{R_2}$$

$$\Rightarrow V_O = 2V_{EB} + \frac{R_2}{R_1} V_S - \frac{3R_2}{R_1} V_{EB}$$

$$= \frac{R_2}{R_1} V_S + \left(2 - \frac{3R_2}{R_1}\right) V_{EB}$$

For $V_O = \frac{1}{3} V_S$, select $\frac{R_2}{R_1} = \frac{1}{3}$

$$R_2 = \frac{R_1}{3} = \frac{50}{3} = \underline{16.7 \text{ k}\Omega}$$

To keep the gain unchanged, we must change R_3 so that

$$\frac{2R_2}{R_3} = 50$$

$$R_3 = \frac{2 \times (50/3)}{50} = \frac{2}{3} = \underline{0.67 \text{ k}\Omega}$$

14.45 Refer to the circuit in Fig. 14.30 **14**

$$V_{B1} \simeq 0$$

$$V_{E1} \simeq +0.7 \text{ V}$$

$$V_{E3} \simeq +1.4 \text{ V}$$

$$V_{C10} = 20 - 0.7 = 19.3 \text{ V}$$

$$I_{E3} = \frac{19.3 - 1.4}{50} = 0.358 \text{ mA}$$

$$I_{B3} = I_{E1} = \frac{0.358}{21} = 17.05\ \mu A$$

$$I_{B1} = \frac{17.05}{21} = 0.81\ \mu A$$

$$V_{B1} = 0.81\ \mu A \times 150 \text{ k}\Omega = 0.122 \text{ V} \simeq 0 \checkmark$$

i.e. $I_{E1} = I_{E2} \simeq \underline{17\ \mu A}$

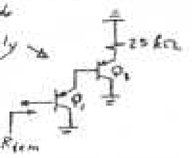
$$I_{E3} = I_{E4} \simeq \underline{358\ \mu A}$$

$$I_{E5} \simeq I_{E6} = \frac{20}{21} \times 358 = \underline{341\ \mu A}$$

$$I_{R_3} = I_{R_2} = 358\ \mu A$$

$$V_O = 0.12 + 1.4 + 25 \text{ k}\Omega \times 0.358 \text{ mA} = \underline{10.5 \text{ V}}$$

$\boxed{14.46}$ The equivalent common-mode half circuit is approximately →



From the results of Problem 14.45 above: $I_{E1} = 17\ \mu A$

$I_{E3} = 0.358\ mA$

$r_{o1} = \frac{|V_A|}{I_{C1}}$

$\boxed{14}$

Assume $|V_A|$ for all transistors = 100 V.

$r_{o1} \simeq \frac{100}{17\ \mu A} = 5.9\ M\Omega$

$r_{o3} = \frac{100}{0.358} = 279.3\ k\Omega$

Assume $r_\mu = 10\beta r_o$

$r_{\mu1} = 10 \times 20 \times 5.9 = 1180\ M\Omega$

$r_{\mu3} = 10 \times 20 \times 279.3 = 55.9\ M\Omega$

Now,
$2R_{icm} = r_{\mu1} // (\beta+1)[r_{o1} // r_{\mu3} // (\beta+1)(25 // r_{o3})]$

$= 1180\ M\Omega // 21\,[5.9 // 55.9 // 21(25//279.3)\times10^{-3}]\ M\Omega$

$\simeq 21 \times 21 \times (25//279.3)\ k\Omega$

$= 10.1\ M\Omega$

If we neglect all r_o's and r_μ's

$2R_{icm} \simeq (\beta+1)^2 \times 25\ k\Omega$

$= 11\ M\Omega$ (close enough!)

$R_{icm} = \underline{\underline{5.5\ M\Omega}}$

To find R_{id} we use the equivalent differential half-circuit →

$R_{id}/2 = (\beta+1)[r_{e1} + (\beta+1)(r_{e3} + 0.5//25)]$

$\boxed{14}$

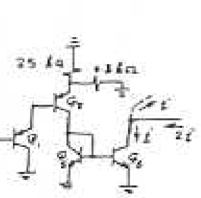
$r_{e1} = \frac{25\ mV}{17\ \mu A} = 1.47\ k\Omega$

$r_{e3} = \frac{25\ mV}{0.358\ mA} = 69.8\ \Omega$

$R_{id} = 2 \times 21\,[1.47 + 21\,(0.0698 + 0.49)]$

$= \underline{\underline{556\ k\Omega}}$

To find $G_{m1} = \frac{2i}{v_{id}}$



$i_{c3} = \alpha_3 i_{e3}$

$= \alpha_3 \frac{v_{id}/2}{(0.5//25) + r_{e3} + \frac{r_{e1}}{\beta+1}}$

$= \frac{20}{21} \times \frac{1}{2} \frac{v_{id}}{(0.5//25) + 0.069 + \frac{1.47}{21}}$

$= 0.757\ v_{id}$

$i = \frac{i_{c3}}{1 + \frac{2}{\beta_{pnp}}} = 0.98\ i_{c3} = 0.98 \times 0.757\ v_{id}$

$G_{m1} = \frac{2i}{v_{id}} = 2 \times 0.98 \times 0.757 = \underline{\underline{1.5\ mA/V}}$

$\boxed{14.47}$ Using Fig. 14.32 (p1269) for 8 Ω load, we see that $V_S = 16$ V allows more than 1.5 W power dissipation for some input signals. Thus we use

$V_S = \underline{14\ V}$

For THD = 3%, $P_{max} = \underline{1.9\ W}$

$1.9 = V_o^2/R_L = V_o^2/8$

$\boxed{14}$

$V_o = \sqrt{8 \times 1.9}$

Peak-to-Peak output sinusoid $= 2\sqrt{2}\,\sqrt{8\times1.9} = \underline{\underline{11\ V}}$

$\boxed{14.48}$ $f_t \simeq \frac{G_m}{2\pi C}$

$= \frac{1.6 \times 10^{-3}}{2\pi \times 10 \times 10^{-12}} = \underline{\underline{25.5\ MHz}}$

With feedback that results in a closed-loop gain of 50,

$f_b = \frac{25.5}{50} = \underline{\underline{509\ kHz}}$

$\boxed{14.49}$ Refer to Fig. 14.33

For $i_L = 1$ A, $i_{C5} \simeq 1$ A and $i_{B5} = \frac{1\ A}{50} = 20$ mA

For $i_L = 20$ mA, $i_{E3} = 0.9 \times 20 = 18$ mA,

10%

$i_{C5} = 2$ mA $i_{B5} = \frac{2}{50} = 0.04$ mA

$i_{C3} = \frac{50}{51} \times 18 = 17.65$ mA

$i_{R3} = i_{C3} - i_{B5} = 17.61$ mA

Thus, $R_3 = \frac{0.7}{17.61} = 39.8 \simeq \underline{\underline{40\ \Omega}}$

Similarly, $R_4 = \underline{\underline{40\ \Omega}}$

Since $i_{B5} \le 20$ mA, $i_{C3} \le \frac{0.7\ V}{40\ \Omega} + 20$ mA

i.e. $i_{C3} \le 37.5$ mA

$i_{B3} \le \frac{37.5}{50} = 0.75$ mA

$\boxed{14}$

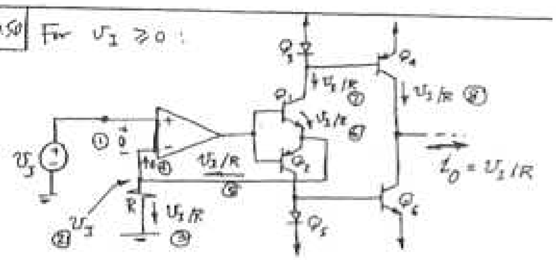
Allowing for a factor of safety of 2, we select R_1 so that the current through it is 1.5 mA. Now, for $V_o = 11$ V, $V_{E1} = 11.7$ V,

$R_1 = \frac{13 - 11.7}{1.5} = \underline{\underline{2.2\ k\Omega}}$

Similarly,

$R_2 = \underline{\underline{2.2\ k\Omega}}$

14.50 For $v_I \geq 0$:



For $v_I \leq 0$, Q_2, Q_5 and Q_6 take the roles of Q_1, Q_3 and Q_4, respectively.

For $\beta = 100$, $i_{C1} = 0.99\, v_I/R$ and

$$i_{C4} = i_{C1}\frac{1}{1+\frac{2}{\beta}} = 0.99\frac{v_I}{R}\frac{1}{1+\frac{2}{100}} = 0.97\frac{v_I}{R}$$

That is, the current is lower than the ideal value by approximately $\underline{3\%}$.

14.51 Refer to Fig. 14.34 | 14

$$\frac{v_O}{v_I} = 2K = 2\left(1+\frac{R_2}{R_1}\right) = 10$$

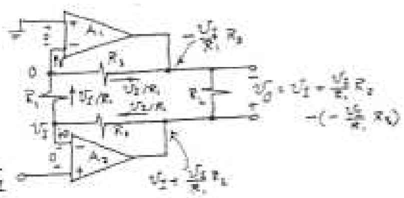
$$\Rightarrow \frac{R_2}{R_1} = 4$$

$$R_2 = \underline{40\ k\Omega}$$

Also, $K = \frac{R_4}{R_3} = 5$

$$\Rightarrow R_4 = \underline{50\ k\Omega}$$

14.52



$$\frac{v_O}{v_I} = \underline{1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}}$$

The largest sine-wave output is obtained when the voltage at the output of one op amp (say A_2) is $+13$ V and that at the output of the other op amp (A_1) is -13 V, resulting in a $\underline{26\text{-V}}$ peak output ($\underline{52\ V}$ peak-to-peak).

14

For $\frac{v_O}{v_I} = 10 = 1 + \frac{R_2+R_3}{R_1}$

and selecting $R_1 = 1\ k\Omega$

$$R_2 + R_3 = 9\ k\Omega$$

Now to keep the outputs complementary

$$\frac{R_3}{R_1} = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow R_3 = 1 + R_2$$

Thus $R_2 = \underline{4\ k\Omega}$ & $R_3 = \underline{5\ k\Omega}$

14.53 Normally, (Eq. 14.46)

$$i_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS}-V_t)^2$$

For velocity saturation, (Eq. 14.47)

$$i_D = \frac{1}{2}C_{ox} W U_{sat}(v_{GS}-V_t)$$

Equating at the two expressions (at the boundary) gives

$$\mu_n \frac{1}{L}(v_{GS}-V_t) = U_{sat}$$

$$\Rightarrow \underline{L = \frac{\mu_n(v_{GS}-V_t)}{U_{sat}}}$$

For $\mu_n = 500\ cm^2/V.s.$, $v_{GS} = 5$ V, $V_t = 2$ V, $U_{sat} = 5\times10^6$ cm/s,

$$L = \frac{500\,(5-2)}{5\times10^6} = 3\times10^{-4}\ cm = \underline{3\ \mu m}$$

Velocity saturation begins at

$$i_D = \frac{1}{2}C_{OX} W U_{sat}(v_{GS}-V_t) \quad \boxed{14}$$

$$= \frac{1}{2}\times 400\times10^{-6}\times 10^5\times10^{-6}\times 5\times10^6\times10^{-2}(5-2)$$

$$= \underline{3\ A}$$

At high currents,

$$g_m = \frac{\partial i_D}{\partial v_{GS}} = \frac{1}{2}C_{OX} W U_{sat}$$

$$= \frac{1}{2}\times 400\times10^{-6}\times10^5\times10^{-6}\times5\times10^6\times10^{-2}$$

$$= \underline{1\ A/V}$$

$\boxed{14.54}$ Refer to the circuit in Fig. 14.38

For $I_{Q_N} = I_{Q_P} = 10\text{ mA} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$

$$10 = 100\,(V_{GS} - 2)^2$$

$$\Rightarrow V_{GS} = 2.32\text{ V}$$

$$V_R = 2|V_{GS}| = 4.63\text{ V}$$

For $I_R = 10\text{ mA}$,

$$R = \frac{4.63\text{ V}}{10\text{ mA}} = \underline{\underline{463\ \Omega}}$$

$$V_{BB} = 4.63 + 4 \times 0.7 = 7.43\text{ V}$$

$$I_{R_2} = I_{R_4} = \frac{100}{2} = 50\ \mu A$$

$$R_2 = R_4 = \frac{700\text{ mV}}{50\ \mu A} = \underline{\underline{14\text{ k}\Omega}}$$

Now, since V_{GS} changes by $2 \times -3\text{ mV}/°C$ $= -6\text{ mV}/°C$ while V_{BE1}, V_{BE2}, V_{BE3} and V_{BE4} remain constant, V_{BB} changes by $-6\text{ mV}/°C$. But the voltage across the Q_5 multiplier remains constant. Thus the voltage across the Q_6 multiplier should be made to change by $-6\text{ mV}/°C$ which can be achieved by making

$\boxed{14}$

$$1 + \frac{R_3}{R_4} = 3$$

$$\Rightarrow R_3 = 2R_4 = \underline{28\text{ k}\Omega}$$

The voltage across the Q_5 multiplier is

$$V_{BB} - 3V_{BE} = 7.43 - 2.1 = 5.33\text{ V}$$

Thus, $5.33 = \left(1 + \frac{R_1}{R_2}\right) \times 0.7$

$$\Rightarrow \frac{R_1}{R_2} = 6.61$$

But $R_2 = 14\text{ k}\Omega$, thus

$$R_1 = \underline{92.6\text{ k}\Omega}$$